Chapter 1 Summary

GEOMETRY FOR PHYSICS: ΔXYZ.

DRAW "EXTRA LINES": extensions of existing lines, parallel to existing lines, perpendicular to existing lines, or to form right-triangles.

TOTAL = SUM OF PARTS: If |← x →← 3 →|, then x = 4.
|←—— 7 ——→|

SIMILAR TRIANGLES: If two triangles have the same angles (and thus the same shape), they have the same side/side ratios.

VECTORS have both magnitude and direction.

If you know 1 of [θ, sin θ, cos θ, tan θ], you can find the others. Calculator: θ —("cos")→ ADJ/HYP ratio, and ADJ/HYP ratio —("$\cos^{-1}$")→ θ angle.

cos θ — θ — sin θ
tan θ

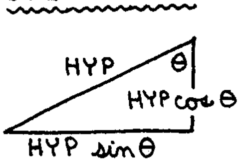
$\cos\theta = \sin(90° - \theta)$, and $\sin\theta = \cos(90° - \theta)$

If you know 2 of the 4 right-triangle variables [ADJ, OPP, HYP, θ/sinθ/cosθ/tanθ] you can find the other 2, by solving the equations that contain the knowns.

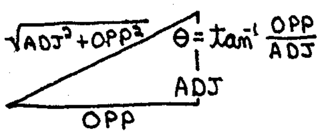
ADJ & OPP are defined "with respect to" θ.

$$\left.\begin{aligned} \sin\theta &\equiv \frac{OPP}{HYP} \\ \cos\theta &\equiv \frac{ADJ}{HYP} \\ \tan\theta &\equiv \frac{OPP}{ADJ} \\ ADJ^2 + OPP^2 &= HYP^2 \end{aligned}\right\}$$

SPLITTING



RECONSTRUCTING



TRUE: $\sin\theta / \cos\theta = \tan\theta$
$\sin^2\theta + \cos^2\theta = 1$

FALSE: $\cos(a + b) \neq \cos a + \cos b$
$\cos(ab) \neq (\cos a)(\cos b)$

To add vectors, A) Draw the vectors head-to-tail, like a "relay race",
B) choose axes and split each vector into x & y components,
C) add x-components to get x_{total}, add y-components to get y_{total},
D) use these x & y components to "reconstruct" the *resultant vector.*

There is a difference between *components* (which are always ⊥) and *originals.*

OPTIONAL: Vector multiplication (dot product & cross product) is explained in Sections 18.51-18.53.

Chapter 2 Summary

the tvvax system

5 variables, 5 equations (each is missing 1 variable)

$v_f - v_i$	$=$	$a\ t$	Δx is missing
$(x_f - x_i)$	$=$	$\frac{1}{2}(v_i + v_f)\ t$	a is missing
$(x_f - x_i)$	$=$	$v_i\ t + \frac{1}{2}a\ t^2$	v_f is missing
$(x_f - x_i)$	$=$	$v_f\ t - \frac{1}{2}a\ t^2$	v_i is missing
$v_f^2 - v_i^2$	$=$	$2a\ (x_f - x_i)$	Δt is missing

These equations are true only if a is constant between i & f.

Step 1: Read carefully [for words, sentence structure, implications], think [creative and logical], draw [form a clear picture-idea, on paper and/or mentally]. <u>Choose initial (i) and final (f) points for a useful constant-a interval</u>.

Step 2: <u>Make a "tvvax table"</u>, to show what you know about the 5 tvvax variables.
Look for zero-v words: from rest, stop, is dropped, peak, maximum height,...
"t" means Δt. You can substitute "$x_f - x_i$" for Δx whenever it's helpful.
Δx depends only on x_i and x_f positions, not on what happens between i & f.

Step 3: <u>Look for a 3-of-5 subgoal</u>: if you know any 3 variables, you can find the other 2. If you can't get 3-of-5, re-read the problem-statement more carefully, look for "links" where the same variable occurs in two tvvax tables [like New Year's Eve {Section 2.6}, semi-known symbols {2.7}, x-time = y-time {2.8},...].

Step 4: <u>Use "1-out strategy" to choose the equation</u> with 3 knowns & the goal-variable, substitute-and-solve. If necessary, use simultaneous equations {2.7}, or a quadratic option {19.7} like the Q-formula {2.6} or 2-step Q-Detour {2.6} or $\sqrt{\ }$ Q-trick {2.7}; for each Q-option you must choose between + and – solutions.
UNITS: Use SI (s, m/s, m/s^2, m). Be careful at start (during substitution), relaxed in middle (algebra solution), careful at end (answering the question).

Step 5: <u>Answer the question</u> that was asked.

<u>FREE FLIGHT {2.5}</u>: If only gravity affects an object and air resistance is ignored, it has $a_x = 0$; a_y = 9.80 m/s per second downward, which is –9.80 m/s^2 if "up" is +.
Don't mix a free flight interval with a "throw" or "impact".

SPLITS: Make a tvvax table for each time interval and/or object and/or direction.

<u>TIME SPLIT {2.6}</u>: Split the action into useful constant-a intervals, separated by special points. Be specific about v-labels; use v_1, v_2, v_3,... (not v_i & v_f), look for New Year's Eve links. Use "total = sum-of-parts" logic, like $\Delta t_{1\text{-to-}2} + \Delta t_{2\text{-to-}4} = \Delta t_{1\text{-to-}4}$.

<u>OBJECT SPLIT {2.7}</u>: Translate the words of a problem into a clear picture that helps you define tvvax knowns and unknowns and semi-knowns (where the same variable-letter is in different tvvax tables). Use "$\Delta x = x_f - x_i$" for each object; at a certain special time (like a passing point) two objects may have the same x-position.

<u>DIRECTION SPLIT {2.8 & 2.9}</u>: Analyze x & y motion independently, make separate x & y tvvax tables for each time interval and object. For free-flight motion,

<u>X: 2-of-3 subgoal.</u>		<u>Y: 3-of-5 subgoal.</u>
$\Delta x = v_x\ \Delta t$	t-LINK if same i & f	$\Delta t =$
$(\quad) = (\quad)(\quad)$		$v_i =$
		$v_f =$
		$a = -9.8$ m/s^2
		$\Delta y =$

Use Section 1.3 methods to split v_i into v_x & v_y, and reconstruct the $(v_{total})_f$ vector.

SYMMETRY: For free-fall motion when $y_i = y_f$, $\Delta t_{\text{before peak}} = \Delta t_{\text{after peak}}$, $\Delta x_{\text{before peak}} = \Delta x_{\text{after peak}}$, $\Delta x_{\text{i-to-f}} = v_i^2 (\sin 2\theta_i)/g$, and $(v_y)_i = -(v_y)_f$.

RELEASE PRINCIPLE: $v_{\text{just-before-release}} = v_{\text{just-after-release}}$ (magnitude & direction).

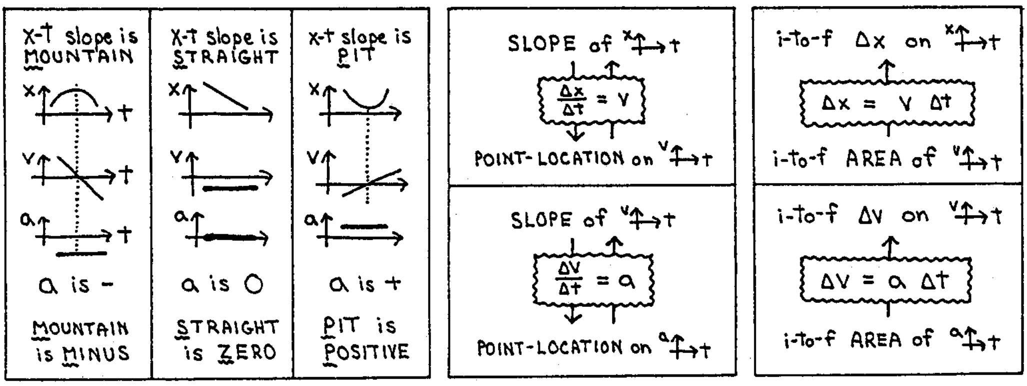
A vector's x (or y) component is + if the vector points in the x (or y) direction you've chosen to be +. The x-component is – if it points in the opposite direction.

$\Delta \mathbf{x} = \mathbf{v}_{\text{average}}\ \Delta t$, so $\Delta \mathbf{x}$ and $\mathbf{v}_{\text{average}}$ always point in the same direction.
$\Delta \mathbf{v} = \mathbf{a}_{\text{average}}\ \Delta t$, so $\Delta \mathbf{v}$ and $\mathbf{a}_{\text{average}}$ always point in the same direction.

v and **a** can have different directions; examples occur in Sections 2.2 & 2.10-Shapes.

While number-line **v** is increasing, a is +; while number-line **v** is decreasing, a is –.
While speed ↑, **v** & **a** have the same ± sign; while speed ↓, **v** & **a** have opposite ± signs.

MOTION GRAPHS {2.0}: Point, Slope, Shape (Concavity), Area.



Average Slope: Use actual graph-points for i+f, calculate RISE/RUN.
Instantaneous Slope: draw tangent line, choose i+f, calculate RISE/RUN.

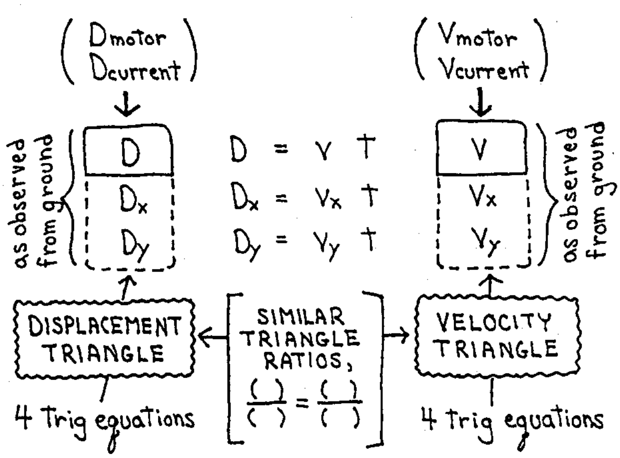
RELATIVE MOTION (2.11)

$v^{\text{train}}_{\text{ground}}$ {v of tr, obs from gr} + $v^{\text{runner}}_{\text{train}}$ {v of ru, obs from tr} = $v^{\text{runner}}_{\text{ground}}$ {v of ru, obs from gr}

$v^{\text{train}}_{\text{ground}} = -v^{\text{ground}}_{\text{train}}$, vectors can be added in any order: $v^{\text{train}}_{\text{ground}} + v^{\text{runner}}_{\text{train}} = v^{\text{runner}}_{\text{train}} + v^{\text{train}}_{\text{ground}}$.

Use consistent reference frames: $\Delta x^{\text{runner}}_{\text{ground}} = v^{\text{runner}}_{\text{ground}}\ \Delta t^{\text{runner}}_{\text{ground}}$, but $\Delta x^{\text{runner}}_{\text{train}} \neq v^{\text{runner}}_{\text{ground}}\ \Delta t^{\text{runner}}_{\text{ground}}$.

Here is a TOOL SUMMARY for 2-dimensional "boat & plane problems":



Chapter 3 Summary

An F-diagram (and its corresponding F=ma) always refer to <u>one specific object</u>; draw a "free body" F-diagram, or use separating-lines (as in Problem 3-B), or colors.

You can define a combination of matched-motion objects as a "system-object" (or just add the F=ma's of individual objects) to make *internal forces* cancel. This canceling can be good (if F_{int} is unknown) or bad (if F_{int} is known).

Make a force-diagram: Draw picture, choose object, imagine you're the object, say "I am <u>being</u> pushed & pulled by __ and __ and ....", then draw and label these forces.

$w/g = m$

If "matched motion", all objects have same v, a, Δx.
If v is constant (whether v is 0 or $\neq 0$), $a = 0$.
If sliding (or) $a_{\perp} = 0$.
Solve tvvax for a, use it.

$$F_{total} = m\,a$$

X & Y motion is independent, so choose axes, split F's (and a) into x & y components. For each F-component, decide whether direction should be represented by a + or – sign.
F-letters represent only magnitude: $-mg = -m(9.80)$, not $-m(-9.80)$.

$$F_x = m\,a_x \qquad \text{and} \qquad F_y = m\,a_y$$

Force NAME	cause	FORCE MAGNITUDE	FORCE DIRECTION
GRAVITY: weight, w	pull of earth	$mg \approx m(9.80)$ {GMm/r^2; see Chptr 5G}	PULL, "down" toward center of earth
TENSION, T or F_T	string, rope,...	There is no magnitude formula for T or N. (find T or N magnitude by solving F = ma)	PULL, in direction the rope points.
NORMAL, N or F_N or....	surface contact		PUSH, $\perp$ to surfaces' plane-of-contact
FRICTION f_k / f_s	surface contact	$f_k = \mu_k N$, $f_s \leq \mu_s N$ (f_s can vary from 0 to $\mu_s N$) f_s is EQUAL-AND-OPPOSITE to $F_{non\text{-}friction}$	f_k opposes sliding motion; f_s opposes "would-be" motion
SPRING	coiled spring	MAGNITUDE + DIRECTION is $-k(x_f - x_e)$, or (if $x_e \equiv 0$), $-kx$	like "homing pigeon", PUSH or PULL toward x_e
OTHER FORCES include air resistance, fluid pressure and buoyancy (in Ch. 6), electrostatic (Ch. 11), magnetic (Ch. 13), and nuclear (Ch. 15).			

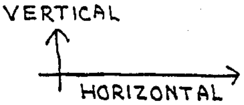
A third law force-pair involves <u>two</u> EQUAL-AND-OPPOSITE relationships:

1) Third law forces are EQUAL IN SIZE and OPPOSITE IN DIRECTION, and also
2) EQUAL IN "KIND OF FORCE" and OPPOSITE IN "MUTUAL SYMMETRY" {for example, "If ground pushes block, then block also pushes ground."}.

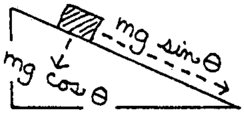
Partners in a third law force-pair don't act on the same object, so they never appear together on the F-diagram for an object. {For example, equal-and-opposite N & mg forces are not a third law pair.}

LINKS:
- third law (mutual-interaction F-partners appear in F=ma for two objects)
- massless rope (pulls object at both ends with equal T, so T is in 2 F=ma's)
- a-link ("a" appears in tvv<u>a</u>x and F=m<u>a</u>; link works in both directions)
- matched motion (if two objects have the same "a")
- split-link (if an F is split into F_x & F_y, that F is in $F_x = ma_x$ & $F_y = ma_y$), N-link (if one F=ma has N, and another F=ma has "friction = μN")

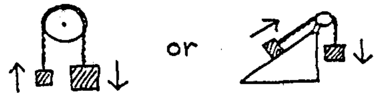
If possible, choose object & axes to get a 1-unknown equation. Some axes-options are:



common "x" & "y"



⊥ and ∥ to plane



DIRECTION-OF-MOTION, with "BENT AXIS"

This flowchart organizes the fundamentals of friction into a useful strategy:

Question #1: Is the object sliding now?

If the answer is YES, ↓

KINETIC FRICTION, f_k

f_k magnitude is $\mu_k N$.
f_k direction is opposite to the direction of sliding velocity.

If the answer is NO, ↓

Question #2: Ask "Will the object begin to slide", and answer by using this 3-step process:

Ⓐ Find the net $F_{non\text{-}friction}$. ($F_{non\text{-}f}$ wants answer to be YES!)
Ⓑ Find $\mu_s N$, the upper limit of f_s. (f_s wants the answer to be NO!)
Ⓒ Compare $F_{non\text{-}f}$ and $\mu_s N$ magnitudes. Who wins? Will the object slide?

If $F_{non\text{-}f}$ is larger, YES!, the block starts sliding. ↑ (to Kinetic Friction)

If $\mu_s N$ is larger, NO!, $\vec{f_s}$ is able to prevent movement by doing just enough to cancel $\vec{F_{non\text{-}f}}$ (no more, no less). ↓

STATIC FRICTION, f_s

f_s magnitude equals $F_{non\text{-}f}$
f_s direction is opposite to the direction of $F_{non\text{-}f}$.

$\boxed{f_s \text{ and } F_{non\text{-}f} \text{ are EQUAL-and-OPPOSITE}}$

Only at the breakaway limit, you can substitute

$$F = ma$$
$$f_s + \text{OTHER FORCES} = m(0)$$
$$\mu_s N + \text{OTHER FORCES} = 0$$

coefficients of friction are μ_k & μ_s, but friction *forces* are f_k & f_s (or $\mu_k N$ & $\mu_s N$).
μ_k & μ_s are approximately independent of surface-contact area and sliding speed.
{ There are many interesting friction problems in Section 3.91. }

Chapter 4A Summary

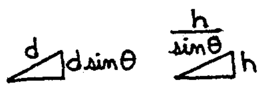
There are two TWE formats: the one shown below, and the one derived in Section 4.3.

	INITIAL ENERGY			+ INPUT	=	FINAL ENERGY			+ WASTE
ENERGY ACCOUNTABILITY	KE_i	$+ PE_i$	$+ PE_i$	$+ W_{other}$	=	KE_f	$+ PE_f$	$+ PE_f$	$+ \lvert W_{fr} \rvert$
	$\frac{1}{2}mv_i^2$	$+ mgh_i$	$+ \frac{1}{2}kx_i^2$	$+ F_{other}\, d \cos\phi$	=	$\frac{1}{2}mv_f^2$	$+ mgh_f$	$+ \frac{1}{2}kx_f^2$	$+ \lvert \mu_k N d \rvert$
CONSERVATION: If $W_{oth} = 0 = W_{fr}$, $(KE+PE)_i = (KE+PE)_f$	KE_i	$+ PE_i$	$+ PE_i$	$+ 0$	=	KE_f	$+ PE_f$	$+ PE_f$	$+ 0$

$\underbrace{mgh}$ ← h or d sinθ $\qquad$ d or $\frac{h}{\sin\theta}$ → $d\,\mu_k N$

W_{other} can be a rope pull, person's push or pull, car engine or brakes (thru tire-friction),...

For mgh, you must define "up" to be +.



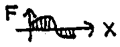
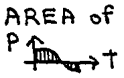
N is often (but not always) mg or mg cosθ.

$W_{friction}$ can also be simply "$f_k\, d$".

$|W_{fr}|$ is always +.

Eliminate the TWE parts you don't need, then choose i & f, h≡0 & x≡0.
Substitute for W_{fr}, cancel m's if every term contains m, remember v② & x②.

EVERY BOX EQUALS EVERY OTHER BOX.

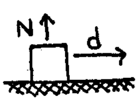
AREA of F–x graph 	If F_{total} is constant, $F_{total}\, d \cos\phi$	$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$
OPTIONAL CALCULUS Section 18.31	W_{total}	$KE_f - KE_i$
AREA of P–t graph 	If P_{total} is constant, $P_{total}\, \Delta t$	ΔKE

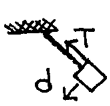
For straight-line motion with constant i-to-f force,

$W = F_{parallel}\, d$

$W = F\, d \cos\phi$

Work = 0 if <u>F = 0</u>, or <u>d = 0</u> (no movement), or <u>cosφ = 0</u> (F & d are ⊥).

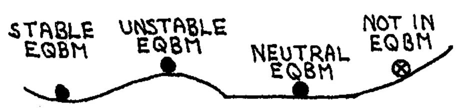




$P = \frac{W}{\Delta t} = F v \cos\phi$

$\frac{F\, d \cos\phi}{\Delta t}$

two different ways to "group" $F d \cos\phi / \Delta t$



If ⊗ (not in equilibrium) is released with $v_i = 0$, it responds so its PE ↓; $F_{parallel}$ & **d** are in same direction, so W is + and KE ↑ (as PE ↓).

If $v_i \neq 0$, object can move "uphill" in PE; W is – so KE ↓ as PE ↑.

If F_{par} & d point same direction, W is +. If F_{par} & d point opposite directions, W is –.

Totals & Partials: $F_{total} = ma$ & $W_{total} = \Delta(\frac{1}{2}mv^2)$, but $F_{part} \neq ma$ & $W_{part} \neq \Delta(\frac{1}{2}mv^2)$.

$F_{total} = F_A + F_B + \ldots,\quad W_{total} = W_A + W_B + \ldots,\quad P_{total} = P_A + P_B + \ldots$

F causes $m(\Delta v / \Delta t)$, $\qquad$ $F\,\Delta x$ causes $\Delta(\frac{1}{2}mv^2)$, $\qquad$ $F\,\Delta t$ causes $\Delta(mv)$.

Chapter 4B Summary

You can define any combination of objects as a "system".

$F_{internal}$ doesn't cause $\Delta(mv)$, because the $F_{int}\,\Delta t$ acting on one part of a system is canceled by an equal-and-opposite $F_{int}\,\Delta t$ (from a "third law mutual-force" partner) that acts on another part of the system; this cancellation makes $(F_{int}\,\Delta t)_{total} = 0$.

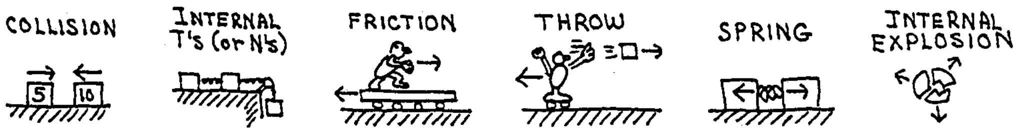
$$\mathbf{F}_{external}\ \Delta t \ = \ (\mathbf{mv})_f - (\mathbf{mv})_i$$

<u>You can ignore all internal forces</u>; the equation only asks for $F_{external}$!

For each force, ask "Is this F caused by another system-object (making it $F_{internal}$) or by something outside the system (which makes it $F_{external}$)?"

Some external forces are gravity, air resistance, and external friction or T or N.

Some internal forces are internal friction or T or N, a throw or gunshot of a system-object by another system-object, an internal spring-force or internal explosion.



CONSERVATION OF MOMENTUM: If $F_{ext} = 0$, mv is conserved, and $(mv)_i = (mv)_f$.
MOMENTUM IS "ALMOST CONSERVED, $(mv)_i \approx (mv)_f$, if $\Delta t \approx 0$ causes $F_{ext}\,\Delta t \approx 0$.

KE retention in a **collision**: If it is 100%, **elastic**. If it is less than 100%, **inelastic**. When the objects "stick", it is the minimum % (not necessarily 0), **totally inelastic**.

For a *1-dimensional elastic collision* of objects 1 & 2, you can use these formulas:

$$(v_1)_f = \frac{m_1 - m_2}{m_1 + m_2}(v_1)_i + \frac{2\,m_2}{m_1 + m_2}(v_2)_i \qquad (v_2)_f = \frac{m_2 - m_1}{m_1 + m_2}(v_2)_i + \frac{2\,m_1}{m_1 + m_2}(v_1)_i$$

- XY independence ⇒ [x & y equations on left & right sides of page]; <u>don't mix x & y</u>!
- If $F_{ext} = 0$, $(mv)_i = (mv)_f$ [i & f on left & right sides of equation]; <u>don't mix i & f</u>!
- Check to be sure you have an mv term for each system-object at the i & f times.

The *center-of-mass* for a symmetric uniform-density object is at its center.

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \dots}{m_1 + m_2 + \dots}$$

If a system has $(v_{cm})_i = 0$ and $F_{ext} = 0$, its c-of-m doesn't move, and $(x_{cm})_i = (x_{cm})_f$.

Optional: system-adaptions of some (but not all) motion equations is discussed in Section 4.11.

How to Choose a Useful Equation

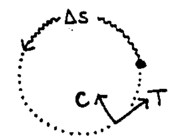
two tvv<u>a</u>x equations	F = m<u>a</u> "a link"	$F\,\Delta x = \Delta(\frac{1}{2} m v^2)$ $F\,\Delta t = \Delta(mv)$

Choose an equation with the goal-variable and lots of "knowns".

For an interval with no F, use one of the 5 tvvax equations (which don't contain F). F=ma has no i or f or Δ; F=ma shows what is happening at a specific instant of time. If the accelerating effects of F_{total} accumulate during an interval, use $F\Delta x = \Delta KE$ if Δx is either given or asked for, and use $F\Delta t = \Delta p$ if Δt is given or asked for.

If an object changes height between i & f, use "$mg\Delta h$" in $W = \Delta KE$.
If "internal force" is involved, simplify things by using $F_{ext}\,\Delta t = \Delta p$.

Chapter 5 Summary



The centripetal (radial) axis points toward the circle's center, along a radius-line. The tangential axis points along the direction of motion (straight out the "front windshield" or "rear window").

At any instant of time, the centripetal and tangential axis-directions are perpendicular ($\perp$) to teach other.

A car's speedometer shows v (for straight-line motion) or v_T (if motion is along a curve). Δs is the distance an object actually travels (see picture above).

Centripetal (Radial) Acceleration: At any instant of time, whether v_T is constant or changing, a_c is $\perp$ to v_T and points toward the circle-center, with magnitude $v_T{}^2/r$.

Cause $\rightarrow$ Effect: A force "F_c" causes "m" to move in a circle with acceleration "a_c",

$$F_c = m\frac{v_T{}^2}{r} \quad \text{—(by substituting "} v_T = r\omega \text{" from 5C)}\!\!\rightarrow \quad F_c = m\, r\, \omega^2$$

F_c toward center is +, F_c away from center is –; the F_T component doesn't cause a_c.

"Centripetal" is a direction (like "x" or "y"), not the name for a new kind of force.

F_c is caused by real objects; "circular motion" or "acceleration" don't cause force.

$F_{gravity}$ = **GMm/r^2**, center-to-center attraction; $G = 6.67 \times 10^{-11}$ (in SI units).

Near the earth's surface, $F_{gravity} = mg$, straight down toward earth's center; $g \approx G M_{earth}\, m/r^2$.

$F_{gravity}$ extends into "space"; it can cause one object to *orbit* around another object,

$$\frac{GMm}{r^2} = m\frac{v_T{}^2}{r} \qquad \frac{GM}{4\pi^2}T^2 = r^3 \qquad GM = r^3\omega^2$$

Angular-Motion Definitions, and Connecting Equations

$$\boxed{\begin{array}{c} \Delta s = r\,\Delta\theta \\ v_T = r\,\omega \\ a_T = r\,\alpha \end{array}}$$

$$\frac{\Delta s}{\Delta t} = v_T \text{ — } v_T = r\,\omega \text{ — } \omega \equiv \frac{\Delta\theta}{\Delta t}$$

$$\frac{\Delta v_T}{\Delta t} = a_T \text{ — } a_T = r\,\alpha \text{ — } \alpha \equiv \frac{\Delta\omega}{\Delta t}$$

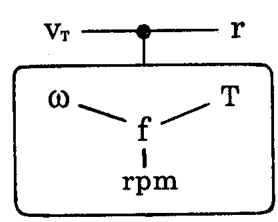
Angular Velocity Units

If you know 1 of 4 (ω , f , rpm , T) you can find the others:

2π rads = 1 rev = 360°,

60 seconds = 1 minute,

$\omega = 2\pi f$; $1/f = T$, $f = 1/T$.



If you know 1 of 3 (v_T , r , ω/f/rpm/T) you can find the others:

$v_T = r\,\omega$

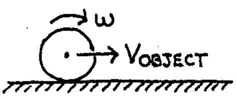
$v_T = 2\pi r/T$

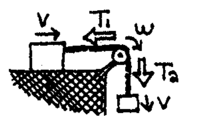
FOUR KINDS OF ACCELERATION

	a LINEAR	a_T TANGENTIAL	α ANGULAR	a_c (or a_R) CENTRIPETAL
SITUATIONS: LINEAR MOTION	a is "REGULAR" ACCELERATION (as in Chapters 2-4)	not used	not used	0
SITUATIONS: CONSTANT-SPEED CIRCULAR MOTION	not used	0	0	a_c shows the rate-of-change of $\vec{v}$-direction as object moves along a curve.
SITUATIONS: CHANGING-SPEED CIRCULAR MOTION	not used	a_T shows v_T's rate-of-change	α shows ω's rate-of-change	
F = ma EQUATION	$F_x = m a_x$ $F_y = m a_y$	$F_T = m a_T$	$\tau = I \alpha$	$F_c = m a_c$
MAGNITUDE	$\frac{\Delta v}{\Delta t}$	$\frac{\Delta v_T}{\Delta t}$ $a_T = r \alpha$	$\frac{\Delta \omega}{\Delta t}$	v_T^2 / r and $r \omega^2$
DIRECTION	If speed ↑, out "front window". If speed ↓, out "rear window". { Along direction of motion. }		α has same $\pm$ sign as a_T. { see * below }	toward center of circle (on radial-line)
What is changing?	Magnitude of v	Magn. of v_T	Magn. of ω	Direction of v vector

There is one kind of non-angular acceleration: **a-vector** $\equiv$ **Δ(v-vector)** / Δt. **a**, $\mathbf{a_c}$ & $\mathbf{a_T}$ are just convenient categories that describe the Δv /Δt for three common situations. To get the circular-motion $\mathbf{a}_{\text{total}}$ vector, add $\mathbf{a_c}$ and $\mathbf{a_T}$ (which are always ⊥) as vectors

COMBINED MOTION: TRANSLATION + ROTATION





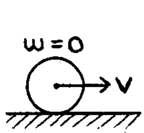
$$\begin{array}{lclcl} (\Delta x)_{object} & =^* & (\Delta s)_{rim} & = & r_{rim}\ \Delta\theta \\ v_{object} & =^* & (v_T)_{rim} & = & r_{rim}\ \omega \\ a_{object} & =^* & (a_T)_{rim} & = & r_{rim}\ \alpha \end{array}$$

$$\begin{array}{lclclcl} (\Delta x)_{blocks} & = & (\Delta x)_{rope} & =^* & (\Delta s)_{rim} & = & r_{rim}\ \Delta\theta \\ v_{blocks} & = & v_{rope} & =^* & (v_T)_{rim} & = & r_{rim}\ \omega \\ a_{blocks} & = & a_{rope} & =^* & (a_T)_{rim} & = & r_{rim}\ \alpha \end{array}$$

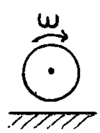
* These ='s are true only if the object (or rope) moves across the floor (or pulley) without slipping.

All points on a spinning plate have the same ω;
but if two points have different r's, they will (because $v_T = r\omega$) have different v_T's.

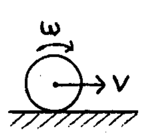
KINETIC ENERGY (translational & rotational)



only KE_{tran}, $\frac{1}{2} m v^2$



only KE_{rotn}, $\frac{1}{2} I \omega^2$



$KE_{tran} + KE_{rotn}$, $\frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

If object is at same point after 1 rev (like rock-on-string), it has one kind of KE; this can be calculated as $\frac{1}{2}mv^2$ or $\frac{1}{2}I\omega^2$. If object has moved after 1 rev (like a rolling sphere), it has KE_{trans} & KE_{rotn}, and if rolling is non-slip so $v_T = r\omega$, $KE_{total} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + X(\frac{1}{2}mv^2)$.

Linear, Tangential & Angular Variables, and CONNECTING EQUATIONS

LINEAR	t	Δx	v	a	F	m
TANGENTIAL	t	Δs	v_T	a_T	F_T	m
ANGULAR	t	$\Delta\theta$	ω	α	τ	I
CONNECTING EQUATIONS	—	$\Delta s = r\,\Delta\theta$	$v_T = r\omega$	$a_T = r\alpha$	$F_T = \frac{\tau}{r}$	$m = \frac{I}{r^2}$

Linear, Tangential and Angular Equations

Linear equations (like $F = ma$) have "tangential analogies" ($F_T = m\,a_T$). If *connecting equation substitutions* are made for each tangential variable ($F_T = \tau/r$, $m = I/r^2$, $a_T = r\alpha$), every "r" will cancel, as shown in Parts 1, 3 & 4 of Chapter 5F. The overall result is that linear variables change to tangential and then angular, even though (as discussed in 5D Part 4) tangential & angular variables are not equal.

LINEAR EQUATIONS → **TANGENTIAL EQUATIONS**

tvvax (if a is constant)

$$v_f - v_i = a t$$
$$\Delta x = \tfrac{1}{2}(v_i + v_f)t$$
$$\Delta x = v_i t + \tfrac{1}{2} a t^2$$
$$\Delta x = v_f t - \tfrac{1}{2} a t^2$$
$$v_f^2 - v_i^2 = 2 a \Delta x$$

Use Chapter 2 strategies.

→

$t\omega\omega\alpha\theta$ (if α is constant)

$$\omega_f - \omega_i = \alpha t$$
$$\Delta\theta = \tfrac{1}{2}(\omega_i + \omega_f)t$$
$$\Delta\theta = \omega_i t + \tfrac{1}{2}\alpha t^2$$
$$\Delta\theta = \omega_f t - \tfrac{1}{2}\alpha t^2$$
$$\omega_f^2 - \omega_i^2 = 2\alpha\Delta\theta$$

Use analogies of tvvax strategies.

$$F_{total} = m a \quad \rightarrow \quad \tau_{total} = I\alpha$$

$F_{ext}\,\Delta t$ — Δp — $p_f - p_i$ — $(mv)_f - (mv)_i$

LINEAR + ANGULAR MOMENTUM cannot be "mixed"

→

$F_{ext}\, r_\perp\, \Delta t$ / $\tau_{ext}\, \Delta t$ — ΔL — $L_f - L_i$ — $\begin{pmatrix} I\omega \text{ or} \\ m v r_\perp \end{pmatrix}_f - \begin{pmatrix} I\omega \text{ or} \\ m v r_\perp \end{pmatrix}_i$

$F d \cos\phi$ / $F_x\, \Delta x$ ⋮ W_{trans} + $F r_\perp \Delta\theta$ / $\tau\, \Delta\theta$ ⋮ W_{rotn} — W_{total}

OPTIONAL: $(F v \cos\phi + \tau\omega)\,\Delta t$ / $P_{total}\,\Delta t$ — W_{total}

$W_{tr} = \Delta KE_{tr}$ | $W_{ro} = \Delta KE_{ro}$

$$\Delta KE_{trans} + \Delta KE_{rotn}$$
$$\left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) + \left(\tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2\right)$$

$$\Delta(KE_{total})$$
$$(KE_{total})_f - (KE_{total})_i$$
$$\left(\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2\right) - \left(\tfrac{1}{2}mv_i^2 + \tfrac{1}{2}I\omega_i^2\right)$$

tvvax strategies (as in Section 2.21 or Chapter 2's Summary) can be used for $t\omega\omega\alpha\theta$: read/think/draw, choose i & f points for a constant-α interval, make a $t\omega\omega\alpha\theta$ table, look for 3-of-5, choose a 1-out equation, substitute and solve, answer the question.

UNITS: For $t\omega\omega\alpha\theta$, just be consistent; use all rads-and-s, or all revs-and-s, or... <u>For other rotational-motion equations</u>, <u>use only radians</u> for $\Delta\theta$, ω and α.

<u>I: Moment of Inertia calculation</u>

For a system of several objects, I_{total} = sum of I's for the individual objects.
$I = m r^2$ for a point-object, $I = X m r^2$ for a large object (get X-value from a table).
Optional: *parallel-axis* ($I = I_{cm} + mh^2$) & *radius of gyration* ($I = m r_g^2$), Problems 5-## & 5-##.

<u>How to calculate Torque, τ</u>

1) Choose object, draw F-diagram with each F acting at F-point (where F is applied).
2) Choose a specific τ-axis (τ is always calculated "with respect to" a specific axis).
3) Use either of the τ-formulas shown below. (I recommend that you learn both formulas.)

<u>$\tau = \pm r F \sin\theta$</u>	$\tau = \pm r_\perp F$
r is a vector from τ-axis to F-point θ is angle between r and F	To find $r_\perp$, a) DRAW the F-extensions, b) find closest approach to τ-axis (at 90°); c) this extension-to-axis distance is $r_\perp$.

4) To find direction-sign of τ, a) POINT pen in r-direction, b) HOLD pen at τ-axis, c) PUSH/PULL pen with F at F-point, d) DECIDE (usually is defined to be +).
5) τ_{total} = sum of individual τ's.

<u>Angular Momentum (L)</u>

<u>Angular Momentum (L)</u> is calculated almost like τ; just substitute mv for **F**.
r is vector from object-location to L-axis, $r_\perp$ is shortest shortest distance from v-extension to L-axis.
Linear motion: $L = m v r_\perp$. Rock-on-string: $m v r_\perp$ or $I\omega$. "$I = X m r^2$" object: $L = I\omega$.

Separate τ into τ_{int} and τ_{ext}; ask "Is the τ-causer inside or outside the system?". Some examples of τ_{int} are "analogies to $F_{internal}$", and an ice skater's arm-extension.

CONSERVATION OF ANGULAR MOMENTUM: If $\tau_{external} = 0$, $L_i = L_f$.
ALMOST-CONSERVATION: If $\Delta t \approx 0$ causes $\tau_{external}\Delta t \approx 0$, $L_i \approx L_f$.

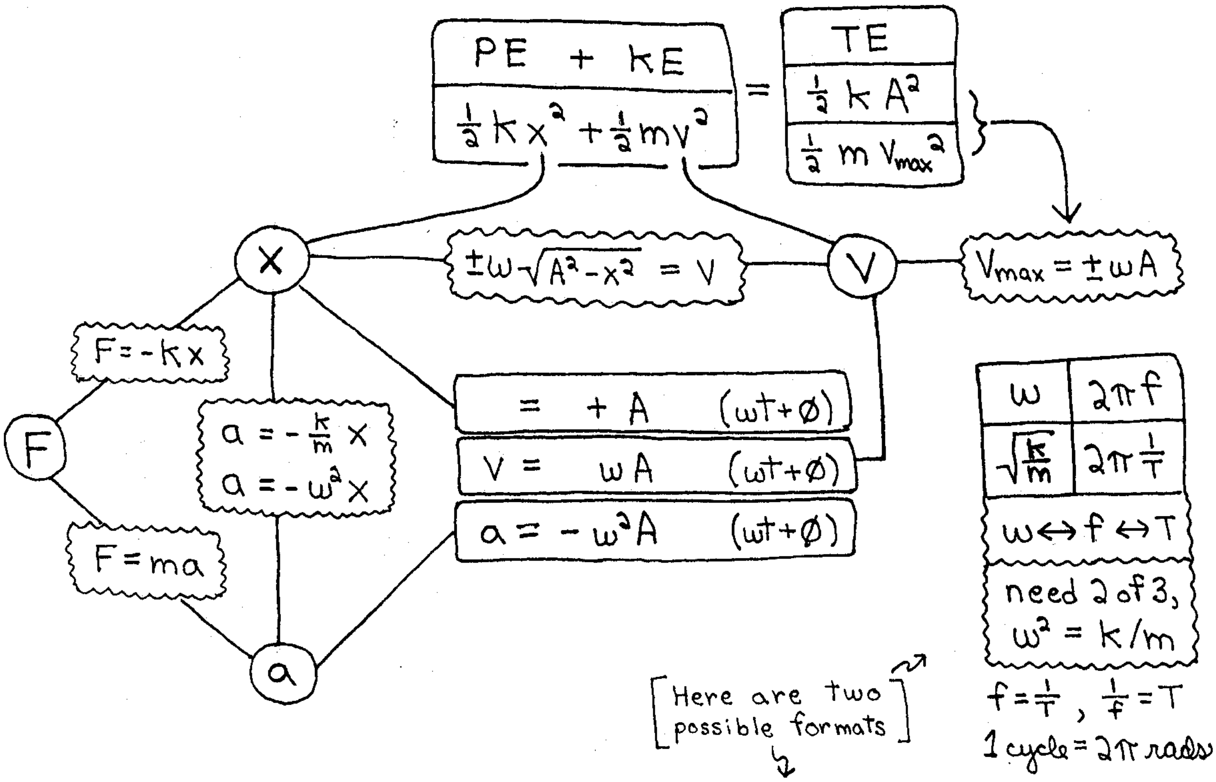
<u>Torque-Equilibrium Problems</u>

If an object is "static" (remaining at rest), it has $F_x = 0$ and $F_y = 0$ and $\tau = 0$.
For $\tau = 0$, choose τ-axis anywhere. (If an F-extension goes through a τ-axis (so $r_\perp = 0$), this F disappears from "$\tau = 0$" for that axis. Try to get a 1-unknown equation, or 2-unknowns/2-equations.)

Each F causes a τ; there are 4 τ-decisions for $\tau = \pm F r \sin\theta$, 3 for $\tau = \pm F r_\perp$.

Chapter 8 Summary

This summary is explained in Section 8.3.



$\omega \leftrightarrow f \leftrightarrow T$

2 of 3: $\omega^2 = k/m$

ω	$2\pi f$
$\sqrt{\frac{k}{m}}$	$2\pi \frac{1}{T}$

$f = \frac{1}{T}$, $\frac{1}{f} = T$

1 cycle = 2π rads

constant-variables:	TE	A	k, m	v_{max}, ω, f, T		ø
changing-variables:	PE, KE	x	F, a	v	t	θ

$$x_{max} = -A \qquad x = 0 \qquad x_{max} = +A$$
$$F_{max} = -k(-A) \qquad F = 0 \qquad F_{max} = -k(+A)$$
$$a_{max} = -k(-A)/m \qquad a = 0 \qquad a_{max} = -k(+A)/m$$
$$PE_{max} = \tfrac{1}{2}kA^2 \qquad PE = 0 \qquad PE_{max} = \tfrac{1}{2}kA^2$$

$$KE = 0 \qquad KE_{max} = \tfrac{1}{2}m(\omega A)^2 \qquad KE = 0$$
$$v = 0 \qquad v_{max} = \pm\omega A \qquad v = 0$$

10.8 A Summary of Electrostatic Relationships

The magnitude formulas marked with • are correct only for point-charges or spherically symmetric charge distributions.
All other formulas and relationships are always true.

The SI value of "k" is $9.00 \times 10^9\ Nm^2/s^2$.
k can also be written as "$1/4\pi \epsilon_0$", where $\epsilon_0 = 8.85 \times 10^{-12}\ s^2/Nm^2$.

$F = ma$, etc.

• $\vec{F} = k \dfrac{Qq}{r^2}$

Alike charges repel, opposite charges attract.

$F_{on\ q} = q\, E_Q$

If q is +, $\vec{F}$ is in direction of $\vec{E}$.
If q is −, $\vec{F}$ is opposite to $\vec{E}$.
For any q, F = 0 in direction ⊥ to $\vec{E}$.

• $\vec{E}_Q = k \dfrac{Q}{r^2}$

Away from +, Toward −.

Directions of $\vec{F}$ (or $\vec{a}$) and movement ($\vec{\Delta x}$ or $\vec{v}$) don't have to be same.

If F_{EL} and Δx are in same direction, W_{EL} is +.
If F_{EL} and Δx are in opposite directions, W_{EL} is −.
If F_{EL} is ⊥ to Δx, $W_{EL} = 0$.

If q is +, F is toward "more −" V.
If q is −, F is toward "more +" V.

$\vec{E}$ points "downhill" in number-line voltage:
In direction E points, V ↓.
Opposite E direction, V ↑.
⊥ to E, V doesn't change.

If E is constant,
$\Delta V = -E\, \Delta x$
(analogous to $W = F\Delta x$)

• $W_{EL} = -k \dfrac{Qq}{r}$

if q begins very far away from Q.

• $W_{EL} = -kQq\left(\dfrac{1}{r_f} - \dfrac{1}{r_i}\right)$

for any r_i and r_f.

$W_{EL} = -q\, \Delta V$

$\Delta PE_{EL} = q\, \Delta V$

V at a point

• $V = k \dfrac{Q}{r}$

if $V = 0$ when $r_i = \infty$.

• $\Delta V_Q = kQ\left(\dfrac{1}{r_f} - \dfrac{1}{r_i}\right)$

for any r_i and r_f.

SIGN: If Q is +, V_Q is +. If Q is −, V_Q is −.

MAGNITUDE: Close to Q, V_Q is large. Far from Q, V_Q is small.

$W_{EL} + W_{OTHER} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$
If constant speed, ΔKE = 0
If "unconstrained", $W_{other} = 0$

In the left-side formulas:
F and W are caused by the mutual interaction between Q & q,
so Q & q both appear in their formulas.

In the right-side formulas:
E_Q & V_Q (the E-field and voltage caused by Q) don't depend on q,
so neither has q in its formula.

Here are some differences between the top-row and bottom-row formulas.
Top formulas (F and E) have 1/r, are vectors (with magnitude and direction).
Bottom formulas (W and V) have $1/r^2$, are non-vectors (with magnitude and ± sign).

For F and E, ignore the ± sign of Q & q; use visual logic and think "vector direction".
For W and V, substitute the ± sign of Q & q, then be careful as you solve the algebra.

Optional Section 18.# discusses the calculus relationships between F & W and between E & V: $\Delta V = \int E\,dr$, $E_x = dV/dx$, etc.

The **PRINCIPLE OF SUPERPOSITION**:
the F, E, W & V caused by one charge is independent of that caused by other charges, so F_{total}, E_{total},... is the sum of the F's, E's,... caused by all charges in a system.

$\mathbf{F}_{total}$	=	sum of **F**'s	$\mathbf{E}_{total}$	=	sum of **E**'s
W_{total}	=	sum of W's	$(\Delta V)_{total}$	=	sum of (ΔV)'s

To find total **F** or **E**, split-add-reconstruct vectors; for total W or V, use + and – signs.

A charge is not affected by interaction with its own E-field:
the F_{el} acting on a charge = (its own charge)(E-field caused by all other charges).

SI-UNIT SUMMARY: the most common SI units for F, E, W & V are circled below.

$$F \text{ is in } N = \frac{kg\,m}{s^2} \qquad E \text{ is in } \frac{N}{C} = \frac{kg\,m}{C\,s^2} = \frac{V}{m}$$

$$W \text{ is in } J = N\,m = \frac{kg\,m^2}{s^2} \qquad V \text{ is in } V = \frac{N\,m}{C} = \frac{J}{C} = \frac{kg\,m^2}{C\,s^2}$$

I've included combinations like "kg m^2/C s^2" for completeness, but you'll probably never use them*. Instead, just be sure every substitution you make is in SI units, and you'll know that any variable you solve for will be in the appropriate SI units.
{ * For example, a battery's voltage will almost always be given as "12 V", not "12 kg m^2/Cs^2".}

[with some ideas cut and others added]

REVIEWER: This was originally part of Section 10.7. It will be modified some when it is made into a "chapter summary", but many of the main ideas are on these two pages.
{ As usual, each idea in the summary is explained in the regular chapter. }

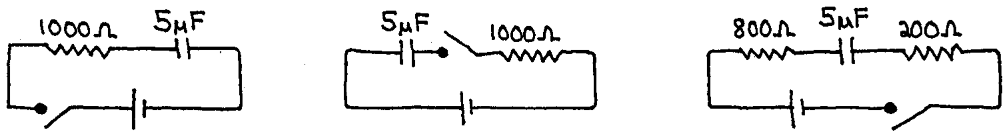
To "liberate" t from the exponent of $e^{-t/RC}$, take the *natural logarithm* ("ln") of both equation-sides; the reason for this strategy is explained in Section 19.6.

$$\begin{aligned}
\Delta V &= \Delta V_{max}\, e^{-t/RC} \\
Q &= Q_{max}\, e^{-\frac{t}{1000(5\mu)}} \\
.75\, \cancel{Q_{max}} &= \cancel{Q_{max}}\, e^{-t/.005} \\
\ln(.75) &= \ln(e^{-t/.005}) \\
\frac{-.288}{1} &= \frac{-t}{.005} \\
.0014\, s &= t
\end{aligned}$$

$$\begin{aligned}
\Delta V &= V_{max}\, e^{-t/RC} \\
\Delta V &= 12\, e^{-\frac{.0005}{1000(5\mu)}} \\
\Delta V &= 10.9 \text{ Volts}
\end{aligned}$$

[The -ww- and -||- both have the same ΔV of 10.9V.]

In a series circuit, the order of battery, switch, resistors & capacitors doesn't matter. For example, each circuit below behaves the same during the process of charging:



A simple *parallel RC circuit* is analyzed in Problem 11-##, using V-logic principles. If your class studies "parallel" circuits or if you're curious, look at this problem.

The principles of RC-analysis are visually organized in the Chapter 11 Summary.

Here is a ROUGH preview of the "RC Summary" for Section 11.10. It will be revised some (I ~~we~~ will PROBABLY change the placement of $Q = \Delta V C$, for example). There will also be a SHORT explanation, referring back to the CHARGE + DISCH problems that began 11.10.

I don't have flash cards finished.

• If $t \ll RC$, $Q_{BEFORE} = Q_{AFTER}$. Whatever Q was, it still is, so $(\Delta V_c)_{BEF} = (\Delta V_c)_{AFT}$.

• If $t \gg RC$, $I = 0$ to and from the -||-, and $\Delta V_R = 0$.

$Q = \Delta V\, C$

Trace V, $\Delta V = V_{HI} - V_{LO}$

$\Delta V = I R$

for exponential decrease, $\Delta V = (\Delta V)_{max} \{ \ e^{-t/RC} \ \}$

for exponential increase, $\Delta V = (\Delta V)_{max} \{ 1 - e^{-t/RC} \}$

Chapter 19 Summary

{ This is a rough sketch of ideas, not a polished finished product.}

There will be some discussion of these ideas in Ch.19 (as in Section 8.3).

three equivalent EQUATION-FORMATS

DERIVATIVE		DIFFERENTIAL		INTEGRAL
$\frac{dv}{dt} = a$	$\rightleftarrows$	$dv = a\,dt$	$\rightleftarrows$	$\int_{v_i}^{v_f} dv = \int_{t_i}^{t_f} a\,dt$

two types of useful INTEGRAL-EQUATION SETUPS, using "$dv = a\,dt$"

$$dv = a\,dt$$
$$\int_{v_i}^{v_f} dv = \int_{t_i}^{t_f} a\,dt$$
$$v_f - v_i = \int_{t_i}^{t_f} a\,dt$$
$$v_f = v_i + \int_{t_i}^{t_f} a\,dt$$

$$v_f = v_i + \Delta v$$
$$v_f = v_i + \int dv$$
$$v_f = v_i + \int a\,dt$$
$$v_f = v_i + \int_{t_i}^{t_f} (a\text{-equation})\,dt$$

{Maybe also refer to the relationships between x, v, a equations and x-t, v-t, a-t graphs, as discussed in Sections 2.10, 19.1 and 19.2.}

POINT
SLOPE
CONCAVITY
AREA

x-equation | $x = x_i + \int (v\text{-eqn})\,dt$

$\frac{dx}{dt} = v$ —— $dx = v\,dt$ —— $\int dx = \int v\,dt = \Delta x$

v-equation | $v = v_i + \int (a\text{-eqn})\,dt$

$\frac{dv}{dt} = a$ —— $dv = a\,dt$ —— $\int dv = \int a\,dt = \Delta v$

a-equation

$$\begin{array}{ccccccc}
x & & & & & & \theta \\
\vec{dx} = \vec{v}\,dt & & & & & & \vec{d\theta} = \vec{\omega}\,dt \\
v & \xrightarrow{(\times m)} & p & \xrightarrow{(\times r_\perp)} & L & \xrightarrow{(\div I)} & \omega \\
\vec{dv} = \vec{a}\,dt & & \vec{dp} = \vec{F}\,dt & & \vec{dL} = \vec{\tau}\,dt & & \vec{d\omega} = \vec{\alpha}\,dt \\
a & \xrightarrow{(\times m)} & F & \xrightarrow{(\times r_\perp)} & \tau & \xrightarrow{(\div I)} & \alpha
\end{array}$$

$$\boxed{\vec{F}\cdot d\vec{r} = dW} \xrightarrow{\text{CAUSES}} \boxed{dK}$$

$$d\vec{r} = \vec{v}\,dt \qquad dW = P\,dt$$

$$\boxed{\vec{F}\cdot\vec{v} = P}$$

Area of F vs. r —

$\int F \cos\phi \; dr$

$\int \vec{F}\cdot d\vec{r}$

$\int dW$

W_{TOTAL} $\xrightarrow{\text{CAUSES}}$

$\int dK$

ΔK

$K_f - K_i$

$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

Area of P vs. t —

$\int P\,dt$

$\int \vec{F}\cdot\vec{v}\,dt$

TYPES OF EQUATIONS	example
multiplication by scalar:	$d\vec{L} = \vec{\tau}\;dt$
dot-product multiplication:	$dW = \vec{F}\cdot d\vec{r}$
non-vector:	$dW = P\;dt$

COMBINATIONS?

$d\vec{L} = \vec{\tau}\;dt \qquad W = P\;dt$

$m\,d(\vec{r}\times\vec{v}) = (\vec{r}\times\vec{F})\,dt \qquad (\vec{F}\cdot d\vec{r}) = (\vec{F}\cdot\vec{v})\,dt$

derivatives w.r.t. TIME

$$\begin{pmatrix} x & & & \theta \\ v & p & L & \omega \\ a & F & \tau & \alpha \end{pmatrix}$$

and others

$d\vec{L} = \vec{\tau}\;dt$

derivatives w.r.t. POSITION

$$\begin{pmatrix} \text{POTENTIAL ENERGY}, & \text{POTENTIAL} \\ F_{conservative} & U \end{pmatrix}$$

F_{cons}: gravity, electric, ...

$dU = -\vec{F}\cdot d\vec{r}$

other derivative Types

$d(\text{FLUID PRESSURE}) = \rho\, g\, dh$

$X_{cm} = \frac{1}{M}\int x\;dm$

$I = \int r^2\;dm$

and others

An example of "other" types of derivatives is SUPERPOSITION-PRODUCTION of the following things by electric charges of these properties: ELECTRIC FIELD (E), POTENTIAL (V).

Gravity can also produce analogous properties.

Magnetic field production is described by BIOT-SAVART LAW: $dB = \frac{\mu_0 I}{4\pi}\,\frac{d\vec{l}\times\vec{r}}{r^3}$

$\vec{E} = \int d\vec{E} = \int \frac{1}{4\pi\epsilon_0}\frac{1}{r^2}\,dq$

The dE's must be added as vectors.

$V = \int dV = \int \frac{1}{4\pi\epsilon_0}\frac{1}{r}\,dq$

TIME-SPACE CONVERSIONS: $a = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = \frac{dv}{dx}\cdot v$

U_{EL} — POSITION DERIVATIVE ↓ ↑ POSITION INTEGRAL — F_{EL}

V — POSN DER ↓ ↑ POSN INT — E

V and E (or F_{EL} and U_{EL}) are related by POSITION DERIVATIVES and POSITION INTEGRALS.

$$\boxed{V} \xrightarrow{\text{DERIVATIVE wrt POSITION: } -\frac{dV}{dr} = \vec{E}} \boxed{E} \qquad \boxed{E} \xrightarrow{\text{POSITION-INTEGRAL: } \Delta V = -\int \vec{E}\cdot\vec{dr}} \boxed{V}$$

$$\boxed{U_{EL}} \xrightarrow{\text{POSN-DERIV: } -\frac{dU}{dr} = \vec{F}} \boxed{F_{EL}} \qquad \boxed{F_{EL}} \xrightarrow{\text{POSN-INT: } \Delta U = -\int \vec{F}\cdot\vec{dr}} \boxed{U_{EL}}$$

DOT PRODUCTS

$$dW = \vec{F} \cdot \vec{dr}$$

$$dW = F \cos\phi \; dr$$

$$dW = \int [F_x\,dx + F_y\,dy + F_z\,dz] \text{ for any type of F.}$$

$$\left.\begin{aligned} dU &= -\int [F_x\,dx + F_y\,dy + F_z\,dz] \\ &= -\left\{ \int F_x\,dx + \int F_y\,dy + \int F_z\,dz \right\} \end{aligned}\right\} \text{ if F is conservative.}$$

TAKING PARTIAL DERIVATIVES $\Big\}$ $-\frac{\partial U}{\partial x} = F_x, \; -\frac{\partial U}{\partial y} = F_y, \; -\frac{\partial U}{\partial z} = F_z$

ANALOGOUS

$$dV = -[E_x\,dx + E_y\,dy + E_z\,dz]$$

$$-\frac{\partial V}{\partial x} = E_x, \; -\frac{\partial V}{\partial y} = E_y, \; -\frac{\partial V}{\partial z} = E_z$$

MULTIPLICATION by a SCALAR

$\int \vec{a}\,dt = \int \vec{dv}$ ($\vec{a}$ causes $\vec{\Delta v}$), where $\vec{\Delta v} = (v_{xf} - v_{xi})\hat{i} + (v_{yf} - v_{yi})\hat{j} + (v_{zf} - v_{zi})\hat{k}$

$= \Delta v_x\,\hat{i} + \Delta v_y\,\hat{j} + \Delta v_z\,\hat{k}$

$\int a_x\,dt = \int dv_x$ (a_x causes Δv_x)

$\int a_y\,dt = \int dv_y$ (a_y causes Δv_y)

$\int a_z\,dt = \int dv_z$ (a_z causes Δv_z)

(miscellaneous)

Time-derivatives of Q: $I = \frac{dQ}{dt}$ for —||— being charged. $\quad I \neq \frac{dQ}{dt}$ for —\/\/\/— at steady state. $\quad I = \oint \vec{j}\cdot\vec{dS}$ for current "flow" through surface.

AESOP'S PROBLEMS

for 2.10, 19-A to C: examples of $x \leftrightarrow v \leftrightarrow a$, graphs $\leftrightarrow$ calculus

4.11, 19-D: Making Variables Match

5.3, 19-E: Tangent Line Approximation

19-F: SUPERPOSITION, $F = \int dF$, adding vector-components

5.6, 19-G: RATIO LOGIC, $dm = \left(\begin{smallmatrix}\text{VOLUME}\\ \text{FRACTION}\end{smallmatrix}\right) M_{total}$

DENSITY, $dm = \rho \; dV$

10.2, 19-H: variable-matching flexibility, $\left.\begin{array}{l} x\text{-and-}dx \text{ (failure)} \\ \theta\text{-and-}d\theta \text{ (success)} \end{array}\right\}$ IN THIS PROBLEM

MAXWELL'S EQUATIONS

Notice the uses of $\oint dV$ *and* $\oint dS$ *for an enclosed volume, and*
$\oint dS$ *and* $\oint dl$ *for a surface that is not closed.*

GAUSS'S LAWS: $\frac{1}{\epsilon_0} \oint \rho \, dV = \oint \vec{E} \cdot \vec{dS}$ (for Φ_E)

$$0 = \oint \vec{B} \cdot \vec{dS} \quad (\text{for } \Phi_B)$$

AMPERE'S LAW: $\mu_0 \left[\epsilon_0 \frac{d \oint \vec{E} \cdot \vec{dS}}{dt} + \oint \vec{j} \cdot \vec{dS} \right] = \oint \vec{B} \cdot \vec{dl}$

FARADAY'S LAW: $-\frac{d \oint \vec{B} \cdot \vec{dS}}{dt} = \oint \vec{E} \cdot \vec{dl}$

Here are three commonly-used (in PHYSICS) integrals:

$\int x \, dx \rightarrow \frac{1}{2} x^2$

$\int \frac{1}{r^2} dr \rightarrow -\frac{1}{r}$

$\int \frac{1}{r} dr \rightarrow \ln r$

These (from 19.11) are not arranged in any special order yet.

$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

magnitude of $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Is vector multiplication "reversible"?

$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$ $\qquad$ $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

also, $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

MANY-SIDED EQUATIONS: $\vec{A} \cdot \vec{B} = A B \cos\phi = A_x B_x + A_y B_y + A_z B_z$

$$\vec{A} \times \vec{B} = A B \sin\phi = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - B_y A_z)\hat{i} + (B_x A_z - A_x B_z)\hat{j} + (A_x B_y - B_x A_y)\hat{k}$$

If $\vec{A}$ *and* $\vec{B}$ *are parallel,* $\vec{A} \cdot \vec{B}$ *is* MAXIMUM, $\vec{A} \times \vec{B} = 0$,
If $\vec{A}$ *and* $\vec{B}$ *are perpendicular,* $\vec{A} \times \vec{B}$ *is* MAXIMUM, $\vec{A} \cdot \vec{B} = 0$.
I'll give examples of each: $\vec{F} \cdot \vec{d}$, $\vec{r} \times \vec{F}$, *and so on.*

Find direction of $\vec{A} \times \vec{B}$ *with "right-hand rule"* (19.11b or 12.3).

MEMORY TRICK *from Problem 19-#* (*for ± signs* $\hat{i} \times \hat{j}, \ldots$)

$i \;\; j \;\; k \;\; i \;\; j$ $\quad \ominus$ (arrows right to left) / $\oplus$ (arrows left to right)

For + or −, vectors must be "same kind".
For × or •, vectors can be different kinds.* $\quad$ **must be??*