17.91 Problems

for Section 17.1,

17-1: How much energy is carried by one 700 nm red photon? one 480 nm blue photon?

What has more energy, 1000 red photons or 500 blue photons? If a beam of red light and blue light each carry the same energy, which beam contains more photons?

17-2: Show that E_{photon} (in eV) and λ (in nm) are related by $\mathbf{E\lambda = 1240}$. { This can be useful for making quick calculations. }

17-3: == bulb

17-4: Sunlight shines on a rock, raising its temperature, and on magnesium metal, knocking electrons from its surface. What is the "time dependence" of each process?

17-5: Sodium metal has a work function of 2.36 eV. What range of light wavelengths is able to eject electrons from sodium?

With 400 nm light, what is the maximum kinetic energy of the ejected electrons? Do all of the electrons have this much KE? What is maximum speed for the electrons? What potential difference (between the sodium and "electron collector") is needed to totally stop the flow of photoelectrons?

Which light beam ejects more photons: high intensity 600 nm orange, medium intensity 500 nm green, or low intensity 400 nm violet? Which beam ejects electrons with the most KE?

17-6: Compare the threshold frequency-and-wavelength for sodium (W_o = 2.36 eV) and silver (W_o = 4.7 eV). If 200 nm light shines on both metals, which one ejects electrons with higher KE_{max}?

for Section 17.2,

17-7: Which has a longer wavelength: 1) A bullet or an electron, if they both have the same speed? 2) A fast electron or slow electron? 3) A 10 eV proton or 10 eV proton?

17-8: What is the wavelength of a .057 kg tennis ball moving at 50 m/s? { 112 miles/hour, the speed of a fairly fast serve. }

What is an electron's wavelength after it accelerates through a potential difference of 75 V? Could such an electron be "diffracted" by an atomic crystal? If a neutron has this same λ, what is its speed? If a photon has this λ, what is its frequency and energy?

{ $m_{electron} = 9.11 \times 10^{-31}$ kg, $m_{neutron} = 1.67 \times 10^{-27}$ kg }

17-9: Show that "$h = \lambda p$" can be derived for photons, using equations in Einstein's 1905 articles on relativity and photoelectric effect: $E = mc^2$, $E^2 = p^2c^2 + m_o^2 c^4$, $m_{o(photon)} = 0$ (Section 16.4), $E = hf$ (17.1), and $f\lambda = c$ (9.2). { Hint: There are two easy ways to derive it! }

And in 1905 Einstein postulated half of wave-particle duality's "symmetric duo": photon-waves with a particle-nature. Why do you think it took so long (until 1923) for DeBroglie to propose the logical conclusion that particles have a wave-like nature?

for Section 17.3,

17-10: **Was Bohr a Fool?**

What are the good and bad points of the Bohr Model of the atom? Was it a mistake for Bohr to propose his model?

17-11: **does God play dice?**

Einstein felt that quantum mechanics is "incomplete" and we would eventually find a better theory. One way he expressed his displeasure at the limitation of answers given in terms of probabilities is that "God does not play dice". What do you think?

17-12: **Is it all a lucky coincidence?**

In Section 17.3, Guillemin discusses how life depends on wave-particles, quantization, and "h". Scientists are discovering that the laws of nature (the basic force laws, charge and mass of electrons,...) seem to be "just right" for a wide variey of life-permitting phenomena: stable nuclei, fusion and star-formation, strength of hydrogen bonds,... Here is one example [out of many possible]: if the strong nuclear force was a few percent weaker or stronger than it is, stable nuclei and sunshine could not exist.

What are some possible explanations for the fact that the laws of nature are "just right" for intelligent life?

for Section 17.4,

17-13: Does the Uncertainty Principle set a limit on how accurately position can be measured? or accuracy of measuring momentum? What limit does it set?

17-14: If a tennis ball (.057 kg) and electron (9.1×10^{-31} kg) each travel at 50 m/s, what is the "uncertainty" in their positions?

17-15: Electrons passes through a slit with width "w", as shown below. Use basic principles & equations ($w \sin\theta = m\lambda$, $d = vt$, $h = \lambda p$, $p \int mv$, and "$\sin\theta \approx s/L$ for small angles") to show that $\Delta y \, \Delta p_x \approx h$. { Hint: Why does an electron diffract (as described in Section 15.1) into the "shadow region"? } [as always, the picture is missing]

17-16: An electron "wants" to be close to a proton, due to electrostatic attraction. Show that, if an electron in contact with a proton (with their centers $\approx 10^{-15}$ m apart) to form a "± clump", it cannot remain there. { Hint: Find the Δp_x and Δv_x required by the U.P. and the KE this produces, then compare it with the electrostatic potential energy that is available to "hold" the electron at this distance from the proton. }

Find the electron-to-proton distance [for an electron in an atom] where the "Uncertainty Principle KE" equals $PE_{electrostatic}$.

{ Hint: Assume that the atom-diameter is the electron-Δx. }

For an electron with the speed predicted by Bohr's model of the atom (2.2×10^6 m/s), what is the minimum Δx that is compatible with the Uncertainty Principle?

17-17: Using principles from Section 4.1, derive $\mathbf{\Delta E\ \Delta t \geq h / 2\pi}$ from $\mathbf{\Delta x\ \Delta p \geq h / 2\pi}$.

Another derivation [for a photon] is based on Section 16.4's $E^2 = p^2c^2 + m_o c^4$ and $m_{o(photon)} = 0$, and the length of a photon's wave-train ($\Delta x_{wave\text{-}train} = c\ \Delta t$) if light is "emitted" during a time interval of Δt.

If an atom radiates during a time of 10^{-9} s, what is the "energy width" of its emission?

17-18: **Einstein versus Bohr**

Einstein never accepted the fundamental philosophy of quantum mechanics, and in the 10 years following the discovery of QM he proposed many "thought experiments" in an effort to escape the limits set by the Uncertainty Principle. Bohr was often a "friendly adversary" who found reasons why Einstein's schemes would not work. Do you think Einstein ever discovered a way to avoid the U.P. limitations?

17-19: If you look at a tree in the forest, does your "act of observation" affect the tree?

17-20: **Schrodinger's cat**

Schrodinger is famous for his "wave equation" and, to a lesser extent, for this thought-experiment: A live cat is placed in a box with a bottle of poison gas and some radioactive atoms. If an atom decays within a one-hour period (a 50% possibility) a geiger counter will detect it and trigger a hammer to break the poison bottle and kill the cat.

Atomic decay is a probabilistic quantum event that, like an electron traveling between the slits and screen, isn't "completed" until an observation is made*. At the end of the "hour of danger", unless we look inside the box we cannot know whether the cat is dead or alive, but can we say that "the cat is either dead or alive"? If we later open the box and find a dead cat, did the cat die at the moment when we "observed" its state?

* you can imagine the 50% quantum event as if it was a double-slit electron that hits the wall either below the center line (a hammer breaks the bottle) or above it (the cat lives)

17-21: In what ways do you think a philosophy that "each person creates his own reality" is valid? If there is a report that the local theater is on fire, can Joe and Tom (who hear the report) affect its "reality"?

17.92 Solutions

17-1: $E_{red} = \dfrac{(6.63 \times 10^{-34}\text{ J s})(3.00 \times 10^{8}\text{ m/s})}{700 \times 10^{-9}\text{ m}}$ $= 2.84 \times 10^{-19}$ J $= (2.84 \times 10^{-19}\text{ J})(1\text{ eV}/1.60 \times 10^{-19}\text{ J}) =$ 1.78 eV. Similarly, $E_{one\ blue\ photon} = hc/(480\text{n}) = 4.14 \times 10^{-19}$ J $= 2.59$ eV.

A blue photon has 1.46 times more energy, but the red group (twice as many photons) has more E_{total}.

In order to have the same energy as blue light, red light must contain more photons.

17-2: The equation (with conversion factor): E =

$$\frac{(6.626 \times 10^{-34}\text{ Js})(2.998 \times 10^{8}\text{ m/s})}{\lambda \times 10^{-9}\text{ m}} \; \frac{1\text{ eV}}{1.602 \times 10^{-19}\text{ J}}$$

17-3: ==bulb

17-4: The rock's T begins increasing immediately and continues until it reaches an "equilibrium T" and the incoming radiant energy equals the outgoing heat losses (as described in Section 7.6) by conduction, convection and radiation.

Magnesium begins to lose electrons immediately and continues to do so as long as light shines on it. {The metal can regain some electrons by grabbing them from air molecules that collide with its surface.}

17-5: The "threshold wavelength" is

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34}\text{ Js})(3.00 \times 10^{8}\text{ m/s})}{(2.36\text{ eV})(1.60 \times 10^{-19}\text{ J/eV})} =$$

527 nm. Or use "$E\lambda = 1240$" from Problem 17-2: $\lambda = 1240/2.36 = 525$ nm [this is more accurate than 527 nm because "1240" is accurate to 4 figures].

525 nm light-photons have just enough energy to eject electrons, and $E_{photon} \neq$ as λ ¬, so photons with $\lambda \leq 525$ nm have enough energy to eject photons.

$KE_{max} = E_{photon} - E_{to\ eject\ electron} = hc/\lambda - W_o =$ $(6.63 \times 10^{-34})(3 \times 10^{8})/(400\text{ n}) - (2.36)(1.6 \times 10^{-19}) =$ $(4.97 \times 10^{-19}) - (3.78 \times 10^{-19}) = 1.19 \times 10^{-19}$ Joules.

In eV, $KE_{max} = 1240/400 - 2.36 = .74$ eV.

For reasons discussed in Section 17.1, some electrons have less KE than this.

$KE = \frac{1}{2}\ m v^2$ can be solved for $v = \sqrt{2\ KE/m} = \sqrt{2(1.19 \times 10^{-19})/(9.11 \times 10^{-31})} = 5.11 \times 10^{5}$ m/s. This is fast, but it is only .0017c, much less than the speed of light, so we don't have to use Section 17.4's formula for relativistic KE. {Notice that to use SI units consistently, KE must be in J, not eV. }

From Section 10.5: $W_{electric} = -q\Delta V$. To stop the fastest electrons, W_{el} must lower their KE from KE_{max} to zero: $-(-1.6 \times 10^{-19})\ \Delta V = (-1.19 \times 10^{-19})$, $\Delta V = -.74$ Volts. The – shows that, as common sense also demands, the electron collector must be at a more negative voltage than the sodium.

By using the definition of *electron-Volt,* we get the same answer: (1 electron)(.74 V) gives W_{el}-and-ΔKE = .74 eV. Have you noticed that it is easier to do calculations with eV (instead of J) in many situations?

A 600 nm photon doesn't carry enough energy to eject an electron, no matter how many photons the high-intensity beam contains. Green & violet photons can both eject electrons. The higher intensity green beam has more photons which eject more electrons. But violet photons have more energy, so they eject electrons that have a higher KE_{max}.

17-6: Silver has higher $f_{threshold}$, lower $\lambda_{threshold}$, lower KE_{max} for electrons ejected by 200 nm light.

17-7: $h = \lambda p$ [help], so $h = \lambda m v$, and λ ≠ when either m or v ¬ . For #1 & #2: the electron (smaller mass) and slow electron (smaller v) have larger λ.

For #3, the situation is a little more complicated. A proton has larger mass by a factor of 1836, but (if they both have the same KE) slower v by a factor of $1/\sqrt{1836}$ = 1/42.85. The proton has more momentum by a factor of (1836)(1/42.85) = 42.85, so the electron has a larger λ.

17-8: $\lambda_{ball} = h/mv = (6.63 \times 10^{-34})/(.057)(50) = 2.3 \times 10^{-34}$ m.

From Section 10.5: $q\Delta V = \frac{1}{2} m v^2$, $v = \sqrt{2qV/m} = \sqrt{2(1.6 \times 10^{-19})(75)/(9.11 \times 10^{-31})}$ = 5.1×10^6 m/s. $\lambda = h/(9.11 \times 10^{-31})(5.1 \times 10^6)$ = 1.4×10^{-10} m = 1.4 Å . This is about the same as distance between atoms in a crystal, so (as discussed in Section 15.1) these electrons could be diffracted by a crystal.

$v_{neutron} = h/m\lambda = h/(1.67 \times 10^{-27})(1.4 \times 10^{-10})$ = 2800 m/s. $f_{photon} = c/\lambda = c/(1.4 \times 10^{-10}) = 2.1 \times 10^{18}$ Hz; this is in the x-ray region of the electromagnetic spectrum. $E_{photon} = hf = 1.4 \times 10^{-15}$ J = 8750 eV.

Analysis of the diffraction patterns of neutrons (or x-rays) can provide valuable information about the structure of a crystal. ==nec?

17-9: Both derivations use E=hc/λ, derived by combining E=hf and fλ=c. For the second derivation, $m_{o(photon)} = 0$, $E^2 = p^2c^2 + 0^2c^4$, $E^2=p^2c^2$, E=pc.

$$\begin{aligned} E &= mc^2 & E &= pc \\ hc/\lambda &= mc^2 & hc/\lambda &= pc \\ h &= \lambda mc & h &= \lambda p \\ h &= \lambda p & & \end{aligned}$$

Both derivations have equivalent starting-equations because $E = \mathbf{mc^2} = mcc = \mathbf{pc}$.

Why was the idea of *matter waves* delayed until 1923? Because it isn't easy to think in such a radical new way" that corresponds with nature. Most radical new ideas -- like this one -- turn out to be wrong, and it's difficult to know which ones are the rare ones that are correct. The scientific community is cautious, so they want to verify. And it's easier to see clearly in hindsight than in foresight.

It's easy to be a Monday Morning Quarterback and say "it should have been so easy to see the correct way to do think." But it takes real genius to invent/discover a new idea. For example, **now** I can explain the main features of special relativity in 10 minutes, but in **1905** inventing this idea was a leap of genius for Einstein.

17-10: Bohr's model (I'll assume you about it from your text) was the best available from 1913 until quantum mechanics was developed in 1925. Some QM advantages are: it predicts correct atomic energy levels (so does the Bohr Model), correctly predicts angular momentum and electron location (BM makes "definite but wrong" predictions about both), and QM explains the periodic table's structure (and BM doesn't).

The BM is an interesting way to think about atoms. As with any model, though, you should separate the aspects that are "analogous to reality" from those that are not. {This essential thinking skill is discussed in the "Comparing Pictures" part of Section 20.7.}

Every scientific theory should be "held with a light grip", with the attitude that "if there is evidence that another theory is a better explanation of nature, I will support its acceptance". In 1925 Niels Bohr did not say "I will fight for the model that bears my name." Instead, he rejoiced at the new theory that brought scientists closer to the truth, and worked with them to develop it. Far from being regarded as a fool because "his model" was replaced, Bohr's flexible attitude and many valuable contributions to QM made him one of the most respected scientists of his era.

17-11: Despite Einstein's greatness, most experts in physics and theology think he was wrong about quantum mechanics and God. Einstein's view of QM is discussed later, in 17-#$'s "Einstein versus Bohr". The focus of the present discussion is on theology.

Einstein believed in "a God who reveals himself in the harmony of all that exists, not in a God who concerns himself with the fate and actions of men". His God was nature-and-science: an orderly system obeying rules that could be discovered & understood by humans. But in theistic religions (like Christianity, Judaism, Moslem) God is much more than "nature" and is not limited by natural laws He designed into the universe. QM claims there is a limit to what we can know about the universe [the Bible agrees] but says nothing, one way or the other, about God's ability to know what is happening in His universe. If God can do the actions recorded in the Bible, He is in control of (and is not limited by) natural processes.

17-12: Here are three of the more popular theories. 1) The *anthropic priciple* says: So what? If natural laws didn't allow intelligent life, we wouldn't be here to be amazed at it. 2) Maybe there are a huge number of universes [existing simultaneously or sequentially] and we happen to be in one where nature allows life. 3) The "watchmaker" conclusion: our universe is the product of extremely intelligent design.

What explanation do you think is most plausible?

17-13: No; the U.P. sets no limit on measurement of position or momentum*. It does limit the accuracy of the $\Delta x\, \Delta p_x$ combination; the more accurate one is, the less accurate the other one is. * technology will, of course, limit the accuracy of measurements

17-14: For each object, $\Delta x \geq (h/2\pi)/m$:
$\Delta x_{t\text{-ball}} \geq (6.63 \times 10^{-34}/2\pi)/(.057) = 1.9 \times 10^{-33}$ m,
$\Delta x_{electron} \geq (h/2\pi)/(9.1 \times 10^{-31}$ kg$) = .00012$ m.

Δx for the ball is trivial (10^{-31} of its diameter), but for an electron at this speed Δx is a million times larger than a typical atomic diameter. The U.P. is important for atomic-level phenomena, but (except for the "Existence of Life" ideas discussed at the end of Section 17.3) it does not affect everyday life.

17-15: An electron can pass through any part of the slit, so its initial Δy-uncertainty is "w". It can move into the shadow region because, instead of the $v_y = 0$ that is implied by $\xrightarrow{v_x}$, there is some uncertainty in its y-momentum and y-velocity (Δp_y and Δv_y). In a Δt of L/v_x, v_y causes $\Delta y = v_y\, \Delta t$.

For an electron that diffracts from the center of the slit to the edge of the central diffraction maximum (a Δy distance of "s"),

$$\begin{array}{cccc} w & \sin\theta & = & m\ \lambda \\ \Delta y & s/L & \approx & 1\ \ h/p_x \\ \Delta y & \Delta v_y\, \Delta t / L & \approx & h/mv_x \\ \Delta y & \Delta v_y\, (L/v_x)/L & \approx & h/mv_x \\ \Delta y & m\, \Delta v_y & \approx & h \\ \Delta y & \Delta p_y & \approx & h \end{array}$$

Some electrons do travel further out, of course, to regions with $m = 2, \ldots$ But for purposes of this rough derivation, the simplified treatment above is adequate.

17-16: $\Delta x\, \Delta(mv) \geq h/2\pi$, so Δv_x (the minimum v an object can have) is $h/(2\pi\, m\, \Delta x)$. If an electron is "localized" to $\Delta x \approx 10^{-15}$ m because it is in contact with the proton,
KE $\int \frac{1}{2} mv^2 = \frac{1}{2} m(h/2\pi m \Delta x)^2 = h^2/(8\pi^2 m\, \Delta x^2) = h^2/([79][9.1 \times 10^{-31}][10^{-15}]^2) = 6.1 \times 10^{-9}$ J.

If an electron is brought to 10^{-15} m, from infinitely far away from the proton, its decrease in electrostatic PE is $(9 \times 10^9)(1.6 \times 10^{-19})^2/(10^{-15}) = 2.3 \times 10^{-13}$ J.

This is less than 6.1×10^{-9} J by a factor of 27000, and is not enough to hold an electron at 10^{-15} m.
{This would be like trying to trap a ball with KE = 27000 J in a pit with mgh = 1 J.} The U.P. thus predicts that protons and electrons do not form "±clumps" that would be useless for supporting life.

Using the same logic as above and substituting for h, $m_{electron}$, $\Delta x \approx 2r$ [for an electron "trapped" within the diameter of an atom], k ($= 9.0 \times 10^9$), $Q_{electron}$, q_{proton}, and an electron-to-proton distance of r,

$$h^2/(8\pi^2 m\, [2r]^2) \leq k\, Q\, q / r$$
$$h^2/(32\pi^2 m\, k\, Q\, q) \leq r$$

we can solve for $r \geq .066 \times 10^{-10}$ m. This says "an atom must have a radius at least this large*, or the U.P. KE will exceed the $PE_{electrostatic}$ that is available to hold the electron near the proton". {U.P. KE varies as $1/r^2$, and PE_{el} as $1/r$, so U.P. KE dominates if r is extremely small.}

This value of r can be compared with the "atomic radius", calculated from the Bohr Model or quantum mechanics, of approximately $.53 \times 10^{-10}$ m ($\approx .53$ Å .). {We should expect only "order of magnitude" estimates from our use of the U.P., not exact answers, especially when we consider that there are several ways to write the U.P. (for example, $h/2\pi$ is sometimes written as h or $h/4\pi$).} ==[is this too much about this?]

$\Delta x\, \Delta p_x \geq h/2\pi$, $[2r][(9.1 \times 10^{-31})(2.2 \times 10^6) \geq h/2\pi$, and $r \geq .26 \times 10^{-10}$ m. This is even closer (than in the calculation above) to the *Bohr radius* of .53 Å .

17-17: These derivations aren't "mathematically rigorous" but they may help you understand that the same wave/quantum aspect of nature causes both U.P. limitations: $\Delta x\, \Delta p_x$ and $\Delta E\, \Delta t$. {There are other "U.P. pairs" but $\Delta x \Delta p$ & $\Delta E \Delta t$ are the most common.}

On the left: if we imagine that Δp is caused by $F\, \Delta t$, we can also imagine that $F \Delta x$ causes ΔE. On the right: if $m_o = 0$ (for a photon), $E^2 = p^2c^2 + (0)^2c^4$, $E = pc$.

$$\begin{array}{lcl|lcl} \Delta x\ \ \Delta p_x & \geq & h/2\pi & \Delta x\ \ \Delta p_x & \geq & h/2\pi \\ \Delta x\ (F\, \Delta t) & \geq & h/2\pi & (c\, \Delta t)(\Delta E/c) & \geq & h/2\pi \\ (\Delta x\ F)\, \Delta t & \geq & h/2\pi & \Delta t\ \ \Delta E & \geq & h/2\pi \\ \Delta E\ \ \Delta t & \geq & h/2\pi & \Delta E\ \ \Delta t & \geq & h/2\pi \end{array}$$

$\Delta E \geq h/2\pi\, \Delta t \geq h/2\pi(10^{-9}) \geq 1.1 \times 10^{-25}$ J $\geq 6.6 \times 10^{-7}$ eV. An emission at 500 nm (with energy = 2.5 eV) has a "width" of (500 nm)$[6.6 \times 10^{-7}/2.5] = 1.3 \times 10^{-4}$ nm. ==[there is unclear difference between the location & width of the line]

17-18: Despite Einstein's clever arguments, Bohr seems to have defended QM & the U.P. successfully. {But there is always the chance that, as has happened so many times in the past, our current scientific theories will turn out to be wrong.}

There are still, however, unanswered questions like "Are there levels-of-nature that QM doesn't describe?" and "What is the connection between our knowledge of events and their reality?". These questions are discussed briefly in Problems 17-## and 17-##: **does God play dice**, and **defining Reality**.

{The 1935 EPR (Einstein-Podolsky-Rosen) article, one of Einstein's more interesting ideas, has sparked interest among scientists and philosophers (especially since the 1980s when experiments showed that EPR were wrong in their claims) but EPR questions aren't directly relevant for most claims that are made in "mystical physics" books. }

17-19: No. Your passive observation is not the active observation (in which there is energy exchange) that is described in the U.P. If you shine light on the tree so you can see it more clearly, the light-photons will affect the tree's atoms, although you (the human "observer") do not. Or if you walk up to the tree and touch its surface in "active observation-probing" the tree will, of course, be affected.

17-20: A non-solopsistic interpretation of quantum mechanics says: until we look we don't know if the cat is alive, but the cat's fate has been determined by a particle hitting the geiger counter (this "observation" completes the quantum event) or the hour-of-danger coming to an end. { for my analysis in 2003, check THIS PAGE }

A popular "mystical physics book" by Gary Zukav says "According to quantum mechanics ... the fate of the cat is not determined until we look inside the box." Do you see the logical error in this statement?

Imagine that we put two weeks of air, water, food (and kitty litter) inside the box and don't look inside it until two weeks after the danger-hour is over. When the box is finally opened, how would Zukav explain the two weeks of eating or rotting unless the cat's fate was determined two weeks earlier — at a time before there was any "obervation by human consciousness"?

17-21: Joe and Tom may have opinions (Joe says "yes, there is a fire," while Tom says "no, there isn't, it's just those ornery kids who have turned in false alarms all week," but their yes-or-no arguments can't change the reality of whether there is or isn't a fire.

Beliefs do affect actions, which then become reality: will a person go to the theater to fight the fire (even though there may be none) or ignore the alarm (even though it may or may not be real)?

{ Our attitudes may also have indirect effects; most religions believe that 1) spiritual entities exist, and 2) these entities can respond to the prayers of people and cause changes in our physical world.} ==[cut this? redo? or cut whole problem and use some in earlier problem?]