16.91 Optional Problems

for Section 16.2,

16-1: An observer on the ground shoots a beam of light toward the mirror so it comes straight back to him: ◊. What does the beam look like to an observer on a train moving ∅. Is either of these observers "proper"?

16-2: The fastest commercial airplane, the Concorde SST, flies at 690 m/s [this is faster than the 464 m/s rotational speed of a person at the equator]. If a clock flew at this speed for 100 years, would it run "slow" or "fast"? By how much? { Hint: Use the tools below. }

Here are some useful arithmetic tools.

If $x \ll 1$,

$\sqrt{1-x} \approx 1 - \frac{1}{2}x$ $\quad\quad$ $\sqrt{1+x} \approx 1 + \frac{1}{2}x$

$1 - 2x \approx (1-x)^2$ $\quad\quad$ $1 + 2x \approx (1+x)^2$

$\frac{1}{1-x} \approx 1 + x$ $\quad\quad$ $\frac{1}{1+x} \approx 1 - x$

For example,

$\sqrt{.9996} = \sqrt{1-.0004} \approx 1 - \frac{1}{2}(.0004) \approx .9998$,

$[1 + (4 \times 10^{-9})]^2 \approx 1 + 2(4 \times 10^{-9}) \approx 1 + (8 \times 10^{-9})$,

and $1/(1 - (7 \times 10^{-15}) \approx 1 + (7 \times 10^{-15})$.

These tools are used in Problems 16-2 and 16-##.

16-3: Is it possible to have more than one proper observer? improper observer? Do all proper observers measure the same i-to-f Δt? Do all improper observers measure the same Δt?

Arrange the Δt's, from smallest to largest, for these observers watching a 15-second interval on the clock:

for Section 16.3,

16-4: A block of gold has lengths of .50 m, .40 m & .30 m in the x, y & z directions. If the block moves at .90 c in the x-direction, what does a person who is standing on the ground observe for the block's 1) size, 2) mass, and 3) density? { At rest, gold density is 19300 kg/m^3. }

16-5: A spectator and car-driver record Δt for a race. Is either observer "proper"? If each records Δt between the car's front and rear crossing the finish line, who is proper?

16-6: If you travel at 2.9×10^8 m/s to a star that is, observed from earth, 100 light-years away, how long did the trip take (according to a wrist watch you wore during the trip), and how far did you travel? According to a person who stays on earth, how long does the trip take? { A light-year measures distance, not time; it is how far light travels in 1 year. }

16-7: If you measure a 36-inch yard stick to be 18 inches, what is the stick's speed?

16-8: In Section 16.3's SIMULTANEITY example, if G says "the A-to-B and A'-to-B' distances are both 50.00 m", what length does T measure for A-to-B and A'-to-B'? { T's speed is .5c, and $\sqrt{1-(.5c)^2/c^2} = .866$. }

Draw a simple picture to show what is seen by T when the lightning strikes B/B' and when it strikes A/A'.

16-9: Rocket Rhonda says "You are in a 1750 kg car (3.0 m long, 1.5 m high) moving backward [◆] at .60c, and I am in a 1500 kg car (4.0 m long, 1.5 m high) at rest." What do you say about your car and her car?

16-10: At rest, half of a group of *muon* particles would decay in 1.5×10^{-6} seconds.

If 100 muons move past you at .97c (as you stand on the ground), how far will you see them move before 50 of them have decayed?

for Section 16.4,

16-11: At rest, a 20 kg object object is 20 m long. At a speed when the object's mass is 80 kg, what is its length. { Hint: you can solve this by finding v, but it is easier to use ratio logic. }

When the object's energy is 4 times as large as its rest energy, what is its length?

16-12: At what speed will an object have
1) a mass that is 5/2 times larger than its rest mass,
2) a total energy 5/3 times its kinetic energy, and
3) a kinetic energy 3/2 times its rest energy.

16-13: If an electron starts from rest and accelerates through a potential difference of 1×10^6 Volts, its kinetic energy increases by $q\,\Delta V = 1 \times 10^6$ eV = 1 MeV = 1.6×10^{-13} Joules.

What is the electron's change of mass? What is its final mass, total energy, speed & momentum? { An electron's m_o is 9.11×10^{-31} kg. }

If an electron is pushed with a force equal to 10 times its weight [this gives it an acceleration of 98 m/s^2 when it begins to move], how long will it take to reach a speed corresponding to 1 MeV of kinetic energy? How far will it travel during this time?

If an electron and proton start from rest and accelerate through 10^6 V, do they have the same change of mass? Do they have the same % increase in mass? { proton's m_o is 1.67×10^{-27} kg }

16-14: What fraction of an object's total energy is kinetic energy when its speed is .01c? when its speed is .99c?

16-15: 1 kg of gasoline (about 1480 cm^3, or .39 gallon) at 20° C is burned inside a container, releasing 48×10^6 J of KE and raising the temperature above 20° C. If no energy or matter escapes from the system, what is the system's change in 1) total energy, 2) rest mass, 3) mass ?

If we then allow heat (but not matter) to leave the system until T returns to 20° C, what is the system's overall change in 4) total energy, 5) rest mass, 6) mass ?

16-16: How much work is needed to change the speed of a 10 kg object by .001c,
1) from 0 to .001c, or 2) from .998c to .999c?
Why is there such a large difference?

16-17: To get a Δm of 3 mg, how fast must a 1000 kg car travel? { Hint: Use the "x « 1" arithmetic tools from Problem 16-2. }

16-18: If 1 g of rest mass was converted entirely into usable energy, how many 75 W light bulbs could be run for 1 year?

In ^{235}U fission (the process involved in the first atomic bomb), approximately .1% of the ^{235}U's rest mass is converted into KE. If 1 kg of ^{235}U reacts, and we assume 100% efficient use of the released energy, how many 75 W bulbs can be run for 1 year?

Exploding 1 ton of TNT releases 4.1×10^9 J. How much TNT would have to be exploded, to equal the fission energy of 1 kg of $_{235}$U ?

16-19: Show that, according to relativity theory, if anything moves at the speed of light, it must have a rest mass of zero.
{ Hint: If v = c, how can mass be non-infinite? }

16-20: A proton (1.67×10^{-27} kg, 1.60×10^{-19} Coulomb) enters a 3.0 T magnetic field at a speed of 1.5×10^8 m/s. What is the radius of its circular path. Hint: An equation for this motion is derived in Section 12.#: $q B r = m v$.

for Section 16.5,

16-21: You watch Rebecca ride a rocket rightward at .8c. She has a gun that shoots bullets at .6c. She shoots one bullet ∅ and another ♦. What bullet velocities do you see?

16-22: Rocket Robert rides ∅ at .8c. He chases Rocket B that, by your measurement, is moving ∅ at .7c. Robert asks: **a)** "What do I observe for B's velocity?" **b)** What does a B-rider observe for my velocity? **c)** I shoot a bullet and see it move ∅ at .6c; what bullet-v does a B-rider see? **d)** B shoots a bullet [from the same kind of gun] ∅ at .6c; what bullet-v do I observe? **e)** The same type of .6c bullet is shot ♦ from Rocket C and I see the bullet-v as .7c ♦ ; what velocity do I observe for C ?

16-23: Linda moves ♦ at .8c and shoots flashlight beams in the ♦ and ∅ directions. Show that v-addition predicts that you, at rest on earth, see each beam move at "c".
==[she sees moving at c --- nec?

16-24: You see Rick move ∅ at .8c. He sees a .6c car move ∅ through a 400m race course. What is the course-length, velocity and race-Δt that is seen by Rick, the driver, and you. Are any of you a proper observer? Does the time-dilation formula correctly predict the difference in observed Δt's?

16-25: Becky, moving $\xrightarrow{.8c}$ with respect to earth, sees Tom [on a rocket ahead of her] shoot a bullet at $\xrightarrow{.5c}$ [wrt him] and says "I see the bullet's velocity as $\xrightarrow{.1c}$ ". What velocity do you [on earth] observe for Tom ? { Hint: Find Tom's v wrt Rebecca, then wrt earth. }

for Section 16.6,

16-26: In an earth laboratory, a certain line in the hydrogen spectrum is at 122 nm. Hydrogen in a distant star emits this same 122 nm light, but it is 400 nm when it reaches us. Is the star moving away from the earth, or toward us? What is its velocity?

for Section 16.7,

16-27: When an elevator is at rest, a ball shot from a gun hits the wall at "e". Where will the ball hit the wall if it is shot while the elevator has a v [or a] of **a)** constant 5 m/s ≠, **b)** 5 m/s^2 ≠, **c)** 5 m/s ¬ , **d)** 10 m/s ¬ , **e)** 15 m/s ¬ , **f)** free fall [cable is cut] ?

{ Use a value of g = 10 m/s^2 for earth's gravity. }

If the elevator is in space, with negligible gravity, where does the ball hit the wall if the elevator is **g)** constant 5 m/s ≠ , **h)** 5 m/s^2 ¬ **i)** 5 m/s^2 ≠ , **j)** 10 m/s^2 ≠ ?

==cut Gravity force causes a free-flight ball to follow a "curved path". As shown in Problem 16-#, general relativity predicts that light will also follow a slightly curved path, due to gravity force. This prediction has been confirmed by experiments.

16.92 Optional Solutions

16-1: An observer on a ∅ train sees the ground-observer & light source "p" moving toward the ♦ :

If the i-to-f event is light going to & from p on the ground, an observer on the ground is "proper". But if the event is light going to & from a p on the train, an observer on the train is proper.

16-2: The SST clock is a "space twin" that runs slow by a factor of $\sqrt{1 - v^2/c^2}$.

$v^2/c^2 = (v/c)^2 = (690/[3 \times 10^8])^2 = 5.29 \times 10^{-12}$. This is a lot smaller than 1, so $\sqrt{1 - (690\text{ m/s})^2/c^2} \approx 1 - \frac{1}{2}(5.29 \times 10^{-12})$.

100 y = (100 y)(365.25 days/y)(24 x 3600 s/day) = 3.16×10^9 s. A "slow motion" SST clock doesn't move fast enough to record this many seconds; it says "$\Delta t = (3.16 \times 10^9\text{ s})(1 - 2.645 \times 10^{-12}) = (3.16 \times 10^9 - .0084)$s". After 100 years the supersonic clock has lost only .0084 seconds!

16-3: There can be many observers of either kind. All proper observers get the same Δt, but improper observers can disagree. C & D are proper observers for their own clock, so they measure the smallest Δt. Then come B, A & E; as the relative velocity between observer and event increases, so does the difference between proper & improper Δt.

16-4: In the x-direction, the block seems to shrink to $(.50\text{ m})\sqrt{1-[.9c]^2/c^2} = (.50\text{ m})(.19) =$ .095 m. Since you don't see the block moving in the y or z directions, these dimensions are unchanged.

The rest mass is (.5m)(.4m)(.3m)(19300 kg/m^3) = 1160 kg. Its mass is $(1160\text{ kg})/\sqrt{1-[.9c]^2/c^2} =$ (1160 kg)/.19 = 6100 kg.

The block's observed volume is (.095m)(.4m)(.3m) = .0114 m^3, and its density is (6100 kg)/(.0114 m^3) = 535,000 kg/m^3.

This result can be understood by using ratio logic. Mass [on top in mass/volume] increases by a factor of $1/\sqrt{1 - v^2/c^2}$ = 1/.19, and volume [on the bottom] decreases by a factor of .19, so density changes by a factor of $(1/.19)/.19 = (1/.19)^2 = 27.7$, from 19300 to 19300(27.7) = 535,000 kg/m^3.

16-5: For the race, the driver says "i & f occur at the front of my car; I am proper". The spectator says "i occurs at the finish line to my left, f occurs at the finish line to my right; I am improper".

For front-to-rear Δt, the driver says "i & f occur at the front & rear of my car, in front of & behind me; I am improper". The spectator says "i & f both occur at the finish line, at the same location to the right of me; I am a proper observer".

16-6: You see the earth-to-star distance moving (like the race course in Section 16.3) at 2.9×10^8 m/s, which is .967c. This makes the distance contract to $(100\text{ l-y})\sqrt{1 - [.967c]^2/c^2} =$ 25.5 light-years. You see the earth move backward at .967c and the star come toward you at .967c , so the trip takes $t = \Delta x/v$ = (25.5 l-y) (365.25 days/yr) (86400 s/day)(3×10^8 m/s) / (2.9×10^8 m/s) = 8.32×10^8 seconds = 26.4 years.

{Or use this intuitive way to get the same result: if it takes light 25.5 years to reach a star 25.5 l-y away, it will take you (traveling at only .967c) a little longer: your Δt is 25.5 years(1.000c/.967c) = 26.4 years.}

How far did you travel? You say "25.5 light years" but people on earth say "100 light-years" and measure your Δt as (100 l-y)(1.000c/.967c) = 103.4 years.

You are a proper observer for the i-to-f Δt because you are present at "initial" (leaving earth) and "final" (reaching the star). We expect an improper observer (the earth person) to measure a longer Δt: Δt_{earth} =
$t_o / \sqrt{1 - v^2/c^2}$ = (26.4 yrs)/(.255) = 103.5 years, the same result we calculated above.

You also have a shorter Δx-and-Δt for the return trip. Because you accelerate at the start and finish of each part of the trip, you are in a proper-time "slow motion" reference frame going both directions. When you return home you have "aged" less than people who stayed on earth.

16-7: Solve

$$
\begin{aligned}
(18\,\text{in}) &= (36\,\text{in})\sqrt{1 - v^2/c^2} \\
.50 &= \sqrt{1 - v^2/c^2} \\
(.50)^2 &= (\sqrt{1 - v^2/c^2}\,)^2 \\
.25 &= 1 - v^2/c^2 \\
v^2/c^2 &= .75 \\
v^2 &= .75\,c^2 \\
v &= .866\,c
\end{aligned}
$$

Notice the logical step-by-step algebra solution.

If the multiplying factor for length is Y, where $Y \int \sqrt{1 - (v/c)^2}$, you can use this Y-v symmetry: $v/c = \sqrt{1 - Y^2}$. In this problem, Y = .866, v/c = .500, $.866 = \sqrt{1 - .500^2}$ and $.500 = \sqrt{1 - .866^2}$
== use γ ? (is it $\sqrt{1 - v^2/c^2}$ or $1/\sqrt{1 - v^2/c^2}$?)

16-8: G sees L_o for A-to-B (stationary ground), L for A'-to-B' (moving train). <u>But T sees A-to-B (on moving ground) as $L = L\sqrt{1 - v^2/c^2}$ = (50 m)(.866) = 43.3 m</u>] <u>and A'-to-B' (on his stationary train) as $L_o = L/\sqrt{1 - v^2/c^2}$ = (50 m)/.866 = 57.7 m</u>].

When T sees B (on the ground, moving ♦ at .5c) reach B', the first lightning bolt hits at B/B'. Later, when A reaches A', lightning strikes A/A'. <u>Notice how length differences</u> [G sees A-B & A'-B' the same length, but
T sees A-B (43.3 m) as shorter than A'-B' (57.7 m)] <u>are related to non-simultaneity</u>.

16-9: $\sqrt{1 - (.60c)^2/c^2}$ = .80, and you say "my car is 1400 kg [=1750(.8)], 3.75 m long [=3/.8] and 1.5 m high [unchanged] at rest, while Rhonda's car is 1875 kg [=1500/.8], 3.2 m long [=4.0(.8)] and 1.5 m high, moving forward [∅] at .60c.

16-10: muon decay-events are proper for a muon, but you observe an improper half-life t and Δx: $t_{1/2}$ =
$(1.5 \times 10^{-6})/\sqrt{1 - (.97c)^2/c^2} = 6.2 \times 10^{-6}$ s, and
$\Delta x_{1/2}$ =
$(.97 \times 3.0 \times 10^8\ \text{m/s})(6.2 \times 10^{-6}\ \text{s}) = 1800$ m.

A non-relativistic calculation predicts that $\Delta x_{1/2}$ is
$(.97 \times 3.0 \times 10^8\ \text{m/s})(1.5 \times 10^{-6}\ \text{s}) = 440$ m, if there is no "time dilation". <u>Experimental results confirm that the relativistic prediction ~~is~~ seems to be correct</u>; the decay process of muons really does "slow down" (as it is measured by an improper earth-observer).

16-11: As speed ≠, m ≠ and L ¬. At a speed that makes m change by a "$1/\sqrt{1 - v^2/c^2}$ " factor of 4/1, from 20 kg to 80 kg, the L-multiplying factor [which is $\sqrt{1 - v^2/c^2}$] is 1/4, causing L to change from 20 m to (20 m)(1/4) = 5 m.

A longer method: solve $m = m_o\sqrt{1 - v^2/c^2}$ for v = .968, substitute this into $L = L_o \sqrt{1 - v^2/c^2}$ and solve for L = 5 m.

Write the problem-statement as an equation, substi-tute for E_{total} & E_{rest}, for m, solve for $\sqrt{1 - v^2/c^2}$, then use the same ratio logic as above and get the same answer. When m ≠ by a factor of 4, so does E.

Write an equation,	E_{total}	=	4 E_{rest}
substitute for E's,	$m\,c^2$	=	4 $m_o\,c^2$
substitute for m,	$\{m_o/\sqrt{1 - v^2/c^2}\}\,c^2$	=	4 $m_o\,c^2$
and solve.	1/4	=	$\sqrt{1 - v^2/c^2}$

16-12: Turn "Part 1" into an equation, solve it:

$$
\begin{aligned}
m &= 2.50\,m_o \\
m_o/\sqrt{1 - v^2/c^2} &= 2.50\,m_o \\
.40 &= \sqrt{1 - v^2/c^2}
\end{aligned}
$$

Using the algebra strategy in Solution 16-# (square both sides,...) you can solve for v = .917c.

$\Delta m = m - m_o = 2.50\,m_o - 1.00\,m_o = 1.50\,m_o$. Ratio of $\mathbf{m_o}$, $\mathbf{\Delta m}$ (= 1.50 m_o) and $\mathbf{m}$ (= 2.50 m_o) is 2 : 3 : 5. Because energy is proportional to mass (just multiply by c^2 to get $m_o c^2$ = rest energy, $\Delta m\, c^2$ = kinetic energy, mc^2 = total energy), the **RE : KE : TE** ratio is also 2 : 3 : 5.

Look at the descriptions of "Parts 1, 2, 3". Do you see that they all describe the same situation?

16-13: $\Delta m = KE/c^2 = (1.6 \times 10^{-13}\ J)/(3 \times 10^8)^2 = 1.78 \times 10^{-30}$ kg. {Notice that SI units (not eV) are used when substituting for KE.}

After it is accelerated, the electron's mass is $m = m_o + \Delta m = (9.11 \times 10^{-31}\ kg) + (17.8 \times 10^{-31}\ kg) = 26.9 \times 10^{-31}$ kg. Its TE is $mc^2 = (26.9 \times 10^{-31})c^2 = 2.4 \times 10^{-13}$ J. To find v, substitute m and m_o into "$m = m_o/\sqrt{1 - v^2/c^2}$" and solve, as shown in Problem 16-##, for v = .94 c. The electron's momentum is $p = mv = (26.9 \times 10^{-31}\ kg)(.94 \times 3.0 \times 10^8\ m/s) = 7.6 \times 10^{-22}$ kg·m/s.

Another option, if your class uses this equation:

$(\mathbf{E_{total}})^2$	=	$\mathbf{m_o^2\, c^4}$	+	$\mathbf{p^2\, c^2}$
$[(26.9 \times 10^{-31})c^2]^2$	=	$(9.11 \times 10^{-31})^2\, c^4$	+	$p^2\, c^2$
58.6×10^{-27}	=	6.72×10^{-27}	+	$p^2\, c^2$
51.9×10^{-27}	=			$p^2\, c^2$
7.6×10^{-22} kg·m/s	=			p

With a "1000 mg" push, we can find Δt and Δx by using $\mathbf{F\, \Delta t = \Delta p}$ and $\mathbf{F\, \Delta x = \Delta KE}$: $\Delta t = \Delta p/F = (7.6 \times 10^{-22})/(10)(9.11 \times 10^{-31})(9.80) = 8.5 \times 10^6$ s [about 100 days], and $\Delta x = \Delta KE/F = (1.6 \times 10^{-13})/(8.9 \times 10^{-29}) = 1.8 \times 10^{15}$ meters.

Let's do non-relativistic "rough checking" of "$a\,\Delta t = \Delta v$" and "$v\,\Delta t = \Delta x$"? $a\,\Delta t = (98)(8500000) = 8.3 \times 10^8$ m/s = 2.8c [impossible], not the .94c we calculated above. Why? {think about it, then look at the answer in the next paragraph} And $v_{average}\,\Delta t = \frac{1}{2}(0+.94c)(8.5 \times 10^6) = 1.2 \times 10^{15}$ m, not 1.8×10^{15} m. Why? {think about it, then look at the explanation}

<u>If</u> m is constant, a = F/m = (1000mg)/m = 1000g = 9800 m/s^2. But as v ≠, m ≠, so a isn't constant during the i-to-f interval. F must produce an increase in v <u>and</u> m. For relativistic motion, $(F/m_o)\,\Delta t \neq \Delta v$!

As v ≠, m ≠ and a ¬. We cannot use a constant-a equation if a isn't constant. The first graph shows the v-versus-t area [$v\Delta t$, which equals Δx] if a is constant. The second graph shows the real situation, with large a at the start [when $m \approx m_o$] and smaller a later [when $m \gg m_o$]. Do you see why this "visual graph-logic" predicts that Δx will be larger than 1.2×10^8 m?

Since the electron and proton have the same KE, they have the same $KE/c^2 = \Delta m = 1.78 \times 10^{-31}$ kg.

The electron's mass begins at 9.11×10^{-31}, changes by 17.8×10^{-31} kg. The % m-increase is $100(\Delta m)/m_o = 100(17.8 \times 10^{-31}\ kg)/(9.11 \times 10^{-31}\ kg) = 195\%$.

But a proton has larger m_o, so its m-increase % is only $100(17.8 \times 10^{-31}\ kg)/(1.67 \times 10^{-27}\ kg) = .107\%$, even though it has the same Δm as the electron.

A proton has larger m_o, so it reaches 1 MeV of KE at slower speed. Because a 1 MeV proton has smaller v, it has a smaller $1/\sqrt{1 - v^2/c^2}$ and m/m_o ratio.

16-14: At .01c, $m = m_o/\sqrt{1 - (.01c)^2/c^2} = 1.00005\, m_o$, and $\Delta m = 1.00005 m_o - m_o = .00005 m_o$.

At .99c, $m = m_o/\sqrt{1 - (.99c)^2/c^2} = 7.09 m_o$, and $\Delta m = 7.09\, m_o - 1.00\, m_o = 6.09\, m_o$.

$KE/TE = \Delta m\, c^2/mc^2 = \Delta m/m$.
At .01c, $KE/TE = .00005 m_o/1.00005 m_o = .00005$. At .99c, $KE/TE = 6.09 m_o/7.09 m_o = .86$.

16-15: 1) No energy or matter crosses the system's boundaries; it is like a "miniature universe", and total energy is conserved. 2) To produce 48×10^6 J of KE, some rest mass is "converted" into KE, and the rest mass decreases by $KE/c^2 = (48 \times 10^6)/(3 \times 10^8)^2 = 5.3 \times 10^{-10}$ kg = $5.3 \times 10^{-10} \times 10^3$ g = $.53 \times 10^{-6}$ g = .53 μg. 3) Total energy [$=mc^2$] is conserved and, since c^2 is a constant, so is m. m is the product of two factors, m_o and $1/\sqrt{1 - v^2/c^2}$; the system's total m_o decreases, but the ≠ in v [when T & KE ≠] causes the $1/\sqrt{1 - v^2/c^2}$ factor to ≠ by just enough to offset ¬ in m_o, and mass is conserved.

ΔTE = 0, Δm_o = –.53 μg, Δm = 0. ==[nec?]

4) T is proportional to KE, so [to restore T and KE to their initial values] 48×10^6 J of energy leaves the system as heat. 5) This process does not affect rest mass, so Δm_o is still –.53 μg. 6) m_o has ¬, but $\sqrt{1 - v^2/c^2}$ is the same as initially, and Δm = –.53 μg. {The "surroundings" has gained mass & energy. The universe (system +

surroundings) has not changed its mass, total energy, or "E_{rest} + KE".} ==[is this analysis ok?]

Energy (in the form of light-radiation,...) leaves our sun continually. Since it is an *open system,* it is (like the second gas-burning system in this problem) losing total energy, rest mass and mass, at the rate of --==-- kg/s. Eventually [in about ----- years] the sun will run out of fuel to burn and will ----. [I didn't find the "==" data.]

16-16: Find KE at each speed, use ΔKE = W. When v = 0, KE = 0. When v = .001c, KE = $10c^2\,[1/\sqrt{1-(.001c)^2/c^2}\; - 1] = 10c^2\,[.0000005]$ = 4.5×10^{11} J. Similarly, when v = .998c the KE is m_oc^2 [14.8] = 1.33×10^{19} J, and when v = .998c the KE is $10c^2$ [21.4] = 1.93×10^{19} J.

From 0 to .001c, W = ΔKE = (4.5×10^{11} J) – 0 = 4.5×10^{11} J. From .998c to .999c, W = ΔKE = (1.93×10^{19} J) – (1.33×10^{19} J) = $.60 \times 10^{19}$ J.

Why does it take more energy (13 million times as much) to go from .998c to .999c? If we calculate KE non-relativistically, $\Delta(\frac{1}{2}mv^2)$ for .998c-to-.999c is 2000 times as large as $\Delta(\frac{1}{2}mv^2)$ for 0-to-.001c. This is not nearly enough to explain a factor of 13 million!

Between .998c and .999c, v doesn't increase much but [because v is so close to c] this Δv causes a large change in mass: m increases from 158 kg [= 10(15.8)] to 224 kg. This Δm of 66 kg, when multiplied by c^2, leads to the huge ΔKE of $.60 \times 10^{19}$.

16-17: m_o = 1000 kg, m = [1000 + (3×10^{-6})] kg.

$$m = m_o / \sqrt{1 - v^2/c^2}$$

$$\sqrt{1 - v^2/c^2} = \frac{1000}{1000 + (3 \times 10^{-6})}$$

Multiply fraction's top & bottom by 1/1000.

$$\sqrt{1 - v^2/c^2} = \frac{1}{1 + (3 \times 10^{-9})}$$

$$\sqrt{1 - v^2/c^2} = 1 - (3 \times 10^{-9})$$

$$(\sqrt{1 - v^2/c^2}\,)^2 = [\,1 - (3 \times 10^{-9})\,]^2$$

$$1 - v^2/c^2 = 1 - (6 \times 10^{-9})$$

$$6 \times 10^{-9} = v^2 / c^2$$

$$(7.75 \times 10^{-5})\,c = v$$

16-18: E = mc^2 = (.001 kg) c^2 = 9×10^{13} J.

In 1 year, a bulb uses (75 J/s)(365 x 24 x 3600 s) = 2.37×10^9 J. The bulbs per gram of mass is: (9×10^{13} J)/(2.37×10^9 J/bulb) = 38,000 bulbs.

The mass of ^{235}U converted to KE is (.1%)(1 kg) = (.001)(1 kg) = 1 gram, the same as above, so the answer is still "38,000 bulbs".

$$\frac{9 \times 10^{13}\ \text{J}}{4.1 \times 10^{19}\ \text{J/ton of TNT}} = 22 \times 10^3 \text{ tons of TNT}$$

This energy of "22 kilotons" is about the same as that released by the first atomic bomb in 1945.

16-19: If v = c, m = $m_o/\sqrt{1 - c^2/c^2}$ = $m_o/0$. If $m_o \neq 0$, m = $m_o/0 = \infty$. The only way to avoid infinite mass is for m_o to be zero, because "m = 0/0" can give a non-∞ value for mass.

16-20: Use m = (1.67×10^{-27} kg) $/\sqrt{1 - (.5c)^2/c^2}$ = 1.93×10^{-27} kg. Substitute and solve: r = mv/qB = (1.93×10^{-27})(1.5×10^8)/(1.63×10^{-19})(3) = .59 m.

16-21: $\xrightarrow{.8c}$ rocket, $\xrightarrow{.6c}$ bullet, $\xrightarrow{.8c}$ rocket, $\xleftarrow{.6c}$ bullet,

$$v_{obs} = \frac{+.8c + .6c}{1 + (.8c)(.6c)/c^2} \qquad v_{obs} = \frac{.8c - .6c}{1 - (.8c)(.6c)/c^2}$$

v_{obs} = .946c, ∅ v_{obs} = .385c, ∅

16-22: **a)** Using non-relativistic freeway logic, Robert is .1c faster than B. With "correction",

$$v = \frac{(.8c) - (.7c)}{1 - (.8c)(.7c)/c^2} = .227c \text{ in } \blacklozenge$$

direction

b) As you would expect, if Robert sees B moving ♦ at .227c, B sees him moving ∅ at .227c

c) A B-rider sees Robert and bullet both move ∅. Freeway logic says "add". **d)** Robert sees B move $\xleftarrow{.227c}$ [**a**-info] and shoot a bullet $\xrightarrow{.6c}$. Freeway logic says "subtract" and "the bullet will move ∅".

$$\frac{.227c + .600c}{1 + (.227c)(.6c)/c^2} \qquad \frac{.600c - .227c}{1 - (.600c)(.227c)/c^2}$$

.728 c toward ∅   .432 c toward ∅

e) Robert says "I see a $\xleftarrow{.6c}$ bullet come at $\xleftarrow{.7c}$, so Rocket C must be moving toward me, ∅. To find its v,
I subtract (.7c – .6c); the relativistically corrected v is (.7c – .6c)/[1–(.7c)(.6c)/c^2] = .172c ∅.

16-23: Combine v's for 1.0c "bullets" ♦ and ∅:

$$\frac{.8c + 1.0c}{1 + (.8c)(1.0c)/c^2} \qquad \frac{.8c - 1.0c}{1 - (1.0c)(.8c)/c^2}$$

± 1.0 c (choose –) ± 1.0 c (choose +)

16-24: Calculations for Rick and the driver are analogous to those for Section 16.3's race.

Rick sees a .6c car on a 400 m track, so $\Delta t = L/v = 400/.6c = 2.22 \times 10^{-6}$ second.

The driver sees the race course moving ♦ at .6c, contracted to a length of $(400\,m)\sqrt{1-(.6c)^2/c^2} = 320\,m$, and $\Delta t = L/v = 320/.6c = 1.78 \times 10^{-6}$ s.

To analyze what you (an observer on the earth) will see, use step-by-step logic. You see the course move ∅ at .8c, with a length of $(400\,m)\sqrt{1-(.8c)^2/c^2} = 240\,m$.
The car moves .6c∅ on a .8c∅ rocket, with $v = (.6c + .8c)/[1 + (.6c)(.8c)/c^2] = .946c$.
You see it move through the race course at .146c (the difference between .946c and .8c), and measure a Δt of $L/v = 240/.146c = 5.48 \times 10^{-6}$ s.

The driver is present at i (crossing the starting line) and f (car at finish line) so she is a proper observer and measures the shortest Δt. Rick and you see her move at .6c & .946c, so you see Δt's that are longer:
$\Delta t = (1.78 \times 10^{-6}\,s)/\sqrt{1-(.6c)^2/c^2} = 2.22 \times 10^{-6}$ s,
$\Delta t = (1.78 \times 10^{-6})/\sqrt{1-(.946c)^2/c^2} = 5.49 \times 10^{-6}$ s,
respectively, the same Δt's we calculated earlier!

16-25: As in Problem 16-A, temporarily forget that Becky is moving .8c wrt earth. She sees the ∅ bullet move .4c slower than .5c because Tom is moving ♦ at $v = (.5c - .1c)/(1-[.5c][.1c]/c^2) = .421c$.

Becky (.8c ---->) sees Tom move <-.421c------ because, as seen from earth, she is moving faster ∅ than he is. Tom's v is $(.8c - .421c)/(1-[.8c][.421c]/c^2) =$ .571c ------> .

16-26: The starlight is *red-shifted* toward longer λ (higher f) because the star is moving away from us.

Substitute for f (= c/λ), and solve for v:

$$f_{observed} = f_{source}\,\frac{\sqrt{1+v/c}}{\sqrt{1-v/c}}$$

$$\frac{c}{122 \times 10^{-9}} = \frac{c}{400 \times 10^{-9}}\,\frac{\sqrt{1+v/c}}{\sqrt{1-v/c}}$$

$$\frac{(400)^2}{(122)^2} = \frac{1+v/c}{1-v/c}$$

$$160000 - 160000\,v/c = 14884 + 14884\,v/c$$

$$.83\,c = v$$

{If $f_{obs} = f_{source}\,[(1+v/c)/\sqrt{1-v^2/c^2}\,]$ is used, to find v you must solve a quadratic equation!}

16-27: **a)** motion is identical whether v = 0 or v is constant; the ball still hits at "e",
b) ≠ acceleration is equivalent to ¬ gravity, so the total g is 10 m/s² ¬ [due to gravity] + 5 m/s² [due to unequal acceleration] = 10 m/s² ¬, and the ball hits at "f".
c) g_{total} = 10¬ + 5≠ = 5¬, hitting at "d".
d) The ¬ g due to gravity is cancelled by the ≠ g due to acceleration; the ball travels straight across to "c".
e) g = 10¬ + 15≠ = 5≠, ball hits at "b". **f)** This is the same situation as in Part d, so it hits "c".

{If the gun is aimed horizontally with $v_{iy} = 0$, $\Delta y = v_i t + \frac{1}{2} a t^2 = \frac{1}{2}(g)\,t^2$. Δy is proportional to g, so when g is –15, –10, –5, 0 and +5, the hitting-spots will be evenly-spaced along the wall.}
==[necessary?]

{Your textbook may discuss how light also follows a "curving path"due to gravity or acceleration.} ==[discuss light-with-mass, ball-analogy,...?

g) There is no g due to gravity or [because v is constant] due to acceleration; the ball hits "c".
h) g is 5 m/s² ≠ because a is 5 m/s² ¬, "b".
i) g = 5¬, "d". **j)** g = 0 + 10¬ = 10¬, like normal earth-gravity, "c".
==[check these for correctness]