# 15.91 Problems

## for Section 15.1,

**15-1**: You can hear a person "around the corner" in another room, even though you can't see him. Why?
Hint: The wavelength of audible sound varies from .02 m to 20 m, while visible light varies from 400 x $10^{-9}$ m to 700 x $10^{-9}$ m.

**15-2**: Green light (wavelength 550 nm) strikes two slits whose centers are .02 mm apart. On a screen 80 cm away, what are the angles from the central maximum to the second bright spot and to the second dark spot on one side? If the center-line is $x \equiv 0$, what is the x-position of these bright and dark spots? What is the angle between the second bright spots on each side of the center-line maximum?

**15-3**: Two slits are too close to make a direct measurement of their separation. The first-order maximum is at 21.4° for the 633 nm red light from a helium-neon laser. What is the separation of the slits?

**15-4**: A diffraction grating that is 5.0 cm wide has 20000 lines. Over what angular range will it spread visible light (with a range of approximately 400 nm to 750 nm) for maxima with orders of 0, 1, 2 and 3. Is there any "overlap" of these spectra?

**15-5**: If the single slit of a 3-slit apparatus is moved slightly upward, so light from the top slit is now 1/4 cycle ahead of light from the bottom slit, what happens to the central maximum, and to the distance between it and the first-order maxima?
Answer these questions if the single slit is restored to its center position, and a thin piece of cellophane in front of the bottom slit changes the phase of its light by 1/4 cycle.

**15-6**: What happens to θ and x for the second-order maximum in Problem 15-2 if the entire apparatus is put into water instead of air?

## for Section 15.2,

**15-7**: What is the angular separation between the dark spots on each side of the central bright region when 500 nm light passes through a slit that is $10^{-6}$ m wide?
Is the central maximum wider for 500 nm light passing through a slit whose width is 1000 nm, or 2000 nm?
Is the bright region wider in front of an opening whose width is .10 m, or .20 m?

**15-8**: How many diffraction minima occur when 440 nm light passes through a single slit whose width is 1640 nm?
What range of slit widths will let 440 nm light pass through without producing any diffraction minima?

**15-9**: For the double-slit wall below, how many interference maxima occur within the central diffraction maximum? For 610 nm orange light, how many interference maximums are "missing"?

Is θ between interference maxima ever larger than θ between diffraction maxima?

**15-10 Diffraction: Rayleigh's Criterion**
Your textbook will explain the limitations placed on optical systems by diffraction, and the *Rayleigh criterion* that is described by:
**$w\,\theta \approx w \sin\theta = 1.00\,\lambda$ for a thin slit**, and
**$D\,\theta \approx D \sin\theta = 1.22\,\lambda$ for a circular opening**
[$\theta \approx \sin\theta$ if θ is in radians, and is small].
Rayleigh's criterion: point-sources are "barely resolved" if the central maximum of one source coincides with the $m_w = 1$ minimum of the other.

If an eye that is "working perfectly" has a pupil opening (lens diameter) of 4 mm, can it resolve two point sources of 550 nm light (direct or reflected) that are 3 mm apart and 20 m away. Does the eye's resolving ability improve if the pupil enlarges to 5 mm?

## for Section 15.3,

**15-11**: To minimize reflection of green light at 550 nm (around the middle of the visible spectrum), a glass lens (n = 1.52) is coated with magnesium fluoride (n = 1.25). What is the thinnest film that can be used?
Why is it best to use a film whose "n" is approximately halfway between 1.00 & 1.52?
What is the color of light that is reflected?
What happens to light that is not reflected due to the film? Is this light (and its energy) absorbed by the film?

**15-12**: If a film of benzene (n = 1.50) floats on water (n = 1.33), is the reflected light at a maximum or minimum near the edge of the drop where the film thickness is much less than the wavelength of light?
Is the edge of an acetone film (n = 1.36) on top of glass (n = 1.52) bright or dark?

**15-13**: Two flat glass plates, 10.0 cm long, are separated at their right ends by a thin wire with diameter = .020 mm. How many bright bands (alternating with dark bands) appear between the left & right ends of the plates when they are illuminated by 500 nm green light? How far apart are the bands?

If both plates are perfectly flat, all of the bright bands will be vertical. If you see the pattern below, and you know the top plate is flat, is the irregularity in the bottom plate a small "hill" or a small "valley"?

**15-14**: For Problem 15-C, answer "What is the actual thickness of the film?" by using math & logic instead of trial-and-error. Hint: write two equations, one for the maximum and another for the minimum, then sove them.

**15-15**: A soap bubble (with n = 1.33) has a reflection maximum for 540 nm green light. What is the minimum possible thickness for this bubble?

OPTIONAL: If the bubble is actually three times this thick, are there any reflection minimums for visible light?

**15-16**: What amount of mirror movement in an *interferometer* (check your textbook for the details of construction & operation) will make 500 bright-bands of 640 nm light move across the viewing screen?

**15-17**: Light that goes to (and from) one mirror one mirror of an interferometer passes through a 1.000 m chamber filled with air (n = 1.0002782). Air in the chamber is gradually pumped out to make a vacuum (n = 1.0000000). How many bright-bands of 546 nm light move across the viewing area during this process?
Hint: convert time-difference into path-difference.

**15-18**: Wave sources at A and B emit equal-intensity waves with $\lambda = 2$ m. If waves at A and B are emitted in-phase, is the resultant wave intensity larger at Point #1, #2 or #3? { A-1-B-2 are on a horizontal line, and 1-3 are on a vertical line. #1 is midway between A and B, which are 3 m apart; #2 and #3 are equal distances from B. } Is the resultant intensity zero at #1, 2 or 3? { If necessary, clarify your answers by specifying average, maximum, or instantaneous intensities. }

Then answer these same questions if A and B are $\frac{1}{2}$ cycle out of phase.

**for Section 15.4**,

**15-19**: You have three polarizing sheets: their "preferred polarizing directions" are 0° (vertical), 90° (horizontal), and 30°. By using one or more of these polarizers, how many final intensities can you produce from an initially unpolarized beam?

**15-20**: Using two polarizing sheets, how can you reduce the intensity of an initially unpolarized beam of light by 90%?

How does your answer change if the beam is initially polarized?

**15-21**: A light beam's intensity is $I_o$. 60% of the light is unpolarized, and 40% is polarized vertically (at 0°). What is the final intensity if this beam passes through a polarizer oriented a) at 0°, b) at 90°?

**15-22**: Brewster's Angle ==nec?

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## 15.92 Solutions

**15-1**: Sound waves, with $\lambda \approx .02\text{-}20$ m, have $\lambda$'s of roughly the same size as a door opening, so they diffract around the corner into the "shadow region", but light-wave $\lambda$'s are <u>too small</u> to be diffracted.

Similarly, you hear FM (but not AM) radio when your car is inside a tunnel because FM waves (≈ 3 m) are close to the tunnel-opening size, but AM waves (≈300 m) are <u>too large</u> to be significantly diffracted.

**15-2**: The second maximum has a path difference of $m\lambda = 2\lambda$. The second minimum has $\Delta\text{path} = m\lambda = 1\frac{1}{2}\ \lambda$. Substitute into $d \sin\theta = \Delta\text{path}$ or $d\, x / L = \Delta\text{path}$, and solve for $\theta$ or $x$:

For the second maximum,

$$(.02 \times 10^{-3}) \sin\theta = (2)(550 \times 10^{-9})$$
$$\theta = 3.15°$$

$$(.02 \times 10^{-3})\, x / (.80) = (1\tfrac{1}{2})(550 \times 10^{-9})$$
$$x = .069 \text{ m}$$

For the second minimum,

$$(.02 \times 10^{-3}) \sin\theta = (1\tfrac{1}{2})(550 \times 10^{-9})$$
$$\theta = 2.36°$$

$$(.02 \times 10^{-3})\, x / (.80) = (1\tfrac{1}{2})(550 \times 10^{-9})$$
$$x = .052 \text{ m}$$

There is 3.15° from the center-line to each second order maximum, so there is 6.30° between them.

**15-3**: $d = m\lambda / \sin\theta = (1)(633 \times 10^{-9}) / (\sin 21.4°) = 1.73 \times 10^{-6}$ m = 1730 nm = .00173 mm.

**15-4**: $d = (.05\text{ m})/(20000\text{ lines}) = 2.5 \times 10^{-6}$ m.

Here are solutions of "$\theta = \sin^{-1}(m\lambda/d)$" for the two extremes of $\lambda$, for each of the four orders:

| | m = 0 | m = 1 | m = 2 | m = 3 |
|---|---|---|---|---|
| 400 nm | 0° | 9.2° | 18.7° | 28.7° |
| 750 nm | 0° | 17.5° | 36.9° | 64.2° |

For m = 0 all of the light, from violet (400 nm) to red (750 nm) goes to the same 0° spot, so the central maximum is "white", not separated into colors.

There is "overlap" of the spectra for m=2 (which ends at 36.9° for red light) and m=3 (which begins at 28.7° for violet light).

**15-5**: Because the top-slit light is 1/4 cycle ahead, it must travel a slightly longer distance (a "handicap") to put the top and bottom waves back in phase. The central (n = 0) maximum shifts a tiny bit downward, to below the center line. Every maximum moves down the same amount, so the pattern (including all peak-to-peak distances) is the same as before.

As explained in Section 14.2, light moves faster in air than in materials like plastic. Cellophane will delay (not advance) the bottom-slit light by 1/4 cycle. This is the same situation as above; if bottom-slit light is 1/4 cycle behind, top-slit light is 1/4 cycle ahead.

**15-6**: n increases by a factor of 1.33, from 1.00 (in air) to 1.33 (in water). As explained in Section 14.2, v and $\lambda$ both decrease by a factor of 1/1.33. $\sin\theta$ is proportional to $\lambda$, and for small angles (like those in 15-2) $\theta$ and x are approximately proportional to $\lambda$.

$\sin\theta$, $\theta$ and x all decrease by a factor of 1.33, so the n=2 maximum occurs at $\theta_{max} = 3.15°(1/1.33) = 2.37°$, and $x_{max} = .069(1/1.33) = .052$ m.

**15-7**: You are asked to find the central maximum's angular width. For the $m_w = 1$ minimum, $\theta = m\lambda/w = (1)(500 \times 10^{-9})/(10^{-6}) = 30°$. The central maximum spans 60°, from the first minimum on one side (at –30°) to the first minimum on the other side (at +30°).

The central maximum spans 60° for a 1000 nm slit (as shown above), and 29° for a 2000 nm slit. The narrower slit produces a wider central maximum.

When w » $\lambda$, diffraction (at the edges) is negligible compared with the vast majority of light that passes straight through without even "knowing" that it has gone through a slit. The bright regions (these are not central maximums) in front of each opening are .10 m and .20 m, respectively. The wider opening produces the wider bright region.

**15-8**: The entire screen goes from 0° (center line) out to 90°. Substitute into $w\sin\theta = m_w\lambda$, and solve for $m_w = (1640 \times 10^{-9})(\sin 90°)/(440\text{ n}) = 3.7$. There are 3 minima on each side of the center-line (3.7 is almost 4, but not quite), so there are 6 minima, plus the one out at ∞ on each end:

If w is less than 440 nm, $\sin\theta$ (= $m\lambda/w$) is greater than 1.00 for the m = 1 minimum. This is impossible, indicating that there is no minimum. The central maximum extends over the entire screen.

**15-9**: The center-to-center slit separation "d" is (.0005 + .0010 + .0005) m = .0020 m, and the slit width "w" is .0010 m. Use the method shown in Problem 15-B to find that the central maximum (between $m_w$ = –1 and +1) contains five interference maxima (at m = –2, –1, 0, +1 and +2).

Two double-slit interference maxima that would usually occur (for m = –3 & +3) are missing because they coincide with single-slit diffraction minima (for $m_w$ = –1 and +1) that don't let light come to those particular points on the screen. Using the methods of Problem 15-##, we know that diffraction minima also occur for $m_w$ = ±2, ±3 and ±4. These eliminate the interference maxima that would have occurred for m = ±6, ±9 and ±12. 8 interference maxima are missing.

{With the same w but different d, we might find less than 8 missing maxima, if some (or all) of the 8 diffraction minima don't coincide with locations of interference maxima.}

Even if the .0020 mm "center bar" between the slits was reduced to .0001 mm, d (= ½ w + .0001 mm + ½ w) would still be larger than w. Because $\theta$ increases if d (or w) decreases, for the same "order" (like m = 2 and $m_w$ = 2) the $\theta_{diffraction}$ (with relatively small w) will be larger than $\theta_{interference}$ (with the larger d), so the single-slit diffraction pattern is always "wider" than the double-slit interference pattern.

**15-10**: The eye is round, and θ is so small that $\theta \approx \sin\theta \approx \tan\theta \approx (.003\text{ m})/(20\text{ m}) = .00015$ radian. If the eye "works perfectly" and the only limitation is diffraction, $\theta = 1.22\lambda/D = 1.22(550 \times 10^{-9})/.004 = .00017$ rad. The eye can distinguish .00017 rad, but the point-sources are closer than this (.00015 rad) so the eye cannot "resolve" them.

If the pupil opening increases to .005 m, the eye's *resolving power* increases because it can now resolve features with a smaller θ-separation. The eye now has $\theta_{Rayleigh} = 1.22\lambda/D = 1.22(550 \times 10^{-9})/.005 = .00013$ rad, so (as judged by Rayleigh's criterion) it can see lights that are .00015 rad apart as coming from two separate sources, not one.

**15-11**: The first reflection (at air/film, 1.00/1.25) changes the light-wave phase by $\frac{1}{2}$ cycle. The second reflection (film/glass, 1.25/1.52) also flips, so flips keep the waves in-phase. To put them out-of-phase and minimize the reflection, we need $2t = \frac{1}{2}\lambda_{film}$, and $\mathbf{t = \frac{1}{4}\lambda_{film}} = \frac{1}{4}(550 \times 10^{-9})/1.25 = 1.1 \times 10^{-7}$ m.

As discussed in Section 14.3, the reflection-% at an interface increases as the interface n-change increases. Because this *quarter-wave coating* has an index of refraction about halfway between that of air and glass, reflection is minimized <u>and</u> it is approximately equal at the two interfaces, so the two reflections almost totally cancel. {Out-of-phase waves with unequal amplitudes will cancel each other only partially.}

The film is most effective at reducing the reflection of green light near the middle of the visible spectrum. At the red and violet ends of the spectrum, more light reflects. This combination of red-plus-violet makes the reflected light look purple.

As emphasized in Section 15.1 and Solution 15-C, diffraction-and-interference redistributes energy, rather than destroying or producing it. The film doesn't reduce reflection by absorbing light energy; it just lets a higher % of the light be transmitted.

**15-12**: The first reflection (air/benzene, 1.00/1.33) flips the light-wave phase, the second (benzene/water, 1.50/1.33) doesn't. The flips are different, so there is minimum reflection (a dark band) near the edge where the path-difference is ≈ 0 because thickness ≈ 0.

For acetone-on-glass, both reflections are flipped, so reflections at the edges (where path difference ≈ 0) are in-phase, to produce a bright maximum.

**15-13**: As we move rightward, the path difference [and phase difference] between these two reflections changes, thus causing alternating dark & bright bands:

By themselves, phase-flips put the reflections out-of-phase; the left end, where extra distance traveled through air is ≈ 0, is a dark band [minimum]. At the wire, path difference $= 2(.02 \times 10^{-3}\text{ m})/(500 \times 10^{-9}\text{ m}) = 80\lambda$. Bright bands [maxima] occur at locations where the path difference is $\frac{1}{2}\lambda$, $1\frac{1}{2}\lambda$, $2\frac{1}{2}\lambda$,..., $79\frac{1}{2}\lambda$. There are 80 bright bands in the 10.0 cm between the left and right ends, so the distance between bands is 100 cm / 80 bands = .125 cm per band.

In the "irregular region" we must move further to the right to get a maximum-producing path difference, because the bottom plate contains a small "hill".

**15-14**: Since "flips" put the waves out-of-phase, maxima occur for $2t = (m \pm \frac{1}{2})\lambda_{oil}$, and minima for $2t = (m)\lambda_{oil}$. Half-integers ($\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$ ,...) can be generated by either $m+\frac{1}{2}$ or $m-\frac{1}{2}$ ; do you see why ratio logic requires that we use $m-\frac{1}{2}$ in this equation?

$$2t\text{ [for 600 nm max]} = 2t\text{ [for 450 nm min]}$$

$$(m - \tfrac{1}{2})\frac{600 \times 10^{-9}}{1.45} = (m)\frac{450 \times 10^{-9}}{1.45}$$

$$600m - 300 = 450m$$

$$m = 2$$

$m-\frac{1}{2}$ is smaller than m, and $m+\frac{1}{2}$ is larger than m. To make the equation-sides equal, we must "balance" the left side's relatively large number (600 > 450) with a relatively small number ($m-\frac{1}{2} < m$). After we know that m = 2, we can substitute into "$2t = (m)\lambda_{oil}$" and solve for $t = (2)(450 \times 10^{-9}/1.45)/2 = 3.10 \times 10^{-7}$ m.

**15-15**: Outer surface (air/bubble, 1.00/1.33, flip), inner surface (bubble/air, 1.33/1.00, no flip). For the smallest thickness, $2t = (0 + \frac{1}{2})(540 \times 10^{-9})/1.33$, and $t = 1.0 \times 10^{-7}$ meter.

If the actual thickness is $3.0 \times 10^{-7}$ m, the path difference for 540 nm light is 3 times $\frac{1}{2}\lambda$, or 1.5λ. Minima will occur for path differences of 1λ, 2λ, 3λ,... Ratio logic: if the path difference is only 1λ for this thickness, light must have a λ that is larger by a factor of (1.5λ / 1λ). And a Δpath of 2λ occurs for light with λ that is shorter by a factor of (1.5λ / 2λ). These wavelengths will be (1.5/1)(540 nm) = 810 nm [this is infrared, not visible] and (1.5/2)(540 nm) = 405 nm [this is violet, visible].

**15-16**: To change from one bright-band maximum to the next, there must be a round-trip change of 1λ, caused by a mirror movement of $\frac{1}{2}\lambda$:

$$(500\text{ bands})(\tfrac{1}{2} \times 640 \times 10^{-9}\text{ m/band}) = .16 \times 10^{-3}\text{ m}.$$

Do you see that an interferometer is a very sensitive instrument for measuring small distances?

**15-17**: Light moves faster through the vacuum, by a factor of 1.0002782/1, so the "effective path length" is shortened to 1/1.0002782 m = .9997219 m. This is .0002781 m shorter than 1 m. Light passes through the chamber twice (coming and going) so this change is equivalent to moving the mirror .0002781 m. Each mirror movement of $\frac{1}{2}$ λ causes a movement of one band, so (.0002781 m)/($\frac{1}{2}$ x 546 x $10^{-9}$ m) = 1018.7 bands move across the viewing area.

**15-18**: #1 is equidistant from A and B (as is #3) so the waves remain in-phase at #1 and #3, producing a maximum. Since #1 is closer to the wave-sources, it has a larger average intensity than #3.

A and B are 3 m apart; λ = 2 m, so A & B are separated by $1\frac{1}{2}$ λ, and waves from A & B are $\frac{1}{2}$ cycle out of phase at #2. But intensity at #2 is not zero. Why? The B-wave is stronger because #2 is closer to B, so the two waves don't totally cancel each other even though they are totally out-of-phase.

At a time when an A-peak & B-peak reach #1, the resultant wave has maximum instantaneous intensity. But later in the wave cycle, the A & B waves reaching #1 have temporarily-zero intensity that add to give zero intensity at #1. But #2 and #3 have a non-zero intensity at this instant because they are further away and the zero-intensity part of the wave hasn't reached them yet. #1 has the largest maximum (and average) intensity, but not always the largest instantaneous intensity. At any instant, B-wave intensity is equal at #2 and #3, which are equally far from B. The A-wave always partially cancels the B-wave at #2, but adds to it at #3, so at every instant the total intensity is larger at #3 than at #2.

If A-waves and B-waves are emitted out-of-phase, they remain out of phase at #1 and they have equal intensity, so they totally cancel to give zero intensity at every instant. For the same reasons, intensity is always zero at #3.

But at #2, the $1\frac{1}{2}$ λ path difference between A & B puts their waves back in-phase. #2 obviously has larger average intensity than #1 or #3, and (except at instants when it is temporarily zero) it also has larger instantaneous intensity.

**15-19**: With any single sheet, $I = \frac{1}{2} I_o$.
0° then 90° (with or without 30°), I = 0.
0° and 30°, $I = \frac{1}{2} I_o (\cos^2 30°) = .375 I_o$.
30° and 90°, $I = \frac{1}{2} I_o (\cos^2 60°) = .125 I_o$.
0°, 30° and 90°, $I = \frac{1}{2} I_o (\cos^2 30°)(\cos^2 60°) = .094 I_o$.
Other possibilities (90°- 0°, 30°- 0°, ...) are duplicates.

**15-20**: $.10 I_o = \frac{1}{2} I_o (\cos^2 \theta)$, $.20 = (\cos\theta)^2$, $.4472 = \cos\theta$, $63.4° = \theta$. Orient the polarizers so the angle between their preferred directions is 63.4°.

Find the polarization direction of the beam, then use either polarizer with $\theta = \cos^{-1}\sqrt{.10} = 71.6°$. Or use both polarizers, in any combination whose $\cos^2\theta$ factors multiply to give .10; for example, θ's of 60.0° and 50.8° have $\cos^2\theta$ factors of .25 and .40, which multiply to give an overall reduction-factor of .10.

**15-21**: The 0° polarizer cuts the unpolarized light ($.6 I_o$) in half (to $.3 I_o$) and doesn't affect the polarized light ($.4 I_o$); $I_{final} = .3 I_o + .4 I_o = .7 I_o$.
The 90° sheet cuts unpolarized light in half, and eliminates the 0° polarized light, so $I_{final} = .30 I_o$.

**15-22**: == cut?