14.91 Problems

for Section 14.1,

14-1: What is the relationship between the directions of the E, B and v vectors? { Look at the picture in Section 14.1, and use the "hint" after it.}

14-2: An EM radio wave has a wavelength of 3.0 m. Is it AM (550 kHz to 1600 kHz) or FM (88 MHz to 108 MHz)?

What are the wavelengths of 1.5×10^{10} Hz microwaves and 6.0×10^{18} Hz x-rays?

for Section 14.2,

14-3: For the plastic below, n = 1.45. $n_{vacuum} \equiv 1$, and $n_{air} = 1.0002782$. What is light speed in the air, plastic and vacuum?

If light has a wavelength of 550 nm in air, what is its λ and f in the plastic?

14-4: Answer the final question of Section 14.2: Which color moves faster? What is the % difference in their speeds?

14-5: In a piece of glass, the speed of light is 1.83×10^8 m/s. If the light has $\lambda_{vacuum} = 500$ nm, what is its wavelength in the glass?

14-6: A Light Race

Two beams of light have a 100 m race. One beam goes through air (n = 1.0002782), the other through vacuum ($n \equiv 1$). Which one wins the race, and by how much?

for Section 14.3,

14-7: **a)** A light beam moves downward in air at an angle of 55° from vertical. It enters a liquid and continues downward at 33° from vertical. What is the liquid's "n"?

b) What is light speed in the liquid below?

14-8: Two plane mirrors are joined at 80°. After this light ray reflects from them, what angle does it make with horizontal?

14-9: A large glass pan with a horizontal bottom is filled with water. A downward light beam makes a 40° angle with vertical. Draw a picture showing angles of reflection and transmission for this light at the first three interfaces: air-water, water-glass, and water-air.

14-10: A light beam begins in a liquid and hits the surface at a 47° angle, but no light is able to escape into the air. What can you say about the liquid's index of refraction?

14-11: What range of θ-angles will allow total internal reflection of light within this "light pipe" made of lucite (n = 1.491)?

14-12: A light ray enters a prism made of quartz glass (n = 1.46) that is shaped into an equilateral triangle. The light and prism base are horizontal: light → Δ prism.

What angle does the emerging ray make with horizontal? { Hint: Use Section 1.1 methods and expect a challenge, but one you can meet. }

14-13: If horizontal green light is "bent" to 40°-below-horizontal by Problem 14-##'s prism, will red light be bent more or less than 40°? What about blue light?

If this prism is immersed in water (n = 1.33), is horizontal green light bent more or less than 40°? What happens if the prism is immersed in benzene (n = 1.50)?

14-14: You're stranded in the wilderness and you're hungry. You make a spear and go to the river, where you see a fish. To stab the fish, should you aim your spear at it, above it, or below it?

14-15: *Fermat's principle of least time* states that when light travels between two points it follows the path that requires the least possible time.

Use this principle to explain (in words, without equations) why light moves along a "bent" path between the two •-points instead of on the - - - line that is a shorter distance.

for Section 14.4,

Except for 14-##, most of these problems are appropriate whether your class begins its study of optics with lenses or mirrors.

14-16: If you are 1.74 m tall, standing 2.0 m from a .70 m high plane mirror whose top is even with your eyes (1.60 m above the floor), how much of your body can you see in the mirror? Will your answer change if you stand closer to (or further from) the mirror?

How far away do your eyes (in the image) appear to be? How far away is the lowest part of your image?

14-17: **Depth-Perception Cues and Optical Illusions.** Your eyes can use "triangulation logic" (plus eye-muscle biofeedback that lets you know how much your eyes are focusing inward from straight-ahead) to judge the distance to nearby objects. For further-away distances, however, you have to rely more on "non-binocular" cues. These perceptual cues are explained in detail in introductory psychology textbooks; I'll just give a brief summary here. They are based on logical assumptions like "constancy of size, shape, brightness and color" and on relationships.

They include line-convergence and height on the plane ( ), overlap ( ), relative motion "parallax" (nearby telephone poles seem to go by quickly while faraway poles barely move), relative size (usually, larger is closer) and brightness (bright tends to be closer) and clarity/gradation-of-texture (nearby objects usually form clear images with more detail).

These subconscious cues are used by artists to give the "appearance of depth" to two-dimensional drawings and paintings. Interpretation of cues is needed to make sense of what we see, but in some cases our eyes-and-brains can lead us astray.

Study the pictures below: are the lines equal size? the box-tops? would the lines "meet" if they were extended? When the moon is near the horizon it looks larger; is it? Are mirages and hallucinations "real"?

14-18: In these pictures, - - - - is the center line. Decide whether each lens (or mirror) is converging or diverging:

14-19: There are 16 types of ray-tracing diagrams: 8 of these are "real" objects that are far (outside the focus) and near (inside the focus) real-objects for a lens or mirror that is converging or diverging.

As shown below, 5 of these were drawn in Section 14.4, and diagrams for a diverging lens (or mirror) are essentially the same no matter where the object is located:

	c-lens	d-lens	c-mirror	d-mirror
far:	14.4	14.4	14.4	14.4
near:	14.4	same	?	same

There are eight analogous situations for "virtual" objects; two of these are drawn in Problem 14-#.

Draw the one real-object situation that has not been done already: a converging mirror with near object. Use d_o = +5 cm, f = +10 cm.

14-20: You have all four types of lenses and mirrors, and a tiny point-source of light. How can you make a parallel beam of light? Hint: the paths of light rays are "reversible".

14-21: This table shows object & image distances for a diverging & converging lens (or mirror), calculated using "$1/d_o$ + ...".

diverging, f = +10		converging, f = –10		
d_o	d_i	d_o	d_i	
+∞	–10.0	+∞	+10	
+20	– 6.7	+20	+20	
		+10.1	+1010	+10
– 5.0	+10.0	± ∞ ?		
		+ 9.9	–990	
+ 5	– 3.3	+ 5.0	–10	
+.1	–.099	+.1	–.101	

Study each table and draw a simple diagram to show the changes in image-location that occur when a real-object moves from ∞ in toward each type of lens and mirror.

14-22: Under what circumstances is an image larger than the object? When is it smaller? Hint: use the "d_o d_i" tables above.

14-23: By filling in the blanks in the four mini-problems below, you should be able to convince yourself that **if you know any 2 of these 4 (d_o, d_i, f, m) you can find the others** using "$1/d_o + 1/d_i = 1/f$" and "$m = -d_i / d_o$".

	d_o	d_i	f	m
Situation A:	–5	?	–4	?
Situation B:	+3	?	?	+4
Situation C:	?	+6	?	–2
Situation D:	?	?	+15	+3

14-24: In the "Real Images and Virtual Images" part of Section 14.4, answer the questions about whether you can see the object or image if your eye is at Δ or ▲, and if an image can form a photograph or [an extra question] heat a cup of water.

14-25: The moon has a diameter of 3500 km and is 3.84×10^5 km from earth. What size is the moon-image formed by a camera whose lens has a focal length of 35 mm? Is the moon's image upside down? What kind of lens is being used, converging or diverging?

What is the image size if a telephoto lens with a focal length of 600 mm is used?

What is the image size and mirror-type if a 600 mm mirror forms a real image?

14-26: What kind of lens or mirror can be used to form an upright (not inverted) image?

To form an image that is 30 cm from a lens and is twice as large as the object, what kind of lens (or mirror) could you use, and where should you place the object? Hint: Use the 2-of-4 substitute-and-solve tool from Problem 14-##, or Problem 14-##'s tables (plus ratio logic).

14-27: Trace the P, C and F rays through a lens ($f = -40$ cm) for a "virtual object" located at –20 cm. Hint: Light still comes "from the left", but instead of coming from the object the rays are aimed as if they were going to the object.

Also trace rays [and locate the image] for a virtual object (at –40 cm) for a concave mirror with $f = +40$ cm.

for Section 14.5,

14-28: A lens of crown glass ($n = 1.52$) has a focal length of 20 cm. What is its radius-of-curvature if both surfaces have the same shape? if one of the surfaces is flat?

Make a rough sketch of these lenses.

When an object is placed 15 cm from either lens, a virtual image forms 60 cm from the lens. What do the lenses look like?

14-29: Find f for this quartz ($n = 1.46$) lens:

14-30: What is the radius-of-curvature for a concave mirror with $f = +36$ cm, and the "f" of a silver-covered sphere whose diameter is 100 cm?

14-31: The lenses and mirrors below have $f = +50.0$ cm and –2.0 cm. If each optical system is used to view a person (1.7 m tall) who is 100 m to the left, where is the final image (that is seen by the eye), is it real or virtual, and is the person upside down?

What is m and M?

14-32: What is the maximum angular magnification of a magnifying glass with $f = +8.0$ cm, looking at a penny (diameter = 1.9 cm), if the person's eye cannot (with or without the lens) focus on an object or image that is closer than its "near point" of 24 cm?
Hint: Use a step-by-step process. As explained in the solution, maximum M occurs when the image forms at 24 cm. You know d_i, f and h_o, so you can find d_o and h_i. Then find $ø_{image}$, d $ø_{object}$, and M.

14.92 Solutions

14-1: Use a modified version of Section 12.#'s *straight right-hand rule:* your fingers still point in the B-direction, but your thumb points in the direction of the electric field (instead of electric current), and your palm faces in the direction of v (not F).

14-2: As explained in Section 14.1, radio waves and microwaves and x-rays are all EM-waves.

Substitute and solve: $v_{light} = f\lambda$, so $f = v_{light}/\lambda = (3 \times 10^8 \text{ m/s})(3 \text{ m}) = 1 \times 10^8 \text{ s}^{-1} \equiv 1 \times 10^8 \text{ Hz} = 100 \times 10^6 \text{ Hz} = 100$ MHz. It's FM.

For the EM-microwaves, $\lambda = c/f = (3 \times 10^8 \text{ m/s})/(1.5 \times 10^{10} \text{ cycles/s}) = .02$ m/cycle = 2 cm/cycle.

For the EM x-ray wave, $\lambda = (3 \times 10^8)/(6 \times 10^{18}) = 5 \times 10^{-11} \text{ m} = 50 \times 10^{-12} \text{ m} = 50$ pm.

{Technically, the SI units of λ and f are m and 1/s, but I like to think in terms of m/cycle and cycles/s.}

14-3: In air, on the 1st and 5th parts of the journey shown, $v = c/n = (299792458 \text{ m/s})/1.0002782 = 299709080$ m/s; this speed was used in Section 9.3.

In the plastic, $v = c/n = (2.9979 \times 10^8 \text{ m/s})/1.45 = 2.07 \times 10^8$ m/s, before and after the vacuum.

In a vacuum, $v = c/n = c/1 = 299{,}792{,}458$ m/s.

In air, $f = v/\lambda = (2.9971 \times 10^8 \text{ m/s})/(550 \times 10^{-9} \text{ m}) = 5.45 \times 10^{14}$. f doesn't change when light enters a new medium and, using the v from above, $\lambda = v/f = (2.07 \times 10^8)/(5.45 \times 10^{14}) = 3.80 \times 10^{-7}$. This is the same λ we get from $\lambda_{plastic} = \lambda_{air}(n_{air}/n_{plastic}) = (550 \times 10^{-9})(1.0003/1.45) = 3.79 \times 10^{-7}$.

14-4: $v = c/n$, so $v \downarrow$ as $n \uparrow$. n is larger for blue light than for red light (in quartz glass, 1.4636 versus 1.4561) so blue light moves more slowly in the glass, even though both colors move at "c" in a vacuum.glass has $n = 1.4561$ for red light (with $\lambda = 670$ nm), and $n = 1.4636$ for blue light.

The % difference in v equals the % difference in n: $100 \times (1.4636 - 1.4561)/(1.46) = .51$ %.

14-5: $n_{glass} = c/v_{glass} = (2.998 \times 10^8)/(1.83 \times 10^8) = 1.638$. λ_{glass} is smaller than λ_{vacuum} by a factor of $1/n_{glass}$: $\lambda_{glass} = \lambda_{vacuum}(1/n_{glass}) = (500 \text{ nm})/1.638 = 305 \text{ nm} = 305 \times 10^{-9}$ meter.

14-6: The vacuum-light finishes in $\Delta t = \Delta x/v = (100 \text{ m})/(299792458 \text{ m/s}) = 3.33564 \times 10^{-7}$ s. $\Delta t_{air} = (100)/(299792458/1.0002782) = 3.33656 \times 10^{-7}$ s. The vacuum-light wins the race by .00092 s. Another way to answer the question "by how much" is to find $\Delta x = v\,\Delta t = (299792458/1.0002782)(.00092 \times 10^{-7}) = .028$ m = 2.8 cm; this is the slim margin of victory.

14-7: **a)** If you translate the problem-words into a picture (in your mind or on paper) you'll see that 55° and 40° are θ_1 and θ_2. Then just substitute and solve:

$$\begin{aligned} n_1 \quad \sin\theta_1 &= n_2 \quad \sin\theta_2 \\ 1.000 \quad \sin 55° &= n_2 \quad \sin 33° \\ 1.50 &= n_2 \end{aligned}$$

b) Light hits the surface at 35° so θ_1 (the angle light makes with the normal-to-the-surface) is 55°. You can use trig, as in Section 1.3, to find $\theta_2 = \tan^{-1}$(opp/adj) = $\tan^{-1}$(3 cm / 4 cm) = 37°. Then solve for n, as above: $n_2 = n_1 \sin\theta_1 / \sin\theta_2$ = (1)(sin 55°)/(sin 37°) = 1.36, and v = c/n = (2.998 x 10^8)/1.36 = 2.20 x 10^8 m/s.

14-8: Draw a picture and use geometry principles from Sections 1.1 (ΔXYZ, draw extra lines) & 14.3:
a) The 80° and 40° angles were given. b) Incident and reflected angles are equal, so this angle is also 40°.
c) Δ-angles add to 180°, and 180° – 80° – 40° = 60°.
d) 60°, because incidence-angle = reflection-angle.
e) Draw a vertical line "V" that makes a 10° angle with the mirror. f) Draw a horizontal line "H"; V-H angle is 90°, we know 10° & 60°, so this angle [and the answer] is 20°.
An alternate method: Do Steps a-d as above, then draw the H-line. Because of Section 1.1's Z (parallel lines, both horizontal, are cut by a transverse line) the mirror-to-H angle is 80°, so subtracting 60° [the angle we know] gives the answer of 20°.

14-9: In the diagram below, reflected & transmitted light is marked R & T. You can calculate the angles yourself by using "$\theta_{incidence} = \theta_{reflection}$" for each R and "$n_1 \sin\theta_1 = n_2 \sin\theta_2$" for each T.
The light begins at "•". When this light reaches the first interface "a", some is reflected (R) and some is transmitted (T). By using the fact that a Δ's angles add up to 180°, you can calculate that the T-ray makes an angle of 55° with the surface at the second interface "b", where some is reflected and some is transmitted. And at the third interface "c", some light is reflected and some is transmitted.
Notice that light transmitted at the second interface (it is also marked •) travels in the same direction as the original light, because the top & bottom interfaces are parallel, but the ray is "displaced" from where it would be if it had not been refracted by the water.

14-10: Angle-to-surface is 47°, so angle-to-normal is 43°. There is total internal reflection, so we know that n_2 sin 43° > (1) sin 90°, and n_2 > 1.47. n_{liquid} cannot be less than 1.47 because an n-change of 1.46-to-1.00 wouldn't be large enough to prevent light transmission. But we don't know whether there also might be total internal reflection for θ's of 42° or 41°, ... (which would show that n_{liquid} must be larger than 1.47) so we can only say that "n_{liquid} is equal to or greater than 1.47" or "$n_{liquid} \geq 1.47$".

14-11: Hint: work backwards. {The full answer is after Solution 14-##.}

14-12: Draw a diagram like the one below, with horizontal lines and surface-normals shown by - - - - lines, then use Section 1.1's ΔXYZ principles and the "n sinθ = n sinθ" refraction equation.
a) given, b) Z, c) Y-90°, d) n sinθ = n sinθ, e) X and 20° + ? = 30°, f) Z and Y-180°, g) Δ, h) Y-90°, i) n sinθ = n sinθ, j) for same reasons as in steps a-c, k) 30° + ? = 70°. The initially horizontal ray emerges at 40° below horizontal.

This problem requires discipline, tool-knowledge, imagination in drawing "extra" lines, and fluency in exploring strategies; I tried several before finding one that worked, especially in Steps e-h and j-k.

(14-11) First find θ_1, then θ_2 and θ_3. For TIR, 1.491 sin θ_3 > 1.000 sin 90°, and θ_3 > 42.1°. Then use "Δ-angles add to 180°" to find that θ_2 < 47.9°, and try to solve " 1.000 sinθ_1 = 1.491 sin 47.9° " for $\theta_1 = \sin^{-1}$(1.11) = ? There is no solution because light that hits the pipe-end at <u>any</u> angle will be "trapped" inside the pipe by total internal reflection.
It can be shown that TIR occurs for a pipe made of any substance with n > 1.414 (= 1/sin45°).

14-13: As explained in Section 14.2, a medium's "n" depends on light frequency: $n_{blue} > n_{green} > n_{red}$.
Because its n is less, red light is bent less than 40°. And blue light is bent more than 40°.
This spreading is called *dispersion.* It can be used to separate ordinary *white light* [which contains a wide range of frequencies and colors] into components: IR, red, orange, yellow, green, blue, violet, UV.

In water, the interface n-difference decreases from 1.00/1.46 to 1.33/1.46, so light is bent less. A prism in benzene bends light the opposite direction because the n-difference (1.50/1.46) is reversed.

14-14: Since light comes into your eyes from that direction, the fish appears to be somewhere along the - - - - line. To hit the real fish, not its refracted image along - - - -, you must aim your spear in the · · · · direction, below the apparent position of the fish.

14-15: Light moves fast (speed ≈ c) through air and slow (at ½ c) through the n=2 medium. Even though the actual refraction-path [→] is longer than the - - - path, → takes less time (in fact, the least possible time) because it lets light travel a larger distance through air (where it is fast) and a smaller distance through the n=2 medium (where it is slow).

The mathematical laws of refraction & reflection can be derived using the least-time principle and calculus or (for reflection) basic logic-and-geometry.

14-16: A plane mirror forms an image that is exactly opposite the object, an equal distance away. In the drawing below, notice the similar triangles that let you use ratio logic: $(h_{image} / 4.00) = (.70 / 2.00)$, so the image height is 1.40 m. In the mirror you can see 1.40 m of your body, from your eyes (+1.64 m) down to .24 m above the floor.

If you are 5 m [or any other distance] from the mirror you'll still see a 1.40 m image, because the image (at 10 m) is still twice as far away as the mirror (at 5 m), thus causing the image (1.40 m) to be twice as high as the mirror (.70 m).

Your eyes appear to be 4.00 m away. The lowest part of the image is 4.24 m (the hypotenuse) away.

14-17: Yes, the lines and box-tops are the same size (check them), and the lines will meet.

A horizon-moon looks larger because of "contextual cues" that are lacking when it is higher in the sky.

When you see a mirage your eye-and-brain work correctly to perceive the light rays "as they are". But refraction (of the same kind that occurs with a lens) causes an image to form at a certain location even though the object "really isn't there". Your eyes are fine, but reality isn't what it appears to be.

A hallucination, however, is a total invention of the perceiver. His eyes-and-mind go a bit haywire and he "sees" (or hears) something that doesn't really exist.

14-18: If you ask "After interaction with the lens (or mirror) does light move toward the center-line or away from it?", you'll get four wrong answers.

Instead, look at the change of direction caused by the lens (or mirror). The first lens bends light "more toward the center line than it would have traveled without the lens" so it is converging, even though the ray ends up moving away from the center line. Similar logic shows that the second lens is diverging, and the mirrors are converging & diverging.

14-19: The "$1/d_o$ +.." equation shows that a virtual image forms at –10 cm, and so does the ray-tracing:

14-20: For an object at ∞ distance, parallel rays that come toward a converging lens or mirror are focused at the focal point. To get a parallel outgoing beam, use the reverse process; put a point-source at the focal point of a converging lens or mirror.

14-21: As a real-object moves from +∞ toward a diverging lens or mirror, a virtual image begins at the focal point and moves toward the lens (where $d_i = 0$). A memory trick: "fuzz", f z , focal then zero.

For a converging lens, there are two phases:

1) FAR: as an object moves → from +∞ to +f, its real image moves ← from +f to +∞, and

2) NEAR: as an object continues moving → from +f to 0, its virtual image moves from –∞ to 0.

FAR and NEAR phases also occur for a converging mirror, but a real image is now on the same side as the light, and a virtual image is on the opposite side:

A memory trick: "fu ± zz", f p n z , focus then positive-∞ (FAR) and negative-∞ then zero (NEAR).

14-22: The key relationship is $h_i / h_o = -d_i / d_o$: size is proportional to distance.

For a diverging lens [or mirror] the image is always closer (and therefore smaller) than the object. For a converging lens [or mirror] the image is smaller when the real-object is further away than 2f, but the image is closer (and larger) when the object has $d_o < 2f$.

14-23: a) Solve $\mathbf{1/d_o}$ for $d_i = -20$, and then **m** for $m = -(-20)/(-5) = -4$. The "real-is-inverted, virtual-is-erect" principle at the end of Section 14.4 cannot be used here, because the object is virtual (not real), thus making it possible to get an image that is virtual (d_i is –) yet inverted (m is –).

b) Solve **m** for $d_i = -12$, and $\mathbf{1/d_o}$ for $f = +4$.

c) Solve **m** for $d_o = +3$, and $\mathbf{1/d_o}$ for $f = +2$.

d) Solve **m** for $d_i = -3\,d_o$, $\mathbf{1/d_o}$ for $d_o = +10$ (you must convert the left side to a "common denominator" of $3d_o$ during the solution), and **m** for $d_i = -30$.

$m \equiv h_i/h_o$, so there is a close link between h_i, h_o and the 2-of-4 situation involving d_o, d_i, f and m.

14-24: The object can be seen at Δ or ▲, virtual image only at ▲, real image at neither place.

If the eye is replaced by film, the object and virtual image won't form a "photo" but the real image will. Similarly, a cup of water placed at the location of a real image absorbs light-energy because lots of real light (not just - - - tracings) is focused at that point.

14-25: To form a real image, a lens (or mirror) must be converging, so $f = +35 \times 10^{-3}$ m = +.035 m. $d_o = 3.84 \times 10^5$ km $= 3.84 \times 10^8$ m. Solve "$1/d_o$ ..." for $d_i = +.035$ m, and "$h_i / h_o = -d_i / d_o$" for $h_i = -h_o(d_i/d_o) = -(+3500 \times 10^3\text{ m})(+.035)/(+3.84 \times 10^8)^* = -3.2 \times 10^{-4}\text{ m} = -.32 \times 10^{-3}\text{ m} = -.32$ mm.

The – sign shows that the image is inverted, as it must be for a real image formed by one lens (or mirror).

* For this equation, h_i & h_o should be in the same units, and d_i & d_o should be in the same units.

h_i is proportional to d_i, so when f (and thus d_i for an object at $d_o \approx \infty$) increases from 35 mm to 600 mm, h_i increases by a multiplying factor of 600/35: $h_i = (-.32\text{ mm})(600/35) = -5.5$ mm. This larger image will show more of the moon's surface details.

The real image formed by a 600 mm converging (concave) mirror is also 5.5 mm high, and inverted.

14-26: An upright image formed by one lens [or mirror] must be virtual. This could be formed by any of the 4 types: C-lens, D-lens, C-mirror, D-mirror.

If $h_i = 2\,h_o$, $d_i = -2\,d_o$; a double-size image is twice as far from the lens [or mirror] as the object. $d_o = -2d_1/2 = -(-30)/2 = +15$ cm. Then solve "$1/(+15) + 1/(-30) = 1/f$" for $f = +30$ cm. Use a converging lens or mirror, with the object 15 cm away.

Problem 14-##'s table shows that a diverging lens [or mirror] always forms a virtual image that is closer (and thus smaller) than the object; this is not what you need. But the converging table has one entry with a virtual double-sized image: if $d_o = \frac{1}{2}\,f = +5$, $d_i = -10$. To get $d_i = -30$ cm (3 times larger than –10), d_o must be 3 times larger than +5 cm (making it +15), yet it must still equal $\frac{1}{2}$ f (so f = +30 cm).

14-27: Check: $1/d_o$ ... predicts that a real image will form at +40 cm (at the pseudo-focus) and it does:

$1/d_o$ predicts that ▲ forms an image (Δ) at +20 cm:

14-28: Substitute into the lens equation and solve,

$$\frac{1}{20} = (1.52 - 1)\;\frac{1}{R} + \frac{1}{R} \qquad \frac{1}{20} = (1.52 - 1)\;\frac{1}{\infty} + \frac{1}{R}$$

$$\frac{1}{(20)(.52)} = \frac{2}{R} \qquad \frac{1}{(20)(.52)} = \frac{1}{R}$$

$$R = 20.8\text{ cm} \qquad R = 10.4\text{ cm}$$

If one surface is flat (with R = ∞), the other surface must curve more sharply (with a radius of 10.4 cm instead of 20.8 cm) to match the refracting power of a symmetric lens with two curving surfaces.

Here is what the two lenses could look like:

After solving "$1/(+15) + 1/(-60) = 1/f$" for $f = +20$ cm (not –20 cm), we know the lenses are converging and they look like and either or .

14-29: The left & right surfaces are diverging (R = –10) and converging (R = +20). Substitute & solve: $1/f = (1.46 - 1)[\,1/(-10) + 1/(+20)\,]$, $f = -43$ cm.

14-30: $f = \frac{1}{2}$ R, so (36 cm) $= \frac{1}{2}$ R, and R = 72 cm.

The sphere has $R = \frac{1}{2}$ (100 cm) ; $f = \frac{1}{2}$ (50 cm) $= -25$ cm. The ± sign must be – because a sphere is a "diverging" mirror.

14-31: In each system, the ⇑ object is so far away that the converging lens [or mirror] forms a real image at the focal point of +50.0 cm, 2.2 cm past the second lens. This first-lens [or mirror] image, ⇓, is a virtual object (with $d_o = -2.2$ cm) for the second-lens.
Solve $1/(-2.2) + 1/d_i = 1/(-2)$ for $d_i = -22$ cm. There is a virtual ⇑ image at the ∗-spot on the problem-drawing. The person is right-side up, not inverted like the tree in Problem 14-B's telescope. To decide that the image is ⇑, use the "reversed" inversion principles at the end of Section 14.4, solve $h_i/h_o = -d_o/d_i$ (be careful with ± signs), or trace rays (as in Problem 14-##).

Substitute-and-solve to find h_i, m and M. Answers are identical for the lens-lens and mirror-lens systems.

h_i [first lens] = –(+1.7)(+.50/+100) = –.0085 m,
h_i [second lens] = –(–.0085)(–.22/–.022) = +.085 m,
lateral magnification "m" is (+.085)/(+1.7) = +.05.
{You cannot use "$m = -d_i/d_o$" for the final image and initial object: m = –(–.22)/(+100) = +.0022, wrong!}

The angular magnification "M" is

$$M \equiv \frac{ø_{image}}{ø_{object}} = \frac{\tan^{-1}(.085/.22)}{\tan^{-1}(1.7/100)} = \frac{21.1°}{.974°} = 21.7$$

14-32: As explained in Section 14.4, the image you see through a magnifying glass is virtual.
Solve "$1/d_o + 1(-24) = 1/(+8)$" for $d_o = +6$.
$h_i = h_o(-d_i/d_o) = 1.9(-[-24]/[6] = +7.6$ cm.
$ø_{image} = \tan^{-1}(7.6/24) = 17.6°$. Without the lens, you see $ø_{object} = \tan^{-1}(1.9/24) = 4.53°$. The angular magnification is $ø_{image}/ø_{object} = 17.6°/4.53° = 3.89$.

Optional: Many textbooks derive formulas for the magnification of a lens, using the assumption of "an image that forms at infinity when the eye is relaxed". The following analysis will help you understand what this means.

M can also be calculated for d_i = 20 m = 2000 cm; for purposes of "focusing", this is ≈ ∞, which lets the eye-focusing muscles relax. If $d_i = -2000$, d_o must be +7.968, and $h_i = 1.9(2000/7.968) = 476.9$.

$$M = \frac{\tan^{-1}(476.9/2000)}{\tan^{-1}(1.9/24)} = \frac{13.41°}{4.53°} = 2.96$$

Many textbooks give these formulas: M = 1 + N/f (if the image is at the eye's near-point N), or M = N/f (if the image forms at ∞, which lets the eye-focusing muscles relax). If the lens is placed so the image is at –24 cm, M = 1 + N/f = 1 + 24/8 = 4; this is almost the same as the 3.89 we calculated above. If the lens is placed so an image forms at ∞, M = N/f = 24/8 = 3, almost the same result as the 2.96 above. {The 1 + N/f and N/f formulas are derived by using the approximation that $ø_{image}$ and $ø_{object}$ are so small that tan ø can be replaced by ø [in radians], so we expect that there will be some difference between the M's calculated by $ø_{image}/ø_{object}$ and by 1+N/f or N/f.}

If the lens is placed so an image forms at 12 cm (for a person whose near-point is 12 cm), d_o must be +4.8, and $h_i = 1.9(12/4.8) = 4.75$.

$$M = \frac{\tan^{-1}(4.75/12)}{\tan^{-1}(1.9/12)} = \frac{21.6°}{9.00°} = 2.40$$

If the eye can focus at 12 cm instead of 24 cm, $ø_{image}$ increases from 17.6° to 21.6° (a factor of 1.23), $ø_{object}$ increases from 4.53° to 9.00° (a factor of 1.99). The penny-image is now larger, but the lens doesn't cause as much ø-change as it did before (M decreases from 3.89 to 2.40) because $ø_{image}$ is multiplied by a smaller factor than $ø_{object}$.