11.91 Problems

for Section 11.1 R,

• **11-1**: You have a knife, a long wire, a battery, and a light bulb that glows when current flows through its filament -ΛΛ- . Draw a picture showing how you connect the battery to make the bulb "light up".

11-2: A current of 4 A flows for 6 minutes. How much charge (in SI units) flows past each circuit point? How many electrons?

If current is 12 A, how long will it take for this amount of charge to flow?

11-3: What is the resistance of a 3.0 m long piece of cylindrical copper wire ($\rho = 1.7 \times 10^{-8}\ \Omega$m) whose diameter is 1.5 mm?

If this wire is cut in 3 equally long pieces that are bundled together side-by-side to form a "cable", what is its resistance?

11-4: What can you do to make a -ΛΛ- with low resistance? high resistance?

11-5: When the temperature of a resistor increases, so does its resistance. If α is the material's *temperature coefficient,*

$$\rho_{high} = \rho_{low} \{ 1 + \alpha\, \Delta T \}$$

If a copper wire has R = 100 Ω at 20° C, at what temperature is its R 10% larger?
{ The temperature coefficient of copper is .0068 $C^{\circ\,-1}$.}

11-6 optional: **drift velocity**

In copper, there are 8.2×10^{28} "conduction electrons" per m^3. If current in a wire (with 2.0 mm diameter) is 3.0 C/s →, what is the electrons' average sideways "drift velocity"? { Hint: Convert I to electrons/s, draw a cylinder with length L and cross-section A, use "visual logic". }

Now solve this problem using $\mathbf{I = v_d\, A\, n\, e}$, where **I** is the current in C/s, $\mathbf{v_d}$ is the *drift velocity* (average net speed of charge carriers as they move through a wire), **A** is the wire's cross-section area, **n** is the number of charge carriers per m^3 of wire, **q** is the charge per carrier (this is usually 1.6×10^{-19} C, because the carriers of charge are usually electrons).

If "n" isn't given, estimate it by assuming one electron "charge carrier" per atom, and using copper's density of 8930 kg/m^3 and atomic mass of 63.54 g/mole.

for Section 11.1 C,

11-7: One plate of a 40 μF capacitor has a charge of +800 μC, and the other plate carries –800 μC. What is the potential difference between the two plates?

11-8: How much charge flows through a 12 V battery when it is connected to a 30 nF capacitor?

11-9: What can you do to make a ⊣⊢ with high capacitance? low capacitance?

11-10: What is the capacitance of square plates, 10.0 cm on each side, 2.0 mm apart, separated by a piece of glass whose dielectric constant is 7.0?

Use ratio logic to find C if the glass is removed, while the plates' separation is cut in half and its area triples. Which changes tend to make C larger?

for Section 11.2 R,

11-11: Which way does "conventional current" flow in each of these circuits?

11-12: What is the electric field in an iron wire (resistivity = $10 \times 10^{-8}\ \Omega$m, diameter = 2.0 mm) with a current of .10 A →? If the right end of a 5.0 m section of this wire is at 10.00 V, what is the potential of its left end?
{ Hint: combine formulas from Sections 10.7 & 11.1. }

for Section 11.2 C,

11-13: For each capacitor, which plate has + charge?

11-14: 4 μF and 12 μF ⊣⊢'s are charged to 20 V and 10 V, respectively. After their batteries are removed, these capacitors are connected with their + and – plates together:

What is the final Q and ΔV of each capacitor?

11-14: 4 μF & 12 μF capacitors are charged to 20 V and 10 V, respectively. After their batteries are removed, the capacitors' + plates are connected together, and their – plates are connected together. What is the final charge and voltage of each capacitor?

==choose

11-15: In air, "sparking" occurs when the electric field exceeds a strength of about 3×10^6 V/m. If the air gap of a 5 μF capacitor is 2.0 mm, how much charge can be stored before a spark occurs? { Hint: Combine formulas from Sections 10.7 and 11.1. }

If the ⊣⊢'s air gap is decreased to 1.0 mm, does its maximum Q and ΔV change?

For plexiglass ($\kappa = 3.40$), sparking occurs at 100×10^6 V/m. If a piece of plexiglass is placed between the ⊣⊢ plates, what is the capacitor's maximum Q and ΔV?

for Section 11.3 R,

11-16: Use $\Delta V = IR$ and the basic principles of circuits to derive the R_{total} formulas for three resistors (R_1 R_2 R_3) in series, and for three resistors in parallel.

11-17: A circuit contains one 50 Ω resistor. How can you change the circuit's total resistance to 70 Ω? 30 Ω?

11-18: Find the total resistance between "a" and "e", if every -\/\/\- has R = 5 Ω:

11-19: Redraw these circuits to clearly show the series and parallel relationships, then find R_{total}:

11-20: Use $Q = \Delta V C$ and circuit principles to derive C_{total} formulas for three capacitors (C_1 C_2 C_3) in series, and for three capacitors in parallel.

11-21: A circuit contains a 50 nF capacitor. How can you change the circuit to make its total capacitance 90 nF? 10 nF?

11-22: If each -\/\/\- in Problem 11-18 is a ⊣⊢ with C = 5 nF, what is C_{total} between a & e? #

for Section 11.4 R,

11-23: What is the current in each -\/\/\- ?

11-24: If the 25 Ω resistor in the above circuit is replaced by 100 Ω, how does this affect current through each other resistor.

Don't do calculations (unless you want); just use ratio logic to decide whether I increases or decreases.

11-25: Find the value of R:

11-26: What is current in these circuits?
{ Each -\/\/\- is 48 Ω, and each battery is 12 V. }

11-27: For each circuit, find I_{total} and V at points a through e, if V ≡ 0 in the usual way (at the low-V side of the battery) for the first circuit and at *ground* for the second circuit. { "Grounds" are discussed in Problem 10-##. }

11-28 optional (use it if you study meters):

A *galvanometer* measures the current flowing through itself. Let's imagine we have a galvanometer (with R = 100 Ω) that reads "full scale" when I = 200 μA, and a circuit with a 20 V battery and two resistors (200 Ω and 200 Ω) connected in series.

Two common ways to use galvanometer are to measure I (in an *ammeter)* or V (in a *voltmeter).*

The diagram below shows an ammeter inside the : a galvanometer and -\/\/\- in parallel. It is connected to the main circuit in series, so all of the current that passes through this part of the circuit must also pass through the ammeter.

The diagram below shows a voltmeter inside the : a galvanometer and -\/\/\- in series. This voltmeter has the same ΔV as the 200 Ω resistor because they are connected in parallel.

Summary: An ammeter is "internally parallel" but is connected in series. The roles are reversed for a voltmeter, which is "internally in series" but is connected to the circuit in parallel.

To get an ammeter that reads full-scale when I = 50 mA, what value of R_a is needed?

To get a voltmeter that reads full-scale when ΔV = 5 V, what value of R_v is needed?

How much is this circuit "disrupted" by the ammeter? the voltmeter? { Compare the I and ΔV in the "unmeasured circuit" with the I and ΔV that are read by the meters. } To get minimum disruption, do you want R to be high or low for an I-meter? a V-meter?

for Section 11.4 C,

11-29: If the -\/\/\-'s in Problem 11-23 are replaced by ⊣⊢'s with C = 10μF, 20 μF, 100 μF and 25 μF, what is the charge on each ⊣⊢?

11-30: If the 25 μF capacitor in Problem 11-29 is replaced by 100 μF, does the charge on each capacitor increase or decrease? {Use ratio logic first, then do calculations to check your intuitive conclusions.} #

11-31: The -/\/\/-'s in Problem 11-25 are replaced by ⊣⊢'s with C = 8 pF, 4 pF, 6 pF, and "C". If Q = 26.9 pC, what is C? #

for Section 11.5 R,

11-32: What is the potential difference for the 40 Ω resistor, and the potential at • ?

11-33: Write an equation that expresses the relationship between R_1, R_2, R_3 and R_4 when no current passes through R_5. Does the same amount of current pass through R_1 and R_3?

11-34: A group of series resistors has R_{total} and ΔV_{total}. Resistor #2 has R_2 and ΔV_2. Show that R-fraction = ΔV-fraction: $R_2 / R_{total} = \Delta V_2 / \Delta V_{total}$.

11-35: This circuit shows that
**for two (and only 2) parallel resistors,
the R-ratio and I-ratio are "reversed".**

For example, the R_{top} / R_{bottom} ratio is 10/30 = 1/3, and I_{top} / I_{bottom} is 6/2 = 3/1, which is just "1/3" flipped upside down. 3/4 of the total current goes through the top branch, and 1/4 goes through the bottom branch.

Because moving charges prefer "the path of least resistance" the parallel branch with small R has large I, and vice versa.

Use ratio logic to answer these questions:
1.8 A goes into a parallel pair of resistors (47Ω and 28Ω). What is I in each branch?
Parallel resistors (12Ω and 19Ω) have I_{total} = .93 A. What is I and I-% for each branch?

11-36: When a battery with $\mathcal{E}$ = 6.00 V is connected to a 1.5 Ω resistor, the current is 3.529 C/s. With a 15.0 Ω resistor, I = .3947 A. What is the battery's internal resistance and terminal voltage?
In the circuit, + charge moves (at least we pretend that it does) from the + terminal to –terminal. Inside the battery, + charge moves from the – to + terminal. Why?

for Section 11.5 C,

11-37: Three capacitors in parallel (2 nF, 4 nF, 8 nF) hold a total charge of 560 nC. How much charge is on the 4 nF capacitor?

11-38: A group of parallel capacitors has C_{total} and Q_{total}. Capacitor #2 has C_2 and Q_2. Show that C-fraction = Q-fraction:

$$C_2 / C_{total} = Q_2 / Q_{total}.$$

11-39: This circuit shows that
**for two (and only 2) series capacitors,
the C-ratio and ΔV-ratio are "reversed".**

For example, the C_{left}/C_{right} ratio is 10/30 = 1/3, and the ratio of $\Delta V_{left} / \Delta V_{right}$ is 15/5 = 3/1, which is just "1/3" flipped upside down. 3/4 of the total ΔV is across the small-C ⊣⊢, and 1/4 of ΔV is across the small-C ⊣⊢.
Each series ⊣⊢ has the same Q. It is more difficult to hold this Q on a capacitor that has small C, so a small-C ⊣⊢ requires more ΔV.

Use ratio logic to answer these questions:
Series capacitors (16 nF and 27 nF) have ΔV_{total} = 15 V. What is ΔV across each ⊣⊢ ?
A pair of series capacitors (34 μF, 21 μF) have ΔV_{total} = 8 V. What is ΔV and ΔV-% for each ⊣⊢ ?

11-40: A capacitor with air between the plates is connected to a 12 V battery.
a) The battery is then disconnected, and a piece of glass (κ = 5) is placed between the plates, completely filling the gap. By what factors do C, Q and ΔV change?
b) The battery is then reconnected. By what factors have Q, C and ΔV change, compared with their initial values?
After you've studied stored energy (in Secion 11.6): By what factor does the ⊣⊢'s stored energy change from initial to "a", and from initial to "b"? Do you have to "push" or "restrain" the dielectric when it is first inserted? if you remove it after the battery is reconnected? How much work do you do while removing it at constant speed?

for Section 11.6 R,

11-41: How much energy is one kW·hour?
If electric energy is 7.4¢ per kW·hour (this was the average U.S. price in 1988), how much does it cost to run a 75 W bulb continuously for 1 year?

11-42: If a heater connected to a 120 V source uses 252 kJ of electrical energy in 5 minutes, what current runs through it?

11-43: Find V_a, R, and P dissipated by R.

11-44: What are the resistances of 120 V light bulbs rated at 40 W and 100 W? Which bulb is brighter if both are connected to a 120 V source in parallel? in series? What is the power dissipation for each type of circuit? Why is there is a difference?

11-45: How much energy is stored in a 12 V, 60 amp-hour battery? How long can this battery last if the car's headlights (each requires 45 W) are left on? { Hint: To answer these questions, one important [and questionable] assumption must be made. }

11-46: A power station delivers 150 kW to a factory through 5.0 Ω lines. How much less power is wasted if the electricity is delivered at 50,000 V instead of 5,000 V?

11-47: A stereo speaker has an "effective resistance" of 4 Ω, and can safely handle 70 W. Is it protected by a fuse (in series with the speaker) that melts when the current through it exceeds 4.0 A?

for Section 11.6 C,

11-48: The 20 μF capacitor stores .81 mJ of energy. What is the capacitance and stored energy of the other capacitor?

11-49: How much energy is stored in each capacitor if 15 V is connected across two ⊣⊢'s (20 μF and 30 μF) in series? in parallel?
Which circuit stores more energy? Why?

11-50: Show that the energy density (J/m^3) stored in an electric field is $\frac{1}{2} \epsilon_0 \kappa E^2$. { Hint: Analyze two capacitor plates, using formulas from Sections 10.7, 11.1 and 11.6, plus basic geometry. }

for Section 11.7 R-and-C,

11-51: A battery is connected across a circuit containing two resistors. Will the circuit have a larger R_{total}, I_{total} and P_{total} if the resistors are connected in series, or in parallel? Why?
A battery is connected across a circuit with two capacitors. Does the circuit have a larger C_{total}, Q_{total} & PE_{total} if the capacitors are connected in series, or in parallel? Why?

11-52: What can you say about R_{total} if resistors R_1, R_2 and R_3 (with $R_1 < R_2 < R_3$) are connected a) in series? b) in parallel?
What can you say about C_{total} if capacitors C_1, C_2 & C_3 (with $C_1 < C_2 < C_3$) are connected a) in series? b) in parallel?

11-53: If three resistors (10Ω, 20Ω, 30Ω) are available, how many circuits can you make? Which has the smallest R_{total}? largest R_{total}? { Hints: Use your imagination, don't assume any restrictions that aren't stated, and you only have to do two simple calculations. }
With 3 capacitors (10 nF, 20 nF, 30 nF), how many circuits can you make? What is the smallest C_{total}, and the largest?

for Section 11.8 (only C),

• **11-54**: An electron between the plates of a capacitor (1 mm gap with "$\kappa = 4$" dielectric, 20 cm square) feels a force of 1.28 x 10^{-16} N. What is the charge and energy stored by the capacitor?

• **11-55**: If it takes 4.0 J of work to move 2.0 mC of charge from one plate of a capacitor (air gap, 150 cm^2 area) to the other, and the electric field between the plates is 8.0 x 10^4 V/m, how much charge is on the plates?

11-56: A capacitor (circular with 4.0 cm diameter, air gap) holds 1.0 nC of charge. What is the capacitor's electric field? { Hint: Be persistent. }

11-57 optional [requires use of equations inside the 's]: A capacitor with .04 m^2 area holds 2.0 nC of charge. What is the energy density in the air between the plates?

for Section 11.9 (only R),

11-58: Write equations for every junction in this circuit, for the left, center and right loops, and for the "big loop" that goes around the whole circuit. Then add 3 of the junction equations together. What do you discover? Now add the left, center and right loops. If every V and R was given, would you have enough equations to find all of the I's?
{ So you'll get the same equations as the "solution", walk clockwise around each loop, and assume that I goes either ↓ or ← through the circuit elements. }

11-59: How many "useful" junction & loop equations could be written for each circuit?

11-60: How many of these circuits can be analyzed using "series-and-parallel" strategy?

11-61: Find the current through each part of this circuit by using Kirchoff's Rules, and then by using series-and-parallel strategy.

11-62: [==is this necessary?] Explain why energy conservation requires Kirchoff's Loop Rule. { Hint: Think about what would happen if ΔV around a loop was not zero. }

Soln: If a loop has $\Delta V \neq 0$, W_{el} (= q ΔV) $\neq 0$,... Moving around the loop in one direction ---- ??

for Section 11.10 (RC circuits),

11-63:

11-64:

11.92 Solutions

11-1: Cut the wire in two pieces and make a "loop" so current can flow through both battery and .

11-2: 4 A = 4 C/s, and 5 minutes = 360 s, so (4 C/s)(360 s) = 1440 C [in SI units].

(1440 C)(1 el / 1.60×10^{-19} C) = 9×10^{21} electrons.

(**Q** C) = (**I** C/s)(**t** s), I and t are inversely proportional. If a battery moves 1 A for 60 hours, it can run 5A for 12 hours.

If charge "Q" is constant, current "I" and time "t" are inversely proportional: (**I** C/s)(**t** s) = (**Q** C), or **It = Q**. If I increases by a factor of 3, t is decreased to a factor of 1/3: t = (1/3)(6 minutes) = 2 minutes.

11-3: $R = \rho L/A = (1.7 \times 10^{-8})(3) / (\pi [.00075]^2) = .029\ \Omega$.

Ratio logic: The copper's ρ stays the same, L (on top of the R-formula) is multiplied by 1/3, and A (on the bottom of the R-formula) is multiplied by 3. The overall effect of these two factors is (x 1/3) / (x 3) = (x 1/9), and the new R is (1/9)(.029 Ω) = .0032 Ω.

Check: New L is 1.0 m, new A is $3(\pi[.00075^2] = 5.3 \times 10^{-6}$. $R = (1.7 \times 10^{-8})(1) / (5.3 \times 10^{-6}) = .0032\ \Omega$.

11-4: $R = \rho L / A$. To make R low, choose material that is a good conductor (with low ρ), and form it into a shape that is short (small L) and fat (large A).

For high R: it is more difficult to "push" electrons through a poor conductor (high ρ) with a shape that is long (large L) and thin (small A).

11-5: $R = \rho L/A$. If the resistor shape is assumed constant (we'll ignore the small increases in L and A), R is proportional to ρ. When R increases by 10%, $R_{high} = R_{low}\ \{1.10\}$, and $\rho_{high} = \rho_{low}\ \{1.10\}$.

Compare this equation with $\rho_{high} = \rho_{low}\ \{1 + \alpha \Delta T\}$ and you'll see that $1.10 = 1 + \alpha\ \Delta T$,
$1.10 = 1.00 + .0068(T_{high} - 20)$.
Solve for "34.7° C = T_{high}".

11-6: In 1 s, the number of electrons that pass each point in the circuit is (3.0 C/s)(1 el / 1.60×10^{-19}) = 1.9×10^{19}. These electrons occupy a volume of (1.9×10^{19} els)(1 m^3 / 8.2×10^{28} els) = 2.3×10^{-10} m^3. V = LA, so the wire length that contains this number of electrons is $L = V/A = (2.3 \times 10^{-10}) / (\pi\ .001^2) = 7.3 \times 10^{-5}$ m. If electrons move at 7.3×10^{-5} m/s, all of the 1.9×10^{19} electrons in this length will pass a given point during one second. {The "conventional current" is →, so electrons actually move ←.}

$v_d = I/(Anq) = 3 / [\pi\ .001^2][8.2 \times 10^{28}][1.60 \times 10^{-19}] = 7.3 \times 10^{-5}$ m/s. It is easier to get "the answer" by using this formula, but you may learn some valuable skills (that will be useful to you later) by struggling with the more "intuitive" logic in the first paragraph.

If "n" is not given, estimate it using this process:
If "n" is not given, you can estimate its value: ===

$$1\ m^3\ \frac{8930\ kg}{1\ m^3}\ \frac{1000\ g}{1\ kg}\ \frac{1\ mole}{63.54\ g}\ \frac{6.02 \times 10^{23}\ atoms}{1\ mole}\ \frac{1\ charge\ carrier}{1\ atom} = 8.5 \times 10^{28}\ \text{charge carriers in } 1\ m^3.$$

This is higher than the experimentally determined value, $n = 8.2 \times 10^{28}$, because the average number of "charge carriers per atom" is not exactly 1.

11-7: Q = 800 μC (not 1600 μC), and $Q = \Delta V\ C$, so $\Delta V = Q / C = (800 \times 10^{-6}) / 40\mu = 20$ Volts.

11-8: The question asks you to find Q. $Q = \Delta V\ C = 12(30\mu) = 360 \times 10^{-6}$ Coulombs = .00036 C.

11-9: $C = \epsilon_o\ \kappa\ A/d$. To make C high, use a dielectric with a large κ-value, and big plates (large A) that are close together (small d).

00-0: $C = \epsilon_o\ \kappa\ A / d$. To make C high, use plates with large area, and a narrow gap (small d) filled with high-κ material. ==[choose

To get a low C, use a small-κ dielectric (like air, with κ = 1.0005) and small plates that are far apart.

11-10: $C = \in_o \kappa A / d = (8.85 \times 10^{-12})(7).10^2 / .002 = 3.1 \times 10^{-10}$ F = 31 nF. Do you see why C's are usually given in units of μF, nF or pF? {To get C = 1 F if only plate-area is changed, this capacitor would need square plates 5.7 km (3.5 miles) on each side! }

κ and A (on top) are multiplied by 1/7 and 3, while d (on the bottom) is multiplied by 1/2. The new C is (old C) [multiplying factor] = (31 nF)[(1/7)(2)/(1/2)] = 27 nF. The A-↑ and d-↓ tend to make C larger, but these factors are overcome by the κ-↓ when glass is replaced by air.

11-11: Conventional current (+ charge movement) is toward V that is "more –", in the direction of lower number-line V. In both situations shown, this is →.

11-12: If a section of wire has length L, it has ΔV = E L (from 10.7) and R = ρL/A (from 11.1). These can be substituted into ΔV = IR (from 11.1) to get (EL) = I(ρL/A), so $E = I\rho/A = 10(10 \times 10^{-8})/(\pi\ .001^2)$ = .32 V/m = .32 N/C. Positive charge moves in the direction of E; I is →, so is E.

ΔV = Ed = (.32 V/m)(5.0 m) = 1.60 V. V decreases in the direction of I, so if V is 10.00 V at the right end, it must have been (before the 1.60 V drop) 11.60 V at the left end.

11-13: Positive charge builds up on the "more +", higher-V side of the plate; this charge is what gives it + potential. For these two situations, +Q builds up on the +20 V and 0 V sides, –Q builds up on the +12 V and –7 V sides.

11-14: Initially, the capacitors' charges are Q = VC = 20(4μ) = 80 μC, and 10(12μ) = 120 μC. When they are connected together, charge is conserved: combined charge on the + plates is (+80μ) + (+120μ) = +200μC. Similarly, combined charge on the – plates is –200μC. If the 3 μF ⊣⊢ has Q = x (+x and –x on its + and – plates), the 12 μF ⊣⊢ has a charge of ±(200μ – x) on its + and – plates.

"Tracing V's" shows that each ⊣⊢ has the same V:

$$
\begin{aligned}
V_{3\mu F} &= V_{12\mu F} \\
Q / C &= Q / C \\
x / 3\mu &= (200\mu - x) / 12\mu \\
12\mu\, x &= 600\mu^2 - 3\mu\, x \\
15\mu\, x &= 600\mu^2 \\
x &= 40\mu
\end{aligned}
$$

The 3 μF ⊣⊢ has a charge of x = 40 μC. The 12 μF ⊣⊢ has Q = 200μ – x = 160 μC. The ⊣⊢'s have the same ΔV, but the 12 μF ⊣⊢ has 4 times as much Q (160μ versus 40μ) because its C is 4 times as large.

11-15: ΔV = Ed (from 10.7), so Q = ΔV C = Ed C. If we assume E is at the "sparking limit" of 3×10^6 V/m, Q = Ed C = $(3 \times 10^6)(.002)(5 \times 10^{-6})$ = .03 C.

$\Delta V_{max} = Q_{max} / C = .03 / 5\mu = 6000$ V. An extra substitution may help you see the factors that do (and don't) affect the maximum ΔV: $\Delta V_{max} = Q_{max} / C = (E_{max}\, d\, C) / C = E_{max}\, d = (3 \times 10^6)(.002) = 6000$ V.

ΔV = Ed, so if E is to remain at 3×10^6 V/m when d is cut in half, ΔV must also be cut in half. And $C = \in_o \kappa A/d$, so C doubles when d is cut in half. Finally, Q = ΔV C: the multiplying factors for ΔV (it is x 1/2) and C (it is x 2) cancel each other, so the ⊣⊢ still holds the same maximum charge.

With the plexiglass, κ increases by a factor of 3.40 (from 1.00 to 3.40) and so does C: with plexiglass, the ⊣⊢'s capacitance is 3.40(5 μF) = 17 μF.

$Q_{max} = E_{max}\, d\, C = (100 \times 10^6)(.002)(17\mu) = 3.4$ C,
and $\Delta V_{max} = Q_{max} / C = 3.4 / 17\mu = 200{,}000$ V.

Bonus: $\mathbf{Q_{max}} = E_{max}\, d\, C = E_{max}\, d\, [\in_o \kappa A/d] = \mathbf{E_{max} \in_o \kappa A}$, and $\mathbf{\Delta V_{max} = E_{max}\, d}$. Do you see why 1) increasing a ⊣⊢'s plate-A increases Q_{max} but doesn't affect ΔV_{max}, 2) decreasing its d decreases ΔV_{max} but doesn't affect Q_{max}, and 3) changing the dielectric [which affects κ and E_{max}] changes both Q_{max} and ΔV_{max}?

11-16: In series, all -\/\/-'s have the same current (so $I_{total} = I_1 = I_2 = I_3$), and their ΔV's add:

$$
\begin{aligned}
\Delta V_{total} &= \Delta V_1 + \Delta V_2 + \Delta V_3 \\
I_{total}\, R_{total} &= I_1 R_1 + I_2 R_2 + I_3 R_3 \\
R_{total} &= R_1 + R_2 + R_3
\end{aligned}
$$

In parallel, all -\/\/-'s have the same ΔV (so $\Delta V_{total} = \Delta V_1 = \Delta V_2 = \Delta V_3$), and their currents add:

$$
\begin{aligned}
I_{total} &= I_1 + I_2 + I_3 \\
\Delta V_{total} / R_{total} &= \Delta V_1 / R_1 + \Delta V_2 / R_2 + \Delta V_3 / R_3 \\
1 / R_{total} &= 1 / R_1 + 1 / R_2 + 1 / R_3
\end{aligned}
$$

Do you see why we can divide out all I's (for the series circuit) and all ΔV's (for the parallel circuit)?

11-17: If a 20 Ω -\/\/- is added in series, R_{total} increases to 70 Ω. To reduce R_{total} to 30 Ω, add 75 Ω in parallel: solve "1/30 = 1/50 + 1/R" to get "75 = R". { Punch "30 1/x – 50 1/x = 1/x" [notice the "–"]. }

11-18: b-to-c has R = 1.67 Ω, the top & bottom branches of c-to-d are 10 Ω & 7.5 Ω, the entire c-to-d has R = 22.5 Ω, and a-to-e has R_{total} = 5 + 1.67 + 22.5 + 5 = 34.17 Ω

11-19: Here are the circuits, redrawn to show series and parallel relationships. The top circuit has R_{total} = 2.49 Ω; starting with the 8 Ω -\/\/- and continuing through the 1 Ω -\/\/-, the R's are 4.29, 9.29, 2.80, 5.80, 1.49, and 2.49.

The middle circuit has R = 12.6 Ω.

In the bottom circuit, the diagonal wire provides an easy *short circuit* path (with R ≈ 0) for the current; it "ignores" five resistors (they don't contribute to R_{total} so they aren't drawn on diagram below) and R_{total} = 11.9 Ω.

You'll be able to analyze many circuits without redrawing them. {I find that re-drawing is often helpful, especially for the more complicated circuits.}

11-20: In series, all of the ⊣⊢'s have the same charge (so $Q_{total} = Q_1 = Q_2 = Q_3$), and their ΔV's add:

$$\begin{aligned} \Delta V_{total} &= \Delta V_1 + \Delta V_2 + \Delta V_3 \\ Q_{total} / C_{total} &= Q_1 / C_1 + Q_2 / C_2 + Q_3 / C_3 \\ 1 / C_{total} &= 1 / C_1 + 1 / C_2 + 1 / C_3 \end{aligned}$$

In parallel, all ⊣⊢'s have the same ΔV (so $\Delta V_{total} = \Delta V_1 = \Delta V_2 = \Delta V_3$), and their charges add:

$$\begin{aligned} Q_{total} &= Q_1 + Q_2 + Q_3 \\ \Delta V_{total}\, C_{total} &= \Delta V_1\, C_1 + \Delta V_2\, C_2 + \Delta V_3\, C_3 \\ C_{total} &= C_1 + C_2 + C_3 \end{aligned}$$

Do you see why we can divide out the Q's (for the series circuit) and the ΔV's (for the parallel circuit)?

11-21: To increase C_{total} to 90 nF, add a 40 nF capacitor in parallel. To reduce C_{total} to 10 nF, add a 12.5 nF ⊣⊢ in series {solve "1/10 = 1/50 + 1/C"}.

11-22: b-to-c is 15n, the top & bottom branches of c-to-d are 2.5n & 3.33n, c-to-d is 10.83n, a-to-e has $1/C_{total} = 1/5 + 1/15 + 1/10.83 + 1/5$, $C_{total} = 1.79$ nF.

11-23: **a)** the parallel combination is 20 Ω, then add 30 Ω to get $R_{total} = 50$ Ω. **b)** trace V's of 30 [to a] and 0 [to f and g]. **c)** Solve V=IR for the entire circuit: 30 = I(50), $.60 = I_{total}$. **d)** Solve V=IR for "10Ω+20Ω": ΔV = .60(30), ΔV = 18. **e)** V across "10Ω+20Ω" drops by 18, from 30 to 12. **f)** Trace "12 V" to d & e. **g)** Solve V=IR for "100Ω": (12–0) = I(100), .12 = I. **h)** I = .60 A goes into the parallel junction; if the top branch has .12, the bottom must carry .48. **i)** As a check, solve V=IR for "25Ω": (12–0) = .48(25), OK because both sides are equal.

Answers: The 10, 20, 100 and 25 Ω resistors carry currents of .60, .60, .12 and .48 Coulombs/second.

11-24: $R_{parallel}$ and R_{total} increase, so I_{total} decreases. There is less I through the battery, 10Ω, 20Ω and "parallel combination". But the top branch now gets a larger fraction of $I_{parallel}$ (it was getting only 20%, now it gets 50%) so I through the top branch's 100Ω actually increases. I_{bottom} decreases, because $I_{parallel}$ has decreased and the bottom branch's I-fraction has dropped from 80% to 50%.

I_{top} has conflicting factors: $I_{parallel}$ decreases but the top's I-fraction increases. To check which factor is more important, find $I_{total} = .375$ A (as predicted, this is lower than the original .60 A) and $I_{top} = .1875$ A. This is larger than the original I_{top} of .12 A, thus confirming our educated guess that the I-fraction is the more important of the two conflicting factors.

11-25: **a)** Trace 7 V to a, 0 V to f & g. **b)** Solve V=IR for "8Ω": ΔV = .5(8) = 4. **c)** V drops by 4 across "8Ω", from 7 to 3. **d)** Trace 3 V to c & d. **e)** Solve V=IR for "4Ω+6Ω": (3–0) = I(4+6), .3 = I. **f)** The entire parallel region carries .5 A; .3 A goes to the top, so .2 A goes to the bottom. **g)** Solve V=IR for "R": (3–0) = .2 R, so 15 = R. **h)** A check: The only V=IR we haven't used yet is "for the entire circuit", so find $R_{total} = 8 + 6 = 14$, and substitute to get (7–0) = 14(.5), 7 = 7, OK.

11-26: The first picture shows that each battery increases V by 12 V, from 0 to 12, and then from 12 to 24. It doesn't matter where the batteries are located; their combined ΔV is, as shown in second picture, still 24 V.

In the third circuit the batteries oppose each other, and the combined ΔV is 0.

When equal-V batteries are in parallel, V tracing shows that their combined ΔV is 12 V. The overall ΔV is the same as with one battery, but half as much current goes through each battery (because the I_{total} of .25 A is split between the top & bottom branches) so the batteries will last longer. {If parallel batteries have unequal V, a circuit must be analyzed using the *internal resistance* of batteries (Section 11.5) and *Kirchoff's Rules* (Section 11.9).}

11-27: Both circuits have the same V=IR [of 12 = I(48)] and the same I_{total} [of .25 C/s] through every part of the circuit.

You might think that all current will go "into the ground" after it passes through the 16Ω -/\/\/-. But if I = 0 through the 32Ω -/\/\/- (after all current has gone into the ground) there would be 0 C/s going into the battery and .25 C/s going out of it. The battery would have to "create" charge (not just move it from one place to another); this would violate the "conservation of charge", and it does not occur. {Using *Kirchoff's Rules* (Section 11.9) leads to the same conclusion: that I is the same throughout the entire circuit, whether there is a "ground" or not.}

This cannot occur because huge electrostatic fields (due to piled-up charge) would quickly cancel the battery's voltage and prevent the further flow of current in the circuit. ==[combine this with --- above?

==[is this analysis correct? what about lightning rods? ac electrocutions?

These diagrams show V at the 5 locations. Notice that V at a location is different on the two diagrams, but ΔV across each circuit element (and this ΔV is what actually produces the "action") is the same.

A "ground" can affect a circuit. For example, if a /\/\/ is put in parallel with a ground, it provides a "short circuit" path for current. In both of these circuits, there will be a == nec?

11-28: We want G [with R = 100 Ω] to read full-scale when 50 mA goes through the ammeter. When these (full-scale meter, 50 mA) occur simultaneously, **a)** G [which reads full-scale] has I = 200 μA and ΔV = IR = (200μ)(100) = .02 Volts, **b)** tracing V's shows that R_a [in parallel with G] also has ΔV = .02, **c)** 50 mA goes into the ammeter and 200 μA goes into the top branch so the rest [50m – 200μ = .0498] goes through the bottom branch, **d)** Solve V=IR for the bottom branch: .02 = .0498 R, .402 = R.

a) G reads full-scale so it has I = .0002 A, and ΔV = IR = .0002(100) = .02. **b)** I in R_V is also .0002 A. **c)** ΔV across R_V is .50 – .02 = .48. **d)** Solve V=IR [.48 = .0002 R_V] for R_V = 2400 Ω.

Our ammeter has R_{total} = .4003 Ω. Without the ammeter, the circuit has I = 20/1000 = .02 A. With it, I = 20/1000.4 = .019992 A. The ammeter's R is very low [because it has a small R in parallel with G's 50Ω] so it has a negligible effect on the system.

{If a circuit had ΔV = .02 V and R = 1.0 Ω (smaller than in the original circuit) it would have I = .0200 A without the ammeter, and I = .02/1.4003 = .0143 A with it. Now there is significant disruption!}

Our V-meter has R = 2450 Ω, and the I-meter/200Ω part of the circuit has R = 185Ω. Without this V-meter the 200Ω resistor has ΔV = 4.000 V. With it the circuit has I = ΔV/R = 20/(185+800) = .0203, and the I-meter/200Ω section has ΔV = IR = .0203(185) = 3.76 V. The V-meter disrupts the circuit a little bit, making V increase from .0200 A to .0203 A, but the "measured ΔV" is much lower than the "undisturbed ΔV" because R changes from 200Ω to 185Ω.

11-29: **a)** C_{series} = 6.67μ, C_{par} = 125μ, C_{total} = 6.33μ. **b)** Trace 12 V to a, 0 V to f and g. **c)** Solve Q=VC for the entire circuit: Q = (30–0)(6.33μ), Q = 189.9μ.
d) Solve Q=VC for "10μ+20μ": both ⊣⊢'s carry Q = 189.9μ , and so does the combination, 189.9μ = ΔV(6.67μ), 28.5 = ΔV.
e) V drops by 28.5 across "10μ+20μ", from 30 to 1.5.
f) Trace 1.5 V from c to d & e.
g) Solve Q=VC for "100μ": Q = (1.5–0)(100μ) = 150μ.
h) If both branches together have Q = 189.9μ and the top branch has 150μ, the bottom branch must have 39.9μ.

i) As a check, solve Q=VC for "25μ": 39.9μ = 1.5(25μ). {39.9μ versus 37.5μ is close, but it isn't exact because of round-offs, especially in subtraction of large numbers (30.0 – 28.5) to give a small number (1.5).} To get a more accurate calculation, use ratio logic. The 100μ & 25μ have the same ΔV, but 100μ holds 4 times as much Q because its C is 4 times as large. The 100μ ⊣⊢ holds 189.9μ(4/5) = 151.9μ, the 25μ holds 189.9μ(1/5) = 38.0μ.

Answers: The 10, 20, 100 & 25 μF capacitors hold charges of 189.9, 189.9, 151.9 & 38.0 μC.

11-30: C_{total} increases, and so does Q_{total} (which equals the Q across "10μF" and "20μF"). $Q_{parallel}$ also increases, but Q_{top} decreases because it originally carried 80% of the $Q_{parallel}$ but now it only gets 50%. Q_{bottom} increases because $Q_{parallel}$ increases and the bottom branch's % increases.

Check: calculate C_{total} = 6.45μ F, Q_{total} = $Q_{parallel}$ = 193.5μ C, and (after the 50-50 split) Q_{top} = 96.75μ C. This is smaller than the original Q_{top} of 151.9μ C, confirming our guess about the "conflicting factors": that the I-fraction decrease (from 80% to 50%) is more important than $Q_{parallel}$ increase (from 189.9μ C to 193.5μ C).

11-31: **a)** Trace 7 V to a, 0 V to f & g.
b) Solve Q=VC for "8p": 26.9p = ΔV(8p), 3.3625 = ΔV.
c) V drops 3.3625 across "8p", from 7 to 3.6375.
d) Trace 3.6375 to c & d. **e)** Solve Q=VC for "4p+6p": Q = (3.6375–0)(2.4p) so Q = 8.73p.
f) The parallel region holds Q = 26.9p; if the top branch has 8.73p the bottom must have 18.17p.
g) Solve Q=VC for "C": 18.17p = (3.6375–0) C, 5.0p = C.
h) Do a check: The only Q=VC we haven't used yet is "for the entire circuit". C_{top} = 2.4, $C_{parallel}$ = 7.4, C_{total} = 3.844p, substitute to get 26.9p = (7–0)(3.844p), 26.9 ≈ 26.91, OK. {To minimize the rounding-off errors that caused trouble in 11-29, this solution kept many significant figures (more than were "justified") for the intermediate calculations.}

11-32: The /\/\/ combination has R = 200Ω and ΔV = 15 V. The 40Ω /\/\/ has ΔV = (40/200)(15 V) = 3 V. The V at • is 8 V {3 V higher than 5 V}.

11-33: If I in R_5 is 0, $V_b = V_c$, and (subtracting V_a from both sides) $V_b - V_a = V_c - V_a$. Use ratio logic:

$$V_b - V_a = V_c - V_a$$

$$\frac{\mathbf{R_1}}{\mathbf{R_1 + R_2}}(V_d - V_a) = \frac{\mathbf{R_3}}{\mathbf{R_3 + R_4}}(V_d - V_a)$$

$$R_1 R_3 + R_1 R_4 = R_1 R_3 + R_2 R_3$$

$$\frac{\mathbf{R_1}}{\mathbf{R_2}} = \frac{\mathbf{R_3}}{\mathbf{R_4}}$$

The ratio relationship can be expressed in two ways: $R_1/(R_1 + R_2) = R_3/(R_3 + R_4)$ and $R_1/R_2 = R_3/R_4$.

In the example below, the top & bottom branch each have an R_{left}/R_{total} ratio of 1/4 (either 3/12 or 5/20) because R_{left}/R_{right} is 1/3 (3/9 or 5/15). Do you see the connection between these two kinds of ratios?

11-34: Resistor #2 has the same current as every other  in the series: $I_2 = I_{total} = \Delta V_{total}/R_{total}$.
$\Delta V_2 = I_2 R_2$, so $\Delta V_2 = (\Delta V_{total}/R_{total}) R_2$.
This can be rearranged to give $\Delta V_2/\Delta V_{total} = R_2/R_{total}$, or **ΔV-fraction = R-fraction**.

11-35: The 47Ω (high-R) branch gets less current than the 28Ω (low-R) branch. The 47Ω branch has $I = (28/75)(1.8\text{ A}) = 1.13\text{ A}$, and the 28Ω branch has $I = (47/75)(1.8\text{ A}) = 1.13\text{ A}$.

For 12Ω (low R) branch, $I = (19/31)(.93) = .57\text{ A}$.
For 19Ω (high R) branch, $I = (12/31)(.93\text{ A}) = .36\text{ A}$.
I-%'s: $(19/31)(100) = 61\%$, $(12/31)(100) = 39\%$.
Or use $(.57/.93)(100) = 61\%$, $(.36/.93)(100) = 39\%$.

11-36: Draw a diagram as in Section 11.5, write V=IR equations for each circuit, and solve them:

$$6 = 3.529\,(1.5 + r) \qquad 6 = .3947\,(15 + r)$$
$$5.2935 + 3.529\,r = 5.9205 + .3947\,r$$
$$r \text{ (internal resistance)} = .200\ \Omega$$

A battery's terminal-V varies because it depends on "context". In the first circuit, internal V-loss is $\Delta V = IR = Ir = 3.529(.2) = .706$: $V_{terminal} = 6.00 - .706 = 5.29\text{ V}$. But $V_{terminal} = 6.00 - .3947(.200) = 5.92\text{ V}$ in the context of the second circuit.

As discussed in Section 11.1, a battery can "make things happen" in a circuit. The circuit is <u>passive</u>, so + charges are repelled away from the + terminal and attracted toward the – terminal, as you expect. But chemical reactions inside a battery (your physics or chemistry text may discuss the details) make it <u>active</u> so it can push charge in the "unnatural direction" from the – terminal to + terminal.

11-37: Q on "4 nF" is $(4n/14n)(560\text{ nF}) = 40\text{ nF}$.

11-38: Every series capacitor, including #2, has the same ΔV: $\Delta V_2 = \Delta V_{total} = Q_{total}/C_{total}$. $Q_2 = \Delta V_2 C_2$, so $Q_2 = (Q_{total}/C_{total}) C_2$. This can be rearranged to $Q_2/Q_{total} = C_2/C_{total}$: **Q-fraction = C-fraction**.

11-39: The 16 nF (low-C) capacitor requires more ΔV. It has 27/(16+27) of the total ΔV of 15 V, or 9.42 V. The 27 nF (high-C) capacitor has $\Delta V = (16/43)(15\text{ V}) = 5.58\text{ V}$.

The 34 μF (high C) capacitor has $(21/55)(8\text{ V}) = 3.05\text{ V}$. The 21 μF (low C) capacitor has $\Delta V = (34/55)(8) = 4.95\text{ V}$.
ΔV-%'s: $(21/55)100 = 38\%$, $(34/55)100 = 62\%$.
Or calculate $(3.05/8)100 = 38\%$, $(4.95/8)100 = 62\%$.

11-40: With the battery disconnected, <u>Q is constant</u> but ΔV can change. Use ratio logic to determine what happens to C & ΔV when κ increases by a factor of 5:

$$\begin{array}{ccccc} (\times 5) & = & \times 5 & \text{same} & \\ C & = & \kappa & \epsilon_o A/d & \end{array} \qquad \begin{array}{ccccc} \text{same} & = & (\times 1/5) & \times 5 \\ Q & = & \Delta V & C \end{array}$$

C and ΔV change by factors of x5 and x 1/5.

When the battery is reconnected <u>ΔV returns to its initial value of 12 V</u>, but (since the circuit loop is re-established) Q can change. As above, use ratio logic:

$$\begin{array}{cccc} (\times 5) & = & \times 5 & \text{same} \\ C & = & \kappa & \epsilon_o A/d \end{array} \qquad \begin{array}{ccccc} (\times 5) & = & \text{same} & \times 5 \\ Q & = & \Delta V & C \end{array}$$

C and Q both increase by a factor of x 5.

We know the factors of Q (x1), ΔV (x.2) and C (x5) for i-to-a. To find the PE_{el} factor, use any equation:

$$\begin{array}{ccccccccc} (\times .2) & = & \times 1 \;\; \times .2 & = & \times 5 \;\; \times .2^2 & = & \times 1^2 / \times 5 \\ PE_{el} & = & \tfrac{1}{2}\, Q \;\; V & = & \tfrac{1}{2}\, C \;\; V^2 & = & \tfrac{1}{2}\, Q^2 / C \end{array}$$

For i-to-b the factors are Q (x5), ΔV (x1), C (x5), and

$$\begin{array}{ccccccccc} (\times .2) & = & \times 5 \;\; \times 1 & = & \times 5 \;\; \times 1 & = & \times 5^2 / \times 5 \\ PE_{el} & = & \tfrac{1}{2}\, Q \;\; V & = & \tfrac{1}{2}\, C \;\; V^2 & = & \tfrac{1}{2}\, Q^2 / C \end{array}$$

Here is a PE diagram for "i", "a" (lower than i because of the x.2 factor) and "b" (higher than i because of the x5 factor). When you insert the slab, thus taking the system "downhill" from i and a, you must restrain the dielectric. And when you remove the dielectric, taking the system downhill from b to i, you must restrain it.

high PE_{el} at **b**

medium PE_{el} at **i**

low PE_{el} at **a**

For b-to-i the electrical work is $\Delta PE = PE_f - PE_i = 5PE_i - PE_i = 4\,PE_i = 4(\tfrac{1}{2}\, C_i V_i^2)$, and the work you do is $-4\,PE_i = -4(\tfrac{1}{2}\, C_i V_i^2)$. If you knew the numerical values of C and V (all you know is $\kappa_i = 1$ and $V_i = 12$) you could calculate the value of work.

11-41: 1 kW·hr is 1 kW of power used for 1 hour: $(10^3 \text{ J/s})(3600 \text{ s}) = 3.6 \times 10^6$ J.

$$\frac{75 \text{ J}}{1 \text{ s}} \; \frac{365 \times 24 \times 3600 \text{ s}}{1 \text{ year}} \; \frac{1 \text{ kW·hr}}{3.6 \times 10^6 \text{ J}} \; \frac{.074 \text{ dollar}}{1 \text{ kW·hr}}$$

= 49 dollars! (are you shocked at the high cost?)

11-42: $P = (252 \times 10^3 \text{ J})/(300 \text{ s}) = 840 \text{ J/s} \equiv 840$ W. $P = IV$, so $I = P/V = 840/120 = 7 \text{ C/s} \equiv 7$ A.

11-43: Add "$P = IV = I^2R = V^2/R$" to your box of circuit-analysis tools from Section 11.4. **a)** Solve $P=V^2/R$ for "200Ω": $.32 = (V_a - 4)^2/200$, $8 = (V_a - 4)$, $12 = V_a$ [the circuit has a 12 V battery]. **b)** Solve $P=I_2R$: $.32 = I^2(200)$, $.04 = I$. **c)** .04 C/s also flows through "R". **d)** Solve V=IR for "R": $(4–0) = .04R$, $100 = R$.
e) Solve P=IV for "R": $P = .04(4–0) = .16$ J/s, or [to check] $P = I^2R = .04^2(100) = .16$ W, or $P = V^2/R = (4–0)^2/100 = .16$ J/s.

{Other solution-orders are possible. For example, Steps a and b above solve $P=V^2/R$ and $P=I^2R$, but you could also solve $P=V^2/R$ & V=IR (in that order), or $P=I^2R$ and V=IR (in that order).}

11-44: $P = V^2/R$, so $R = V^2/P$.
A 40 W bulb has $R = 120^2/40 = 360\,\Omega$.
A 100 W bulb has $R = 120^2/100 = 144\,\Omega$.

{Bulbs in flashlights use the DC (direct current) we are studying in this chapter. A 40 W bulb for a "wall outlet" uses AC (alternating current) but we can still analyze it using V=IR & $P=IV=I^2R=V^2/R$. Chapter 13 gives a more complete analysis of AC circuits.}

Connected in parallel, each bulb has $\Delta V = 120$ V. P-dissipations are $P=V^2/R = 120^2/360 = 40$ J/s, and $P = 120^2/144 = 100$ W. These calculations are just the reverse of those done above.

In series, $R_{total} = 360\Omega + 144\Omega = 504\,\Omega$. Each bulb has $I = V/R = 120/504 = .238$ C/s. The "40W" has $P = I^2R = .238^2(360) = 20.4$ J/s, and "100 W" has $P = .238^2(144) = 8.2$ W. The 40 W bulb is now brighter! {This analysis ignores the fact that each bulb's -/\/\/- filament now has lower T and thus, as discussed in Problem 11-##, lower R. But even if this R-decrease is taken into account, we still reach the same conclusion: the 40 W bulb is brighter.}

In parallel both bulbs have the same ΔV, so P ($=V^2/R$) is larger for the low-R 100 W bulb. But in series both bulbs have the same I, so P ($= I^2R$) is larger for the high-R 40 W bulb. {For each circuit, $IV = I^2R = V^2/R$, but it is easier to use "ratio logic" by choosing the formula that has R and the variable (either ΔV or I) that is equal for both bulbs.}

In parallel, P_{total} = 40W+100W = 140 W. But in series, P_{total} is only 20.4W+8.2W ≈ 29 W*. Why? In parallel each -/\/\/- has the entire 120 V, but in series the 120 V is "split"; each bulb has a smaller ΔV and thus a smaller V^2/R power.

* Because R is lower at the lower temperatures that occur in the series circuit, P_{total} is larger than the 29 W we've calculated, but it is still much less than 140 W.

11-45: A necessary-but-questionable assumption: a battery's ΔV is constant while it is "running down".

The units of "amp-hour" [(C/s)(s) = C] show that it is a unit of charge: 60 amp-hours = 60(1 C/s)(3600 s) = 216000 Coulombs. If ΔV = 12 V while this charge is moved, work = $q\Delta V = (216000)(12) = 2.6 \times 10^6$ J.

Or imagine the battery providing a 3 C/s current for 20 hours*. If ΔV = 12 V, P = IV = 3(12) = 36 W, energy = (36 J/s)(20 hrs)(3600 s/hr) = 2.6×10^6 J.
* it could also provide 4 A for 15 hours, or...

If headlights are connected to the battery in parallel, each light has ΔV = 12 V, and I = P/ΔV = 45/12 = 3.75 C/s. The two lights draw 7.5 A from the battery, and Δt = Q/I = (60 amp-hours)/(7.5 amps) = 8 hours.

11-46: To solve this problem, you must think about this 3-step process: electrical energy-and-power is generated at the station, moved over power lines, and used in the factory. R = 5Ω for the movement phase, P = 150 kW and ΔV = 50000 V (or 5000) at the factory*, and current is the same for both phases. The question asks about the P dissipated during the movement phase.
{ * Before it is used in a factory, voltage is reduced by a *transformer.*}

If 50000 V is used, 150 kW at the factory requires $I = P/V = (150 \times 10^3)/(50000) = 3.0$ A. The power dissipated in the 5Ω lines is $P = I^2R = 3^2(5) = 45$ W.

But with 5000 V, I must be 150k/5000 = 30 A, and the line's P-waste is $30^2(5) = 4500$ W.

"How much more power is wasted?" can be answered by subtraction, division, or %-of-whole.

Subtraction: 4500 W – 450 W = 4455 W less waste.

Division: (45 W)/(4500 W) is 1/100 as much waste.

%-of-whole: At 50000 V, the plant must generate P = (150000+45) W, waste is (45/150045)100 = .03%. At 5000 V, % waste is (4500/154500)100 = 2.9%.

Do you see why high-V lines are used for transport?

==[refer to Problem 12-# (or 13-#) for advantages of AC:
generation as sine wave, motors (no rings nec),
transformers possible (and high-V for less P loss),
== V-gen, V-up, transmit(up to 750kV),
V-down (≈2400 V thru neighborhood lines), 120 V

11-47: If speaker has I = 4.0 A, $P = I^2 R = 4^2(4) = 64$ Ω, a safe amount for the speaker (but maybe not for your ears). If a fuse allowed a current larger than 4.2 A, P would be larger than 70 W and the speakers might be damaged. To protect speakers (expensive), you want a fuse (cheap) that melts at a current below 4.2 A. {Don't replace a blown fuse with a new fuse until you know what made it melt, and whether it is safe to use the circuit again.}

11-48: Combine "$PE_{el} = \frac{1}{2} QV = \frac{1}{2} V^2C = \frac{1}{2} Q^2/C$" with the circuit-analyzing tools from Section 11.4. **a)** For "20μF", solve $PE = \frac{1}{2} Q^2/C$: $.81 \times 10^{-3} = \frac{1}{2} Q^2/(20 \times 10^{-6})$, $1.8 \times 10^{-4} = Q$. **b)** The ⊣⊢s are in series, so "C" also has this Q. **c)** Solve Q=VC for "C": 180μ = ΔV(20μ), 9 = ΔV. **d)** ΔV_{total} is 15 V and "20μF" has a ΔV of 9 V, so "C" has ΔV = 6 V. **e)** Solve Q=VC for "C": 180μ = 6C, 30μ = C. **f)** "C" has a stored PE of
$\frac{1}{2}$ QV = .5(180μ)(6) = 5.4×10^{-4} J, or
$\frac{1}{2} V^2C = \frac{1}{2}(6^2)$ 30μ = 540 μJ, or $\frac{1}{2} Q^2/C = .5(180\mu)^2/30\mu$ = .54 mJ. **g)** Check: PE_{total} = .81mJ + .54 mJ = 1.35 mJ, $PE_{total} = \frac{1}{2}$ QV = .5(180μ)15 = 1.35 mJ, OK.

11-49: C_{total} = 12 μF. Solve Q=VC for the whole circuit: Q = 15(12μ) = 180 μC. Solve $PE = \frac{1}{2} Q^2/C$ for each ⊣⊢: $PE = \frac{1}{2}(180\mu)^2/20\mu$ = .81 mJ, and $\frac{1}{2}(180\mu)^2/30\mu$ = .54 mJ. Together, PE_{total} = .81 mJ + .54 mJ = 1.35 mJ.

For a parallel circuit, each ⊣⊢ has ΔV = 15 V.
$PE = \frac{1}{2} V^2C = \frac{1}{2} 15^2(20\mu)$ = 2.25 mJ,
$\frac{1}{2} 15^2(30\mu)$ = 3.375 mJ, and
PE_{total} = 2.25 mJ + 3.375 mJ = 5.625 mJ.

The parallel circuit stores more energy because each ⊣⊢ has the whole 15 V across it. {In series, the 15 V is split; "20μF" has 9V, and "30 μF" has 6 V.

11-50: PE- density = $PE_{electric}$ / Volume
$= \frac{1}{2}\ \Delta V^2\ C\ / d\,A$
$= \frac{1}{2} (Ed)^2 (\epsilon_o \kappa A/d) / d\,A$
$= \frac{1}{2} \epsilon_o \kappa E^2$

11-51: The series circuit has larger R_{total}, but the parallel circuit has larger I_{total} & P_{total}.

The parallel circuit has larger C_{total}, Q_{total} & PE_{total}.

Section 11.7 shows why, if "all other things are equal", I and Q are larger for parallel circuits.

For the same reasons, power and $PE_{electric}$ are also larger for parallel circuits. As discussed in Problems 11-43 (for resistors) and 11-48 (for capacitors), in parallel the entire ΔV is across each circuit element, but in series the ΔV must be split across the circuit elements. And if ΔV is smaller, so is the I-and-P (or Q-and-PE) "action" that is produced by ΔV.

11-52: In series, R_{total} is larger than R_3 (largest R). In parallel, R_{total} is smaller than R_1 (the smallest R).

In series, C_{total} is smaller than C_1 (the smallest C), and in parallel, C_{total} is larger than C_3 (the largest C).

11-53: 3 circuits have one -\/\/\-, 6 circuits have two -\/\/\-'s, and 8 circuits have three -\/\/\-'s:

The smallest R_{total} (5.45 Ω) is 3 resistors in parallel. The largest R_{total} (60 Ω) is for all 3 resistors in series.

Three capacitors can make the same 17 circuits.
Smallest C_{total} (5.45 nF) is 3 capacitors in series, and largest C_{total} (60 nF) is 3 capacitors in parallel.

11-54: Search your memory (or the Section 11.8's diagram) to find useful equations, solve for what you can, use the results (substitute), solve, substitute,....

$$E = F/q = (1.28 \times 10^{-16})/(1.6 \times 10^{-19}) = 800 \text{ N/C},$$
$$\Delta V = E\,d = 800(.01) = 8 \text{ V},$$
$$C = \kappa \epsilon_o A/d = (4)(8.85p)(.20^2)/.01 = 141.6 \text{ pF},$$
$$Q = \Delta V\,C = 8(141.6p) = 1.13 \times 10^{-9} = 1.13 \text{ nF},$$
$$PE = \tfrac{1}{2} CV^2 = \tfrac{1}{2}(1.13n)\,8^2 = 3.6 \times 10^{-8} = 36 \text{ nJ}.$$

11-55: $\Delta V = W/q = 4.0/(2 \times 10^{-3}) = 2000$ V,
$$d = \Delta V/E = 2000/(8.0 \times 10^4) = .025 \text{ m},$$
$$C = \kappa \epsilon_o A/d = (1)(8.85p)(150/10^4)/.025 = 5.3 \text{ pF},$$
$$Q = \Delta V\,C = 2000(5.3p) = 1.06 \times 10^{-8} = 10.6 \text{ nF}.$$

11-56: There isn't enough information to find C or ΔV, but you can find E by combining three equations:

$$Q = \Delta V \quad C$$
$$\Downarrow \quad \Downarrow$$
$$Q = E\,d\,(\kappa \epsilon_o A/d)$$
$$Q/(\kappa \epsilon_o A) = E$$
$$(1n)/[(1)(8.85p)(\pi\,.02^2)] = E$$
$$9.0 \times 10^4 \text{ V/m} = E$$

Alternate solution: If you use "$E = \sigma/\kappa\epsilon_o$", find $\sigma = Q/A = 1n/(\pi\,.02^2) = 7.96 \times 10^{-7}$, and then $E = \sigma/\kappa = (7.96 \times 10^{-7})/(1)(8.85p) = 9.0 \times 10^4$ V/m.

11-57: $E = \sigma/\kappa\epsilon_o = (Q/A)/\kappa\epsilon_o = (2n/.04)/[1(8.85p)] = 5650$ V/m. Energy density $= \frac{1}{2} \epsilon_o E^2 = \frac{1}{2}(8.85p)\, 5650^2 = 1.4 \times 10^{-4}$ J/m^2.

11-58: If any three "$I_{in} = I_{out}$" equations are added, they produce the fourth equation. For example,

top-left junction: $I_1 = I_2 + I_3$
top-right junction: $0 = I_1 + I_4 + I_5$
lower-left junction: $I_2 + I_3 + I_6 = 0$
sum of 3 equations: $I_6 = I_4 + I_5$
lower-right equation: $I_4 + I_5 = I_6$

The number of useful junction-equations is always one less than the number of junctions because the final equation doesn't give any "new information" that isn't already contained in the previous equations.

Similarly, combining three of the loop-equations (either adding or subtracting, as needed to give useful cancellation) produces the fourth equation.

left loop: $-V_2 - I_3 R_3 = 0$
center loop: $+I_3 R_3 + I_1 R_1 - I_4 R_4 - I_6 R_6 = 0$
right loop: $+I_4 R_4 + V_5 = 0$
sum of these: $-V_2 + I_1 R_1 + V_5 - I_6 R_6 = 0$
big loop: $-V_2 + I_1 R_1 + V_5 - I_6 R_6 = 0$

This is why the number of useful loop-equations equals the number of small loops. There are 6 loops (including the big & medium-big loops) and 20 ways to "choose 3 loops". All of these ways are acceptable except 4. Two of the "bad choices" are shown below; notice that they omit one circuit element (labeled "x").

Your loop-equations will contain all information that is available if you write as many equations as there are small loops and each circuit element appears in at least one equation. {If you write an equation for each small loop, both of these conditions are satisfied.}

Are there enough equations? Yes. Four junctions (3 eqns) and three small loops (3 eqns) give 6 "useful equations", enough to find the 6 unknown currents.

11-59: **A:** There are 4 junctions (3 j-equations) and 3 small loops (3 l-equations).
B: 3 j's (2 j-eqns) and 3 loops (3 l-eqns).
C: 6 j (5 eqns) and 4 l (3 eqns).
D: 2 j (1 eqn), 4 l (4 eqns).
E: This circuit is the same as D, with 2 junctions (not 6), 1 j-eqn, 4 l-eqns.
F: There are 3 junctions (not 4), 2 j-eqns, 3 l-eqns.

If a circuit section (between apparent junctions) has no circuit elements, it is really just part of the junction. Junctions are shown by • and :

11-60: Series-parallel can be used for A, B, F, G.
A & B are the same circuit, although B shows the series and parallel sections more clearly. Similarly, C, D & G are the same, though they look different. The drawings below show two parallel branches with an "in-between resistor" (represented by - - - -) that prevents the circuit from being separated into "pure series" and "pure parallel" sections: ===
C, D & G are the same, though they look quite different. These drawings show the two parallel branches with an "in-between resistor" (represented by - - - -) that prevents the circuit from being separated into "pure series" & "pure parallel" sections:

As discussed in Problem 11-26, F (series batteries) and G (equal-V parallel batteries) can be easily analyzed, but H (unequal-V parallel batteries) and I (batteries not in series or in parallel) require Kirchoff's Rules and internal resistance.

11-61: **Kirchoff's:** Label I's, guess I-directions (this is easy). These loop-equations are for counter-clockwise walks around the small loops:

junction: $I_1 = I_2 + I_3$
left loop: $+5 - 20 I_2 - 8 I_1 = 0$
right loop: $-30 I_3 + 20 I_2 = 0$

Solve #1 (for $I_1 = I_2 + I_3$), #2 ($.179 - .286 I_3 = I_2$), #3 ($.100 = I_3$), #2 ($.150 = I_2$), and #1 ($I_1 = .250$).

Series-parallel: a) Find $R_{total} = 20\Omega$. b) Solve V=IR for $I_{total} = .25$ C/s. c) Use the ratio logic of Problem 11-## to split I_{total} into $I_{20\Omega} = (30/50).25 = .15$ A, $I_{30\Omega} = (20/50).25 = .10$ A. d) Or find that the ΔV of "8Ω" is .25(8) = 2.00, use "$\Delta V = V_{hi} - V_{lo}$" and V-tracing to find that "20Ω+30Ω" has $\Delta V = 3.00$, solve V=IR for $I_{20\Omega} = \Delta V/R = 3/20 = .15$ C/s, $I_{30\Omega} = 3/30 = .10$ C/s, or find $I_{30\Omega} = .25 - .15 = .10$ C/s.
{I prefer Step c's "ratio logic".}

11-62: [will be done later, if this problem is retained]

11-63:

11-64: