8.91 Problems

for Section 8.1,

• **8-1**: Why does KE increase more in the first quarter-cycle (it goes from 0 J to 3 J) than in the second quarter-cycle (3 J to 4 J).

• **8-2**: How many ways can you think of to change the amplitude, spring constant or mass of a spring-block system so it will double the total vibration energy? What changes will double its maximum speed?

8-3: Compare TE for these systems. Both are at the far-right turning point, and have identical springs.

Which block is faster at the center point? Will each block be the same distance from the wall at the far-left turning point?

How could you change the SHM energy?

8-4: At what position (expressed as a fraction of x_{max}) is the force acting on a block equal to 49% of its maximum value?

At what positions are PE, KE and v equal to 49% of their maximum value?

• **8-5**: If these blocks stick together, what is their maximum speed, and what is the closest they come to the wall? { The spring has x_e = .5 m. Without this information, could you be certain that x_e = .5 m? }

for Section 8.2 (if your class studies 8.2)

8-6: Which object returns to its starting point first? For the circle: diameter = 4 m, v = 6 m/s. For the SHM: A = 2 m, k = 45 N/m, m = 5 kg. If the circle's diameter and SHM's amplitude are doubled, who wins the race?

8-7: Derive the time-dependent formulas for **x** (or **y**), **v** and **a** by using this diagram, Section 1.3 trigonometry, $\theta = \omega t + ø$, $\omega = \sqrt{k/m}$, $v = \pm\sqrt{k/m}\,\sqrt{A^2 - x^2}$, and $a = -(k/m)x$. { If you are using y-format equations, replace x's with y's. }

Or you can derive the v and a-equations by drawing the **v** and **a** vectors at •, splitting them into components, and substituting "v = rω" and "$a_c = r\omega^2$" from Chapter 5.

If you know calculus, you can begin with "x = Acos(ωt + ø)" or "y = Asin(ωt + ø)" and derive the v and a-equations.

8-8: The solid-line graphs below show graphs of x-versus-t (for x-format equations) and y-versus-t (for y-format equations). For the dashed-line graph, does ø have a ± sign that is positive or negative? { Hint: When does the SHM object reach the θ ≡ 0 position? }

For SHM motion with ø = 0, draw graphs of v-versus-t and a-versus-t.

for Section 8.3,

8-9: **a)** A cycle of SHM takes .5 s, and speed at the center is .4 m/s. Find the amplitude.

b) What is the average speed for this cycle? Derive an equation to show the relationship of $v_{average}$ and v_{max}: $v_{average} = x\ v_{max}$, where "x" is a number.

c) A SHM system has 50 J of energy, 5 m/s maximum speed, and 2 cycles/s frequency. What is the spring constant?

8-10: **a)** A 4 N/m spring makes a 200 gram block move with simple harmonic motion. Where is the block, and what is its acceler-ation, when the force on it is .8 N toward the wall where the spring is attached? Is the spring stretched or compressed?

b) With the information given, how many of these quantities can you find? PE, KE, TE, v, v_{max}, ω, T What information would you need to find the others?

c) If you know that A = .40, what is the value for all quantities in Part b?

8-11: A SHM system takes .500 s to finish a full cycle, has an amplitude of 40.0 cm, and 50.0 J of energy. What is the spring constant and the block's mass? When the block is at −.300 m, what is its acceleration, and what force acts on it?

8-12: A certain SHM system has A = .8 m. It takes .60 s for the block to move a distance of .4 m, from far-right turning point at +.8 m to +.4 m, but only .30 s for it to move the same distance, from +.40 m to the center point at 0. Explain why, without using equations.

Section 8.3 optionals: if your class uses time-dependent equations. Choose either the x-format equations (in the left column) or the y-format equations (in the right column).

8-13 optional: find equations for v & a, if

$x = 4\cos(7t - 2)$ $\qquad$ $y = 4\sin(7t - 2)$

8-14 optional: Find A, ω, ø and v_{max}, if

$v = -4\sin(7t - 2)$ $\qquad$ $v = 4\cos(7t - 2)$

8-15 optional: At what value of x (or y) is SHM speed 49% of its maximum value?

8-16 optional: **a)** At $t \equiv 0$ a 15 kg SHM object is at –.4 m [compressing a 375 N/m spring] and has v = 0. Write the v-equation. Define $\theta \equiv 0$ in the standard way, as in Section 8.2.

b) If $t \equiv 0$ at the collision-time in Problem 8-5, write the a-equation. { Hint: Use information from Solution 8-5. }

c) At $t \equiv 0$ a SHM object (T = .4π , A = .4) is 320° past $\theta \equiv 0$. Write the x (or y) equation.

8-17 optional: A SHM object (f = 2.5/π, A = 2) is at +1.5, moving away from the center, when t = 4.00 s. Find its v-equation.

for Section 8.4,

8-18: The block is held (in #3) and released (in #4). What is its total oscillation energy, maximum velocity, maximum acceleration, and how much time elapses before it returns to where it is in #3?

To get the same SHM energy, you hold the block __ from the ceiling and let it drop.

8-19: When a block hangs motionless on it, a spring stretches from 50 cm to 90 cm. When the block is set into motion, what is its oscillation frequency and period?

8-20: Platform and block are not "glued together". For what frequency range will the block and platform always stay in contact with each other?

8-21: With a mass of M, a vertical spring-block system has T = .5236 s. After 2.00 kg is added to M, the system's T is .3927 s. Find M and the spring constant.

8-22: Show that a vertical spring-block system oscillates in SHM about the position where kx = mg.

Optional: Use analogy to show that a floating block (Section 6.4) will oscillate in vertical SHM about the position where $F_{buoyant}$ = mg. When a 50 kg swimmer climbs onto a 400 kg floating raft, it sinks 2.0 cm deeper into the water. When she jumps off, what is it the raft's oscillation frequency?

8-23: If your spaceship lands on a strange planet, describe two ways you could measure the free-fall acceleration. What equipment would you need for each method?

8-24: To determine which of two objects has more mass, would you use a pendulum or vertical spring? Which system could give you information about the change in "g" if you move from New York to Denver?

8-25: On earth, what pendulum length is needed to get a period of 2.00 seconds? How long does it take this pendulum to make the motion shown here: i f ?

To make a 2.00 s pendulum on the moon, would you have to use a shorter or longer string? What is the "multiplying factor"?

On the moon, where g_{moon} = 1.67 m/s^2, what is the period of this earth-pendulum?

On another planet, the earth-pendulum's period is 5.00 s. What is an object's mass if it weighs 98.0 N on this planet? What would its weight & mass be on the earth?

8-26: If a pendulum clock moves from New York (g = 9.803 m/s^2), does it run slow or fast in Denver (g = 9.796 m/s^2)? By how much in one day?

8-27: In these pictures, the same spring and block are used on the earth and a planet:

Use ratio logic to answer this question:
If a pendulum has a period of 2.00 seconds on earth, what is its period on this planet?

8-28: For a pendulum, SHM would occur if the restoring force was –mgθ. The actual restoring force is –mg sinθ. What is the difference between θ (in radians) and sinθ for θ = 1°, 5°, 15° and 30°? Section 8.4 states that $T \approx 2\pi / \sqrt{g/L}$. For large amplitudes (like 30°), will the actual oscillation period be smaller or larger than $2\pi / \sqrt{g/L}$? { Hint: if a spring is strong, does it give a small T or large T ? }

8-29 (optional): Damped & Driven SHM

A SHM system has k = 400, m = 4, and a "damping constant" of b = [[== probably cut this?]]

8.92 Solutions

8-1: F Δx = ΔKE. Δx is equal during both quarter cycles (from 1.0A to .5A, or .5A to 0A), but F is larger in the first quarter cycle when it varies from k(1.0A) to k(.5A) than in the second quarter cycle when it changes from k(.5A) to k(0A), so FΔx and ΔKE are larger in the first quarter cycle.

8-2: TE = ½ kA^2 so TE doubles if you increase k by a multiplying factor of x2, or increase A by x1.414. Changing m does not affect a system's TE.

The "or" in the question implies that one variable changes while the other two remain constant. But if you change two variables at a time there are an infinite number of ways to double TE: multiply k by x1.4 and A by x1.195 (try it), or k by x.5 and A by x2, or...

$\frac{1}{2} kA^2 = \frac{1}{2} mv_{max}^2$, so you can double the block's v_{max} by multiplying k by x4 (when you take the $\sqrt{\ }$ to solve for v_{max}, the factor of x4 decreases to x2), or A by x2, or m by x.7071.

Think about "intuitive ratio logic". Do you see why each change [stronger spring, bigger range of motion, or smaller mass] will make v_{max} larger?

8-3: The identical springs have equal k's and (since they have the same x_e and have been "stretched" to the same length at the far-right turning point) equal A's, so they have equal TE ($= \frac{1}{2} kA^2$).

The less massive 2 kg block has a faster v_{max}.

At the far-left turning point, both blocks are the same distance from the wall. {Symmetry: the center-to-turning distance is the same whether a spring is being stretched or compressed.}

To increase TE for either SHM system, stretch the spring further away from x_e (or compress it further from x_e) before you release it.

8-4: Make an equation and solve it.

F	$= .49\, F_{max}$	PE	$= .49\, PE_{max}$
$-kx$	$= .49\, (-k\, x_{max})$	$\frac{1}{2} k x^2$	$= .49\, (\frac{1}{2} k\, x_{max}^2)$
x	$= .49\, (x_{max})$	x^2	$= .49\, (A^2)$
x	$= .49\ A$	x	$= .70\ A$

F is proportional to x, but PE is proportional to x^2.

There is no "x" in "$KE = \frac{1}{2} mv^2$" but we can use the principle of energy conservation: if KE is 49% of its maximum value, PE is 51% of its maximum. This occurs when $x = \sqrt{.51}\ A = .714\ A$.

We can use similar logic for v. If $v = .49\, v_{max}$, $KE = \frac{1}{2} m(.49\, v_{max})^2 = \frac{1}{2} m(.24) v_{max}^2 = .24\, KE_{max}$, $PE = .76\, PE_{max}$, and $x = \sqrt{.76}\ A = .87\ A$.

8-5: If "$x_e = .5$ m" is not given, the picture could show a static situation with $x_e = .5$ m, or a snapshot taken at an instant when the block has v=0 because it is at either turning point of a SHM cycle.

Reasonable assumptions: the floor is horizontal, and 375 N/m is (notice the units) the spring's k.

External force = 0, so momentum is conserved and we can find "v = 2 m/s immediately after collision". As the blocks move → they compress the spring and, as it pushes ← against them, they slow down; 2 m/s is maximum speed, at the center point. Now we can solve a "TE = TE" equation for A.

$(mv)_i$	$= (mv)_f$	$\frac{1}{2}\, k A^2$	$= \frac{1}{2}\, m\, v_{max}^2$
$5(6) + 10(0)$	$= (15)v$	$(375)A^2$	$= (15)\,(2)^2$
2 m/s	$= v$	A	$= .4$ meter

The 10 kg block moves .4 m leftward, from its initial center position at x_e to the turning point, so it stops .1 m short of the wall.

8-6: The circle-object has $\omega = v/r = 6/2 = 3$ rads/s, the SHM block has $\omega = \sqrt{k/m} = \sqrt{45/5} = 3$ rads/s. Both objects return at the same time.

Or you can calculate the times for a complete cycle. For the circle, $T = \Delta x/v = 2\pi(2)/6 = 2/3\ \pi$ seconds. For the block, $T = 1/f = 1/(\omega/2\pi) = 2\pi/\omega = 2\pi/3$ s.

If r is halved, • returns in half the time: $T = 1/3\ \pi$. For SHM, T doesn't depend on A, so T is still $2/3\ \pi$.

If •'s speed is cut in half so it is 2.5 m/s, $v = r\omega$ is true again, •'s x-motion imitates the block's SHM, and they both return in $1/3\ \pi$ seconds.

8-7: For x-format: $x = A\cos\theta = A\cos(\omega t + ø)$, $\sqrt{A^2 - x^2} = A\cos\theta = A\sin(\omega t + ø)$. For y-format, $y = A\sin\theta = A\sin(\omega t + ø)$, $\sqrt{A^2 - x^2} = A\cos\theta = A\cos(\omega t + ø)$. Beginning with the v and a-equations, substitute for x, $\sqrt{A^2 - x^2}$ and k/m. {As shown later in this solution, the ± sign of v_x is –.}

for x-format equations,	for y-format equations,
$v = \pm\, \omega A \sin(\omega t + ø)$	$v = +\, \omega A \cos(\omega t + ø)$
$a = -\, \omega^2 A \cos(\omega t + ø)$	$a = -\, \omega^2 A \sin(\omega t + ø)$

On the diagram below, **v** points tangentially, has magnitude $v = r\omega = A\omega$. **a** points toward the circle's center and has a magnitude of $a_c = v^2/r = r^2\omega = A^2\omega$. Look at the diagram and use tools from Sections 1.1 (Z & Y-90°) and 1.3 (sine & cosine) to find v_x & a_x (or v_y & a_y). Notice that 3 of the 4 components point in the –x or –y direction. Replace θ with $\omega t + ø$.

$v = -\, \omega A$	$v = +\, \omega A$
$v_x = -\, \omega A \sin(\omega t + ø)$	$v_y = +\, \omega A \cos(\omega t + ø)$
$a = -\, \omega^2 A$	$a = -\, \omega^2 A$
$a_x = -\, \omega^2 A \cos(\omega t + ø)$	$a_y = -\, \omega^2 A \sin(\omega t + ø)$

Optional. Derivatives give the same equations as above: v_x-equation = d(x-equation)/dt, v_y-equation = d(y-equation)/dt, a-equation = d(v-equation)/dt.

8-8: The SHM object that is represented by the dashed-line graph reaches θ≡0 at approximately .2 T, so at t≡0 its θ-value (and thus ø) was –.

{ Be careful. If the graphs were "racing" toward the right (they aren't) the dashed-line graph would be "ahead" and you might think its ø was +. Why is this a wrong interpretation? Because, as emphasized in Section 2.10, a graph is not a "photograph". }

Graphs for x-format: x-t is "positive cosine", v-t is "negative sine", a-t is "negative cosine".

Graphs for y-format: x-t is "positive sine", v-t is "positive cosine", a-t is "negative sine".

For either format, the maximum values of x (or y), v and a are $\pm A$, $\pm\omega A$ and $\pm\omega^2 A$, respectively.

8-9: **a)** We know T (which automatically gives us f, ω and k/m) and v_{max} (v at the center). Look for an equation that contains A and is solve-able: $v_{max} = \omega A$, $A = v_{max}/\omega = v_{max}/(2\pi/T) = (.4)/(2\pi/.5) = .032$ m.

b) $v_{average} = \Delta x/\Delta t = 4A/T = 4(.032)/.5 = .26$ m/s.
$v_{max} = \boldsymbol{\omega A}$, and $v_{average} = 4A/T = 4A/(2\pi/\omega) = (4/2\pi)\boldsymbol{\omega A} = (4/2\pi)\, v_{max} = .637\, v_{max}$. We can use this for our cycle, to find $v_{average} = .637\, v_{max} = .637\,(.4) =0$ .255 m/s, almost the same as the answer above. Which answer do you think is more accurate?

c) We know TE, v_{max} & f (and thus ω, T). Some potentially useful equations with k and the "knowns" are $TE = \frac{1}{2}\, kA^2$ and $\omega^2 = k/m$. They cannot be solved immediately because we don't A or m. Two equally good sub-goals are to find **A** (using $v_{max} = A\omega$) or **m** (using $TE = \frac{1}{2}\, mv_{max}^2$). Then find k by substituting **A** ($k = 2\,TE/A^2 = 2(50)/(.3978)^2 = 631.9$ N/m), or **m** ($k = \omega^2 m = (12.57)^2 (4) = 632.0$ N/m). {Rounding off to one "significant figure" gives k = 600 N/m.}

8-10: If (as is usual) we define the +x direction to be "away from the wall", F is –.8 N.

$$F = -k\,x \qquad\qquad F = m\,a$$
$$(-.8) = -(4)\,x \qquad\qquad (-.8) = (.200)\,a$$
$$+.20 \text{ m} = x \qquad\qquad 4 \text{ m/s}^2 = a$$

The block is at +.20. The spring is stretched .20 m beyond its equilibrium length.

b) You can find PE ($= \frac{1}{2}\, kx^2$), ω ($= \sqrt{k/m}$), and T ($= 2\pi/\omega$). You cannot find KE (need v in $\frac{1}{2}\, mv^2$), TE (need A in $\frac{1}{2}\, k\underline{A^2}$, or v_{max} in $\frac{1}{2}\, mv_{max}^2$), v (need A in $\pm\omega\sqrt{A^2 - x^2}$) or v_{max} (need A in $\pm\omega A$).

c) Substitute & solve: PE = .08 J, $\omega = 4.47$ rads/s, T = 1.41 s, TE = .32 J, v = 1.55 m/s, $v_{max} = 1.79$ m/s, KE = .24 J. Checks: does $\frac{1}{2}\, kA^2 = \frac{1}{2}\, mv_{max}^2$, and does PE + KE = TE ?

8-11: Look at the summary and find equations that contain what you're asked to find (k, m, a, F) and some things you know (T, A, TE, x). Write equations on a piece of paper, substitute the knowns, solve equations if you can, use "links" and see what happens. {After you've solved for the 4 unknowns, check the rest of the solution, after 8-##.}

8-12: During its first quarter-cycle the block moves slowly, starting from rest, so it takes a relatively long time (.60 s) to travel the first .4 m. But it moves faster in the second quarter-cycle, so it takes less time (only .30 s) to travel the second .4 m.

Compare this result with Problem 8-1: when F-and-a are large, v is small, and vice versa.

Optional: Use this if you're studying Section 8.2. In Section 8.1 we analyzed a SHM cycle that was split into equal-distance intervals. This diagram splits half of a SHM cycle into equal-time intervals:

The bottom row shows that • reaches quarter-cycle locations at 0°, 60°, 90°, 120° and 180°. Do you see that equal time doesn't always mean equal distance?

8-13: By comparing the top row general-equation (with letters) and the specific-equation (with numbers) you can find the values of A, ω and ø.

$$x = A\cos(\omega t + ø) \qquad\qquad y = A\sin(\omega t + ø)$$
$$x = 4\cos(7t - 2) \qquad\qquad y = 4\sin(7t - 2)$$

Now substitute $A = 4$, $\omega = 7$, $ø = -2$ into the general-equations for v and a:

$$v = -\omega A\sin(\omega t + ø) \qquad\qquad v = \omega A\cos(\omega t + ø)$$
$$v = -7(4)\sin(7t + [-2]) \qquad\qquad v = 7(4)\cos(7t + [-2])$$

$$a = -\omega^2 A\cos(\omega t + ø) \qquad\qquad a = -\omega^2 A\sin(\omega t + ø)$$
$$a = -7^2(4)\cos(7t - 2) \qquad\qquad a = -7^2(4)\sin(7t - 2)$$

8-14: As in 8-13, $\omega = 7$ rads/s, $ø = -2$ rads. But $\omega A = 4$, $7A = 4$, $A = .57$. $v_{max} = \omega A = 4$ m/s, or substitute "1" (which is the maximum value of $\sin\theta$ or $\cos\theta$) into the v-equation and solve for v = 4 m/s.

(8-11): Here is a "flowchart" for the solution,

$$\underline{TE} = \tfrac{1}{2}\,\mathbf{k}\,\underline{A}^2 \qquad\qquad \boldsymbol{\omega} = 2\pi/\underline{T}$$

$$\mathbf{F} = -k\,\underline{x} \qquad\qquad k/\mathbf{m} = \omega^2 \qquad\qquad \mathbf{a} = -\omega^2\,\underline{x}$$

$$F = m\,a \text{ (optional, as a "check")}$$

Do you recognize the significance of underlining, bold-face type, and arrows?

You don't have to know your entire "plan" before you begin. Sometimes, as discussed in Section 20.2, it is better to go-and-improvise. Do something. If it works, great! If it doesn't, try another strategy.

$$k = 2\,TE/A^2 = 2(50)/(.4)^2 = 625 \text{ N/m}$$
$$\omega = 2\pi/T = 2\pi/(.5) = 12.57 \text{ rads/s}$$
$$F = -k\,x = -(625)(-.3) = +187.5 \text{ N}$$
$$m = k/\omega^2 = (625)/(12.57)^2 = 3.96 \text{ kg}$$
$$a = -\omega^2 x = -(12.57)^2(-.3) = +47.4 \text{ m/s}^2$$

F = ma "check": Does (+187.5) = (3.96)(+47.4) ?

8-15: Substitute "$v = .49\ v_{max}$", "$\omega A = v_{max}$" and "$\omega t + ø = \theta$" into the v-equation, solve for θ, then find the value of x (or y) at this θ where $v = .49\ v_{max}$:

$$v = -\omega A \sin(\omega t + ø) \qquad v = \omega A \cos(\omega t + ø)$$
$$.49\ v_{max} = -v_{max} \sin\theta \qquad .49\ v_{max} = v_{max} \cos\theta$$
$$-.49 = \sin\theta \qquad .49 = \cos\theta$$
$$-.512 \text{ rads} = \theta \qquad 1.059 \text{ rads} = \theta$$

$$x = A\cos\theta \qquad y = A\sin\theta$$
$$x = A\cos(-.512) \qquad y = A\sin(1.059)$$
$$x = .87\ A \qquad y = .87\ A$$

This is, of course, the same answer we found in Problem 8-4.

8-16: To find ø, draw the object's t≡0 position.

$$v = -5(.4)\sin(5t + 3.14) \qquad v = +5(.4)\cos(5t + 3\pi/2)$$
$$a = -5^2(.4)\cos(5t + \pi/2) \qquad a = -5^2(.4)\sin(5t + 3.14)$$
$$x = .4\cos(5t + 5\pi/3) \qquad y = .4\sin(5t + 5\pi/3)$$
$$x = .4\cos(5t + 5.236) \qquad y = .4\sin(5t + 5.236)$$
$$x = .4\cos(5t - 1.047) \qquad y = .4\sin(5t - 1.047)$$

8-17: The object is at +1.5 twice in each cycle; at "o" it moves toward the center (we don't want this) but at "•" it moves away from the center. After solving for θ, decide whether θ must be "adjusted". If you're using x-format: +.723 radians is θ for the o-position, but you want the •-position at –.723 rads. With y-format, +.848 radians is θ for the o-position, but you want the •-position that is .848 rads before π, at +2.294 rads {or you can think of • and o as being symmetric about the 90°/1.571 rad point; o is at .848, .723 rads before 1.571, so • is .723 rads after 1.571, at 2.294 radians}.

$$1.5 = 2\cos(5[4] + ø) \qquad 1.5 = 2\sin(5[4] + ø)$$
$$.75 = \cos(20 + ø) \qquad .75 = \sin(20 + ø)$$
$$+.723 = 20 + ø \qquad +.848 = 20 + ø$$

But for the •-object, $-.723 = 20 + ø$, $-20.723 = ø$ | But for the •-object, $+2.294 = 20 + ø$, $-17.706 = ø$

θ and ø : At 4.00s the x-object is at +.723 rads, at 0.00s it was at –20.723 rads. At 4.00s the y-object is at +.848 rads, at 0.00s it was at –17.706 rads.

The SHM cycle repeats every 2π rads; we can add or subtract 2π from ø without affecting it, because cos(ø) = cos(ø + 2π) = cos(ø – 2π), and sin(ø) = sin(ø + 2π) = sin(ø – 2π). To get ø closer to 0, we add 2π (6.283) several times. For x-format, ø can be –20.723, –14.44 (after adding 6.283 once), –8.157, –1.874, or +4.409. For y-format, ø can be –17.706, –11.423, –5.140 (after adding 6.283 twice), or +1.143. We can use any of these ø's in a v-equation:

$$v = -10\sin(5t - 20.723) \qquad v = -10\cos(5t - 17.706)$$
$$v = -5(2)\sin(5t - 1.874) \qquad v = -5(2)\cos(5t - 5.140)$$
$$v = -10\sin(5t + 4.409) \qquad v = -10\cos(5t + 1.143)$$

8-18: Compare #1 & #2: kx = mg, k(.55 – .35) = (2)(9.8), k = 98 N/m. Comparing #2 and #3: the SHM amplitude is (.80 – .55) = .25 m.
$TE = \frac{1}{2} kA^2 = \frac{1}{2}(98)(.25)^2 = 3.06$ J.
$\omega = \sqrt{k/m} = \sqrt{98/2} = 7$, $T = 2\pi/\omega = 2\pi/7 = .898$ s.
$TE = \frac{1}{2} m v_{max}^2$, $3.06 = \frac{1}{2}(2)\, v_{max}^2$, $v_{max} = 1.75$ m/s.
$a_{max} = F_{max}/m = -kx_{max}/m = -(98)(.25)/(2) = 12$ m/s^2. v_{max} occurs at center, a_{max} at top & bottom.

If you drop the block from the top turning-point, 30 cm below the ceiling, it has the same TE (and T, v_{max}, a_{max}) as when you release it from 80 cm.

8-19: kx = mg, k = mg/x = m(9.8)/.40 = 24.5 m. $\omega = \sqrt{k/m} = \sqrt{24.5m/m}$ = 4.95 radians/second.
f = 2π/ω = 2π/4.95 = 1.27 cycles/second,
T = 1/f = 1/1.27 = .79 second/cycle.

8-20: If the platform is pulled downward with an acceleration of 9.8 m/s^2, it remains in contact with the "free fall" block, but N-force = 0. If the acceleration exceeds 9.8 m/s^2 the block cannot accelerate fast enough to "follow" and it temporarily loses contact. This occurs at the top turning-point, where x = +.30 (amplitude is half of the total motion range of .60 m): $-9.8 < a_{max} = -\omega^2 x_{max} = -(2\pi f)^2(+.30)$, and $f > .83$. If frequency is between 0 and .83 cycles/s, the block stays in contact with the platform.

8-21: Initially, ω = 2π/T = 2π/(.5236) = 12 rads/s. After the 2 kg is added, ω = 2π/(.3927) = 16 rads/s. We have two "$\omega^2 = k/m$" equations: $(12)^2 = k/M$, $(16)^2 = k/(M + 2)$. These equations can be solved, using standard "leapfrog substitution", to get M = 2.57 kg, and k = 658 N/m.

8-22: At the "new equilibrium position" where the spring stretches "x", kx = mg. If it stretches an extra distance y and its total stretch is "x + y", $F_{restoring} = +k(x + y) - mg = +kx + ky - mg = +\mathbf{ky}$ (because +kx – mg = 0). This force is proportional to y (distance from the "new x_e") and it points toward x_e: SHM!

$F_{buoyant} = +\rho_{fluid} V_{object}\, g\, (\mathbf{h_{subm}}/h_{total})$, where h_{subm} is distance below the surface. This is analogous to $F_{spring} = +k\mathbf{x}$, where x is distance below x_e. At the "floating position", $+F_B - mg = 0$. This is analogous to the "new x_e", ≠kx – mg = 0.

The fluid that supports the raft produces a "k" of ΔF/Δx = (50)(9.8)/.02 = 24500 N/m. $f = \omega/2\pi = \sqrt{k/m}/2\pi = \sqrt{24500/400}/2\pi = 1.25$ oscillations/s.

8-23: a) As explained in Problems 8-## to 8-##, you can find g with a pendulum if you have a string of known length (or string & ruler) and a stopwatch. b) With these items you can also measure the time for free-fall from a known height, then calculate g using the "tvvax method" of Sections 2.4-2.5.

8-24: The T of vertical-spring SHM depends on m, but not on g. Pendulum T depends on g, but not m.

8-25: If $T = 2\pi/\sqrt{g/L}$ is squared & rearranged it becomes $T^2 g = 4\pi^2 L$, which is easier to use for "substitute-and-solve". For example, to get a 2.00 s pendulum on earth, $L = T^2 g/4\pi^2 = 2^2 (9.8) / 4\pi^2 =$ .993 meters. The i-to-f motion shown is half of a full T-cycle, so it is completed in $\frac{1}{2}$ (2.00s) = 1.00 s.

To get a 2.00 s moon-pendulum, L must decrease (by a factor of 1.67/9.80) because g decreases.

To answer the last two questions, use the length you calculated for the 2.00s earth-pendulum:

$T^2 g_{moon}$	$= 4\pi^2 \; L$	$T^2 \; g$	$= 4\pi^2 \; L$
$T^2 (1.67)$	$= 4\pi^2 (.993)$	$(5)^2 g$	$= 4\pi^2 (.993)$
T	$= 4.84$ s	g_{planet}	$= 1.57$ m/s^2

The object's mass is $m = w / g = 98 / 1.57 = 62$ kg.
On the earth, m = 62 kg, w = mg = 62(9.8) = 608 N.

8-26: It runs slow by a factor of $\sqrt{9.796/9.803} =$ .99964. In one day it loses (1 – .99964)(24)(3600 s) = 31 seconds.

8-27: The spring stretches 1.5 times as much on the planet (30 cm versus 20 cm) because it supports an Mg that is 1.5 times as large. Since the block's M doesn't change, the planet's "g" is 1.5 times larger: it is 1.5(9.8) = 14.7 m/s^2. We can also use ratio logic on $T^2 g = 4\pi^2 L$; when g increases by a factor of 1.5, T decreases by a factor of $1/\sqrt{1.5}$. The planet's pendulum-T is $(2.00 \text{ s})/\sqrt{1.5} = 1.63$ s.

8-28: (1°)(2π radians/360°) = .0174532 radians.
If θ = 1° = .0174532 rads, sinθ = .0174524, .005%
If θ = 5° = .0872664 rads, sinθ = .0871557, .13%
If θ = 15° = .26180 rads, sinθ = .25882, 1.14%
If θ = 30° = .52360 rads, sinθ = .50000, 4.51%

If θ = 30°, the restoring force is –mg sin(.52360) = –mg(.50000) instead of the –mgθ = –mg(.52360) that is needed for SHM. Because the real force is less than what is needed to produce SHM, the real time is greater than $2\pi/\sqrt{g/L}$.

8-29: ==cut this problem?