5.91 Problems

for Sections 5A & 5B

5-1: In this picture, a 3.0 kg block moves at 2.5 m/s in a circle with a constant radius of 80 cm. What mass "m" is attached to the other end of the string? { Assume there is no friction when the string passes through the hole in the table. }

5-2: This bird's eye view shows a race car moving around a circular track. What is the car's initial velocity and subsequent motion if it hits wet ice (and friction suddenly drops to approximately zero) at a? at b? at c?

5-3: **a Ferris Wheel**

To find the diameter of a ferris wheel, Susan sits on a scale that measures her weight at the bottom (650 N) and top (390 N) of the wheel. She uses her watch to find that the wheel makes one revolution every 15 seconds. What is the diameter?

5-4: If μ_s between tires and road is .60, how fast can a car drive around a curve with curvature-radius = 120 m?

If μ_s between a coin and a 78 rpm phonograph record is .40, how far from the center can the coin rest without sliding?

5-5: Compare these curves & speeds, and decide which one requires the most (and least) friction-force to keep the car on the road. First use "intuitive common sense", then show how these conclusions are supported by physics ratio logic.

5-6: You are driving east → at 15 m/s when you see a north-south wall, | , straight ahead. To avoid hitting the wall, is it better to use the car's brakes or turn its steering wheel? { $\mu_{tire/road}$ = .60 }

5-7: Find the centripetal and tangential forces when an object moving in a vertical circle is at the bottom, side (90° past bottom), top (180° past bottom) and "240° past bottom". Write your answers in terms of m, g, N, and [if necessary] θ. At each point, decide if speed is increasing, decreasing or neither.

5-8: **centrifugal force — is it real?**

Problem 3-## gives several examples of a "pseudo force" that seems (from the point of view of a truck rider) to make a ball deviate from its expected free-fall motion. Similar forces seem to operate when you ride in a car and feel yourself pulled toward the outside of a curve, or if an elevator accelerates upward and you are pushed upward with greater-than-usual force. Are these forces real, illusion, or does youranswer depend on "interpretation"?

5-9: **artificial gravity**

If a space station shaped like a donut with 200 m diameter rotates at 3 rpm, what is the simulated "g" acting on the head and toes of a person who is 1.8 m tall? Can you think of any difficulties this situation might cause?

5-10: Can Tarzan cross a gorge by swinging on a 5.0 m vine that initially makes an angle of 50° with vertical, if his arms can exert a force of 1800 N on the vine and his mass is 80 kg?

5-11: This *conical pendulum* swings in a horizontal circle (shown by) and makes a revolution in time "t". Write useful F=ma equations and solve for t in terms of L and ø.

5-12: **banked curves** (optional)

With the proper *banking angle* (the road's slant toward the inside of the curve) cars can drive around a curve without using friction. For a 60 mi/hr (26.8 m/s) car on a curve with 120 m radius, what is this banking-angle?

Draw a force-diagram and write F=ma equations for a car that is 1) driving this curve too fast, 2) driving too slow.

A block that rests on the side of a rotating funnel moves with speed "v". The side and bird's-eye views are shown below. Write useful F=ma equations for the block if the friction force is zero.

for Section 5C

TABLE of useful information:

$M_{moon} = 7.36 \times 10^{22}$ kg, $r_{moon} = 1.74 \times 10^6$ m
$M_{earth} = 5.98 \times 10^{24}$ kg, $r_{earth} = 6.37 \times 10^6$ m
$M_{sun} = 1.99 \times 10^{30}$ kg, $r_{sun} = 6.96 \times 10^8$ m
moon's orbit: $r = 3.84 \times 10^8$ m, T = 27.32 days
earth's orbit: $r = 1.50 \times 10^{11}$ m, T = 365.26 days

5-13: "Low" and "high" satellites orbit the earth at radii of R & 3R, respectively. Which satellite moves faster? How much faster?

A satellite is a distance r from the earth's center, moving with speed v parallel to the earth's surface (it moves in the tangential direction). Sketch the satellite's motion if v is: 1) $\sqrt{GM_{earth}/r}$, 2) faster than $\sqrt{GM_e/r}$, 3) slower than $\sqrt{GM_e/r}$, 4) zero.

5-14: If you could view the earth from above the North Pole, would you see it rotate clockwise or counterclockwise?

5-15: At what point do gravitational forces due to the earth and moon cancel? { Use the table above.}
At this location, what gravity force do the earth, moon and sun exert on a 1 kg object? What is the net $F_{gravity}$?

Does the weight of a 10 kg block change when it travels from the earth to moon? Does its mass change? If you kick this block when it is "almost weightless", could you hurt your foot?

5-16: If a planet is further from the sun than earth, is its orbiting time shorter or longer than 1 year? On Mars, a "year" (the time to complete an orbit of the sun) lasts 687 days. How far is Mars from the sun? { Use the info-table above.}

5-17: From F=ma, derive $(GM/4\pi^2)T^2 = r^3$.

Optional: Derive $GM = r^3\,\omega^3$, using formulas from Section 5.4.

5-18: Compare the $F_{gravity}$ formulas: GMm/r^2 and mg. What is "g" equal to? Now substitute numbers into GM/r^2 (use the table above). Why don't you get the "official g-value" of 9.807 m/s^2?

At the equator, what weight is measured by the scale below? How can you explain the discrepancies you've discovered? { ==bonus: if weightless, what is T_{rotn}? }

5-19: moon (given ---, find ---) ==[unfinished details on a white page]

5-20: If a comet orbits the sun elliptically, as shown below, is it moving faster at • or x? { Hint: Use "energy conservation" or "work". }

5-21: Will it take more fuel for a rocket trip from earth-to-moon, or moon-to-earth?

5-##: == GMm/r, escape v, ± signs ??

for Section 5D

5-22: If the big wheel spins at 1.8 rads/s, what is the size & direction of ω for the little wheel in the first picture? second picture?

5-23: Which types of acceleration are zero and non-zero if a a car 1) drives in a circle at constnt speed, 2) drives a circle with a speed that steadily decreases. In each situation, what stays constant (velocity? acceleration? force?) and what changes?

5-##: Use suggestions from Section 5.4 to derive $\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$ and $\omega_f^2 - \omega_i^2 = 2\alpha\Delta\theta$ from $\Delta x = v_i t + \frac{1}{2} at^2$ and $v_f^2 - v_i^2 = 2a\Delta x$.

5-24: A bicycle that has wheels with 27 inch (68.6 cm) diameter travels 3.00 km at 8.00 m/s. What is the angular velocity of its wheels [ω, f, rpm & T]? How many revolutions do they make?

If this bike stops in 3.00 s, what is its angular acceleration? What is the angular movement of its wheels during the braking period?

5-25: For the bicycle wheels of Problem 5-##, with bicycle moving →, what is the total velocity (rotation-v + sideways-v) for each of these points?

5-26: If you put oversize tires (with extra-large diameter) on your car and your speedometer reads "55 miles/hour", what is your actual speed?

5-27: If you sit in a chair at the equator, what is your velocity (relative to "fixed stars") and angular velocity due to the earth's rotation? the earth's orbit? { Use information from the table in Section 5.3. }

Do these answers change if your chair is in Los Angeles, at 34° latitude?

5-28: A bullet is fired through these discs, both rotating at 1200 rpm, 1.4 m apart, and there is a 43° separation between the holes. What is the bullet's velocity?

5-29: From rest, a centrifuge accelerates at a constant rate of 55 rads/s per s for $5\frac{1}{2}$ minutes. What is its final angular velocity? How many revolutions does it make during this time?

for Section 5E

5-30: When you pedal a bicycle, at what point during the pedaling cycle do you exert maximum torque? minimum torque?

5-31: What is the torque with respect to •?
What factors would lead you to predict that the larger torque is produced by the 80 N force? by the 70 N force?

5-32: Which screwdriver produces the largest torque? Which wrench?

for Section 5F
Section 5F
5F — Part 1

5-33: What is the rotational inertia of this object if it rotates about the left end? about the center? Assume the rod is massless.

5-34: What is the rotational inertia for the above object (about the left end, and about the center) if the rod's mass is 6 kg?

If all of the system's mass was at one spot, and this "rock on a string" had the same rotational inertia as this system, how far would the mass be from the rotation axis? ==[radius of gyration – nec?

5-35: A loaded cannon (250 kg) is 3.0 m from the center of a merry-go-round (450 kg, 10.0 m diameter, solid cylinder). What is the rotational inertia of this system?

5-36: **parallel axis theorem** (optional)

If $\mathbf{I_{cm}}$ is the rotational inertia about an axis through the object's center of mass, and **I** is the rotational inertia about an axis that is a distance **h** from the center of mass, and **m** is the object's mass,

$$I = I_{cm} + mh^2$$

Use this relationship to derive the I-formula for a rod rotating through one end, starting from "$I = mL^2/12$" for a rod rotating about an axis through its center.

5F — Part 3

5-37: A 5 kg block attached to a pulley (840 grams, 60 cm diameter, solid cylindrical) accelerates at 3 m/s^2. What is the difference in rope tension across the pulley?

Will your answer change if a larger pulley (same mass, but 1.5 m diameter) is used?

5-38: Two strings wrap around the rim of a cylinder. What is its acceleration, and what is the tension in each string?

5-39: Using this system as an example, show that a pulley can be replaced [in F=ma] by an "equivalent mass" of $\underline{Xm}$ (if $I = Xmr^2$) or $\underline{I/r^2}$ (if I is just given as "I"). **To convert I into "equivalent mass", divide by r^2.**

5-##:

5-40: Starting from rest, a cylinder rolls down a ramp (36.9°, 3.0 m high) without slipping. What is its acceleration, and its speed at the bottom of the ramp?

Show that μ_s must be larger than $\frac{1}{3}\tan\theta$, if the cylinder is to roll without slipping.

5F — Part 4

5-41: A solid sperical ball rolls along a circular ramp (r = 70 cm, as in Problem 5-B) without slipping. If its speed is 6 m/s at the bottom, what is its speed at the top?

Compare this result with Problems 5-B and 5-G. Are you surprised? Explain.

5-42: The engine of a 1982 Ford Mustang has maximum torque of 240 ft lb (= 325 N m) at 2400 rpm, and maximum power of 157 hp at 4200 rpm. What is the engine's power at 2400 rpm, and torque at 4200 rpm?

5-43: For Problem 5-##'s merry-go-round (450 kg cylinder with 5 m radius, 250 kg at 3 m), what torque will give it an angular acceleration of .4 revs/s per s? What force (applied tangentially at the rim) is needed? After this torque pushes it for 10 revolutions, what is the system's KE and angular speed, and how much work has been done on it?

5-44: If the ballistic pendulum in Problem 4-K (Section 4.12) is hit "off center" so the block spins, what can you say about the velocity you calculate for the bullet?

If an explosion gives a block 5000 J of KE and sends the block straight up into the air without spinning, will it rise to the same height as a 5000 J off-center explosion that propels it ↑ <u>and</u> makes it spin?

5-##: If a cyclist exerts an average force of 160 pounds (= 712 N) throughout the pedaling cycle, and the pedals rotate in a circle of 18.0 cm radius, the wheels have 34.3 cm radius (this is 27 inch diameter), the front & rear gear sprockets have 52 & 13 teeth (in high gear), what average friction force do the tires exert against the ground? If the bicycle's speed is constant, what can you say about this force?

5-45: What is the answer to Problem 4-##, if the pulley is a solid cylinder with a 6.0 kg mass and 90 cm diameter? Why is your answer different than in 4-##?

5-46: For each situation, find the object's maximum ω, and the maximum v for the fastest-moving part of the object. {Hint: A-D are all variations on the same basic theme. Solve A first, then modify its solution as required.}

In A, friction prevents slipping; the bottom of the rod stays where it begins. In B, there is no friction. In D, the string is massless.

5F — Part 5

5-47: If the string in Problem 5-1 is slowly pulled inward till the circle radius is 40 cm, what is the block's speed? Are kinetic energy and angular momentum conserved? Why?

5-48: A small disk (5.0 kg, .60 m radius, 3 revs/s) drops onto a large disk (10.0 kg, .80 m radius, at rest). Both disks rotate on the same axle. When the disks are rotating at the same angular speed, what is their ω? Is kinetic energy conserved? Why?

5-49: In Problem 5-H, if the man drops the box when he is moving 5 rads/s, what is his final ω? What if he throws the box so (seen from the ground) it travels radially straight away from the disk-axle?

5-50: If this bullet (100 grams, 400 m/s) does not travel all the way through the disk (5.0 kg, 1.20 m diameter), what is ω_f?

5-51: ---[what was supposed to be here?? ==[have a 5-50 variation with bullet passing all the way through the disk? replace the above problem with all-the-way-through ---?

5-52: Starting from rest, what tangential rim-force and torque will give Problem 5-#'s merry-go-round (r = 5 m, I_{total} = 7875 kg m^2) an angular momentum of 1.40 x 10^5 kg m^2/s after 7.07 s of pushing? At this time, what is the angular velocity?

5-53: A solid spherical ball spinning at 20 rads/s drops straight down onto the floor. What is the ball's velocity when it begins to roll without "slipping"?

5-54: The discs of Problem 5-## are now side-by-side on separate axles, , not touching. The small disc (.60 m, .90 kg m^2) rotates at 6π rads/s, the large disc (.80 m, 3.20 kg m^2) is at rest. The discs are brought into contact. What is the rotational speed of each disc when the "slipping" has stopped? { Hint: Use a strategy like that in 5-53, not 5-48. And you can use the "tool" from Problem 5-22.}#

5-55: balance spinning ball? (gyroscope)

5-56: precession formula (also qualitative to find direction?)==

for Section 5G

5-57: A woman (weight W) stands 2/3 of the way up a 6 m ladder (mass M) that makes a 50° angle with the floor and rests against a vertical wall without slipping. Draw a force diagram for the ladder, write F=ma for the x & y directions, and $\tau = I\alpha$ for τ-axes at the bottom, center and top of the ladder.

If you know that m = 12 kg, W = 500 N, and the wall is frictionless, find the minimum μ_{floor} that will keep the ladder in place.

What can the woman do to make the ladder less likely to slip?

5-58: Where should you place the animal if you want to 1) lift an elephant slowly, or 2) send a mouse flying quickly?

5-59: A 210 kg refrigerator is 1.5 m high and .8 m wide; its coefficient of friction with the floor is .50. If you push horizontally at its base with enough force, the refrigerator will slide. But if you push → at the top, it tips. At what "pushing height" does the sliding-to-tipping transition occur?

5-60: The plank below is 8.0 m long, and weighs 196 N. Use principles from Section 4.11 to show that the system's center of mass is at •, and <u>if the system's center-of-mass is chosen as the torque-axis, $\tau = 0$</u>.

5-61: For each situation, write F_x=0, F_y=0, and τ=0 about each •-point. In A, there are two ropes. In B, the plank is marked into six equal lengths. In C, there are four equal lengths. Each plank (in A, B and C) has mass "m".

5-62:

5.92 Solutions

5-1: At one end of the rope: if the circle's radius is constant, "m" doesn't move up down, so T = mg. At the other end, this same rope-T provides centripetal force that causes the ❍ motion.

Write the 3-block F_c=ma_c: +T = m v^2/r. Substitute: +(m[9.8]) = (3) 2.5^2 / .80. Solve: m = 2.4 kg.

5-2: As in Problem 5-A: if F_c = 0, a = 0, and the object continues to move with the same v (magnitude and direction) it had at the instant F_c became zero.

5-3: Draw F-diagrams for the top & bottom points of the motion. At both points, N is ↑ and mg is ↓; for $F_c = ma_c$, toward-the-middle is always + (check the ± signs in the equation below) even though "up" is always + for mgh in W=ΔKE. The ferris wheel motor keeps Susan's speed fairly constant (unlike the vertical-circle object in Problem 5-B) so mv^2/r is equal at the top & bottom, and both F_c's are equal:

$$\begin{aligned} F_c\text{ [at bottom]} &= F_c\text{ [at top]} \\ +650 - mg &= -390 + mg \\ 520 / g &= m \end{aligned}$$

Find Susan's speed: $v_T = \Delta s/\Delta t = 2\pi r/15 = .419\,r$.

Write $F_c = ma_c$ for either point and substitute for N, mg, m & v_T. Solve either equation for r = 14.0 m.

At the top, $-390 + 520 = (520/9.8)(.419\,r)^2/r$

At the bottom, $-390 + 520 = (520/9.8)(.419\,r)^2/r$

5-4: When viewed from above, both situations are essentially the same: an object moves in a circle. For purposes of F=ma analysis, it doesn't matter if an object (the car) is moving with respect to the surface (the road) or if object-and-surface (coin-and-record) move together at the same speed. In both of these examples, the centripetal force is provided by friction that points toward the center of the circle.

Write the car's F=ma { $\mu_s mg = m v^2/r$ }, substitute { $.6(9.8) = v^2/120$ }, and solve: v = 26.6 m/s.

For the coin, two extra steps are needed. 78 rpm is 78 revs/60 s, or 60 s/78 revs = .769 s/rev.
$v \equiv \Delta x/\Delta t = 2\pi r/.77 = 8.17\,r$.

Write $F_c = ma_c$ { $\mu mg = m v^2/r$ }, substitute { $.4(9.8) = (8.17r)^2/r$ }, and solve: r = .059 m = 2.3 inches.

{ If you know "v = rω" from Section 5.4, you can use $\omega = 2\pi f = 2\pi(78/60) = 8.17$ rads/s. $\mu mg = mr\omega^2$ and $r = .4(9.8)/8.17^2 = .059$ m. }

5-5: Common sense says A is easiest, C is hardest.

Ratio logic predicts the same results: $F_c = m a_c = m v^2/r$, so A (low v, large r) requires the least F_c from friction, while C (high v, large r) requires the most.

5-6: To find the distance required to brake to a stop, write W=ΔKE: $(-\mu mg)\,d = 0 - \frac{1}{2} m v_i^2$.
Solve: $d = v_i^2/2\mu g = 15^2/2(.6)(9.8) = 19$ m.

If you turn, your → distance is the radius of the quarter-circle drawn below. Write $F_c = ma_c$: $\mu mg = m v_i^2$.
Solve: $r = v_i^2/\mu g = 15^2/(.6)(9.8) = 38$ m.

The brakes will stop you in a shorter distance.

{ Both calculations assume that friction is used "up to the μN limit, but not over it". If you don't brake [or turn] enough, d will be larger. But if you brake [or turn] too much, the tires "break loose" and the car slides/fishtails/... and maybe (especially if you turn too quickly) it will even flip over. }

5-7: Draw the forces acting at each of the 4 points, as in the right diagram. You must, as shown in the left diagram, split the mg-force at 240° into its F_c and F_T components by using standard trig-and-geometry.

bottom:	$F_c = +N_{bottom} - mg$,	$F_T = 0$
side:	$F_c = +N_{side}$,	$F_T = -mg$
top:	$F_c = +N_{top} + mg$,	$F_T = 0$
240°:	$F_c = +N_{240} + mg\sin 30°$,	$F_T = +mg\cos 30°$

$F_T = 0$ at the bottom point because it is the dividing point between the speeding-up & slowing-down parts of the circle. For the same reason, $F_T = 0$ at the top, and at this instant the object is not changing speed. At the side where the object moves ↑ but F_T points ↓, the object is slowing down. At 240°, speed is increasing.

5-8: There are two ways to view these forces.

Because these forces are not caused by any specific object in the environment, they are *pseudo forces.* If your car is moving ↑ and makes a sudden left turn, an object on the roof continues to move ↑. And so will you, unless some ← force (due to friction from the car seat, restraint by a seat belt, a push by the car door,...) provides the F_c that changes your v-direction.

The usual way to interpret F=ma is "F causes ma", F⇒ma. But the theory of general relativity, discussed in Section 16.7, says we can also think "ma can cause F", F⇐ma. Interpreting from within an accelerated reference frame (a left-turning car), you really do feel a force "pull you to the outside". But this force is not produced by any object in the environment, so it can be classified as "pseudo". Similarly, the truck rider in Problem 3-## observes a "force" that affects the ball's motion. And a person can, as described in the next problem, experience an *artificial gravity force* that is produced by a space station's centripetal acceleration.

==more? (wrap-up, refer to main text, ...?)

5-9: The station radius is 100 m. When the person stands, her head is 1.8 m closer to the center than her feet. Their rotation radii are 98.2 m and 100 m.

Find v as in Problem 5-3:

$v = \Delta x/\Delta t = 2\pi(98.2)/(60/3) = 30.85$ m/s!

$a_c = v^2/r = 30.85^2/98.2 = 9.69$ m/s². Her feet feel $a_c = \{2\pi(100)/(60/3)\}^2/100 = 9.87$ m/s². The "average g" she feels is $\frac{1}{2}(9.69+9.87) = 9.78$ m/s².

This would be quite comfortable, except for 1) the difference in g between the head and feet [this is different than on earth], and 2) the rotation rate might cause discomfort due to the sensitive rotation-detecting apparatus in the middle ear.

5-10: Find Δh as in Problem 4-K, by using "total = sum of parts" logic: $\Delta h = 5.0 - 5.0 \cos 50° = 1.79$ m from his initial position to the bottom of the arc.

Then solve $W=\Delta KE$, $mg\Delta h = \frac{1}{2} m v_f^2 - 0$, for $v_f^2 = 2g\Delta h = 2(9.8)(1.79) = 35.1\ m^2/s^2$. The centripetal force supplied by the vine tension (we're assuming the vine won't break) is $F_c = mv^2/r = 80(35.1)/5.0 = 562$ N. Yes, Tarzan can easily hold on.

5-11: Draw a F-diagram, split T into components:

The circle is horizontal, so $a_y = 0$, and $F_{horizontal}$ is F_c. $r = L \sin ø$, and $v = \Delta x/\Delta t = 2\pi(L \sin\theta)/t$.

$F_y = ma_y$: $+T\cos ø - mg = m(0)$
$F_c = ma_c$: $+T\sin ø = m[2\pi(L \sin ø)/t]^2/(L \sin ø)$

Solve for $T = mg/\cos ø$, substitute T and solve for $t = \sqrt{(4\pi^2/g)\, L \cos ø} = 2\pi\sqrt{L \cos ø/g}$.

{The best way to understand this algebra is to write the equations and solve them yourself}.

5-12: The key to banking problems is to **choose useful axes directions.** In most inclined plane problems an object slides straight up or down the plane, and the best axes choices are $\perp$ and $\parallel$ to the plane surface. But in this problem the car moves sideways on the ramp (not up or down it) and also, as shown in this bird's-eye view, around in a circle:

The best axes choices are horizontal and vertical, because the horizontal axis is also *centripetal* — it points toward the center of the highway-curve circle!

If the car's speed is "just right", no friction force acts on the car. This F-diagram shows a cross-section of the banked road, and the forces that act on the car:

Here is F=ma for the centripetal & vertical directions:

F_c	$= m\ a_c$	F_y	$= m\ a_y$
$+N \sin\theta$	$= m\,(v^2/r)$	$N\cos\theta - mg$	$= m\,(0)$

If the car is not sliding up or down the ramp, $a_y = 0$.

Solve the y-equation: $N = mg/\cos\theta$. Substitute N into $F_c=ma_c$ and, because $\sin\theta/\cos\theta = \tan\theta$, solve for $\theta = \tan^{-1}(v^2/rg) = \tan^{-1}(26.8^2/120[9.8]) = 31°$.

If it goes around the curve too fast, the car tries to slide up the ramp, ↗. Friction tries to prevent this sliding by pointing in the opposite direction, ↙:

Here are the centripetal & vertical F=ma equations:

$$+\mu_s N \cos 31° + N \sin 31° = m\, v^2/r$$
$$-\mu_s N \sin 31° + N \cos 31° - mg = m\,(0)$$

If a car drives on this curve slower than 26.8 m/s, which direction will it tend to slide? Which direction will friction point? Draw the F-diagram for yourself, split the F's into components, and write the F=ma's:

$$-\mu_s N \cos 31° + N \sin 31° = m\, v^2/r$$
$$+\mu_s N \sin 31° + N \cos 31° - mg = m\,(0)$$

For the same reasons as in Problem 5-3, the banking and funnel problems are essentially the same. So is the F-diagram, and these frictionless F=ma equations:

$+N \sin\theta = m\,(v^2/r)$ $\qquad$ $N\cos\theta - mg = m\,(0)$

5-13: For an orbiting object, $GMm/r^2 = v^2/r$, and $\sqrt{GM/r} = v$. $v = \sqrt{GM}\sqrt{1/r}$, so $v \downarrow$ as $r \uparrow$. The high satellite, with a larger r by a factor of 3, moves slower by a multiplying factor of $\sqrt{1/3} = .577$.

If a satellite is in a stable circular orbit, $\sqrt{GM_e/r} = v$. Satellite #1 is in a stable orbit, but #2 (too fast) will move "outward" and #3 (too slow) moves "inward", while #3 will "drop like a rock". Here is a rough sketch:

5-14: Hint: Think about which direction you look (and why) when you watch a sunrise or sunset. {The answer is after Solution 5-##.}

5-15: In this picture, F_{earth} pulls ← and F_{moon} pulls →. Use "total = sum of parts logic" to express r_{earth} & r_{moon}:

Solve for the "x" that makes F_e and F_m equal:

$$\frac{G\,(5.98 \times 10^{24})\, m_{object}}{x^2} = \frac{G\,(7.36 \times 10^{22})\, m_{object}}{\{(3.84 \times 10^8) - x\}^2}$$

$$3.46 \times 10^8 \text{ m} = x$$

ALGEBRA: To avoid a quadratic equation, take the $\sqrt{\ }$ of both sides, then cross-multiply and solve for x.

Use GMm/r^2 to calculate $F_{earth} = .0033\,N = F_{moon}$, in opposite directions. $F_{sun} = .0059$ N. The net $F_{gravity}$ is a .0059 N pull toward the sun. {$F_{gravity}$ due to other planets (Mars,...) are much smaller than F_{sun}.}

Near the earth's surface, $F_{gravity}$ is 98 N ←. F decreases as the block moves away from earth, until F becomes zero at the location calculated above. As the block continues to move →, F increases until it is $GM_m(10)/r_m^2 = 16$ N →. Notice the change of F-direction, from ← near the earth to → near the moon.

Even if the block was "weightless" it would still retain its 10 kg of mass, and you could hurt your foot by kicking it.

{5-14} Imagine that sunshine comes from the right, as in the North Pole bird's eye view below. In the morning the sun "rises" toward the east <u>if</u> the earth is spinning counter-clockwise. About a half-day later the same rotation makes the sun "set" toward the west. {Do you see why sunrise (or sunset) occurs earlier on the U.S. east coast than on the U.S. west coast, and why there are "time zones"?}

5-16: T^2 is proportional to r^3, so T ↑ as R ↑. The far-out planets have a long "year".

You may find it easiest to use the "equation-dividing ratio logic" discussed in Section 18.9 and used in Problem 5-C.

Or you can use this intuitive ratio approach: $T^2 \propto r^3$, so $\{T^2\}^{1/3} \propto \{r^3\}^{1/3}$, and $T^{2/3} \propto r$. If r ↑ by a factor of 687/365 = 1.88, T ↑ by a factor of only $1.88^{2/3} = 1.52$. $r_{mars} = 1.52\ r_{earth} = 1.52(1.50 \times 10^{11}\ m) = 2.28 \times 10^{11}$ m.

Notice that a less-than-one exponent for T (in $T^{2/3}$) means that the r-factor (1.52) is less than the T-factor (1.88).

5-17: Start with F=ma, and substitute: $F_c = GMm/r^2$, $a_c = v^2/r$, $v = 2\pi r/T$. Rearrange the resulting equation.

{Newton did the same derivation, but in different order. He began with F=ma, substituted $a_c = v^2/r$, $v = 2\pi r/T$, and $T^2 = r^3/k$ [this "law" had been discovered by Kepler, through observation of planets with a telescope] and then rearranged the equation to get "$F = 4\pi^2 k(1/r^2)$", to show that $F_{gravity}$ is proportional to $1/r^2$.} ==cut? edit?

Optional: Begin with $(GM/4\pi^2)T^2 = r^3$, substitute $T = 2\pi/\omega$ (from Section 5.4), rearrange to get $GM = r^3\omega^2$.

5-18: If both $F_{gravity}$'s describe the same force, mg = GMm/r^2. Dividing both equation-sides by m: $g = GM/r^2$.

But $GM/r^2 = (6.67 \times 10^{-11})(5.98 \times 10^{24})/(6.37 \times 10^6)^2 = 9.830\ m/s^2$, instead of the expected $9.807\ m/s^2$. Why?

Forces acting on the block are T and GMm/r^2. T points "up", while GMm/r^2 points "down" toward the earth-center. At the equator, the center of the earth's rotational motion is "down", so $F_c = +GMm/r^2 - T$. The block's rotational v is $2\pi r/T$, so $v^2/r = (2\pi r/T)^2/r = 4\pi^2 r/T^2$. The block F=ma is:

$$+GM(1)/r^2 - T = (1)\ 4\pi^2 r/T^2$$
$$9.830(1) - T = (1)\ 4\pi^2(6.37 \times 10^6)/(24 \times 3600)^2$$
$$9.830 - .0337 = T$$

We calculate the scale's weight-reading to be 9.796 N, instead of "mg = (1)(9.807)". There are many reasons for this difference: the earth's radius varies (it is larger at the equator than the poles, and increases on a mountain-top), the earth's density varies, and the "centripetal correction" of mv^2/r decreases as we move away from the equator*. For these reasons (and others), we don't expect our calculations to be exact, just "reasonably close". *when we move away from the equator toward higher latitudes, $v_{rotation}$ decreases (as discussed in Problem 5-##), and the center-of-rotation is not the same direction as "down" toward the earth-center

5-19:

5-20: Energy conservation: PE + KE is constant. Like a rock in a ∩ path near the earth's surface, the comet moves slowly when it is far from the sun (at x) and $PE_{gravity}$ is high. Maximum KE-and-speed occurs when it is closest to the sun (at •) with minimum PE.

Work: From x to •, W is + because F and d point in approximately the same direction, so KE ↑. From • to x, W is – (compare the F & d directions) so KE ↓.

5-21: $M_{earth} > M_{moon}$, so earth pulls the rocket with more gravity-force than does the moon. During an earth-to-moon trip, the rocket must fight the earth's gravity (this wastes a lot of fuel) but is helped by the moon's gravity (this saves a little fuel). For the return trip this is reversed (a little waste, lots of saving) and less fuel is needed.

5-##:

5-22: The edge points of each wheel have the same v_T, and thus the same $r\omega$. The small wheel has a smaller r (by a factor of .3/.5), so its ω is larger (by a factor of .5/.3); $\omega_{small} = \omega_{large}(.5/.3) = 1.8(.5/.3) = 3.0$ rads/s.

Or you can form an equation, then substitute and solve:

$$v_1 = v_2$$
$$r_1\ \omega_1 = r_2\ \omega_2$$
$$.5\ (1.8) = .3\ \omega_2$$
$$3.0\ \text{rads/s} = \omega_2$$

With direct contact, the wheels rotate in opposite directions: . With the belt, they rotate the same direction: .

5-23: **1)** $a_c \neq 0$, but a_T and α (= a_T/r) are zero. The magnitude of v_T and ω are constant, but the direction of v_T changes. F_c and a_c are constant in direction (they always point toward the circle's center) and in magnitude: $F_c = ma_c = m\ v_T^2/r$. F_T, a_T and α are constantly zero.

2) All three accelerations (a_c a_T α) are non-zero. The direction-and-magnitude of v_T changes, and so does ω. F_c and a_c direction is constant (toward center) but their magnitude decreases as v_T decreases. If v_T changes at a constant rate, F_T, a_T and α are constant.

5-##: 3 steps: **convert** the motion equation from linear (tvvax) to tangential (tvvas), **substitute** using connecting-equations, **cancel** the r's.

5-24: If wheels don't slip, $v_T = v = 8$ m/s, and $\Delta s = \Delta x = 3000$ m. $\omega = v_T / r = 8.0/.343 = 23.3$ rads/s. {In other units: 23.3 rads/s = 3.71 revs/s = 222 rpm = .270 s/rev.} $\Delta\theta = \Delta s / r = 3000/.343 = 8746$ rads; 8750 rads (1 rev / 2π rads) = 1390 revs.

We know 3-of-5: $\Delta t = 3.00$ s, $\omega_i = 23.3$ rads/s, $\omega_f = 0$. Solve the "Δθ-out equation": $23.3 - 0 = \alpha\,(3)$, 7.77 rads/s^2 = α. And solve "α-out": $\Delta\theta = \frac{1}{2}(23.3+0)(3)$, Δθ = 35.0 rads or 5.56 revs.

{Another method. We also know 3-of-5 tvvax: $\Delta t = 3$ s, $v_i = 8$ m/s, $v_f = 0$. Solve "Δx-out": $8 - 0 = a\,(3)$, 2.67 m/s^2 = a; if $a_T = a$, and $\alpha = a_T / r =^{*} a/r = 2.67/.343 = 7.78$ rads/s^2. Then solve "θ-out": $\Delta x = \frac{1}{2}(8+0)\,3$, Δx = 12.0 m, and $\Delta\theta = \Delta s / r =^{**} \Delta x / r = 35.0$ rads. [* and ** if there is no "slipping", $a_T = a$, $\Delta s = \Delta x$]

5-25: Point "a" has a $v_{rotation}$ of 8 m/s → <u>and</u> is moving 8 m/s → along with the wheel: it has v_{total} = +16 m/s →.

If "b" is halfway out to the rim, it has v_{rotn} = +4 m/s, and $v_{total} = v_{rotn} + v_{sideways}$ = (+4) + (+8) = +12 m/s.

The center-point "c" has v_{rotn} = 0, and v_{total} = (0) + (+8) = +12 m/s. ==[transl instead of rotn]

"d" has v_{rotn} = –8 m/s ← and $v_{sideways}$ = +8 m/s ←. These cancel each other, and v_{total} = (–8) + (+8) = 0.

"e" has v_{rotn} = 8 m/s ↑, and $v_{sideways}$ = 8 m/a →. v_{total} [the vector sum of these v's] is 11.3 m/s, in the ↗ direction with $\theta = \tan^{-1}(8/8) = 45°$.

5-26: Your actual v is faster than 55 mi/hr. {The speed-ometer measures ω, then converts this (using standard r_{tire}) to the v you see on the dashboard. But if r_{tire} is larger than normal, so is v (which equals rω).

5-27: The earth rotates once a day: f = 1 rev/(24 x 3600 s) = 1.16 x 10^{-5} rads/s, or ω = 7.27 x 10^{-5} rads/s. v = rω = (6.37 x 10^6)(7.27 x 10^{-5}) = 463 m/s. {Alternate method: $v \equiv \Delta x/\Delta t = 2\pi r/\Delta t = 2\pi(6.37 \times 10^6)/(24 \times 3600) = 463$ m/s.}

At 35° latitude, ω is the same, but r is smaller (as shown above), and v = rω = (6.37 x 10^6 x cos 34°)(7.27 x 10^{-5}) = 384 m/s.

The earth orbits the sun once per year. At the equator or in LA, f = 1 rev/(365 x 24 x 3600 s) = 3.17 x 10^{-8}, and ω = 19.9 x 10^{-8} rads/s.
v = rω = (1.50 x 10^{11})(19.9 x 10^{-8}) = 29900 m/s.
{An option: use v = 2πr/Δt.}

5-28: ω = (1200 revs/ 60 s)(2π rads/rev) = 126 rads/s. Δθ = (43 degrees)(2π rads/ 360 deg) = .750 rads.

Δx	=	v	Δt	Δθ	=	ω	Δt
	=		⇓	.750	=	126	Δt
1.4	=	v	.00595	.00595 s	=		Δt
235 m/s	=	v					

5-29: 3-of-5 tωωαθ are given: Δt = 330 s, $\omega_i = 0$, α = 55 rads/s^2. Solve the θ-out and ω_f-out equations for ω_f = 18150 rads/s and Δθ = 2.99 x 10^6 rads = 4.77 x 10^5 revs.

5-30: If we assume the foot can only exert force downward, maximum τ occurs when F is applied ⊥ to r, as in the first picture. At the bottom/top point in the cycle, τ = 0 if there are only ↓ forces*.

* Friction lets your feet exert ← and → forces, so τ ≠ 0 at the bottom/top point. With toe-clips and/or special shoes that are "fixed to the pedal", a rider can exert ↑ force and a larger amount of ← and → force, to get a more uniform torque throughout the cycle. But maximum torque still occurs at the point shown above.

5-31: The 80 N force tries to rotate the object ; we'll define this to be the + direction for τ. The 70 N tries to rotate it the opposite direction, so its τ is –.

$$\tau_{total} = +80(.90)\sin 25° - 70(.60)\sin 130°$$
$$\tau_{total} = +30.4 - 32.2$$

{For the 80 N force, θ can be described by 205°, 25° or 155°. But if we use 205° (larger than 180°) the sinθ is –, which (unless it is ignored) will mess up the decision we made about the ± sign of τ.}

Of the three factors that affect τ-magnitude, two (F and r) say "the 80 N torque is larger", while one (sinθ) says "the 70 N torque is larger".

5-32: If the same F_T is applied to each screwdriver [or wrench], the largest torque is produced by the second and third screwdrivers (it's a tie), and by the second wrench. Do you see why "length" affects torque for the wrench but not for the screwdriver?

5-33: Use visual logic to find distances from the left end and from the center, then use "total = sum of parts" and $I = mr^2$.

L-end: $I = 8(2^2) + 2(10^2) = 32 + 200 = 232$ kg m^2.
Center: $I = 8(3^2) + 2(5^2) = 72 + 50 = 122$ kg m^2.

Do you see why the 2 kg ball contributes more to I if the rotation axis is at the left end, while the 8 kg ball contributes more when the axis is at the center?

5-34: Use mr^2 for the balls, an Xmr^2 table for the rod, and "total = sum of parts" logic.

L-end: $I = 8(2^2) + 2(10^2) + \frac{1}{3}(6)(10^2) = 432\ \text{kg m}^2$.
Center: $I = 8(3^2) + 2(5^2) + \frac{1}{2}(6)(10^2) = 172\ \text{kg m}^2$.

5-35: $I_{total} = I_{cannon} + I_{m\text{-}g\text{-}round} =$
$(250)(3.0^2) + .5(450)(5.0^2) = 2250 + 5625 = 7875\ \text{kg m}^2$.

5-36: There is a distance of .5 L between the rod's center-of-mass (at its center) and an axis at the end.

$$\begin{aligned} I &= I_{cm} + mh^2 \\ I &= mL^2/12 + m(.5L)^2 \\ I &= 1mL^2/12 + 12m(.25L)^2/12 \\ I &= 1mL^2/12 + 3mL^2/12 \\ I &= 4mL^2/12 \end{aligned}$$

The parallel axis theorem predicts $I = mL^2/3$. This is, of course, the same I as in Section 5.6's Xmr^2 table.

5-37: The "5 kg" is an unnecessary decoy.

$$\begin{aligned} \tau &= I \\ rF &= Xmr^2 \\ a/r.30\,(T_2 - T_1) &= .5(.840)(.30^2)\quad 3/.30 \\ T_2 - T_1 &= 1.26\ \text{Newtons} \end{aligned}$$

All of the .30's cancel each other. The large-radius pulley requires the same ΔT.

5-38: Draw the cylinder's F-diagram, define the + direction for a (↓) and α ( ), solve F=ma and τ=Iα:

$$\begin{aligned} F &= ma \\ -2T + mg &= ma \\ &\Downarrow \\ -\tfrac{1}{2}ma + mg &= ma \\ mg &= 1.5a \\ \tfrac{2}{3}g &= a \end{aligned} \qquad \begin{aligned} \tau &= I\alpha \\ +(2T)r &= \tfrac{1}{2}mr^2 a/r \\ 2T &= \tfrac{1}{2}ma \\ &\Downarrow \\ 2T &= \tfrac{1}{2}m(\tfrac{2}{3}g) \\ T &= \tfrac{1}{6}mg \end{aligned}$$

5-39: Write F=ma's & τ=Iα, solve. If $I = Xmr^2$, and the tensions on each side of the pulley are t and T,

$$\begin{aligned} F &= ma \\ +t &= Ma \end{aligned} \qquad \begin{aligned} \tau &= I\alpha \\ (-t+T)r &= Xmr^2 a/r \\ -t+T &= Xm \end{aligned} \qquad \begin{aligned} F &= ma \\ -T+mg &= Ma \end{aligned}$$

Adding all three equations: $mg = \underline{M}a + \underline{Xm}a + \underline{M}a$. This is the same F=ma we get by treating the block-pulley-block as a "matched motion, bent axis system" like the in Problem 3-B, with the pulley having a mass (its "equivalent" mass) of Xm.

Adding all three equations: $mg = \underline{M}a + \underline{Xm}a + \underline{M}a$, $mg = (M + Xm + M)a$. This is the same F=ma we get by treating block-pulley-block as a *matched-motion bent-axis system* like the in Problem 3-B, with the pulley having an "equivalent mass" of Xm.

If I is expressed as "I", the F=ma's and τ=Iα are:

$$\begin{aligned} +t &= Ma \\ -t+T &= (I/r^2)a \end{aligned} \qquad \begin{aligned} (-t+T)r &= I\ a/r \end{aligned} \qquad \begin{aligned} -T+mg &= Ma \end{aligned}$$

Adding these equations: $mg = \underline{M}a + \underline{I/r^2}a + \underline{M}a = (M + I/r^2 + M)a$. The pulley's "equivalent m" is I/r^2.

5-40:

$$\begin{aligned} F &= ma \\ -f + Mg\sin\theta &= Ma \\ &\Downarrow \\ -\tfrac{1}{2}Ma + Mg\sin\theta &= Ma \\ \tfrac{2}{3}g\sin\theta &= a \end{aligned} \qquad \begin{aligned} \tau_{center} &= I\alpha \\ +fr &= \tfrac{1}{2}Mr^2 a/r \\ f &= \tfrac{1}{2}Ma \end{aligned}$$

$\theta = 36.9°$, so $a = 3.92$ m/s per s. We know 3-of-5 tvvax: $v_i = 0$, $a = +3.92\ \text{m/s}^2$, $\Delta x = 3.00/\sin 36.9° = 5.00$ m. Solve the "t-out" equation for $v_f = 6.26$ m/s. This same situation is analyzed in Section 5.6D, using the work→energy principle, and (of course) the same v_f is calculated.

Notice that f, which points up the ramp, makes **a** (and thus v_f) smaller. But it causes **α**, and thus the rotation-and-ω_f of the cylinder. ==[awkward] nec?

If μ_s is reduced to the point where f_s is "just above the breakaway limit", $f_s = \mu_s N = \mu_s Mg\cos\theta$. Since there is enough f_s to prevent slipping, $\alpha = a/r$, and the analysis above [which assumed that $\alpha = a/r$, and produced "$a = \frac{2}{3}g\sin\theta$" and "$f_s = \frac{1}{2}Ma$"] is valid. Using these equations (and $\sin\theta/\cos\theta = \tan\theta$), we can find the value of μ_s at the breakaway limit:

$$\begin{aligned} f &= \tfrac{1}{2}Ma \\ \mu_s Mg\cos\theta &= \tfrac{1}{2}M(\tfrac{2}{3}g\sin\theta) \\ \mu_s &= \tfrac{1}{3}(\sin\theta/\cos\theta) \\ \mu_s &= \tfrac{1}{3}\tan\theta \end{aligned}$$

5-41: As in Problem 5-G, $KE_{total} = KE_{tr} + KE_{rotn} = \frac{1}{2}mv^2(1+X) = \frac{1}{2}mv^2(1+\frac{1}{2}) = .75mv^2$. If we assume that energy is conserved by the "rolling friction",

$$\begin{aligned} PE_i + KE_i &= PE_f + KE_f \\ mg(0) + .75m(6)^2 &= m(9.8)(+1.40) + .75mv_f^2 \\ 4.21\ \text{m/s} &= v_f \end{aligned}$$

Now and in 5-B, ΔPE is $mg\Delta h = 13.7$ J. In 5-B, $W_{gravity}$ ($= -mg\Delta h$) changes KE from 18m to 4.3m, a reduction of 76%. But now KE_i is larger because it is $.75mv^2$, not $.50mv^2$, and KE changes from 27m to 13.3m, a reduction of only 51%. $W_{gravity}$ is the same in both cases, but when KE is larger (when there is KE_{tr} <u>and</u> KE_{rotn}) it is a smaller fraction of KE and causes a smaller change in v. {Since m is not known, KE must be expressed in terms of "m".}==nec?

Comparing 5-B and now, we find smaller v-change when there is KE_{rotn} and rolling friction: 6.00 → 2.93 without KE_{rotn}, while 6.00 → 4.21 with KE_{rotn}. We find the same result in 5-G: v changes from 0 to 6.57 without KE_{rotn} (without f_k it would change from 0 to 7.67), but 0 → 6.48 [or 6.26] with KE_{rotn}.

==[in 5.6 I said this 5-# would show why friction can increase mgh; did I do this here or elsewhere?

5-42: (2400 revs/min)(1 min/60 s)(2π rads/rev) = 251 rads/s. Similarly, 4200 rpm = 440 rads/s.

At 2400 rpm, $P_{rotn} = \tau\,\omega = (325)(251) = 81575$ W = 109 hp. At 4200 rpm, $\tau = P/\omega = (157 \times 746)/440 =$ 266 N m = 197 ft lb. {If τ was constant (it isn't), P would be proportional to ω. And if P was constant (it isn't), τ would be inversely proportional to ω.}

5-43: $I = .5(450)(5^2) + (250)(3^2) = 7875$ kg m^2.

$\tau = I\alpha = 7875(.4 \text{ revs/s}^2)(2\pi \text{ rads/rev}) = 19800$ N m,

$F_\perp = \tau/r = 19800/5 = 3960$ N,

$W = \tau\,\Delta\theta = 19800(10 \times 2\pi \text{ rads}) = 1.24 \times 10^6$ J,

$KE_f = KE_i + \Delta KE = 0 + 1.24 \times 10^6 = 1.24 \times 10^6$ J.

In $\tau = I\alpha$ and $\tau\,\Delta\theta = \Delta\frac{1}{2} I\omega^2$, <u>you must use radians</u>.

$1.24 \times 10^6 = \frac{1}{2}(7875)\,\omega^2$, so $\omega = 17.7$ rads/s.

{Another method: We know ω_i, α and Δθ, so we can solve the t-out tωωαθ equation for ω = 17.8 rads/s.}

5-44: Because linear momentum is conserved, the system's velocity "just after bullet comes to rest within the block" is the same as in 4-K. The bullet-block system also has the same KE_{trans} as before and the same Δh rise. We measure the same "string angle at the peak" and calculate the same initial bullet speed.

But now the system also has KE_{rotn}. How do you reconcile this with energy conservation? Think about this before you read the answer later in this solution.

After the centered explosion, the block has 5000 J of KE_{trans} that will be converted into mgΔh. But after the off-center explosion, part of the 5000 J is KE_{rotn} that doesn't contribute to the block's Δh, so the block doesn't rise as high.

Why does the pendulum rise to the same height but not the block? Most of the KE_i is dissipated when the bullet burrows into the block. A little less KE is lost during the off-center hit, so the bullet/block can have some KE_{rotn} and still have the same KE_{trans} as before. FΔt is the same during both hits (so each has the same Δmv) and FΔx is different (so a different amount of energy is dissipated).

But the explosion has a fixed amount of energy to give the block. If some of the 5000 J causes KE_{rotn}, less goes into KE_{trans}-and-mgΔh.

5-##: τ caused by the rider's feet is transmitted through chain, gears and axle to the rear tire, causing a frictional τ (and force) between tire and ground that "pulls" the bike forward. If the bike moves →, this $F_{friction}$ points → to cause → acceleration or maintain → motion by overcoming the counter-forces of air resistance and "rolling friction". When v is constant, $F_{feet/friction} = F_{counter\text{-}forces}$. Whether v is constant or changing, though, we can use this principle of energy conservation: if the energy wasted by internal friction (in chain, gears,...) is negligible, work done by the rider's feet pushing on the pedals equals the work done by → friction-force on the rear tire,

$$\begin{aligned} W_{feet} &= W_{tire} \\ F_{feet}\ r_{feet}\ \Delta\theta_{feet} &= F_{tire}\ r_{tire}\ \Delta\theta_{tire} \\ (712)\,(.18)\,\Delta\theta_{feet} &= F_{tire}\,(.343)\,[(52/13)\,\Delta\theta_{feet}] \\ 93.4 \text{ Newtons} &= F_{tire} \end{aligned}$$

If the 52-tooth front gear rotates 1 time, the 13-tooth back gear rotates 4 times, so $\Delta\theta_{tire} = (52/13)\Delta\theta_{wheel}$.

High gear is good for going fast (if the foot moves 2π[.18] the tire's rim moves 2π[52/13][.343], more by a factor of 7.62), but force decreases from 712 N to 93.4 N, decreasing by a factor of 1/7.62. During the feet-to-tire transmission process (chain/gears/axle) F ↑ and Δx ↓, but FΔx (= W) stays the same.

In a lower gear (like 40/28 instead of 52/13) there is less Δx-multiplication (only [40/28][.343/.18] = 2.72) but there is more F_{tire} (it is decreases by a factor of only 1/2.72). A low gear is slow but strong; it is good for acceleration when you're moving slowly, or for overcoming "mg sinθ" when you ride up a hill.

5-45: $(KE_{pulley})_i = \frac{1}{2} I\omega^2 = \frac{1}{2}(\frac{1}{2}Mr^2)(v/r)^2 = \frac{1}{2}(\frac{1}{2}M)\,v^2$.

$KE_i = \frac{1}{2}(4)\,2^2 + \frac{1}{2}(5)\,2^2 + \frac{1}{2}(\frac{1}{2}[6])\,2^2 = 24.0$ J. As in 4-##, $\Delta KE = -mg\Delta h = -\{4g(+1.6) + 5g(-1.6)\} = +15.7$ J.

$KE_f = KE_i + \Delta KE = 24.0 + 15.7 = 39.7$ J. This equals $\frac{1}{2}(4 + \frac{1}{2}[6] + 5)\,v_f^2$, and $v_f = 2.57$ m/s.

Because part of the mgΔh energy is converted into KE_{rotn} of the pulley, the KE increase (and KE_f) of the blocks is slightly less than in 4-##.

5-46: Each object reaches maximum ω-and-v at its lowest point: just before impact for A-B, and at the bottom of the swing for C-D. The fastest part of the rods is at their ends. In A & C, the end is a distance H from the rotation-axis (at the other end), while in B (rotation about the center) this distance is only $\frac{1}{2}$ H.

In A-D, the object's center-of-mass falls a distance of $\Delta h = -\frac{1}{2}$ H, and $\Delta PE = mg\Delta h = mg(-\frac{1}{2}\,H)$. For A-D, I is $mH^2/3$, $mH^2/12$, $mH^2/3$ and $mH^2/1$, respectively. For A, we write $-\Delta PE = \Delta KE$, $\frac{1}{2}\,mgH = \frac{1}{2}(mH^2/3)\,\omega^2$, and then solve it: $\sqrt{3g/H} = \omega$. At the end of the rod, a distance H from the axis of rotation, $v = r\omega = H\sqrt{3g/H} = \sqrt{3gH^2/H} = \sqrt{3gH}$.

For B-D, "–PE = ΔKE" gives $\omega = \sqrt{12g/H}$, $\sqrt{3g/H}$ and $\sqrt{1g/H}$, respectively. Distances from the place where v is maximum to the axes of rotation are $\frac{1}{2}$ H, H and H, so $v = r\omega = \sqrt{3gH}$, $\sqrt{3gH}$ and $\sqrt{1gH}$.

{In B, $v = r\omega = \frac{1}{2}\,H\sqrt{12g/H} = \sqrt{(H^2/4)(12g/H)}$.}

5-47: A force pulls the string toward the center; F_c is +. But $F_T = 0$, so $\tau_{ext} = 0$, and $L_i = L_f$. With the given information, it is easier to use $mvr_\perp$ than $I\omega$:

$$\begin{aligned} L_i &= L_f \\ (mvr)_i &= (mvr)_f \\ 3(2.5)(.80) &= 3\,v_f(.40) \\ 5.0 \text{ m/s} &= v_f \end{aligned}$$

Because F_c points toward the center and the block also moves toward the center (as well as tangentially), W is + and KE increases from 9.4 J $\{KE_i = \frac{1}{2}(3)\ 2.5^2\}$ to 37.5 J $\{KE_f = \frac{1}{2}(3)\ 5.0^2\}$.

We can estimate work by taking the average of T: $T_i = mv^2/r = 3(2.5^2)/.8 = 23.4$ N, $T_f = 3(5.0^2)/.4 = 187.5$ N, and $T_{average} = \frac{1}{2}(23.4 + 187.5) = 105.4$ N. $W \approx F_{avg}\,\Delta r = 105.4(.8 - .4) = 42$ J. This is larger than the actual W, but is reasonably close.

To calculate W requires calculus. Optional: set up an integral to find W, evaluate it (to get W = 28.1 N), then check the solution after Problem 5-##.

5-48: **a)** The disks have $I = \frac{1}{2}(5)\ .60^2 = .90\ \text{kg m}^2$ and $I = \frac{1}{2}(10)\ .80^2 = 3.20\ \text{kg m}^2$, respectively.

Friction between the disks is internal. $\tau_{ext} = 0$, so $(I\omega)_i = (I\omega)_f$: $.9(3 \times 2\pi) + 3.2(0) = (.9 + 3.2)\omega_f$, and $4.14\ \text{rads/s} = \omega_f$.

While the disk-speeds are "matching" (small-disk $\omega \downarrow$, large-disk $\omega \uparrow$) there is friction that makes KE $\downarrow$. $KE_i = \frac{1}{2}(.9)(6\pi)^2 = 160$ J, $KE_f = \frac{1}{2}(4.1)\ 4.14^2 = 35$ J.

{5-47} $W = \int F_c\,dr = \int mv^2/r\,dr = \int m\{v_i r_i/r\}^2/r\,dr = m v_i r_i \int 1/r^3\,dr = m v_i^2 r_i^2 \{-\frac{1}{2}(1/r^2)\}$ evaluated: $W = 3(2.5^2)(.80^2)(-\frac{1}{2})\{1/.80^2 - 1/.40^2\} = +28.1$ J.

5-49: The drop doesn't change the box's v_T or L, and L is conserved, so disk-and-man have unchanged L and they still rotate at 2.46 rads/s.

After the throw, the box has $L = mvr\sin\theta = mvr(0) = 0$, so L of the disk-and-man system must increase, and so does ω. For disk-man-box system: $I\omega_i = I\omega_f$, $(2925)(2.46) = (2875)\omega_f + (50)(0)$, $2.50\ \text{rads/s} = \omega_f$.

5-50: $L_i = mv\,r_\perp = mvr\sin\theta = .100(400).60\sin 40° = 15.4\ \text{kg m}^2/\text{s}$. If we assume the bullet ends up $.60\sin 40° = .39$ m from the center, a bullet-and-disk system has $I = \frac{1}{2}(5)\ .60^2 + .1(.39^2) = .92\ \text{kg m}^2$. The bullet/disk force is internal; $\tau_{ext} = 0$, so L_f (= $I_f\omega_f = .92\ \omega_f$) equals L_i (= 15.4), and $\omega_f = 16.7$ rads/s.

5-51:

5-52: $F_T\ r\,\Delta t = L_f - L_i$, $F_T(5)(7.07) = 140000 - 0$, $F_T = 3960$ N. $\tau = F_T\,r = 3960(5) = 19800$ N m.

$\omega_f = L_f/I = 140000/7875 = 17.8$ rads/s.

These are the same answers as in Problem 5-##, because pushing with this F-and-τ for 7.07 s makes the merry-go-round turn through 10 revolutions.

5-53: Before the ball starts to roll without slipping, kinetic friction (the f_k shown in Solution 5-F's first diagram) acts for a time Δt to make v increase and ω decrease: $\mathbf{f\,\Delta t = \Delta(mv)}$, and $\mathbf{r\,f\,\Delta t = \Delta(I\omega)}$.

Slipping stops when the v-increase and ω-decrease are enough to make v and ω satisfy the relationship $\mathbf{v = r\omega}$. If we define the direction of rotation as +, the τ caused by f is in the – direction: $\tau = -r\,f$.

$$\begin{aligned} f\,\Delta t = mv_f - mv_i \quad & (-r\,f)\,\Delta t = I\ \omega_f - I\ \omega_i \\ f\,\Delta t = m\,r\omega_f - 0 \quad & -r\,(f\,\Delta t) = .4mr^2\,\omega_f - .4mr^2\,(20) \\ f\,\Delta t = m\,r\omega_f \quad & \Downarrow \quad = .4mr^2\,\omega_f - .4mr^2\,(20) \\ f\,\Delta t = m\,r\omega_f \quad & -r\,(mr\,\omega_f) = .4mr^2\,\omega_f - .4mr^2\,(20) \\ & -\omega_f = .4\,\omega_f - .4(20) \\ & 5.7\ \text{rads/s} = \omega_f \end{aligned}$$

$v_f = r\,\omega_f = r(5.7)$. There is not enough information to give a numerical answer to "What is v_f?". The best answer we can give is "$v_f = (5.7\,r)$ rads/s".

I found this solution (including the substitutions discussed in the first paragraph) after some trial-and-error, trying different approaches until something worked. {Another option is to substitute "$f\,\Delta t = mv_f$" (from the left equation) and "$\omega_f = v_f/r$" into the right equation, then solve it for "$5.7\,r = v_f$".}

5-54: As shown below, the same kinetic friction "f" acts on each disc (Newton's Third Law). For each disc to remain in place, its f must be balanced by "F" acting on its axle. When we calculate L with respect to the small-disc axle, the small-disc F produces no τ because r = 0, but the large-disc F produces τ_{ext} and the discs' total angular momentum is not conserved. {And it would be difficult to calculate the large disc's L w.r.t. the small-disc axle, anyway; it is not just $I\omega$.}

We can't use L-conservation. But as suggested in the hint, we can use a variation on the strategy-tools from Problem 5-5#. $\Delta\mathbf{L} = \tau\,\Delta t = F_\perp\,r\,\Delta t = \mathbf{f\,r\,\Delta t}$. The same f acts on each disc for the same Δt, so the disc-ΔL's are proportional to their r's. The large disc has larger r, so it has larger ΔL: $\Delta L_2 = -1.333\ \Delta L_1$. {The – sign is needed because ΔL_1 is – and ΔL_2 is +.}

As in Problem 5-##, when the slipping stops both discs have equal rim-speed: $v_1 = v_2$, $r_1\omega_1 = r_2\omega_2$, $.6\ \omega_1 = .8\ \omega_2$, $.75\ \omega_1 = \omega_2$.

$$\begin{aligned} -1.333 \quad & \Delta L_1 = \Delta L_2 \\ -1.333 \quad & \{I_1\,(\omega_f - \omega_i)\} = I_2\,(\omega_f - \omega_i) \\ -1.333 \quad & \{.9\,(\omega_1 - 6\pi)\} = 3.2\,(.75\omega_1 - 0) \\ & 6.28\ \text{rads/s} = \omega_1 \end{aligned}$$

$$\omega_2 = .75\ \omega_1 = .75(6.28\ \text{rads/s}) = 4.71\ \text{rads/s}$$

5-55:

5-56:

5-57: To decide the direction of each friction force, ask "which direction would the ladder slide", then aim $F_{friction}$ in the opposite direction. The F-diagram is:

"without slipping" means that $a_x = 0$, $a_y = 0$, $\tau = 0$:

X: $+F - n = 0$

Y: $+N - mg - W + f = 0$

τ_{bottom}:	N(0)	+F(0)	–mg(3)sin40
	–W(4)sin40	+n(6)sin50	+f(6)sin140
τ_{center}:	–N(3)sin40	+F(3)sin50	+mg(0)
	–W(1)sin40	+n(3)sin50	+f(3)sin140
τ_{top}:	–N(6)sin40	+F(6)sin50	+mg(3)sin140
	+W(2)sin140	+n(0)	+f(0)

{sin 140° can be replaced by sin 40°}

A "frictionless wall" means "f = 0". If μ_{floor} is the minimum that will prevent slipping, the ladder is at its *breakaway limit* with $F = \mu_s N$.

Solve F_y=0: N = mg +W –f = 12(9.8) +500 – 0 = 618 Newtons. F_x=0 now has 2 unknowns (μ_s, n) but τ_{bottom}=0 only has 1 unknown (n),

–12(9.8)3 sin40 –500(4)sin40 +n(6)sin50 +0 = 0, which we can solve: n = 329 Newtons. Now we can solve F_x=0: μ_s (618) –(329) = 0, μ_s = .53.

To prevent ladder-slipping, the woman can try to increase μ_s, make the ladder-angle larger than 50° (but not too much, or the ladder may fall backward away from the wall), or not climb up so high on the ladder.

5-58: Put the elephant on the short (S) end, and push downward on the long (L) end. To catapult the mouse, put it on L and push down quickly on S.

This example shows the usual "leverage trade-off": lifting-capability ↑ as lifting-speed ↓, and vice-versa. ==[wording?

5-59: At the "transition point" the refrigerator is 1) almost sliding [so $f_s = \mu_s N$], 2) almost tipping [so N acts at the same point it would occur if tipping actually occurred, as shown below], and 3) neither movement is actually occurring [so $F_x = 0$, $F_y = 0$, and $\tau = 0$].

If any of the • points is used as an axis, 2 of the 4 τ's will be zero. These 3 equations can be solved for N = 2060 *N*, P = 4120 *N*, **y = .8 meters**:

X: +P –.5N = 0

Y: +N –m(9.8) = 0

τ: P(0) +N(0) –.5Ny +m(9.8)(.4) = 0

For many problems "$\tau = \pm F r \sin\theta$" is easier to use, but for this situation "$\tau = \pm F r_\perp$" is easier.

We get the same P, N & y by using either of these equations (for τ with respect to the other • points):

τ: N(0) +μ_sN(0) +m(9.8)(.4) –Py = 0

τ: mg(0) +μ_sN(0) +N(.4) –Py = 0

5-60: $x_{c\text{-}of\text{-}m} = \dfrac{100(1) + (196/9.8)(4) + 45(7)}{100 + 20 + 45}$

= 3.0 m [this is the location of •]

$\tau_{about\ \bullet}$ = +100g(2)sin90° –20g(1)1 –45g(4)1 = 0

5-61: In A, the rope-tensions are not necessarily equal. We can calculate τ about an axis that, as in B, is not inside the object; this is probably not a "useful" τ-axis, because it doesn't eliminate any variables, but it can be used. In C, the "normal" forces are ⊥ to the surface of the plank; they are not vertical. For writing equations, I'll assume lengths of L, 6 and 4 for the objects in pictures A, B and C.

X: +T cos 30° –H = 0

Y: –Mg –mg +T sin 30° +V = 0

τ: –mg(L/2) sin50° –H(L) sin40° +V(L) sin50° = 0

τ: +Mg(L) sin50° +mg($\frac{1}{2}$ L) sin50° –T(L) sin70° = 0

X: there are no forces in the x-direction

Y: –n +N –mg –Mg = 0

τ: +n(1) sin90° +0 –mg(1)(1) –Mg(3)(1) = 0

τ: +n(6)1 –N(5)1 +mg(4)1 +Mg(2)1 = 0

X: –n cos70° –N cos70° +f sin70° +F sin70° = 0

Y: +n sin70 +N sin70 –mg –f cos70 –F cos70 = 0

τ: +0 +0 –mg(1)sin70 +N(3)sin90° +F(3)sin0° = 0

τ: f(3)(0) –n(3)sin90° +mg(2)sin70° +0 +0 = 0

==[but bottom supports more w (because closer to c-of-m?) and I don't think these equations show this

5-62: