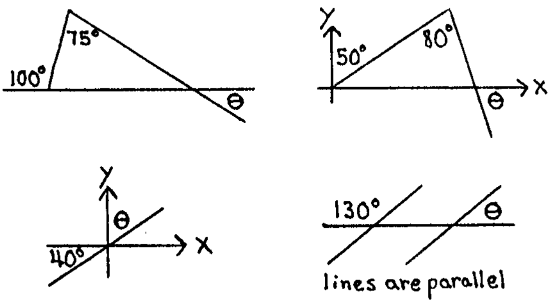
1.91 Optional Problems

The most important problems are marked •

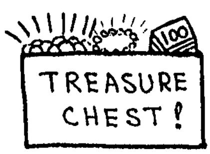
for Section 1.1,

• **1-1**: For each diagram below, find θ.



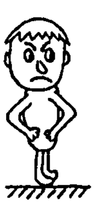
for Section ~~1.2,~~ CUT

1-2: It's been a long difficult journey, but you've finally arrived. Somewhere near the big oak tree is buried treasure — lots of it!

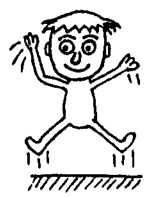


You know it is 5 feet under ground but you don't know the location because you forgot to bring the treasure map. You have a shovel, tape measure, compass, and 25¢ — enough for one phone call to your partner who has the map.

Ring! Ring! "Hello." "Hi, Herman. I'm at the oak tree with a shovel. Look at the map, tell me where to dig, and we'll soon be rich!" Herman is glad to help: he says "It's exactly 100 feet away from the tree", and hangs up.



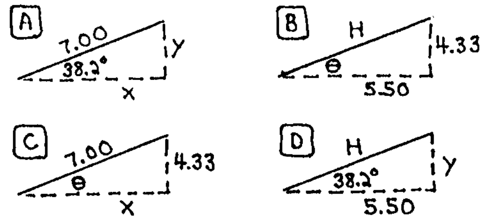
Are you happy with Herman? Do you have enough info to find the treasure? Which of the messages below gives you all of the location information you need?



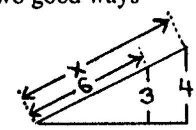
a) Walk 100 feet away and dig.
b) Go 36.87° South of due East, dig.
c) Face 36.87° S of E, walk 100 feet, dig.
d) Walk 80 feet East, then 60 feet South, dig.

for Section ~~1.3,~~ 1.2

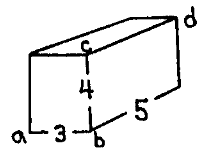
• **1-3**: For each Δ, find the 2 unknowns.



• **1-4**: Find x. (There are two good ways to set up a solution- equation. Even if you get the answer, look at Solution 1-3.)



1-5: For this rectangular box, find the length of the "long diagonal" from a to d.

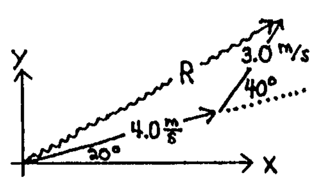


1-6, Part 1: If $\sin\theta = .340$, find θ, $\cos\theta$, $\tan\theta$. **Part 2**: If $y = 8 - 3\sin^2\theta - 3\cos^2\theta$, find y. **Part 3**: If $\cos y = 10x$ and $\sin y = 12x$, find y.

for Section ~~1.4,~~ CUT

• ~~1-7~~: A man walks 4m south, 3m west, and 5m at 30° E of N. What is the magnitude and direction of his *displacement*? What *distance* has he traveled during the walk?

~~1-8~~: Find the magnitude & direction of **R**.



~~1-9~~, **Part 1**: If **A** is a displacement vector (8 m eastward) and **B** is a velocity vector (6 m/s eastward), what is **A + B**? **Part 2**: What is the largest & smallest magnitudes that are possible when adding vectors **C** (3 m/s eastward) and **D** (2 m/s, direction unknown)?

for Section 1.7,

• **1-10**: A car is traveling 60 miles/hour. Convert this speed to ft/s and km/hr, using the conversion factors in Section 1.7.

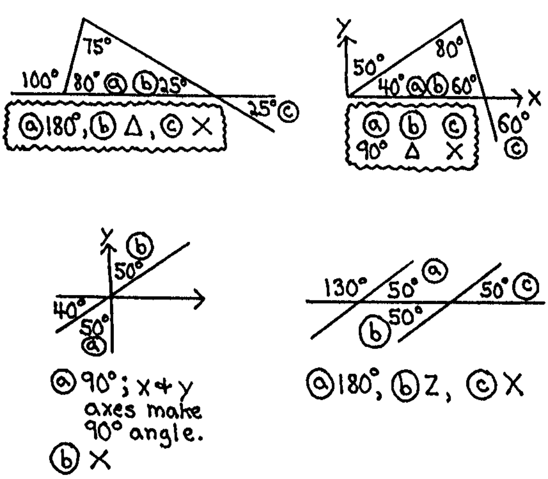
comment: the next two problems were in my original 1.91, but I think they should be cut; I don't use "vector subtraction" at all in this book, and multiplication-of-a-vector-by-a-scalar is covered briefly in Section 1.5 and (in greater depth) in Section 2.2.

1-9xx, optional: VECTOR SUBTRACTION
A has components of $A_x = +2$ and $A_y = 0$. **B**'s components are $B_x = +3$ and $B_y = -2$. Draw a rough sketch of **R** and **S**, where **R** = **A** + **B** and **S** = **A** – **B**. Find the components of **R** and **S**: R_x & R_y, S_x & S_y. {To find **A** – **B**, add "**A** + (–**B**)", where –**B** has the same magnitude as **B** but points in the opposite direction.}

1-10xx: When a woman runs for 2 minutes on a long straight road, she travels 300 m east and 400 m north. What is her total eastward and northward progress if she runs for an additional 6 minutes at this same velocity?

1.92 Optional Solutions

1-1: The step-by-step solutions below use these tools from Section 1.1: Δ X Y(90°, 180°) Z.



1-2: "a" and "b" give only magnitude and direction, respectively. This is not enough information.
"c" (displacement magnitude & direction) and "d" (displacement-components) give enough information.

1-3: For each triangle, you know 2 of the 4 right-triangle variables, so you can easily find the others:

A: $x = 7\cos 38.2°$, $y = 7\sin 38.2°$

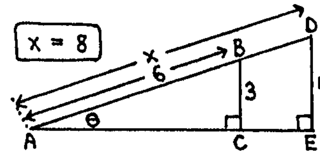
B: $H = \sqrt{5.50^2 + 4.33^2}$, $\theta = \tan^{-1}\frac{4.33}{5.50}$

C: $\sin\theta = \frac{4.33}{7.00}$, $x = \sqrt{7.00^2 - 4.33^2}$

D: $\tan 38.2° = \frac{y}{5.50}$, $\frac{\cos 38.2°}{1} = \frac{5.50}{H}$

1-4: Triangles ACB and AED each have angles of 90°, θ and 90°–θ. This makes them "similar triangles" that have the same side/side ratios.

Two good ways to set up an equation are shown below. Do you see the "ratio logic" of each method?



ABC has angles of θ, 90°, ADE has angles of θ, 90° } triangles are "similar"

RATIO of diagonals = RATIO of verticals: $\frac{x}{6} = \frac{4}{3}$

small-Δ RATIO = large-Δ RATIO: $\frac{3}{6} = \frac{4}{x}$

1-5: Lines cb, ba and ac form a right triangle, so the "ac diagonal" is $\sqrt{3^2 + 4^2}$. Lines dc, ca and ad also form a right triangle (do you see why?), so the "ad diagonal" is $\sqrt{(\sqrt{3^2+4^2})^2 + 5^2} = \sqrt{3^2 + 4^2 + 5^2}$.
A general formula: If a vector **A** has components A_x, A_y and A_z, it has a magnitude of $\sqrt{A_x + A_y + A_z}$.

1-6: Part 1: $\theta = \sin^{-1}(.340) = 19.9°$
$\cos\theta = \cos 19.9° = .940$
$\tan\theta = \tan 19.9° = .362$

Part 2

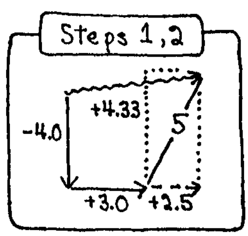
$y = 8 - 3\sin^2\theta - 3\cos^2\theta$
$y = 8 - 3(\sin^2\theta - \cos^2\theta)$
$y = 8 - 3(1)$
$y = 5$

Part 3

$\frac{\sin y}{\cos y} = \frac{12x}{10x}$
$\tan y = 1.20$
$y = 50.2°$

1-7: Draw, split, add x's & add y's, reconstruct.

Steps 1,2



Step 3

$R_x = (0)+(+3)+(+2.5) = +5.5$

$R_y = (-4)+(0)+(+4.33) = +.33$

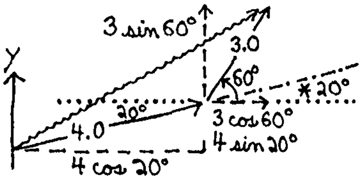
Step 4

$R = \sqrt{5.5^2 + .33^2} = 5.51$ m
$\theta = \tan^{-1}\left(\frac{.33}{5.5}\right) = 3.4°$ N of E

The man walks a *distance* of 12 m (4, 3 and 5), but his *displacement* is only 5.51 m. Do you see the difference between these descriptions of "movement"? For every physics situation & equation, think about which kind of "movement" should be used.

1-8: Draw the ···line parallel to the x-axis, and use Section 1.1's Z & X rules to find the *20° angle and that the angle marked " ↖ " is 20°+40° = 60°.

Then split, add x's & add y's, reconstruct, to get
$R_x = (+4\cos 20°) + (+3\cos 60°) = +5.26$, and
$R_y = (+4\sin 20°) + (+3\sin 60°) = +3.97$.
R-magnitude = 6.59, at 37.0° above the x-axis.



1-9, Part 1: **A** & **B** are different kinds of vectors, so they cannot be added, and there is no "answer".

Part 2: The largest possible **C+D** magnitude (5 m/s) occurs when **C** and **D** point in the same direction. If **C** & **D** point in opposite directions, they add to give the least magnitude: 1 m/s. For **CD** angles between these extremes, **C+D** is between 1 m/s and 5 m/s.

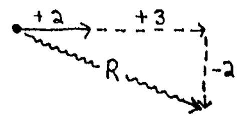
1-10: Use the "standard" conversion factor method.

$$60\,\frac{mi}{hr}\left(\frac{5280\,ft}{1\,mile}\right)\left(\frac{1\,hr}{60\,min}\right)\left(\frac{1\,min}{60\,s}\right) = 88\,\frac{ft}{s}$$

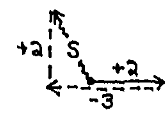
$$60\,\frac{mi}{hr}\left(\frac{5280\,ft}{1\,mile}\right)\left(\frac{12\,in}{1\,ft}\right)\left(\frac{1\,m}{39.37\,in}\right)\left(\frac{1\,km}{1000\,m}\right) = 96.56\,\frac{km}{hr}$$

cut? **1-9** xx: To find the x & y components of **–B**, just multiply **B**'s components by –1. For example, **B**'s x-component (which is +3, →) points in the direction opposite to **–B**'s x-component (which is –3, ←).

A + B



A - B



$R_x = A_x + B_x$
$= (+2)+(+3) = +5$

$R_y = A_y + B_y$
$= (0)+(-2) = -2$

$S_x = A_x + (-B_x)$
$= (+2)+(-[+3]) = -1$

$S_y = A_y + B_y$
$= (0)+(-[-2]) = +2$

cut? **1-10** xx: She will run 4 times as far in 8 minutes (the total time) as in 2 minutes, so her total progress is 4(300m) = 1200m east, and 4(400m) = 1600m north.

Her 2-minute displacement is 500m in a direction with $\theta = \tan^{-1}(300/400) = 36.9°$ E of N. Because her velocity is constant, her 8-minute displacement vector is 4 times as large (2000m) in the same direction, with $\theta = \tan^{-1}(900/1200) = 36.9°$ E of N.

2.91 Optional Problems

The recommended •-problems are either interesting examples of "standard problems" (as in 5, 6, 12, 14, 17, 18, 26 & 30), or they show how to use practical intuitive-thinking tools (as in 8, 19, 20 & 21).

For all problems, assume that friction and air resistance are negligible and can be ignored.

for Sections 2.1 and 2.2,

these ideas were covered in the OLD 2.2

2-1: Car A drives 20 m/s eastward. Car B drives 20 m/s northward. Do both cars have the same velocity? Does a car's speedometer measure velocity?

2-2: a) A man's v changes from 0 to +10 m/s in 2 s. A car's v goes from +20 m/s to +32 m/s in 3 s. Who has the larger v? the larger Δv? the larger a?

b) A woman runs 100 m east at 4 m/s, then runs another 100 m east in 11 s. What is her average velocity for the entire run?

2-3: A car is moving east. What direction does a point if the car's speed is increasing? if its speed is decreasing?

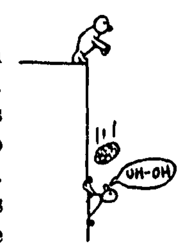
for Sections ~~2.4 and 2.5,~~ 2.3, 2.4

2-4: Simple tvvax Practice-Problems

a) A car travels –120 m in 5 seconds, while accelerating at +3 m/s². What is the car's final velocity? Is it getting slower or faster?

b) At t = 5 s, a car starts from rest at x = 120 m and accelerates at +3 m/s². When t = 15 s, where is the car?

c) A BIG ONE GETS AWAY: Oops! You drop a watermelon from the roof of a building. The runaway food races downward, splattering onto the ground after 1.50 seconds. What was the errant melon's top speed? How high is the building?



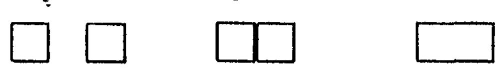
• 2-5: When the front of a 100 m long train is 60 m from a water tower, its speed is 20 m/s; it is accelerating at .5 m/s². What is its speed when the rear of the train passes the tower? { Hint: Draw i & f pictures! }

• 2-6: A man drops a rock from a hot-air balloon that is 20 m above the ground, rising at 4 m/s. When does the rock hit the ground?

2-7: The Great Race, Part I

What happens if you drop 6 identical blocks,

2 blocks are separated	2 are close together	2 are glued to form a big-block



Can you think of an argument, based on this thought-experiment, to show that small and large objects will fall at the same rate?

• **2-8**: RATIO LOGIC
Do Problems 1 & 2 of Section 18.9.

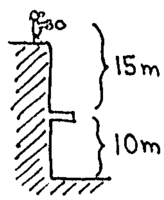
2-9: **More Ratio-Logic Problems**
Part 1) A rock is dropped from the roof of a building. It falls for time "t", and has a just-before-impact speed "v". If the rock drops from a building that is twice as high, what is its falling time and maximum speed?
Part 2) From rest, a car with **a** = 2 m/s^2 drives 400 m in 20 s. Will a car with twice the acceleration (4 m/s^2) finish in half the time?

2-10: A constant-a interval has $v_{average}$ = 20 m/s and Δv = –16 m/s. What is v_f?

for Sections ~~2.6 and 2.7~~, 2.5, 2.6

2-11: A car accelerates from rest to 20 m/s in 5 s, continues at constant speed for 10 s, then brakes at 5 m/s. What total distance does the car travel before it stops?

• **2-12**: A ball is thrown downward at 5.0 m/s. It crashes through a glass plate and loses 3/4 of its speed. Just before the ball hits the ground, what is its velocity?



2-13: A rock falls 15 m during 1 s. How far did it fall during the previous second?

• **2-14**: At t ≡ 0, a stoplight turns green; a 15 m/s constant-velocity truck is 15 m behind a stopped car (in a separate lane). One second later, the car begins to accelerate at 4.0 m/s^2. When does the car pass the truck? {Hint: Form a clear idea of the tvvax situation when the light turns green, 1 second later, and at the passing point.}

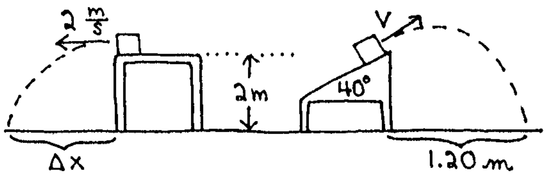
2-15, Part 1: When the front of his 20 m/s car is 52 m from a cliff, the driver brakes and decelerates at 4 m/s. Is the driver safe, or will he go over the edge into "free flight"?
Part 2: The driver of a 72 km/hr car sees a fallen tree 60 m ahead; after .40 s, she steps on the brakes and decelerates at 4 m/s. Will her car hit the tree?

2-16: An elevator's constant upward acceleration is 3.0 m/s^2; when its velocity is "v" downward, a rider drops a coin from 2.0 m above the floor. When does it hit the floor? {Hint: Choose two objects. Use the tvvax information that is seen by an observer standing on the ground.}

for Sections ~~2.8 and 2.9~~, 2.7, 2.8

• **2-17: A Home Run?** A baseball is hit when it is 1.0 m above the ground, giving it a v_i of 35 m/s at 40° above horizontal. Will the ball go over a 3.0 m wall that is 110 m away? {Hint: there are two good ways to find the answer.}

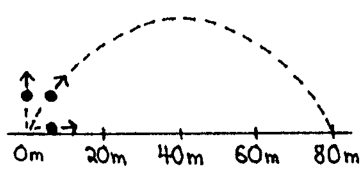
• **2-18**: The left table is horizontal; find Δx. Find v_i as the block leaves the right table.



• **2-19**: Use common sense "ratio logic", not calculations, to answer these two questions.
You throw a rock straight up into the air from ground level and it lands after 2.00 s. .50 s after you release it, the rock is at

This throw's peak height is 4.90 m. How long after release does it reach the 2.45 m "halfway point? a) between 0 & .50 s, b) at .50 s, c) between .50 s & 1.00 s, d) at 1.00 s.

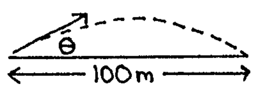
• **2-20: The meaning of "x-y independence".**
3 bullets are fired at the same time, from ground level. One slides horizontally on a frictionless surface with $(v_x)_i$ = +15 m/s, one moves vertically with $(v_y)_i$ = +26 m/s; another has $(v_x)_i$ = +15 m/s and $(v_y)_i$ = +26 m/s:



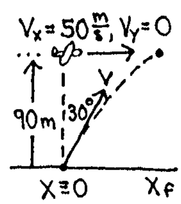
Draw rough sketches (use estimates of the y-position, as in Problem 2-19) that show the position of all 3 bullets when the horizontal bullet is at 0 m, 20 m, 40 m, 60 m and 80 m.

• **2-21: The independence of v and a.**
If an object has **v** = 0, can **a** be non-zero? If **a** is –, can speed be increasing? Can **v** & **a** point in opposite directions? Can there be a 90° angle between **v** & **a**? If you answer YES to any of these questions, give examples.
In Problem 2-D, does H (because it has the larger **a**) always have a larger speed than T?

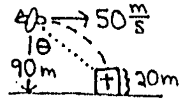
2-22: a) If θ = 20°, find v_i. b) If v_i = 39 m/s, find θ. {Hint: Use a tool from Section 2.9.}



2-23: When an airplane is directly overhead, a man fires a bullet that is aimed 30° ahead of the plane (see picture) and hits the plane. Find the bullet's initial speed, and x_f.



The airplane survives. To drop a medical-supply package onto the hospital roof, what must θ be?



2-24: If a free-flight interval has $y_i = y_f$, what elevation angle θ' gives maximum Δx? Which gives a larger Δx, $\theta_i = 40°$ or $\theta_i = 50°$?

If y_f is lower than y_i, will the θ" that gives maximum Δx be smaller or larger than θ'?

Hints: To find θ', find a useful formula in Section 2.9, and think. After awhile, read the discussion about "conflicting factors" in Section 19.10.

To answer the second question, consider an extreme case where y_f is thousands of meters lower than y_i, and ask "What does this do to the relative importance of the two conflicting factors?".

2-25: Equation-Derivations

When a free-flight object is at "x_i, 0", its velocity is "v_i" at angle "θ" above horizontal. At later times, its position is "x, y", where

$$y = y_i + (\tan\theta_i)\,x - \frac{g}{2\,v_i^2\cos^2\theta}\,x^2$$

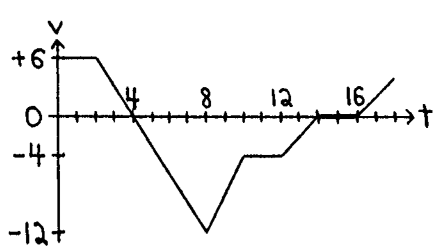
Derive this equation and also, using the fact that "2 sinθ cosθ = sin 2θ", the equation for free-flight motion when $y_i = y_f$:

$$\Delta x = \frac{v_i^2}{g}\,\sin 2\theta$$

Hint: There is no "t" in either equation. Make tvvax tables, then use the main tool from Section 2.3.

for Section ~~2.10,~~ 19.1

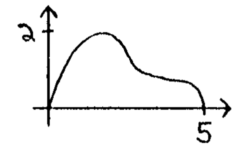
• **2-26**: For the v-t graph below, a) Find a time interval when v is – and speed is ↑ing (increasing). Find t when speed is largest, and when v is largest. b) When are the car's brakes being used? c) Find all t's when **a** is + and speed is ↓ing. Find t's when **a** is – and speed is ↑ing. d) Find t's when x is ↑ing and v is ↓ing. e) Find t's when the x-t graph has a ∩ shape. f) When is the car furthest from its starting point in the – direction and also moving back toward the starting point?



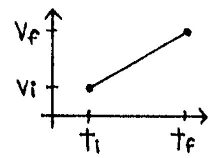
Draw a rough sketch (don't do calculations) of the x-t and a-t graphs that correspond to this v-t graph, if $x_i = -4$.

OPTIONAL: Make more accurate x-t & a-t graphs, by calculating areas and slopes.

2-27: Estimate this area, by drawing Δ's and/or □'s that have approximately the same area as ⌓.

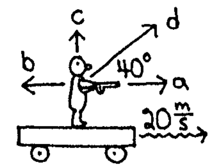


2-28: Use v-areas (try different kinds of areas & substitutions) to derive $\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$ and $\Delta x = v_i \Delta t + \frac{1}{2}a(\Delta t)^2$.



for Section ~~2.11,~~ 2.10

2-29: Four 50 m/s bullets are fired from a 20 m/s train, as shown. For each bullet, what $\mathbf{v}_i$ is observed by a person standing on the ground?



• **2-30**: An ice cream store is directly across a 50 m wide, 3 m/s river. If you aim a 5 m/s boat straight across, how quickly can you cross the river? Where will you land?

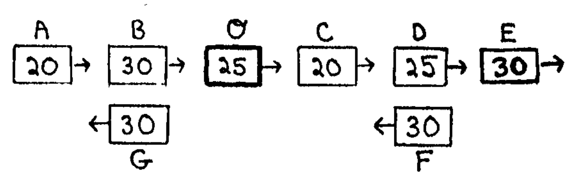
If you want to travel straight across the river so you land in front of the store, what direction should you aim the boat? How quickly will you cross the river?

To cross in the shortest possible time, what direction should you aim the boat?

2-31, Part 1: A plane with an airspeed of 100 m/s flies in a 20 m/s wind blowing from the west. To reach a city 500 km due north, what direction should the plane be aimed? How long will the trip take? { Hint: What do you think "airspeed" and "wind" are analogous to? }

Part 2) If a 20 m/s wind blows from 25° S of W, and you want to go north, what direction should you aim the plane? What will be your velocity with respect to the ground?

2-32: If you are in car "O" and you define your reference frame (car O) as v ≡ 0, what is the v of each other car with respect to you?



As in Section 2.11, a man runs 5 m/s rightward on a train whose v is 20 m/s rightward. A man standing on the ground throws a ball 30 m/s leftward toward the runner. What is the ball's $\mathbf{v}_i$, as observed by the runner?

Optional: If you used common sense to answer these questions, try using Section 2.11's "canceling trick" to find v for car A and for the ball.

2.92 Optional Solutions

2-1: Both cars have the same 20 m/s speed, but their directions differ, so they have different velocities. A speedometer measures speed (v-magnitude) only; it doesn't show v-direction, so it doesn't show velocity.

2-2: a) The car has larger v, and also larger Δv (+10 m/s versus +12 m/s). But the man has larger **a** (+10/2 = +5m/s^2 versus +12/3 = +4 m/s^2).
b) At 4 m/s, she runs the first 100 m in 25.0 s. After the second 100 m, she has run 200 m in a time of 25 s + 11 s = 36 s. $v_{avg} \equiv \Delta x/\Delta t = 5.6$ m/s east.

2-3: If speed is increasing, **a** points in the same direction as **v** (eastward). But if speed is decreasing, **a** points in the direction opposite to **v** (westward).

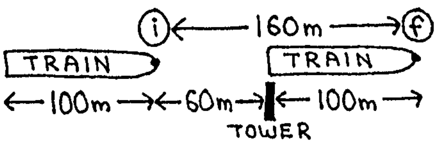
2-4: Make a tvvax table for each problem, look for 3-of-5, choose the "1-out" equation, substitute and solve. Answers are a) –16.5 m/s, **a** and **v** have opposite ± signs so speed is decreasing, b) at +30 m, c) 14.7 m/s, and 11.0 m (about 36 feet).

	Part a	Part b	Part c
$\Delta t =$	5	15 - 5	1.50
$V_i =$		0	0
$V_f =$	( )		( )
$a =$	+3	+3	+9.8
$\Delta x =$	-120	$X_f - (-120)$	( )

$$\begin{aligned} a:&\quad -120 = V_f(5) - \tfrac{1}{2}(+3)(5)^2 \\ b:&\quad X_f - (-120) = 0(10) + \tfrac{1}{2}(+3)(10)^2 \\ c:&\quad V_f - 0 = (+9.8)(1.50) \\ &\quad \Delta x = 0(1.5) + \tfrac{1}{2}(+9.8)(1.5)^2 \end{aligned}$$

Comments: In Part a, "travels" means that –120 is the car's change of location: Δx = –120. But in Part b, "at x = –120" and "...where is the car?" refer to the car's initial & final locations: x_i = –120, and "find x_f".
In Part c, I've chosen "down" to be the + direction.

2-5: This problem is easy if you draw pictures, and difficult if you don't. You know 3 of 5 (v_i = +20, a = +.5, Δx = +160), so you can find v_f = 23.7 m/s.



2-6: The key to this problem is seeing that the man and rock are being carried upward with the balloon, so the rock has v = +4 m/s when it is released.
Do you see why Δy is –12, and not just "12"?

Δt = ?
v_i = +4
v_f =
a = –9.8
Δx = – 12

$$-12 = (+4)t + \tfrac{1}{2}(-9.8)t^2$$
$$+4.9t^2 - 4t - 12 = 0$$
$$t = \frac{-(-4) \pm \sqrt{4^2 - 4(+4.9)(-12)}}{2(+4.9)}$$

The Q-Formula gives t = +2.03 s (the answer) and t = –1.21 s (impossible). Or you can use a "Quadratic Detour"; solve for v_f = –15.85, then find t = +2.03.

2-7: Will the blocks fall faster if the space between them decreases or if they're glued together?
No. All of the blocks, whether they're small or large, fall at the same "free flight" rate.

2-8: The solutions are in Section 18.9.

2-9, Part 1: v_i = 0, so four of the tvvax equations simplify to give $\Delta y = \frac{1}{2} v_f t$, $v_f = a t$, $\Delta y = \frac{1}{2} a t^2$, and $v_f^2 = 2 a \Delta y$.
When the building height doubles, Δy is multiplied by a factor of "x2", and **a** is constant. There is only one unknown multiplying factor in "$\Delta y = \frac{1}{2} a t^2$", and only one in "$v_f^2 = 2 a \Delta y$". As shown below, you can use these equations (with the Section 18.9 logic that "left side multipliers = right side multipliers") to find the multiplying factors for t and v_f: they are both $\sqrt{2}$.

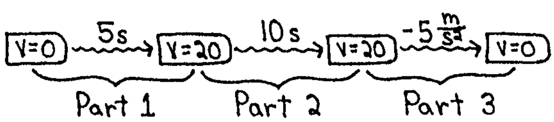
$$\begin{aligned} (\times 2) &= (\times 1)\,(?)^2 \\ \Delta y &= \tfrac{1}{2}\,a\;\; t^2 \end{aligned} \qquad\qquad \begin{aligned} (?)^2 &= (\times 1)(\times 2) \\ V_f^2 &= 2\,a\,\Delta y \end{aligned}$$

Part 2) Δx is constant, so $\Delta x = \frac{1}{2} a t^2$ has only one "multiplying-constant unknown". The multiplier for t is $1/\sqrt{2}$, so t for the second car is $20 \times 1/\sqrt{2} = 14.1$.

Or you can use this alternative solution method: the first car's "$\Delta x = \frac{1}{2} a t^2$" is "$400 = \frac{1}{2}a (20)^2$, so it has "a = 2.0 m/s^2". The second car, with doubled acceleration, has "$400 = \frac{1}{2}(4) t^2$", and t = 14.1.

2-10: **a** is constant, so $v_{average}$ is halfway between v_i and v_f; half of the "–16" change in v occurs on each side of +20. Δv is –, so v is decreasing; v_i was +28 (8 above 20), and v_f is +12 m/s (8 below 20).
Or write equations for Δv and $v_{average}$, "$v_f - v_i$ = +12" and "$\frac{1}{2}(v_f + v_i)$ = +20", and solve them for v_f.

2-11: Split the trip into 3 intervals, make tvvax tables, find each Δx, then add them to get Δx_{total}.



$$\left.\begin{aligned} \Delta x_1 &= \tfrac{1}{2}(0+20)(5) \\ \Delta x_2 &= (10s)(20\tfrac{m}{s}) \\ 0^2 - 20^2 &= 2(-5)(\Delta x_3) \end{aligned}\right\} \quad \begin{aligned} \Delta x_{TOTAL} &= \Delta x_1 + \Delta x_2 + \Delta x_3 \\ &= 50 + 200 + 40 \\ &= 290 \text{ meters} \end{aligned}$$

2-12: Choose the 4 special points shown below. The left table has 3-of-5; you can find $v_2 = +17.86$. The ball loses 3/4 of its speed, so $v_3 = (1/4)\ v_2$. The ball's final velocity is "14.7 m/s downward".

From 1 to 2	From 3 to 4
$\Delta t =$	$\Delta t =$
$V_1 = +5\ m/s$	$V_3 = \frac{1}{4}(17.86)\frac{m}{s}$
$V_2 = [17.86]$	$V_4 =\ ?$
$a = +9.8\ m/s^2$	$a = +9.8\ m/s^2$
$\Delta y = +15\ m$	$\Delta y = +10\ m$

2-13: Choose 3 special points (think about what they should be). For 2-to-3, $\Delta t = 1$, $a = -9.8$, $\Delta y = -15$; find $v_2 = -10.1$. During the "previous second" (the 1-to-2 interval), $\Delta t = 1$, $v_2 = -10.1$, $a = -9.8$; solve for $\Delta y = -5.2$ m.

2-14: We can choose i & f points to be 1 second after the light turns green (when car and truck are both at $x_i = 0$, and the car begins to accelerate) & when the car passes the truck at "$x_f = P$". The tvvax tables and 1-out equations are shown at the left below.

CAR, from 0 to A	TRUCK, from 0 to A	CAR, from 1 to B	TRUCK, from 0 to B
$\Delta t = A$	$\Delta t = A$	$\Delta t = B-1$	$\Delta t = B$
$V_i = 0$	$V_i = +15$	$V_i = 0$	$V_i = +15$
$V_f =$	$V_f = +15$	$V_f =$	$V_f = +15$
$a = +4$	$a = 0$	$a = +4$	$a = 0$
$\Delta x = P-0$	$\Delta x = P-0$	$\Delta x = P-0$	$\Delta x = P-(-15)$

$P-0 = 0(A) + \frac{1}{2}(+4)(A)^2$ and $P-0 = 15(A)$

Solutions: $A=0$, $A=7.5$

$P-0 = 0(B-1) + \frac{1}{2}(+4)(B-1)^2$, and $P+15 = 15B$.

Solutions: $B=1$, $B=8.5$

Method B: We can choose i & f to be when the light turns green (and the truck is at $x_i = -15$) & at the passing point "P". The tvvax tables & equations are shown at the right above. To find solutions, use the Quadratic Formula or Quadratic Detour.

The diagram below shows 3 events, and when they occur, using the time-definitions for Methods A & B.

Method A:	$t = -1$	$t = 0$	$t = A$
	LIGHT TURNS GREEN	TRUCK PASSES CAR	CAR PASSES TRUCK
Method B:	$t = 0$	$t = 1$	$t = B$

Do you see why the two passing-point times $A=0$ and $A=7.5$ are the same as $B=1$ and $B=8.5$?

2-15: The car decelerates (slows down), so v_i and a have opposite ± signs; if v_i is +20 m/s, a is $-4\ m/s^2$.

The problem doesn't ask for a specific thing (like Δx or t or...). You must think about what the tvvax situation is like "if the driver is safe". Two approaches are shown below. A) Find out how far he travels before coming to a stop. If Δx is more than 52 m, he is flying. Solving the "1-out equation" (you can do this by yourself) gives $\Delta x = 50$, so he is safe.

B) Find out what v_f is when he has moved +52 m. If v_f is + he is still moving forward, over the edge. The 1-out equation gives "$v_f^2 = -16$", which can't be solved. We get an impossible equation because the tvvax table is impossible; we assumed that the car reaches the +52m location, and it never does!

METHOD A		METHOD B	
Δt	$=$	Δt	$=$
v_i	$= +20$ m/s	v_i	$= +20$ m/s
v_f	$= 0$	v_f	$=\ ?$
a	$= -4\ m/s^2$	a	$= -4\ m/s^2$
Δx	$=\ ?$	Δx	$= +52$ m

Part 2: By the time she hits the brakes, the car has moved $(.4\,s)(72 \times 10^3\ m/3600\,s) = 8$ m. She is now 52 m from the tree, moving +20 m/s with $a = -4\ m/s^2$, just like in Part 1. She and the tree are safe.

2-16: Choose coin and floor as objects, i & f as the coin-release (when floor is at $x \equiv 0$, coin is at +2.0 m) & just-before-impact (when coin and floor are both at "P"). If "up" is +, floor & coin both have $v_i = -v$ [as in Problem 2-5, and Section 2.9's Release Principle].

After release, the coin is in free flight, and the floor still has "$a = +3.00\ m/s^2$".

COIN		FLOOR	
Δt	$= T$	Δt	$= T$
v_i	$= -v$	v_i	$= -v$
v_f	$=$	v_f	$=$
a	$= -9.8$	a	$= +3.0$
Δx	$= P-(+2)$	Δx	$= P-(0)$

$$\text{COIN: } P-2 = (-v)\,T + \tfrac{1}{2}(-9.8)\,T^2$$
$$\text{FLOOR: } P-0 = (-v)\,T + \tfrac{1}{2}(+3.0)\,T^2$$

Subtracting equations gives $-2 = \frac{1}{2}(-12.8)T^2$, the same equation that would occur if the rider observed a "free flight" coin acceleration of $-12.8\ m/s^2$. The coin has its usual "$a = -9.8$" and the floor accelerates up toward it with an extra $3.0\ m/s^2$, to give the apparent free-flight acceleration of $-12.8\ m/s^2$. $T = .56$ s.

2-17: The problem doesn't ask for a specific thing; you must think about what happens if it is a home run. In the left method below, is it a home run if $y_f = 2.0$ m? if $y_f = 8.0$ m? At the right, does the ball clear the wall if $\Delta x = 100$ m? if $\Delta x = 130$ m? (To understand the algebra, solve the equations yourself.)

When $\Delta x = 110$ m, what is y_f?

x-motion	y-motion
$\Delta x = V_x\, t_x$	$\Delta t =$ SAME AS t_x
$110 = 26.8\, t_x$	$V_i = +22.5$
	$V_f =$
	$a = -9.8$
	$\Delta y = y_f - (+1)$

$y_f = 10.8$ m (>3.0 m), HOME RUN!

When $y_f = 3$, what is Δx?

x-motion	y-motion
$\Delta x = V_x\, t_x$	$\Delta t = t_y$
$\Delta x = 26.8(\ )$ SAME AS t_y	$V_i = +22.5$
	$V_f =$
	$a = -9.8$
	$\Delta y = (3)-(1)$

$\Delta x = 120.6$ m (>100 m), HOME RUN!

2-18: For the LEFT TABLE's y-direction, $v_i = 0$ (release principle, "horizontal"), $\Delta y = -2.0$, $a = -9.8$; solve for $t = .64$ s. This is also t for the x-direction: $\Delta x = (-2.0 \text{ m/s})(.64 \text{ s}) = -1.92$ m.

For the RIGHT TABLE in the x-direction, $+1.20 = (+v_i \cos 40°)\,t$, so $1.57/v_i = t$. This is also t for the y-direction, which has $a_y = -9.8$, $\Delta y = -2$, and $(v_i)_y = +v_i \sin 40°$. We don't know anything about v_f, so we'll use the "v_f out" equation. After substitution, the equation looks complicated, but it is easy to solve:

$$\Delta y = v_i \quad T + \tfrac{1}{2} \quad a \quad T^2$$
$$(-2) = (.643v)\left(\frac{1.57}{v}\right) + \frac{1}{2}(-9.8)\left(\frac{1.57}{v}\right)^2$$
$$-2 = +1.01 - \frac{12.1}{v^2}$$
$$\frac{-3.01}{1} = \frac{-12.1}{v^2}$$
$$v = 2.0 \text{ m/s}$$

2-19: .50 s is 1/2 of the upward flight time. The rock travels further in the first .50 s (when it is still going fast) than in the second .50 s (when it has been slowed by gravity), so after .50 s it has moved more than halfway up. The answer is "c".

It is moving fast at the start, so less than half of the 1.0s zero-to-peak time is needed to reach 2.45 m. "a"

2-20: The horizontal and diagonal bullets have the same x_i, $(v_x)_i$ and a_x (= 0), so they always have the same x-position*, even though their y-motions differ. Why? Because x-motion is not affected by what happens in the y-direction! * As shown below, both are at 0, then both are at 20m, both are at 40m, ...

at 0m at 20m at 40m at 60m at 80m

The vertical and diagonal bullets have the same y_i, $(v_y)_i$ and a_y (= -9.8 m/s^2) so they always have the same y-position [as shown above], even though their x-motions differ. Why? Because y-motion doesn't depend on what happens in the x-direction.

2-21: All answers are YES. At the peak of a vertical throw or start of a dragrace, $v = 0$ but $a \neq 0$.

If a is – and v is – (like during free-flight descent), speed ↑. When speed ↓ during straight-line motion, v & a point in opposite directions.

At the peak of ⌒ free-flight, v is → and a is ↓.

{ Later, in Chapter 5B, you'll see that v and a are also ⊥ for *constant-speed circular motion.* }

No. For example, at 7 s, $v_H = 0$ and $v_T = +14$ m/s. At 11.67s, $v_H = v_T = 23.3$ m/s. After 11.67s, v_H is larger than v_T; eventually (at 19.05s), H passes T.

2-22: The ground is horizontal, so the i & f points are at the same height. If air resistance is neglected, the object's path is "symmetric" and we can use the formula "$\Delta x = v_i^2 (\sin 2\theta)/g$" to solve for v_i or θ.

$100 = v^2 \sin(2 \times 20°)/9.8$ gives $v = 39$ m/s.
$100 = (39)^2 (\sin 2\theta)/9.8$ gives $40° = 2\theta$ and $20° = \theta$.

2-23: Combine the "object" and "2-D" strategies.

30° is the angle away from vertical (not horizontal), so the bullet has $(v_i)_x = v \sin 30°$, $(v_i)_y = v \cos 30°$. Make 3 tvvax tables (x-airplane, x-bullet, y-bullet) and then look for equations that can be solved.

For the airplane's x-direction, $(x_f - 0) = (+50)T$, and for the bullet's x-direction, $(x_f - 0) = (v \sin 30°)T$. Solving "$50T = (v \sin 30°)T$" gives 100 m/s $= v$.

For the bullet's y-direction, the "v_f out" equation is $+90 = (100)(\cos 30°)T + \frac{1}{2}(-9.8)T^2$, which gives $T = 1.1$ s (if the bullet hits on its way up) or $T = 16.6$ s (if the bullet hits on its way down). If $T = 1.1$ s, $x_f = (+50)T = (+50)(1.1) = 55$ m.

To find θ for the hospital roof drop, make tables $\{t_x = T, v_x = +50;\ t_y = T, v_i = 0, a = -9.8, \Delta y = -90\}$, solve for $t = 50/\Delta x$, use the t-link substitution to get "$-70 = (0)T + \frac{1}{2}(-9.8)(\Delta x/50)^2$". Solve this for $\Delta x = 214$; $\theta = \tan^{-1}(\Delta x/\Delta y) = \tan^{-1}(189/70) = 69.7°$. This solution assumes that air resistance is negligible. Do you think this is a valid assumption?

2-24: As discussed in Section 10.10, θ' = 45°. If $y_i = y_f$, angles equally far above & below 45° (like 40° & 50°) give equal Δx; $\sin(2 \times 40°) = \sin(2 \times 50°)$.

If y_f is a lot lower than y_i, you'll get plenty of Δt, no matter what θ is. What you need is a large v_x! The θ" that gives maximum Δx is less than 45°.

Extra: In Problem 2-18, the 2 m/s flat-table has Δx = 1.28m, but Δx for the 2 m/s 40°-table is only 1.20 m. Do you see why the θ that gives maximum Δx must be somewhere between 0° (which is an "extreme") and 40° (which gives a smaller Δx)?

2-25: The hint suggests that you use the "t-link": $(x - 0) = (v_i \cos\theta_i)T$, so $x/(v_i \cos\theta_i) = T$. This T can be substituted into the y-direction's "v_f out" equation:

$$y - y_i = (v_i \sin\theta_i)\left(\frac{x}{v_i \cos\theta_i}\right) + \frac{1}{2}(-g)\left(\frac{x}{v_i \cos\theta_i}\right)^2$$
$$y = y_i + (\tan\theta_i)x - \frac{g}{2 v_i^2 \cos^2\theta_i} x^2$$

The equation-form is "$y = a x^2 + b x^2 + c$", which is the equation-form of a *parabola*. This shows that a free-flight object follows a path that is a parabola.

For the second derivation, $\Delta y = 0$. You can solve $0 = (v_i \sin\theta_i)T + \frac{1}{2}(-g)T^2$ for $T = 2 v_i \sin\theta_i / g$, and use the t-link: $\Delta x = (v_i \cos\theta_i)(2 v_i \sin\theta_i / g)$. Then replace "$\cos\theta_i\ 2\ \sin\theta_i$" with "$\sin 2\theta_i$", and you get $\Delta x = (v_i^2/g)\ \sin 2\theta_i$.

$\Delta x = v_i^2 (2 \cos\theta_i \sin\theta_i)/g$ shows why $\theta_i = 40°$ and 50° give the same Δx: $\cos 40° = \sin 50°$, and $\sin 40° = \cos 50°$, so $(\cos 40°)[\sin 40°] = (\sin 50°)[\cos 50°]$.

2-26: a) When speed is ↑ing, v is getting further away from the "v = 0" line; from 4 s to 8 s, this occurs and v is –. Speed is largest at 8 s, v is largest from 0s to 2s. b) From 2s-4s, 8s-10s, and 12s-14s.

c) a is + (and v-slope is +) from 8s-10s, 12s-14s and 16s-18s. If speed is ↓ing, a and v have opposite ± signs; a is +, so v is –; this occurs from 4s-14s.

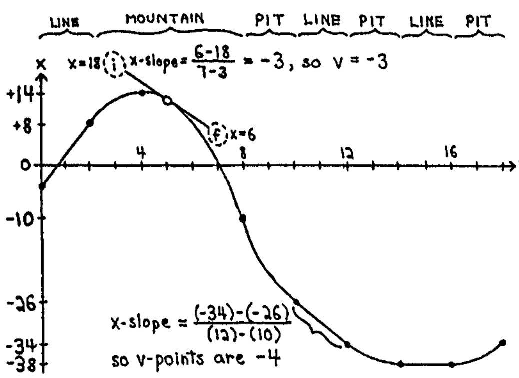
The times when a is + and v is – are 8s-10s and 12s-14s. // a (and the v-slope) is – from 2s-8s. Speed is ↑ing from 4s-8s and 16s-18s. Answer: 4s-8s.

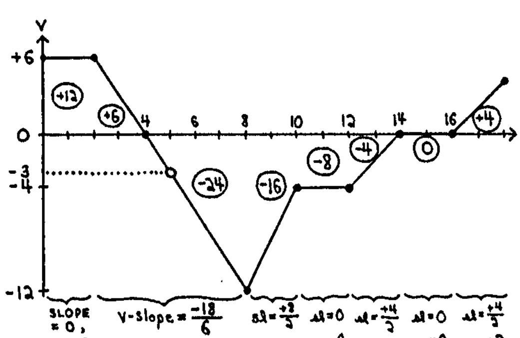
d) From 2s-4s, x ↑ (because v is +) while v ↓.
e) "mountain is minus"; x-t has ∩ shape when **a** is –, from 2s-8s. **f)** Just after 16s, the car is at its largest "negative x-position", moving in the + direction.

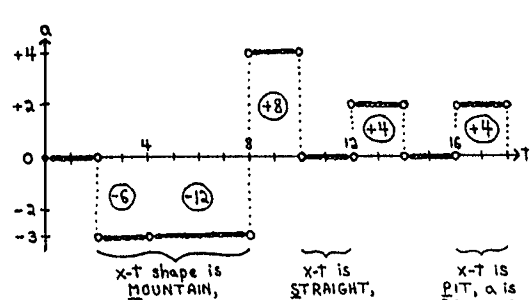
The table below emphasizes that graph relationships are often "package deals". For example, from 2s-4s all of these [v is +, v-area is +, Δx is +, x is ↑ ing] must occur if any of them occurs. Do you see why?

From 0s to 2s,	From 2s to 4s,	From 8s to 10s,
x ↑, Δx is +	x ↑, Δx is +	x ↓, Δx is –
⇓⇑ ⇑	⇓⇑ ⇑	⇓⇑ ⇑
v is +, + v-area	v is +, + v-area	v is –, – v-area
constant v, Δv=0	v ↓, Δv is –	v ↑, Δv is +
⇓⇑ ⇑	⇓⇑ ⇑	⇓⇑ ⇑
a = 0, a-area = 0	a is –, – a-area	a is +, + a-area

Use basic principles (v-slope → a-point, v-point → x-slope or v-area → Δx) to draw the general shapes of the a-t and x-t graphs, as shown below:







OPTIONAL: To use the numerical information on the original v-t graph, look at the times marked "•". For each •-to-• interval, calculate the v-slope (which gives a-points) and the v-area (which gives Δx).

For example, from 8s-10s, the v-slope (= +8/2 = +4) is constant, so all a-points from 8s-10s are +4.

From 8s-10s, v-area = ▭-area + Δ-area = (–4)(2) + ½(–8)(2) = –8 – 8 = –16. This equals Δx; the x-points change by –16, going from –10 to –26.

Notice how each v-area (+12, +6, –24,...) matches the x-change (–4 to +8, +8 to +14, +14 to –10,...). And the v-slopes (0, –3, +4,...) give the a-points.

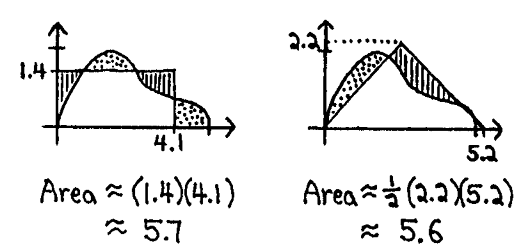
Extra: a-areas (0, –6, –12,...) equal the •-to-• Δv. x-shapes (line/straight, mountain, pit,...) give a-signs (zero, minus, positive).

At 5s (marked "o"), using the i & f points shown, the approximate instantaneous x-slope is calculated:

At 5s (marked "o") the approximate instantaneous x-slope is calculated, using the circled i & f points: x-slope ≈ {6m – 18m}/{7s – 3s} = –6 m/s. This gives the v-point "o" on the v-t graph. From 10s-12s the x-slope is constant at –4, so the v-points are all –4.

If you want extra practice, pretend that you know only the x-t graph above, then derive v-t & a-t. And if you only knew a-t (and also that when t = 0, x = –4 and v = +6), you could draw the v-t and x-t graphs.

2-27: At the left below, notice how the ⁘ area (that isn't included in the ▭ but should be) is approximately matched by the ▥ area (that is included but should not be). The same is true for the Δ-estimate. The ▭-area and Δ-area calculated below are approximately, but not exactly, equal to the true area.



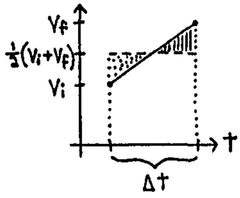
Draw your ▭ (or Δ) estimate with pencil and check the too-much and too-little areas visually to see if they cancel. Then, if necessary, change the shape until you get a good match between the ▭-area & true area.

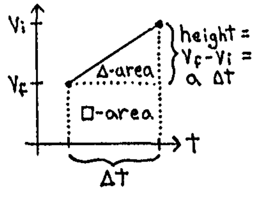
This ▭ and Δ are only two of many shapes (other ▭'s, other Δ's, combinations of ▭'s and Δ's, ...) that could be drawn to approximate the true area.

2-28: At the left, do you see why the ⁘ area and ▥ area cancel, so the ▭-area and ◿-area are equal?

At the right, the Δ-height is "$v_f - v_i = a\,\Delta t$". $\frac{1}{2}at^2$ is the "extra" distance (in addition to $v_i\,t$) that is traveled because v is being increased by acceleration.

Does the second derivation help you see that $\frac{1}{2}at^2$ is the "extra" distance (in addition to $v_i t$) that is traveled because v is being increased by acceleration?



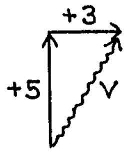


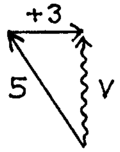
v-area = (height)(width)
$\Delta x = \frac{1}{2}(v_i+v_f)[\Delta x]$

Area = □-area + Δ-area
Area = (ht)(wid) + $\frac{1}{2}$(ht)(wid)
$\Delta x = (v_i)(\Delta t) + \frac{1}{2}(a\,\Delta t)(\Delta t)$

2-29: a) $v_x = (+20) + (+50) = +70$ m/s, $v_y = 0$. b) $v_x = (+20) + (-50) = -30$ m/s, $v_y = 0$. c) $v_x = +20$ m/s, $v_y = +50$ m/s, v = 53.8 m/s with "elevation angle" = $\tan^{-1}(50/20) = 68°$. d) Split the bullet-v into x & y components, then combine: $v_x = (+20) + (+50 \cos 40°) = +58$ m/s, and $v_y = (0) + (+50 \sin 40°) = +32$ m/s.

2-30: For the first trip, the motor & current v's are KNOWN (with both magnitude & direction given). For the second trip, the current v is KNOWN, but the motor v (magnitude given, direction unknown) and resultant v (straight-across direction known, magnitude unknown) are SEMI-KNOWN. First draw all that you know, then solve for what you don't know:





$v = v_y = 5.83$ m/s
$\theta = 31.0°$

$\Delta y = v_y \; \Delta t$
$+50 = 5.0 \; \Delta t$
$10 \text{ s} = \Delta t$

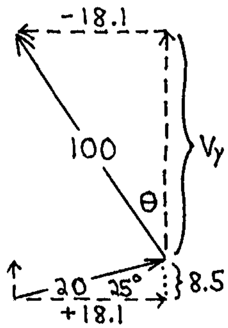
$v_y = 4.00$ m/s
$\theta = 36.9°$

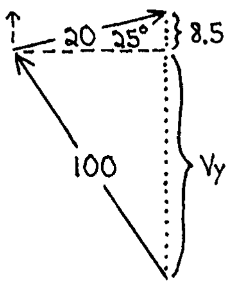
$\Delta y = v_y \; \Delta t$
$+50 = 4.0 \; \Delta t$
$12.5 \text{ s} = \Delta t$

Only the motor-v can move you across the river, because the current-v has no "y component". If you aim the boat straight across, every bit of this 5 m/s motor speed is used for crossing, thus giving the shortest possible time: 10 s.

2-31: "Airspeed" is like "motor v"; it is $v^{\text{plane}}_{\text{air}}$, the speed of the plane with respect to the air. "Wind" is like "current v"; it is $v^{\text{air}}_{\text{ground}}$, the speed of the air with respect to the ground. A wind *from* the west moves *toward* the east. The given info is just like in the 2nd trip of Problem 2-30, and so is the solution method. The aiming direction is 11.5° W of N, and $\Delta t = (500 \times 10^3 \text{ m})/(97.8 \text{ m/s}) = 5112 \text{ s} = 1.42$ hrs.

Part 2) The v-vectors can be added in two ways, as shown below. Here is the key idea: in order to go north, the +18.1 m/s x-component of the wind-v must be canceled by the x-component of the motor-v.





$\theta = \sin^{-1}\left(\frac{18.1}{100}\right) = 10.4°$
$v_y = 100 \cos 10.4° = 98.3$

$v_y = \sqrt{100^2 - 18.1^2} = 98.3$
$\theta = \tan^{-1}\left(\frac{18.1}{98.3}\right) = 10.4°$

GROUND SPEED = Total v_y = +8.5 + 98.3 = 106.8 m/s

2-32: Looking out the back window, you see A moving away (in the – direction) with v = –5 m/s. B is coming toward you (in the + direction) at +5 m/s. Looking out the front window, C comes toward you but (unlike B) it is moving in the – direction with v = –5 m/s. D isn't moving with respect to you, so it has v = 0. E moves away with v = +5 m/s. F is coming head-on toward you at –55 m/s; after it passes you it will look like G, which also has v = –55 m/s.

Observed from the ground, the runner's v is +25 m/s (like car O) and the ball's v is –30 m/s (like car F). The result is the same; the velocity of the ball, as seen by the runner, is –55 m/s.

Using "cancellation", $v^{\text{car A}}_{\text{car O}} = v^{\text{car A}}_{\text{ground}} + v^{\text{ground}}_{\text{car O}} = v^{\text{car A}}_{\text{ground}} - v^{\text{car O}}_{\text{ground}} = (+20) - (+25) = -5$ m/s.

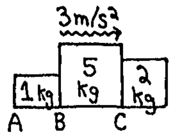
Similarly, $v^{\text{ball}}_{\text{runner}} = v^{\text{ball}}_{\text{ground}} + v^{\text{ground}}_{\text{train}} + v^{\text{train}}_{\text{runner}}$
$= v^{\text{ball}}_{\text{ground}} - v^{\text{train}}_{\text{ground}} - v^{\text{runner}}_{\text{train}}$
$= (-30) - (+20) - (+5)$

3.91 Optional Problems

Many of the •-problems teach important intuitive principles. Or just browse through the problems (read them, look at pictures) to find the ones you think will be be useful.

for Section ~~3.8~~,3.2

3-1: To give these blocks the acceleration shown, what "Normal force" must be exerted at faces A, B and C ?



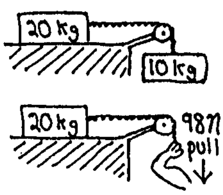
3-2: To jump upward with an acceleration of 20 m/s², what normal-force must a 70 kg man's feet exert against the ground?

3-3: A 200 gram ball is dropped 5.0 m, and rebounds to a height of 4.0 m. What average force acts on the ball during the "bounce" if it is in contact with the ground for .01 s?

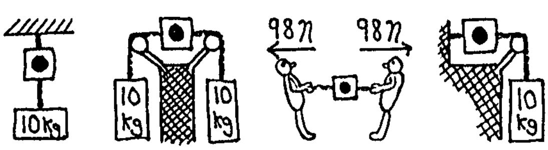
• 3-4: In New York, g = 9.803 m/s², while in Denver, g = 9.796 m/s². If a bowling ball is moved from New York to Denver, will its mass change? Will its weight change? If a ball is dropped from a height of 3.000 m, will its time-of-travel differ in the two cities?

• 3-5: A rope tied to a 10.0 kg block is pulled upward with gradually increasing T. What is N just before the block leaves the ground?

3-6: Does a for the 20 kg block change if the 98*N* weight (top picture) is replaced by a 98.0*N* force that pulls on the rope (bottom picture)?

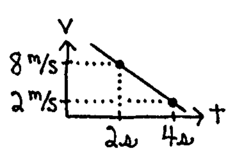


In the first picture below, a "fish weighing" scale (⊡) registers 98*N*. What does the scale register in the other situations?



• 3-7: A force can accelerate a 1000 kg car at a = 1.00 m/s². This same force gives a truck a = .25 m/s². What is the truck's mass?

• 3-8, Part 1: Here is the v-t graph of a 40 kg object. When t = 2.5 s, what net force acts on the object?



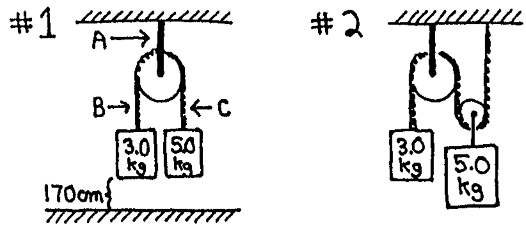
Part 2, CALCULUS (If you're in a non-calculus course, you can skip the rest of this problem.)

If a 50 kg block has "v = 3 t²", what force acts on the block when t = 1.4 s ?

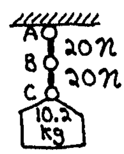
If a 50 kg block has "x = t³", what force acts on the block when t = 1.4 s ?

• 3-9: In the pictures below, the pulleys are massless, with frictionless bearings. For #1, find the tension force in massless ropes A, B and C. When will a block hit the ground?

In #2, the small pulley is free to move up or down; find the blocks' accelerations. (Hint: When the 5.0 kg block moves 10 cm, how far does the 3.0 kg block move?)



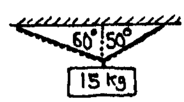
3-10, Part 1: Each rope weighs 20 N. Find the tension-force acting on each of the massless rings A, B and C.



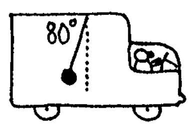
Part 2) Write F=ma for the second rope of Problem 3-B, and show that if the rope has m=0, the T-force at each end must be equal.

2-Dimensional Problems

3-11: Find the T-force in each rope. (Hint: use the rope-knot as "object".)



• 3-12: A ball hanging from the roof of a truck makes an angle of 10° with the vertical. Find the truck's acceleration.



What happens to the ball when the truck moves at constant speed? when it brakes?

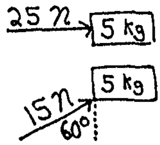
• 3-13: A glass-walled truck drives at 20 m/s down a dark street at night. A passenger drops a brightly glowing ball from 2 m above the truck floor. Describe the general path of the ball (straight, curved,...) as it is seen by a person standing on the curb.

PSEUDO FORCES: What general ball-path does a passenger see if the truck's velocity is constant during the ball's flight? What does he see if the truck suddenly accelerates? if it brakes? if it swerves sharply to the right?

3-14: If Problem 3-3's man pulls a 10.0 kg block with 240*N* at 25.0° above horizontal, what happens?

If this is to be avoided, what is the maximum allowable pulling force? While this maximum force is being applied, what is "N" between the block and sliding-surface?

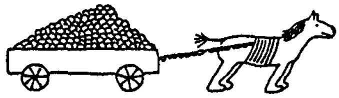
3-15: To give it an acceleration of 3.0 m/s^2 toward the right, what single extra force must be applied to each box?



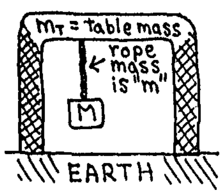
3-16: What is the tension-force at the middle of this 2.0 kg rope? Both ends of the rope are at the same height.

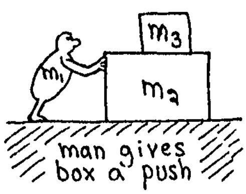
for Section 3.5,

3-17: A Lazy Horse Trick. You ask a horse to pull a wagon, but he refuses to even try, arguing that his best efforts will be futile because "No matter how hard I pull on the wagon, it will pull back on me just as hard. Our forces will cancel, and we won't go anywhere." Could you refute the horse's logic?



• **3-18**: For the situations below, draw a "free body" F-diagram for each object. (There is friction at all surface-contacts in the second picture.) Hint: There are 8 objects, and 17 force-pairs (34 F's).



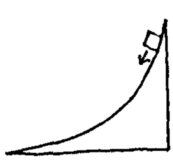


• **3-19**: For each interaction below, decide which of the 2 objects feels the greater force.

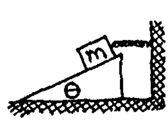
for Section 3.6,

3-20: From rest, how long does it take a box to slide down a 35° frictionless 20 m ramp?

• **3-21**: As the block slides down this curved frictionless ramp, at what point does it have the largest speed? the largest acceleration?



3-22: A horizontal rope keeps this block at rest. Find the rope tension & the block/ramp N-force.



3-23: In Problem 3-A, what is the man's apparent weight if his acceleration is 2.0 m/s^2 downward? if the cable is cut? if a is 11.0 m/s^2 downward (how could this occur)?

3-24: A block starts from rest at the top of a 3 m high frictionless ramp. What is its speed at the bottom of the ramp?

for Section 3.7,

• **3-25**: μ_k is always < (less than) μ_s. Does this mean that f_k is always < f_s?

When a car is moving, is friction between its tire and the road "static" or "kinetic"?

To get maximum acceleration at the start of a car race, should you step on the gas so hard that the tires break loose and spin, or back off a little (how much?) to prevent this? When you brake, should the tires "screech"?

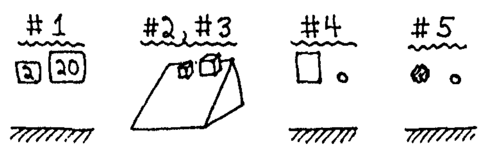
3-26: Redo Problem 3-20, but with $\mu_k = .30$.

• **3-27**: A block on a horizontal surface has $\mu_s N = 29.4 N$, and $\mu_k N = 19.6 N$. Will it move if simultaneous forces of 28.0 N eastward and 8.0 N northward act on it? What if the forces are 28.0 N east & 10.0 N north? In each case, find the magnitude & direction of friction.

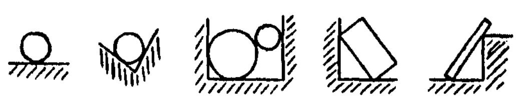
3-28: With a protractor (a tool to accurately measure angles), how can you find the μ_s between a smooth plane and a penny?

• **3-29**: **The Great Race, Part 2**

In each of the five races below, objects are released simultaneously, with their bottoms at the same height: Race #1 matches steel balls with masses of 2 kg and 20 kg. In #2, 2 kg and 20 kg blocks race on a frictionless ramp. In #3, this race occurs on a friction-causing ramp; both blocks have the same μ_k. In #4, a sheet of paper races a penny. In #5, the paper is compressed into a tight ball. Who wins each race-to-the-ground?



3-30: Draw the "N" forces on each ball, and the N & friction forces acting on each block.



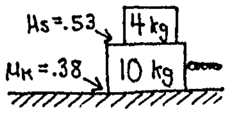
3-31: From 24.6 m/s (55 mi/hr), a car stops in 64.0 m (210 ft) on a dry road and 152.4 m (500 ft) on a wet road. If the driver uses 90% of the available μ_s, find the wet and dry μ_s.

3-32: If P = 70 Newtons and $\mu_s = .40$, will the 4.0 kilogram block move?

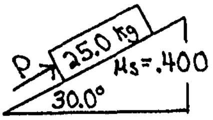
• **3-33**: a) If $\mu_k = .30$, what horizontal pull is needed to keep a 50 kg block moving at 3 m/s? at 6 m/s? b) If the pull is directed 25° above horizontal, what force is needed for constant-velocity motion? (Do you expect this force to be more or less than when the pull is horizontal?)

OPTIMIZATION: What "optimal pulling angle" will minimize the rope force needed to maintain constant speed, if $\mu_k = .30$? Will this angle change if $\mu_k = .43$?

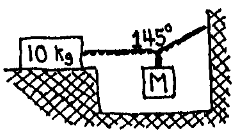
3-34: Piggy Back Blocks. A small block rides on top of a large block. To give the top block maximum acceleration, how hard should you pull on the horizontal rope?



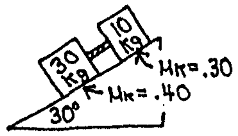
• **3-35**: What range of P-values (minimum and maximum) will keep the block at rest? (Hint: What happens if P is too small? too large?)



3-36: If this system is to remain at rest, what is the maximum value "M" can have? $\mu_s = .60$

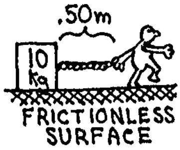


3-37: The blocks are connected by a rigid massless bar. What force does this bar exert on the 10 kg block? (Hint: Would it move faster if the bar was removed?) What will happen if the positions of the blocks are reversed?

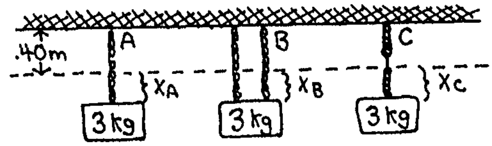


for Section 3.8,

3-38: When a spring (x_e = .40 m) stays stretched to the length that is shown, the block's **a** is .40 m/s². What is the spring's "k"?



3-39: Each spring below has $x_e = .40$ m and k = 150 N/m. A motionless block is supported by one spring (in A), side-by-side springs (B), or two end-to-end springs (C). Find x_A, x_B and x_C. (Pictures are not drawn "to scale".)



3.92 Optional Solutions

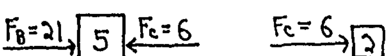
3-1: Draw Force-diagrams for 1-5-2-together, 1-5-together and 1-alone; ignore all "internal forces", as discussed in Problem 3-B. F_B acts between 1 & 5, and F_C acts between 5 & 2. Then solve the F=ma's:

$F_A = (8)(+3)$, $F_A = +24$

$+F_A - F_C = (6)(+3)$ ⇓ $(+24) - F_C = +18$, $+6 = F_C$

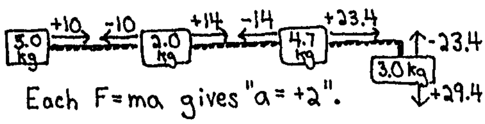
$+F_A - F_B = (1)(+3)$ ⇓ $(+24) - F_B = +3$, $+21 = F_B$

Other combinations of object-choices (for example, 1-2-5, 5-alone & 2-alone) also give these solutions. An optional "check": Will these N-forces, when they act on 5-alone and 2-alone, give a 3 m/s² acceleration?

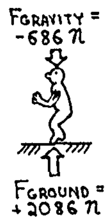


$(+21) + (-6) = 5a$ $(+6) = 2a$

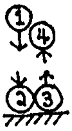
You can do a similar F=ma check for Problem 3-B. Will the net force on each block give it "a = 2 m/s²"?



3-2: The man's F=ma is
$-70g + N = 70(+20)$.
His legs exert 2086*N* downward against the ground so the ground will, because of the third-law mutual interaction, push him upward with 2086*N*. 686*N* overcomes his own mg, and the "extra" 1400*N* gives him an acceleration of +20 m/s².



3-3: Use "time splits" as in Section 2.6.



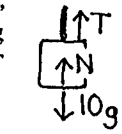
1 to 2	2 to 3	3 to 4
t =	t = .01	t =
v_1 = 0	v_2 = [ ]	v_3 = ( )
v_2 = ( )	v_3 = [ ]	v_4 = 0
a = -9.8	a =	a = -9.8
Δy = -4.0	Δy =	Δy = +3.0

Use tvvax to solve for $v_2 = -8.85$, $v_3 = +7.67$, and $a = \Delta v/\Delta t = \{(+7.67) - (-8.85)\}/.01 = +1652$.
The *a-link* gives $F_{1\text{-to-}2} = ma = (.2)(+1652) = 330N$.

3-4: The ball's mass is the same in both cities, but its weight changes from "mg = m(9.803)" to m(9.796).
The ball's dropping-time will be longer in Denver, by a factor of $\sqrt{9.803/9.796}$, because $\Delta y = \frac{1}{2} a t^2$. (This is discussed in Section 18.9, "Ratio Logic".)
The times are $\Delta t_{NY} = .7823$ s, $\Delta t_D = .7826$ s.

Does mass ever change? The answer to this question (as it is discussed in Section 16-#) may surprise you.

3-5: When the block is on the ground, N & T (upward) combine to cancel 10g (downward). Just before lift-off, when T = 97.9*N*, N = .1*N* (almost zero). When T = 98.0, a = 0 because T and mg cancel; N = 0 even though the block is still in contact with the floor. When T = 98.1*N*, T > mg and the block accelerates upward; after lift-off, N = 0, of course.



3-6: Yes. With a 98*N* force, the rope-T is 98*N*, the 20-block's F=ma is "98 = 20 a", and a = 4.9 m/s^2.
But with a 98*N* weight, T is only 65.4*N* (not 98*N*) and a = 3.27 m/s^2. Prove this to yourself by solving F=ma for 20-and-10 { +T −T +10(9.8) = (20+10) a } and for 20-only { +T = 20 a }.

As far as the scale is concerned, all four situations are the same; a 98*N* force pulls it in one direction, and it is held motionless by the ceiling, counterweight, tug of war partner, or wall. Every scale reads 98*N*.

3-7: Ratio logic: F is constant, so ma is constant. "a" is multiplied by 1/4, so m must be multiplied by 4. $m_{truck} = 4\, m_{car} = 4(1000) = 4000$ kg.

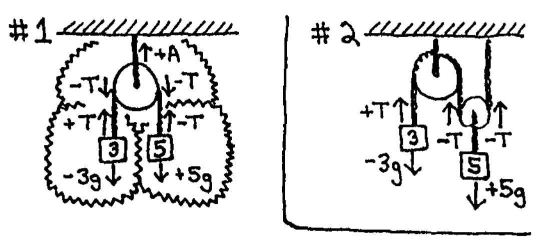
Or solve the car's F=ma for "$\underline{F} = \underline{m_{car}(1.00)}$", then find $m_{truck} = \underline{F}/a_{truck} = \underline{m_{car}(1.00)}/(.25) = 4\, m_{car}$.

3-8: Find the slope of the v-t graph: $\Delta v/\Delta t = a$.

$$F = m a = m \frac{\Delta v}{\Delta t} = 40\frac{(+8)-(+2)}{4-2} = 120\,N$$

Part 2: This problem is easy. Just use the calculus relationship between x, v & a: a = dv/dt = d(dx/dt)/dt = d^{2}x/dt^2. **If you know an object's v-formula or x-formula, you can find its a-formula.**
F = ma = m[dv/dt] = m[d(3t^2)/dt] = m[6t].
Substitution gives F = m[6t] = (50)[6(1.4)] = 420*N*.
The second block has the same v-formula as the first block, v = dx/dt = d(t^3)/dt = 6t, so at 1.4s it feels the same 420*N* force.

3-9, #1: If rope & pulley are massless, $T_B = T_C$. I've chosen "up" to be + for the pulley, and "direction of acceleration" to be + for the blocks. F-diagrams for both pulleys (#1 and #2) are given below, followed by the F=ma solutions for Situation #1.



for 3-block	for 5-block	for pulley
+T − 3(9.8) = 3(2a)	−T + 5(9.8) = 5a	+A − T − T = (0)(0)
T − 29.4 = 3(2.45)	(add equations)	A = 2T
T = 36.8 N	49.0 − 29.4 = 5a + 3a	A = 2(36.8)
		A = 73.6 N

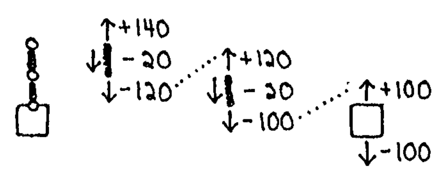
To find Δt, use the *a-link* (tvvax, F=ma). Solve "+1.70 = (0) t + ½ (2.45) t^2" for t = 1.18 seconds.

Study the F-diagram for #2 carefully. Notice that when the 5-block moves <u>.1 m</u> downward, each of the two ropes above it is lengthened by .1 m. The total rope length is constant, so the rope above the 3-block is shortened by .2 m; the 3-block moves <u>.2 m</u> upward! If the 5-block has "Δy", "v" and "a", the 3-block will have "2 Δy", "2v" and "2a". I'll assume the 5-block moves downward, and call this the + direction.

+T − 3(9.8) = 3(2a)	−T − T + 5(9.8) = 5(a)
T = +29.4 + 6 a	−2T + 49 = 5a
T = 29.4 + 6(−.58)	−2(+29.4 + 6a) + 49 = 5a
T = +25.9	−.58 = a

"a = −.58 m/s^2" says the 5-block is pulled upward by the 3-block! It may seem amazing, but this really does happen. This is an example of ***leverage***. <u>You gain something</u> (a relatively small 3g force can lift 5g) <u>by giving up something</u> (to move the 5-block .1 m, the 3-block must move .2 m, twice as far).

3-10, Part 1: You can draw F-diagrams for each rope, as shown below, or use common sense: Ring A supports all of the 140 N weight, while B supports only one rope and the block (120 N), and C supports the 10.2 kg block (100 N).



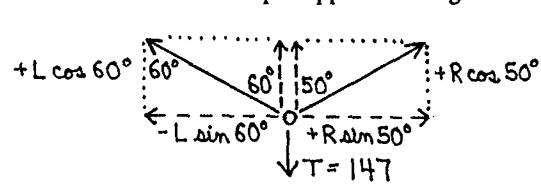
Because each object has a = 0, the net force on the top rope, bottom rope and block should be 0. Is it?

Part 2) If T at the rope's ends is $-B_{left}$ & $+B_{right}$,

$B_{left} \Leftarrow\!\!\sim\!\!\Rightarrow B_{right}$,

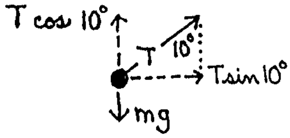
then $a_{rope} = F_{on\ rope}/m_{rope} = (-B_{left} + B_{right})/0$.
If $B_{left} \neq B_{right}$, a = (something)/0 = ∞; this is wrong [the rope has a = 2 m/s^2, just like the blocks] so B_{left} must equal B_{right}. (**This argument can also be used to show that if the rope has m ≠ 0, then $B_{left} \neq B_{right}$.**)

3-11: Solve F=ma (x & y directions) for the knot. T = 147*N* because the rope supports a 15 kg block.



$(F_x)_{TOTAL} = m a_y$	$(F_y)_{TOTAL} = m a_y$
−L sin 60° + R sin 50° = m(0)	+L cos 60° + R cos 50° − 147 = m(0)
R = 1.13 L	L(.5) + R(1.13L)(.643) = 147
R = 1.13(119.8)	L = 119.8

3-12: Draw ball's F-diagram, solve x & y F=ma.



$$F_x = ma_x \qquad F_y = ma_y$$

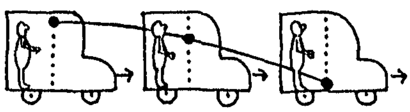
$$+T \sin 10° = ma_x \qquad +T\cos 10° - mg = m(0)$$

$$\left(\frac{mg}{\cos 10°}\right)\sin 10° = ma_x \qquad T\cos 10° = mg$$

$$1.7 \text{ m/s}^2 = a_x \qquad T = \frac{mg}{\cos 10°}$$

If you stand up in a truck, what happens when the truck speeds up? slows down? turns a corner?

3-13: If the truck's v is constant, it and the ball both move forward at 20 m/s. The pictures below show the truck & ball at 3 different times. The ball follows a curved path (as seen from the ground), but it drops straight down (as observed by the passenger).



If the truck suddenly speeds up (so its v exceeds 20 m/s) after the ball is released, a truck rider will see the ball (which still has the 20 m/s "release v") move backward like a slow runner in a race: see 1st picture.

If the truck brakes, a rider sees the ball move forward like a relatively fast runner (2nd picture).

The 3rd picture is a "bird's eye view". It shows that a person looking down from a telephone pole will see the ball move straight ahead. But a truck rider sees the ball move toward the left side of the truck.

PSEUDO FORCES: In each situation, a rider says "When I drop a ball from rest, it should (if it is in free-flight), fall straight down toward the truck floor. The ball isn't falling straight down, so it must (because F=ma) have a sideways force acting on it."

This sideways force has no "cause" (no ropes, no gravity, no...) so it is called a ***pseudo-force***. The rider thinks he sees a force because he is observing the motion from a reference frame that is being accelerated. Pseudo-forces, which also occur in Problems 2-16, 3-12 & 3-23), are discussed in more detail in Problem 5.# and in Section 16.#.

3-14: The upward component of T is 240(sin 25°) = 101.4*N*. This exceeds the block's downward mg of 10(9.8)*N*, so the block is lifted off the ground.

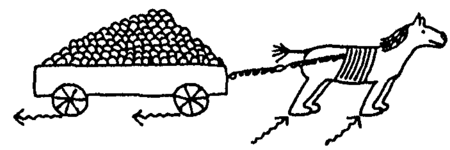
If T = 231.9*N*, downward mg is exactly canceled by upward T(sin 25°), so no upward N is needed and N = 0. The block will not "fly" if T < 231.9*N*.

3-15: To get $a_x = +3$ m/s^2 and $a_y = 0$, the first block must have an extra force of 10*N* toward the left.

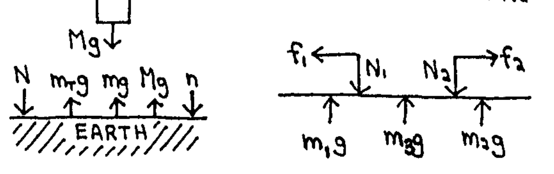
For the second block, split the 15*N* into x & y components of +13.0*N* & +7.5*N*. To get $a_x = 3$ m/s^2 requires $(F_x)_{total} = +15N$; there is 13*N* already, so an "Extra" force of $E_x = +2.0N$ is needed. To get $a_y = 0$, the +7.5*N* must be canceled by $E_y = -7.5N$. Now reconstruct E_x & E_y to get $E_{total} = 7.76N$ with $\theta = 75°$.

3-16: Hint — Treat the left half of the rope as an "F=ma object". {The solution is after 3-19.}

3-17: As you might expect, the horse did not know his physics. If the horse can push against the ground (and thus have it push back against him, as shown by the ↗'s below) with more horizontal force than the friction-etc. pulling back on the wagon (the ←'s), he can make it move. The rope-forces between the wagon and horse (these were the focus of the horse's erroneous argument) are "internal"; they just transfer the ← and ↗ forces within the horse-wagon system.



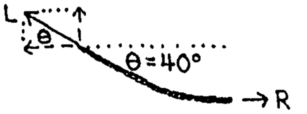
3-18: The forces acting on 8 objects are drawn below. Notice that every F has an equal-and-opposite partner. For example, T_2 (rope is pulled by block, and block is pulled by rope), P (man is pushed by box, box is pushed by man), and so on.



3-19: Every situation involves a "mutual third-law interaction", so each object feels the same force.

Comments: COLLISION: As you might expect, the car driver is more likely to get hurt than the truck driver, even though the F's are equal. {The reason for this is discussed in Problem 4-#.} TACKLE: The tackler's shoulder-pads will hurt less than the tacklee's ribs. PUNCH: With equal F applied to fist and nose, the nose loses. But in fist versus wall, the wall wins.

(**3-16**): The left & right ends of the rope are at the same height so, by symmetry, the middle of the rope is horizontal and so is the R-force pull. {To make the rope horizontal ($\theta = 0$) would require infinite force and an unbreakable rope: $L = mg/\sin(0°) = mg/0 = \infty$.}

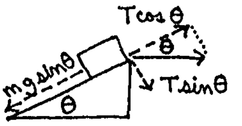


$$F_x = ma_x \qquad F_y = ma_y$$
$$+R - L\cos\theta = m(0) \qquad L\sin\theta - mg = m(0)$$
$$R = \left[\frac{2.0(9.8)}{\sin 40°}\right]\cos 40° \qquad L = \frac{mg}{\sin\theta}$$

3-20: Split mg and solve the down-the-ramp F=ma [+mg sin 25° = m a], then use the a-link and solve "+20 = 0 + ½(+5.6)t^2" for t = 2.7 s.

3-21: The block's "a" is largest at the top, where the ramp is steepest, but its "v" is largest at the bottom. As emphasized in Chapter 2, v and a are different! As the block slides downward, a is decreasing while v is increasing, so a is big when v is small, and vice versa.

3-22: Split T into components down-the-ramp and ⊥-to-the-ramp (use Section 1.1's ΔXYZ tools to find which angle is θ), then solve the F=ma equations.



$$mg\sin\theta - T\cos\theta = m(0)$$
$$mg\,\frac{\sin\theta}{\cos\theta} = T$$

$$+N - T\sin\theta - mg\cos\theta = m(0)$$
$$N = mg\left(\frac{\sin\theta}{\cos\theta}\right)\sin\theta + mg\cos\theta$$
$$N = mg\left(\frac{\sin^2\theta}{\cos\theta}\right) + mg\left(\frac{\cos^2\theta}{\cos\theta}\right)$$
$$N = mg\left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta}\right)$$
$$N = mg\,\frac{1}{\cos\theta}$$

3-23: If a = 2 m/s² downward, the man's F=ma is "–70g +N = 70(–2)", and N = 546; the man weighs less than usual. If the cable is cut, the man & elevator are both in free-flight with a = –9.8 m/s². The man's F=ma is "–70g + N = 70(–9.8)", and N = 0, showing that the man is "weightless".

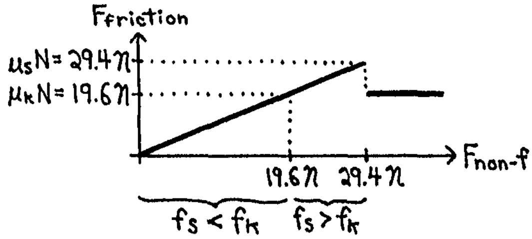
If a = –11.0 m/s², N = –84 Newtons. N is just a magnitude so, as discussed in Section 3.6, the – sign of –84*N* says "you goofed!". Our F-diagram assumed that N points upward; the – sign says "No, you made a wrong choice; N points downward." If the man's shoes are glued to the scale (and it to the floor) he will pull the scale upward. All objects that aren't nailed down will "fall up" toward the elevator's roof.

To get this larger-than-g acceleration, you could either pull the elevator downward, or push it down like a "slam dunked" basketball.

3-24: We have 3-of-5 tvvax; $v_i = 0$, "+mg sin θ = ma" can be solved for a = + g sin θ, and the diagram shows that Δx = 3/sin θ. Now solve "$v_f^2 - 0^2$ = 2(g sin θ)(3/sin θ)" for v_f = 7.67 m/s.

3-25: In a Section 3.7 example, f_k was 19.6*N*, while f_s varied from 0 to 29.4*N*; f_s was 0 and 10.0*N* (these are less than f_k) and 29.3*N*. $\mu_s N$ is always larger than $\mu_k N$; but f_k always equals $\mu_k N$, while f_s can be much less than $\mu_s N$ (and much less than f_k).

As shown below, if you gradually increase $F_{non\text{-}f}$, f_s also increases (to "match" $F_{non\text{-}f}$) and you pass from a region (0 to 19.6) where $f_s < f_k$ to a region (19.6 to 29.4) where $f_s > f_k$. But if you push too hard and exceed $\mu_s N$, friction drops to $\mu_k N$.



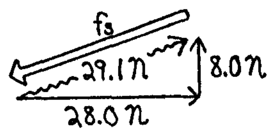
If a car moves at constant speed or with gradual acceleration, its tires move (they are carried forward along with the car and they also rotate) but they don't slide, so friction is static. If a car accelerates or brakes too quickly, its tires screech because there is slipping between tires and ground, and friction is kinetic.

If you step on the gas too hard, tires "break loose" and the f_k available for acceleration is $\mu_k N$. If you let up on the gas, friction becomes static; f_s can vary from 0 (you lose the race!) to $\mu_s N$ (which is larger than $\mu_k N$). Optimal strategy requires control; you want to push f_s all the way to the $\mu_s N$ limit, but not past it.

If the tires break loose when you brake, three bad things happen: you lose some tire-tread (this costs $), f_k is less than $\mu_s N$ (as discussed above), and you also lose some control over the car (when tires are rolling, the car tends to move in the direction the tires point; but when tires slide across the ground, the car may move in an unexpected direction).

3-26: F=ma is now "+mg(sin 25°) – μ_k mg(cos 25°) = m a", and "+20 = 0 + ½(+3.2)t^2" gives t = 3.5 s.

3-27: Add 28*N* eastward & 8*N* northward forces as vectors (in the bird's eye pictures below); the resultant $F_{non\text{-}f}$ vector is 29.1*N*. This is canceled by an equal-and-opposite f_s of 29.1*N*, so the block doesn't move.



28*N* and 10*N* are less than $\mu_s N$, but they combine to make $F_{non\text{-}f}$ = 29.7*N*, which is larger than 29.4*N*. The block slides in the direction of $F_{non\text{-}f}$, so friction changes from static to kinetic; f_k = 19.6*N*, opposite to the direction of sliding.

3-28: Gradually tilt the plane until the penny slides, measure θ, calculate μ_s. {A solution is after 3-29.}

3-29: For race #1, F=ma for the two blocks are:

– (2) g = (2) a – (20) g = (20) a

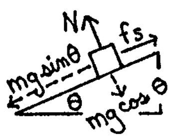
The 2's & 20's can be "divided out", so both equations give "–g = a". The 20 kg block has 10 times as much force acting on it (in the –20g term), but it also has 10 times as much "resistance to being accelerated" (in the 20a term). These factors cancel each other, so the 20 kg block has the same a (and Δt) as the 2 kg block.

In races 2 & 3, "m" also disappears from F=ma:
$+mg\sin\theta = ma \qquad +mg\sin\theta - \mu\, mg\cos\theta = ma$.
The acceleration of a block doesn't depend on its mass; both blocks reach the bottom at the same time.

Races #4 and #5 show the importance of *air resistance* (which we've been ignoring), a force that is explored in Section 6.#.

If you dropped the paper and penny, you did a *reality check,* as discussed in Section 20.#. If you imagined what would happen if you dropped them, you did a *thought experiment,* as discussed in Section 16.1.

(3-28): $f_s = \mu_s N$ only at the "breakaway limit", just before the penny begins to slide. At this instant,



$+N - mg\cos\theta = m(0)$

$N = mg\cos\theta$

ONLY AT "BREAKAWAY"

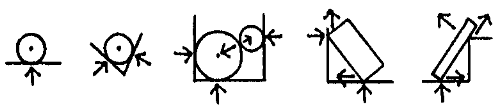
$+mg\sin\theta - \mu_s N = m(0)$

$+mg\sin\theta = \mu_s(mg\cos\theta)$

$\frac{\sin\theta}{\cos\theta} = \mu_s$

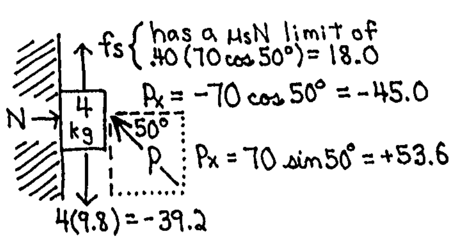
$\tan\theta = \mu_s$

3-30: The "Normal" force for a sphere/plane contact is ⊥ to the plane, and points through the center of the sphere. To find f_s direction, ask the question about "would-be sliding" and then answer it. N and f are ⊥.



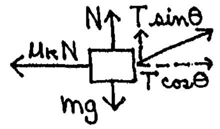
3-31: Friction causes the accelerations. F=ma is ".9 $\mu_s mg = ma$", so $\mu_s = a/(.9\,g) = (v_i^2/2\Delta x)/.9g$. The μ_s's are .54 for dry road, and .23 for wet road.

3-32: Use Section 3.7's "flowchart". The block isn't moving: a) $F_{non\text{-}f} = -39.2 + 53.6 = +14.4$, b) find $\mu_s N = .40(70\cos 50°) = 18.0$, c) because $\mu_s N$ is larger, the block doesn't move and $f_s = -14.4N$; f_s points downward to prevent upward sliding.



3-33: At either speed, $T = \mu_k N = .30(50)(9.8) = 147N$ with a horizontal pull. To keep v constant with a 25° pull, use the F-diagram and F=ma's below to find that $T = .3(50)(9.8)/(\cos 25° + .3\sin 25°) = 142N$.

Do you see why a diagonal pull can require less T than a horizontal pull? Is this a surprising result?



$-mg + T\sin\theta + N = m(0)$

$N = mg - T\sin\theta$

$-\mu N + T\cos\theta = m(0)$

$-\mu(mg - T\sin\theta) + T\cos\theta = 0$

$T = \frac{\mu mg}{\cos\theta + \mu\sin\theta}$

$T = \frac{mg}{\frac{\cos\theta}{\mu} + \sin\theta}$

OPTIMIZATION: Two conflicting factors affect the rope-T needed to maintain constant speed:

1) As the pulling angle "θ" decreases, a higher % of T is aimed in the x-direction to overcome f_k and maintain speed. For example, when θ=0 (a horizontal pull), 100% of T points in the x-direction. But when θ=25°, T cos 25° = .906 T, and .094T is "wasted".

2) As θ increases, N and $\mu_k N$ decrease; there is less f_k to overcome, thus allowing a smaller T. A 25° pull reduces N from 490*N* to 430*N*, reducing f_k from 147*N* to 129*N*. To overcome this f_k, T_x (which is T cos25°) must be 129*N*, and T must be 142*N*.

Factor 1 predicts that a 0° pull will be best, Factor 2 says it should be 90°. The *optimal* θ-angle (that will minimize the T needed to maintain constant v) is the best compromise between these conflicting factors.

For most problems, it is easiest to substitute numerical values and then do algebra. In the solution above, I reverse this order because *solve-and-substitute* gives a "general equation", $T = mg/(\cos\theta/\mu_k + \sin\theta)$. This equation makes it easy to try different θ's and find, by trial-and-error, the θ that gives the smallest T.

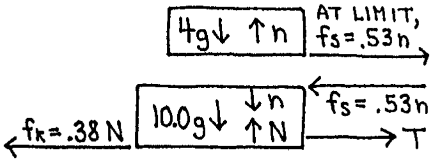
The minimum T occurs when $(\cos\theta/\mu_k + \sin\theta)$, which is on the fraction-bottom, is a maximum. If μ_k = .30, the optimal-θ is 16.7°, and T = 141*N*. Will the optimal-θ be smaller or larger if μ_k = .43? { Hint: What does a large f_k do to the relative importance of Factors 1 & 2? } If μ_k = .43, a 23° pull minimizes T. When μ_k ↑ from .30 to .43, Factor 2 (whose goal is to reduce friction by making θ larger) increases in importance, and the optimal-θ changes from 16.7° to 23°.

"Finding a Maximum or Minimum" is discussed in Section 18.10, and also in Problem 2-24.

3-34: To find the direction of friction acting on the top block, think a) "I'm accelerating toward the right, so f (which is the only horizontal force acting on me) must point toward the right", or b) "If sliding did occur, the bottom block would be pulled rightward out from under me and I would slide (relative to the bottom block) toward the left, so f_s must (to prevent my would-be sliding) point toward the right".

The bottom block says "I could accelerate faster toward the right if friction didn't make me drag the top block along, so the "f" acting on me must point leftward". You can also use equal-and-opposite force pair logic or "prevention of would-be sliding" to reach this conclusion.

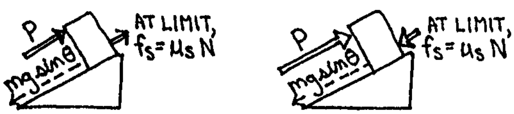
Just under the "breakaway limit", $f_s = \mu_s N$, both blocks have the same a_x, and both have $a_y = 0$.



	x-direction	y-direction
□ :	$+.53n = 4a$	$-4g + n = 4(0)$
▭ :	$-.38N - .53n + T = 10a$	$-10g - n + N = 10(0)$

Find a 1-unknown equation, solve it and substitute in all possible places, then solve for the other variables: $n = 39.2N$, $N = 137.2N$, $a = 5.19\ m/s^2$, $T = 125N$.

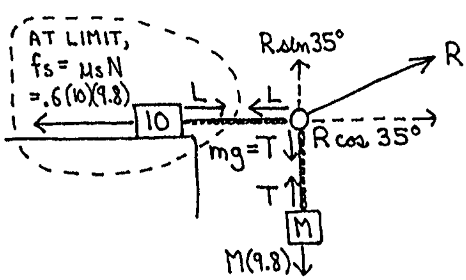
3-35: If P is too small, the block will slide down the ramp. If P is too large, it slides up the ramp. Here are the "breakaway" P-values for each possibility:



fs prevents downslide	fs prevents upslide
$+mg\sin\theta - P - \mu_s mg\cos\theta = 0$	$+mg\sin\theta - P + \mu_s mg\cos\theta = 0$
$mg(\sin\theta - \mu_s\cos\theta) = P$	$mg(\sin\theta + \mu_s\cos\theta) = P$
$25g(\sin 30^\circ - .4\cos 30^\circ) = P$	$25g(\sin 30^\circ + .4\cos 30^\circ) = P$
$37.6\ N = P$	$207.4\ N = P$

Think about what is happening, and you'll see that if $P < 37.6$, □ slides downward. But if P is between 37.6 and 122.5, □ stays at rest and f_s points ↗. If P is $25(9.8)\sin 30^\circ = 122.5$, $f_s = F_{non\text{-}f} = 0$. If P is between 122.5 and 207.4, □ stays at rest and f_s points ↙. If $P > 207.4$, □ slides upward.

3-36: Solve the blocks' F=ma's for L & T, substitute into the knot's x & y F=ma's, then solve for M.



$-.6(10)9.8 + R\cos 35^\circ = m(0)$ → $R = 71.8$

$+R\sin 35^\circ - M(9.8) = m(0)$ → $(71.8)\sin 35^\circ = M(9.8)$ → $4.2 = M$

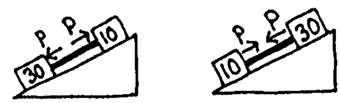
If you treat block-knot-block as a "system object", the only external x-forces are $\mu_s N$ and $R\cos 35^\circ$, the only external y-forces are $R\sin 35^\circ$ and Mg, and you will get the same F=ma's as above.

3-37: If the bar is removed, each block will have $a = g(\sin\theta - \mu_k\cos\theta)$; the 10 kg block moves faster because its μ_k is smaller. As shown on the left diagram below, the bar gives the 10-block an "uphill push" (slowing it down) and the 30-block a "downhill push" (speeding it up) so their speeds will match.

These equations give $a = 1.72$, $P = 6.4\ N$:

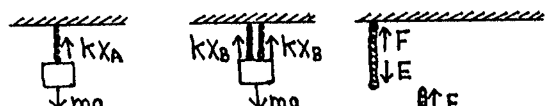
$$+30g(\sin 30^\circ) - (.40)30g(\cos 30^\circ) + P = 30a$$
$$+10g(\sin 30^\circ) - (.30)30g(\cos 30^\circ) - P = 10a$$

If the block-order is reversed, a and P are the same as before. But the bar now "pulls" the 30-block in the downhill direction (as shown on the right diagram), instead of "pushing" it downhill.



3-38: F=ma is $k(.50 - .40) = 10(.40)$; $k = 40\ N/m$.

3-39: Finding x_A and x_B is easy; just draw F's, substitute into F=ma, solve. To find x_C, draw the F's on each object, then think about their relationships.



$+kx_A - mg = 0$	$2kx_B - mg = 0$
$150x_A = 3(9.8)$	$2(150)x_B = 3(9.8)$
$x_A = .196\ m$	$x_B = .098\ m$

Do you see why $3(9.8) = D = E = F$? {Hint: What would happen if these F's aren't equal?} Each spring feels a $29.4N$ pull on both ends, just like in Part A, so each stretches .196 m, and the total x_C is .392 m.

The two side-by-side springs act as if they were one twice-as-strong spring (with $k_{total} = 300\ N/m$), while the end-to-end springs act as if they were one half-as-strong spring (with $k_{total} = 75\ N/m$).