Chapter 16

Relativity

I think you'll find a pleasant surprise in this chapter — that the fundamentals of relativity are easier than you expect. Just read Sections 16.1 to 16.3 in order, then use Sections 16.4 (mass & energy), 16.5 (adding velocities), 16.6 (Doppler shift) and 16.7 (general relativity) when your class studies these topics.

OPTIONAL: Section 16.93 covers Lorentz Transformations.

16.1 The Special Theory of Relativity

Most textbooks explain the history of relativity, from Galileo & Newton through Maxwell & Michelson & Morley to Einstein, so I won't repeat what they've done. Instead, I'll emphasize the simple, easy-to-understand logic of relativity.

The beginning of Section 2.11 discusses relative motion. The results are logical, just what common sense would lead would lead you to expect. For example, if you ride on a 20 m/s train and throw a ball forward at 5 m/s, a person who is standing on the ground will see the ball come toward him with an "additive" speed of 25 m/s. Einstein's theory of relativity uses this same common sense logic, but makes a few simple adaptions so it can be used for objects moving with extremely high speeds.

The SPECIAL THEORY OF RELATIVITY is based on these two postulates:

1) All of the laws of physics are the same for observers in all *inertial* (constant velocity) reference frames.

2) The speed of light in empty space (in a vacuum) always has the same value "c", independent of the relative motion of the source or the observer's reference frame.

Notice that the theory is called "relativity", but it is based on constancy: every inertial frame has the same laws of physics, and sees the same speed of light.

Here are some comments about these two constancies.

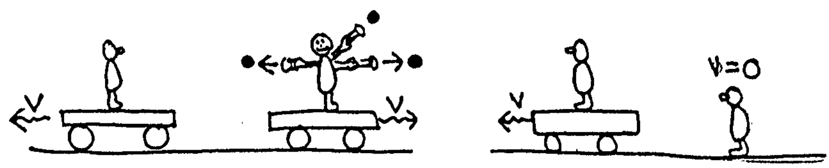
1) Long before Einstein, Galileo and Newton noticed that the laws of physics seem to be identical in all inertial reference frames. For example, a ping pong game is the same whether it's played on the ground or on a constant-velocity train. A physicist will also find that the laws of physics are the same in both environments. {But if the train accelerates, the game suddenly changes! The SPECIAL Theory of Relativity (and many of our intuitive & mathematical "laws of physics") can only be used in constant-velocity reference frames. The GENERAL Theory of Relativity, which is studied in Section 16.7, extends relativity and the laws of physics into all reference frames, even "non-inertial" frames that are being accelerated.}

In the late 1800's, scientists discovered that some laws of physics (Maxwell's electromagnetic equations) change when you move from one constant-velocity reference frame to another. This was confusing for awhile, because the relativity principle and Maxwell's equations both seemed to be correct, but they were in conflict with each other. Eventually Einstein found a way to reconcile them, by postulating the "second constancy".

2) It doesn't seem too amazing that "the speed of light is constant", until you realize that this statement cannot be made about anything else in the universe!

As discussed in Section 9.3, all particle-speeds and all wave-speeds (except for electromagnetic waves) **do** depend on the speed of the source and/or the observer. For example, sound waves travel at a certain speed with respect to a propagating medium (air, water, metal, wood,...), not with respect to the observer. By analogy, scientists expected light to also travel at a certain speed with respect to its propagating medium, which they called the *ether*. In 1887, Michelson & Morley performed a clever experiment (your textbook probably describes it in detail) that would have been able to detect the ether if it really existed. But they couldn't find any evidence for ether because, as explained by Einstein in 1905, it doesn't exist. Light waves travel a certain speed with respect to the observer, not with respect to an ether-medium through which they propagate.

In these pictures, all 4 observers see all 3 light-flashes (•) traveling at speed "c":



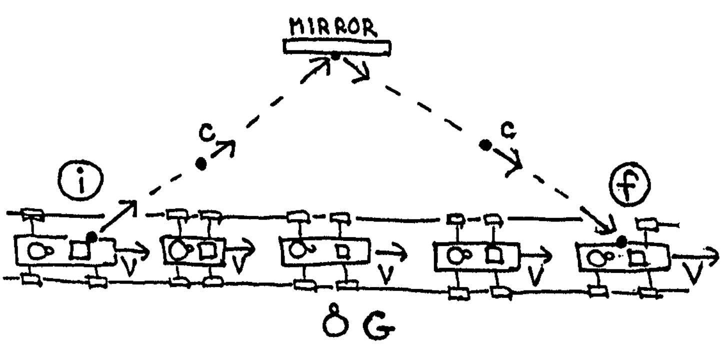
{ As discussed in Section 14.2, the speed of light is "c = 2.9979 x 10^8 m/s" when it travels through a vacuum, but when light moves through other things (air, glass, plastic, water,...) its speed decreases.}

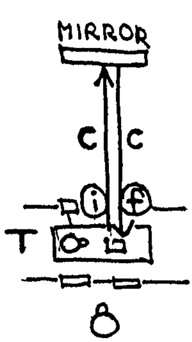
16.2 Time Dilation and Simultaneity

When I first studied relativity, for awhile I accepted it on faith (that Einstein and the other scientists were probably correct) but I was skeptical. I didn't become a true believer until our textbook explained the following experiment. It convinced me that the startling conclusions of relativity are really quite logical. I hope it will make sense to you, too.

In the first bird's-eye picture below, a flash of light " • " is emitted at location "i" from a source "▢" on a train. The light reflects from a mirror and returns to ▢ when the train is at "f". • and the train, which moves rightward with speed "v" as seen by a ground-observer "G", are shown at 5 times: at i, f, and 3 times in-between.

The only way that • can return to ▢ after reflecting from the mirror is if the x-velocity (and thus the x-positions) of • and ▢ are "matched". Because of this x-matching, a train-observer "T" will see • travel straight out and straight back, as shown in the second picture.





G sees • go from i to f, traveling at speed "c", on a path that looks like this: ↗↘ .
But T sees • go from i to f, traveling at speed "c", on a path that looks like this: ↑↓ .

G can calculate the initial-to-final time: Δt = distance/velocity = (↗↘ -distance)/c.
And T can calculate the initial-to-final time: Δt = distance/velocity = (↑↓ -distance)/c.
Because ↗↘ is longer than ↑↓, the Δt observed by G is longer than the Δt observed by T!

It takes time for light to travel from-i-to-G and from-f-to-G. In all discussions and formulas of Chapter 16, this time lag has been correctly taken into account. { I say this to emphasize that the difference in Δt is real; G measures a longer time because he sees the light travel a longer distance, not because of any error we've made in analyzing the situation.}

Our everyday experience seems to indicate that "time is the same for everyone". But this experiment shows that two observers can observe different Δt's for the same initial-to-final event, if we accept the two postulates of special relativity: constant laws of physics (so both observers can use Δt = distance/velocity) and constant speed of light (so both observers see the •-speed as "c"). Are you convinced?

The following definitions are important.

A *proper observer* says "the initial & final events occur at the same location"; he observes a *proper time* "Δt_o" that is the shortest possible time. An *improper observer* says "the initial & final events occur at different locations"; he observes an *improper time* "Δt" that is longer than Δt_o. The fact that an improper observer measures a longer time is called *time dilation.*

For example, T is a proper observer because he says "i & f occur at the same place with respect to my own location, at ☐ a little bit in front of me", so he measures Δt_o. But G is an improper observer who says "i & f are at different places with respect to me; i occurs to the left of me, and f occurs to the right of me", so he measures Δt.

It can be shown (as in Problem 16-#==) that the relationship between Δt and Δt_o is

$$\Delta t = \Delta t_o \frac{1}{\sqrt{1 - v^2/c^2}} \qquad \text{or} \qquad \Delta t \sqrt{1 - v^2/c^2} = \Delta t_o$$

where v is the relative speed of the improper & proper observers.

If $v \neq 0$, $\sqrt{1 - v^2/c^2}$ is less than 1. Dividing by $\sqrt{1 - (.8c)^2/c^2}$ makes Δt larger than Δt_o, just as it should be. And multiplying by $\sqrt{1 - (.8c)^2/c^2}$ makes Δt_o smaller than Δt.

Relativistic Calculations

For objects traveling at speeds that are much slower than c, $1/\sqrt{1 - v^2/c^2}$ is almost equal to 1, so Δt and Δt_o are almost exactly the same. For example, the rocket that carried men for the first moon landing had v = 24300 miles/hour = 10900 m/s, and $1/\sqrt{1 - v^2/c^2}$ = 1.0000000007. For the common speed of v = 60 mi/hr = 26.8 m/s, $1/\sqrt{1 - v^2/c^2}$ = 1.000000000000004. {Problem 16-2 shows an easy way to calculate $1/\sqrt{1 - v^2/c^2}$ when v^2/c^2 is extremely small.}

But as v gets closer to c, the "correction factor" of $1/\sqrt{1 - v^2/c^2}$ becomes significant. For speeds of .10c, .80c, .99c and .9999c, $1/\sqrt{1 - v^2/c^2}$ is 1.005, 1.667, 7.089 and 70.712, respectively. { For practice, try these calculations by yourself. If v is given as a fraction of c, like v = .10c, you don't have to enter the c's into your calculator because the c^2's cancel in $(.10c)^2/c^2$. Just punch "1 − .10 x^2 = $\sqrt{\ }$ 1/x" to get "1.005".}

Problem 16-# shows how to solve ".50 = $\sqrt{1 - v^2/c^2}$" for v, by using basic algebra principles and making progress one easy, logical step at a time.

Does a "proper" observer have the "correct" view of what is happening?

To answer this question, consider the following situation. Rhonda rides a rocket rightward at .8c, while Glenn stands on the ground. Each has an identical good quality clock. {This clock could use any mechanism; pendulum, spring, quartz,...} Think about the results of these two experiments: 1) the second-hand of Glenn's clock moves 15 seconds [1/4 of the way around the 60 second dial], and 2) the second-hand of Rhonda's clock moves 15 seconds [1/4 of the way around its dial].

In Experiment #1, Glenn observes his own clock and says "it is beside me at i & f, so I am a proper observer". But Rhonda is an improper observer because she sees his clock move a large distance between i & f. {She sees it move $\Delta x = (.8 \times 3 \times 10^8 \text{ m/s})(15\text{s}) = 3.6 \times 10^9$ m". To see the clock when it is this far away, Rhonda's eyesight would have to be very good! This is called a "thought experiment" because we can imagine what would happen if we actually did the experiment, and ignore practical difficulties like the limited range of vision.}

In Experiment #2, Rhonda's clock is observed and she is a proper observer who sees the shortest possible time of 15 s. But now Glenn is an improper observer who measures $\Delta t = \Delta t_0 \, [1/\sqrt{1 - v^2/c^2}\,] = (15\text{s})[1/\sqrt{1-(.8c)^2/c^2}\,] = (15\text{s})(1.67) = 25\text{s}$.

<u>EXPERIMENT #1: Glenn's Clock</u>

Glenn is a proper observer.
He says "My clock is running correctly; it took 15 s to move ◔."
Rhonda is an improper observer.
She says "Your clock is running slowly; it took 25 s to move ◔."

<u>EXPERIMENT #2: Rhonda's Clock.</u>

Rhonda is a proper observer.
She says "My clock is running correctly; it took 15 s to move ◔."
Glenn is an improper observer.
He says "Your clock is running slowly; it took 25 s to move ◔."

Some textbooks summarize these facts as "moving clocks run slowly". This is correct, but you must think relativistically when you interpret "moving". You may be tempted to say "Glenn is on the ground; he isn't moving, so his clock is all right". But Rhonda, viewing the action from her own reference-frame, sees Glenn moving ← at .8c, so she measures his moving clock to be "running slowly".

Rhonda knows relativity, so before she decides to criticize Glenn's clock she thinks logically: "When I watch his clock I'm an improper observer. If his clock is running correctly, I <u>should</u> measure a Δt that is larger than 15s. With respect to me, Glenn is moving leftward at .8c, so my Δt should be $(15\text{s})[1/\sqrt{1-(.8c)^2/c^2}\,] = 25\text{s}$, and it is. Glenn's clock is fine!"

A comment: The theory of relativity is sometimes abused by nonscientists who don't understand it, with sloppy analogies like "time is relative (it depends on who measures it), so maybe truth is also relative". But relativity theory applies only to physics, not philosophy or sociology. It says nothing, one way or the other, about absolute truth, relative morality, or other non-physics issues.

Most textbooks discuss two experimentally verifications that <u>time dilation is real, not a motion-induced illusion</u>: 1) When unstable muon-particles travel at extremely high speeds they survive longer than if they are at rest. 2) When two identical clocks were compared after one had ridden around the world on a fast airplane, the "space twin" clock had lost a little bit of time.

I won't discuss these experiments, except to emphasize the important difference between two types of situations. When we compare *Δt observations in two inertial reference frames* (like Glenn-and-Rhonda, or muons racing through the sky versus an observer on earth) there is "symmetric" dilation of time, as each observer says: my clock is fine, yours is slow. But with *earth-and-space twins* the earth-twin is, if we ignore the earth's rotation and orbiting, relatively un-accelerated and inertial, while the space-twin accelerates at the beginning, middle and end of the journey.

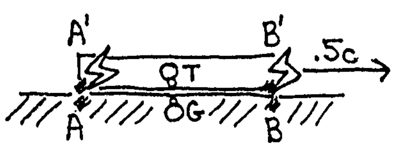
This "assymetry", with one twin inertial and one accelerating, lets the space twin's actions (tick of a clock or beat of a heart) run slower, and at the end of the trip both twins agree: the space twin has aged less than the earth twin.

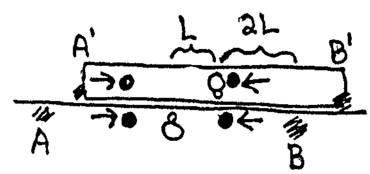
When you read a relativistic time-problem, decide whether it involves two inertial observers [so you think in terms of proper versus improper] or a space twin who is being accelerated [so you think in terms of slowed-down aging processes].

SIMULTANEITY

In the first picture below, a train (carrying observer T and locations A' & B') moves rightward at .5c with respect to the ground (with G, A & B). When T and G are exactly opposite each other, lightning strikes at two locations, leaving scorch marks at A-and-A', and also at B-and-B'. T is midway between A' & B', while G is midway between A & B.

The second picture shows the situation a short time later. Because the .5c train is moving half as fast as the light flash (shown by •), T moves rightward a distance of "L" while • moves leftward "2L". At the instant shown in the second picture, T sees the • flash that was emitted from B/B', but the • flash from A/A' hasn't yet reached him. Now look at G and the • flashes from A/A' and B/B'; G is not moving, so the • from A/A' & B/B' are still equally far away, and will soon reach him simultaneously.





Using simple logic and "$\Delta x = v\ \Delta t$", here is how T and G answer the question "Were the two flashes (at A/A' and B/B') simultaneous?

Here is T's logic: if the lightning struck at A/A' and B/B' simultaneously, the light would reach me simultaneously, because A' and B' are equally far away from me, and each • flash is moving toward me with speed "c". The light from B' reached me first, so the lightning must have struck B/B' before it struck A/A'.

G's logic is the same but his conclusion is different: the • from A/A' and B/B' reach me simultaneously, and their travel-times are identical (because A & B are equally far away and each flash moves toward me at speed "c"), so they must have been emitted at exactly the same time.

G says "the lightning struck at A/A' and B/B' at the same time", but T says "the strike at B/B' occurred first".

This disagreement about simultaneity can be summarized: if events that occur at different x-positions are simultaneous for an observer in one inertial reference frame, they will not be simultaneous for an observer in an inertial reference frame that is moving (in the x-direction) with respect to the first reference frame.

Notice the condition that "if events that occur at different x-positions...". Since the left-side lightning strike occurs when A and A' have the same x-position, G and T can both agree that it occurs at A and A' simultaneously, which is why I've been calling this strike A/A'. For the same reason, G and T can agree that B and B' are struck simultaneously. What they do disagree about (because A/A' and B/B' occur at different x-positions) is whether A/A' and B/B' are simultaneous. Problem 16-# may help you understand the reason for "non-simultaneity" a little more clearly.

THE LIMITS OF STRANGENESS: The logic of relativity theory shows that some familiar ideas about time must be abandonded, but it doesn't say "throw out all the rules". For example, G and T disagree about the timing of the lightning-flashes, but they agree that events in both reference frames (G's ground and T's train) are running "forward in time", as usual. { They won't see any "time reversed" phenomena like the other person's clock running backwards, or gravity pulling their objects upward, or ... }

In the earlier example, Rhonda & Glenn each see things in their own reference frame running normally. They also see things in the other person's reference frame running normally [in correct sequence as time moves forward] but slower than usual; everything runs at 60% of its usual speed.

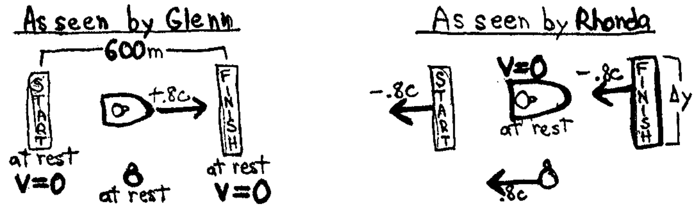
And the space-twin sees spaceship life occur normally [not in slow motion] even though her "biological clock" is, as observed by the earth-twin, running slowly.

16.3 Length Contraction and Mass Increase

Let's return to ground-Glenn and rocket-Rhonda, and you'll soon see why length contraction is a logical result of time dilation, which is itself a logical result of the two "constancy postulates".

The first picture below shows a 600 meter race as it appears to Glenn, who sees the start & finish lines at rest while the rocket moves rightward at .8c. The second picture shows Rhonda's view. If she assumes herself to be at rest, she will see the start & finish lines moving leftward at .8c.

Who is a proper observer when measuring the initial-to-final time for this race?



Glenn sees the initial & final events occur to the left & right of him, respectively, not in the same place, so he is an improper observer. Rhonda says "initial & final occur when the front of my rocket crosses the start & finish lines; i & f both occur at the same location, at the front of my rocket, so I am a proper observer".

Glenn & Rhonda both observe the same speed of .8c, but Rhonda measures a shorter race time because she is a proper observer. When each observer calculates the length of the race course by using "$\Delta x = v\,\Delta t$", Rhonda gets a shorter Δx. { If you do the calculations you'll find that Rhonda, who sees the race course moving, measures its length to be 360 m. Glenn, who sees the race course at rest, measures its length to be 600 m. }

A length perpendicular to motion does not change. For example, the Δy length of the finish line marker is the same whether it is measured by Glenn or Rhonda.

This analysis shows that a moving-length appears to decrease. It can also be shown, although I won't do it here, that a moving-mass appears to increase.

Section 16.2 defined a proper observer of **time**. For **length** and **mass**, a *proper observer* says "the object is at rest"; he measures Δx_{rest} (the *rest length* or *proper length*, L_o) and m_{rest} (the *rest mass*, m_o). An improper observer says "the object is moving"; he measures Δx_{moving} and m_{moving}, which are abbreviated L and m.

The relationships between proper quantities (indicated by zero-subscripts: Δt_0, L_0, m_0) and the corresponding improper quantities (Δt, L, m) are:

$$\Delta t = \Delta t_0 \frac{1}{\sqrt{1 - v^2/c^2}} \qquad L = L_0 \sqrt{1 - v^2/c^2} \qquad m = m_0 \frac{1}{\sqrt{1 - v^2/c^2}}$$

Notice that the same factor, $\sqrt{1 - v^2/c^2}$, is used in each equation. An improper observer measures a larger time and mass, but a smaller distance.

Proper quantities are unique; Δt_0 and m_0 are the smallest time and mass that can be measured, and L_0 is the largest distance that can be measured.

But an improper quantity can have many values. When Rhonda moves at .8c she sees a quarter-cycle of Glenn's clock take $\Delta t = (15\,s)[\,1/\sqrt{1 - (.80c)^2/c^2}\,] = 25\,s$, but an observer on a .99c rocket sees the same event take $\Delta t = (15\,s)[\,1/\sqrt{1 - (.99c)^2/c^2}\,] = 106\,s$.

As usual, it is important to understand similarities <u>and</u> differences.

To determine whether an observer is proper, there are two kinds of questions.
For time, ask: does he see the initial & final events occur at the same location?
But for length or mass, ask: does he see the object at rest?

An observer can be proper for measuring one quantity, but improper for another quantity. For example, Rhonda is a proper observer for the race time (because she sees both i & f occur at the front of her rocket), but she is an improper observer for the race course length (because she sees the race course moving at .8c).

Mass behaves like time in one respect (both increase for an improper observer), but mass is like length in another respect (for both, you determine if an observer is proper by asking "does he see the object at rest").

PROBLEM 16-A

Glenn (on the ground) and Rhonda (on a rocket moving east at .8c) each have a good clock, three meter sticks (east/west, north/south, vertical) and a 100 kg block. Describe each of these 10 objects, as observed by Glenn and by Rhonda.

SOLUTION 16-A: For v = .8c, $\sqrt{1 - (.8c)^2/c^2} = .60$, and $1/\sqrt{1 - (.8c)^2/c^2} = 1.67$.

<u>WHAT GLENN SEES</u>

Glenn says "My objects are at rest.
My clock's quarter-cycle takes 15 s,
all of my meter sticks are 1.00m,
and my block's mass is 100 kg."

He says "Rhonda's objects are moving.
Her clock's quarter-cycle takes 25 s,
her east/west meter stick is .60 m,
her other meter sticks are 1.00 m,
and her block's mass is 167 kg."

<u>WHAT RHONDA SEES</u>

Rhonda says "My objects are at rest.
My clock's quarter-cycle takes 15 s,
all of my meter sticks are 1.00 m,
and my block's mass is 100 kg."

She says "Glenn's objects are moving.
His clock's quarter-cycle takes 25 s,
his east/west meter stick is .60 m,
his other meter sticks are 1.00 m,
and his block's mass is 167 kg."

The other person's east/west stick is parallel to the relative motion so it appears to change size, but the other sticks are perpendicular to motion so they don't change.

16.4 Relativistic Momentum and Energy

As explained in Section 16.3, when an object's speed increases so does its mass. If its speed equaled c, its mass would become infinite $\{m = m_o (1/\sqrt{1-c^2/c^2}\,) = m_o(1/0) = \infty\}$ and so would its momentum $\{p \equiv mv = \infty c = \infty\}$. Using "$F\,\Delta t = \Delta p$" from Section 4.1, we find that to reach infinite momentum requires an infinite amount of time: $\Delta t = \Delta p/F = \infty/F = \infty$. This is impossible, so there is a "natural speed limit": an object with mass can never reach or exceed the speed of light.

{ Light has a rest mass of zero, so it is not governed by this speed limit. There is also a theoretical possibility that particles could travel faster than c; but these hypothetical particles, which are called *tachyons*, could never travel at speeds less than c. No tachyons have ever been detected; if they existed (they may not), it is uncertain whether we could detect them experimentally. }

Substitution of "$m = m_o(1/\sqrt{1-v^2/c^2}\,)$" transforms the classical momentum formula into the correct relativistic formula: $\mathbf{p = mv = m_o\,(1/\sqrt{1-v^2/c^2}\,)\,v}$. But this cannot be done for energy; relativistic KE does not equal $\frac{1}{2} m_o (1/\sqrt{1-v^2/c^2}\,)\, v^2$. Instead, by using the work-energy equation and calculus, Einstein derived this KE formula:

$$KE = mc^2 - m_o c^2$$

$$KE = c^2(m - m_o)$$

$$KE = c^2\left(\frac{m_o}{\sqrt{1-v^2/c^2}} - m_o\right)$$

$$KE = m_o c^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)$$

Notice the close relationship between KE and mass: $\mathbf{KE = c^2(m - m_o) = c^2\,\Delta m}$. When v is large, m (which is $m_o/\sqrt{1-v^2/c^2}$) is much larger than m_o, thus producing a large Δm and correspondingly large kinetic energy.

This led Einstein to conclude that **mass is a form of energy**. Even when an object is at rest with KE = 0, it has mass and thus a *rest energy* of $E_{rest} = m_o c^2$. If the object is moving, its motion gives it "kinetic" energy that causes its mass to increase from m_o to m. Here is the relationship between different forms of energy:

TOTAL	=	SUM OF PARTS		
$\mathbf{E_{total}}$	=	$\mathbf{E_{rest}}$	+	$\mathbf{E_{kinetic}}$
⇓		⇓		
$\mathbf{mc^2}$	=	$\mathbf{m_o c^2}$	+	**KE**

The vertical substitutions (⇓) show that $E_{total} = mc^2$ and $E_{rest} = m_o c^2$.

This 4-sided equation shows that in situations where energy (mc^2) is conserved, so is mass (because mass is just E_{total} divided by c^2, a constant) and the combination of rest energy + *kinetic* energy ($m_o c^2$ + KE). And in any situation, $\Delta E_{total} = \Delta(mc^2) = \Delta m\, c^2 = \Delta KE$; $m_o c^2$ is a constant, so $\Delta(m_o c^2 + KE) = \Delta(m_o c^2) + \Delta KE = 0 + \Delta KE$.

Energy conservation is discussed in Sections 4.4 & 7.7, and in Problem 16-1#.

When v is small (v « c), $E_{kinetic}$ ($\approx \frac{1}{2} m_o v^2$) is much larger than E_{rest} ($= m_o c^2$); most of the total energy is rest energy, not kinetic energy. But as v increases, kinetic energy also increases until, when v > .866c, $E_{kinetic}$ becomes larger than E_{rest}. ==nec?

An object's m_o measures the "amount of matter" in it, while m shows its "resistance to being accelerated" that is summarized in "F=ma" == discussed in Section 3.1's "F = ma ratio logic".
A periodic table lists each element's "m" at normal temperatures (around 20°C) where $\sqrt{1-v^2/c^2}$ is almost exactly 1, so it is safe to say that a periodic table lists m_o's. Only at extremely high speeds does the difference between m_o and m become significant. ==nec?

AN IMPORTANT PRINCIPLE: **If v is much less than c (this is abbreviated "$v \ll c$"), relativistic formulas simplify to give the analogous non-relativistic formula.**

For example, if $v \ll c$, $\sqrt{1-v^2/c^2} \approx 1$, so $t = t_o(1/\sqrt{1-v^2/c^2}) \approx t_o(1/1)$; similarly, $L \approx L_o$ and $m \approx m_o$. The relativistic formulas for *adding velocities* (Section 16.5) and *Lorentz transformations* (optional Section 16.93) also predict "classical results" when $v \ll c$. Using the "binomial expansion" (your text or teacher may show you the details) it can also be shown that if $v \ll c$ the relativistic KE formula simplifies to $KE \approx \frac{1}{2} m_o v^2$, the familiar non-relativistic KE formula.

$F\,\Delta t = \Delta p$ and $F\,\Delta x = \Delta KE = \Delta E_{total}$ are correct, whether v is large or small, if relativistically correct expressions for Δt, p, Δx, KE and E_{total} are used.

{ With appropriate modification, **F = ma** can be applied to relativistic situations, but it is usually not the best way to analyze such situations, so it is rarely used. } ==ok?

The problems in Section 16.91 show how to ==[use ---] solve relativistic energy and momentum problems. ==[here, elsewhere; edit ? "problems" twice?]

OPTIONAL: Some textbooks derive another formula, $(E_{total})^2 = m_o{}^2 c^4 + p^2 c^2$. A wide variety of "energy equations" can be written, formed by equating various parts of the 7-sided equation below. Use the ones your teacher wants you to know.

$$E_{total} = E_{rest} + E_{kinetic} = mc^2 = m_o(1/\sqrt{1-v^2/c^2})c^2 = m_o c^2 + KE$$
$$= \sqrt{m_o{}^2 c^4 + p^2 c^2} = \sqrt{E_{rest}{}^2 + m^2 v^2 c^2}$$

16.5 Relativistic Addition of Velocities

You stand on the ground and see a rocket move rightward at +.8c. It shoots two bullets (one →, the other ←) that, as observed from the rocket, each travel with a speed of .9c. What bullet velocities do you observe?

Use non-relativistic common sense logic to decide that the bullet velocities are ".8c + .9c" in the → direction and ".9c – .8c" in the ← direction, then divide by the "relativistic correction factors" shown in the equations below. When you add speeds the correction factor of 1.72c makes $v_{observed}$ smaller [so instead of impossible faster-than-light 1.7c, you actually see .988c] but if speeds are subtracted the factor of .28c makes $v_{observed}$ larger [instead of .1c, you see .357c], because the fraction's top and bottom are "matched"; **either the top-and-bottom both have +, or they both have –.**

For rocket .8c> and bullet .9c>,

$$v_{observed} = \frac{.8c + .9c}{1 + (.8c)(.9c)/c^2}$$
$$v_{observed} = \frac{1.7c}{1.72}$$
$$v_{observed} = .988c, \rightarrow$$

For rocket .8c> and bullet <.9c,

$$v_{observed} = \frac{.9c - .8c}{1 - (.9c)(.8c)/c^2}$$
$$v_{observed} = \frac{.1c}{.28}$$
$$v_{observed} = .357c, \leftarrow$$

To solve v-addition problems, 1) Develop a clear picture-idea of the velocities that are "given"; think of the objects as cars moving on a freeway. 2) You'll be asked to find a velocity as it is seen by a certain observer; imagine that you are this observer and are "at rest". 3) Use cars-on-the-freeway visual logic to find the nonrelativistic velocity (speed & direction); for practice, do Problem 2-32. It may help to create an analogous problem with cars moving at familiar speeds (like 50 mi/hr, 30 mi/hr,...) and then apply its solution to the higher velocities. 4) Divide by the appropriate correction factor [it contains + if you add speeds, and – if you subtract speeds]: ==rest

If you add speeds,

$$v_{observed} = \frac{u + v}{1 + \frac{u\,v}{c^2}}$$

If you subtract speeds,

$$v_{observed} = \frac{u - v}{1 - \frac{u\,v}{c^2}}$$

Your textbook probably gives a v-addition formula like $v = (u + v')/(1 + uv'/c^2)$, where v and u [without superscripts] are "as seen by the observer" and v' [with the ' superscript] is "as seen by someone who is riding on the object moving at velocity u". In the example above, v = .988c is "bullet-v as seen from the ground", u = .8c is "rocket-v as seen from the ground", and v' = .9c is "bullet-v as seen from the rocket". This formula gives the same correct results as mine, but I think "cars on the freeway" logic is much more intuitive and easier to use than trying to figure out "v, v' and u". ===
Your textbook probably gives a v-addition formula like $v = (u + v')/(1 + uv'/c^2)$, where v and u [without superscripts] are the velocities of two different objects "as seen by the observer" and v' [with the ' superscript] is "as seen by someone who is riding on the object moving at velocity u". In the example above, v = .988c is "bullet-v as seen from the ground", v' = .9c is "bullet-v as seen from the rocket", and u = .8c is "rocket-v as seen from the ground". This formula gives the same correct results as mine, but I think "cars on the freeway" logic is much more intuitive and easier to use than trying to figure out "v, v' and u". ==[two different objects added to this version

PROBLEM 16-A

You watch a super-rocket move rightward at +.9c. You see another rocket move → at .8c; what velocity do its passengers (like Rick) observe for the super-rocket?

Space Cowboy rides a rocket → at .8c (with respect to the earth); he shoots a bullet and watches it move → at .9c. On another rocket, Space Woman sees the bullet move ← at .7c. What is the velocity of SpaceWoman's rocket, as observed by Space Cowboy? Mutual observation: what is the velocity of SC's rocket, as observed by SW?

SOLUTION 16-A

If the situation is .9c> .8c>, Rick sees the SR "gaining on him" at a speed of .1c: he sees .1c>. If it is .8c> .9c>, Rick again sees .1c> as the SR "pulls away from him" at .1c. The correct relativistic speed is, as shown at the left below, .357c →.

(Since we only care about the relative velocities of SC, SW and the bullet, we can ignore the fact that SC is moving .8c> w.r.t. the earth.) There is only one way SW can see the bullet move ← at .7c; she must move → at 1.6c wrt SC (according to non-relativistic logic) so she "outruns" the bullet.

The solution below, which includes the relativistic correction factor, shows the actual situation; even though SC sees the bullet "losing ground" on SW at only .082c (=.982c–.900c), SW observes the bullet moving away from herself at .5c. When three objects (SC, bullet, SW) are involved we find this strange assymetry, but for any two objects the "mutual observation" is simple and symmetric. For example, when SC and SW observe each other, he sees her move .982c> and she sees him move <.982c

$$v_{observed} = \frac{.9c - .8c}{1 - (.9c)(.8c)/c^2}$$

$$v_{observed} = .357c, \rightarrow$$

$$v_{observed} = \frac{.9c + .7c}{1 + (.9c)(.7c)/c^2}$$

$$v_{observed} = .982c, \rightarrow$$

==If SW also moves .8c> w.r.t. earth, so she has v=0 w.r.t. SC, she sees the bullet move at .9c>. If she moves <---- w.r.t. SC, she sees a ----> bullet-v that is larger than

.9c as she "moves toward the bullet". If she moves ----> slowly (at < .9c) she sees a ----> bullet-v of less than .9c. If she moves ----> very fast (at > .9c) she sees

16.6 Relativistic Doppler Effect for Light Waves

Even though the speed of light is not affected by the motion of source or observer, the frequency and wavelength of light are affected by these motions.

In Section 9.4 the Doppler Effect formula for sound waves contains terms for the source-velocity and observer-velocity with respect to the propagating medium. But for light there is no propagating medium and it is impossible to tell whether the source or observer is moving; all we can know is their relative velocity. Because of this, the relativistically correct Doppler Shift formulas for light waves (shown below) contain only relative velocity. The ± sign of $v_{relative}$ is + if the source & observer are moving toward each other, and it is – if they're moving away from each other.

There are two Doppler formulas; some books use one, some the other. Can you derive the second equation from the first equation? Hint: multiply by 1, in the form of "x/x", where x is whatever is needed to turn $\sqrt{1 + v/c}$ (the first fraction-top) into $1 + v/c$ (the second fraction-top). As shown in Problem 16-2#, it is easier to solve for v when using the first equation.

$$f_{observed} = f_{source} \frac{\sqrt{1+\frac{v}{c}}}{\sqrt{1-\frac{v}{c}}} \qquad f_{observed} = f_{source} \frac{1+\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}}$$

RATIO LOGIC. As with sound, $f_{observed}$ is higher than f_{source} if source & observer move toward each other, and lower if they're moving apart. As you would expect, if v is large there is a large change in frequency and wavelength.

16.7 The General Theory of Relativity

CHOICES: Read this if your class studies general relativity, or if you're interested.

In Problem 3-A, the "apparent weight" of a 70 kg rider increases from 70(9.8) to 70(9.8 + 2.0) Newtons because the elevator accelerates upward at 2.0 m/s². If the man drops a coin, as in Problem 2-16, the coin's "apparent free-fall acceleration" changes from (–9.8) to (–9.8 –2.0) m/s² due to the elevator's 2.0 m/s² acceleration.

Do you see that the elevator's acceleration changes the "apparent effects" (either force or acceleration) of gravity? Such observations led Einstein to the theory of general relativity. One of its postulates is the *principle of equivalence:*

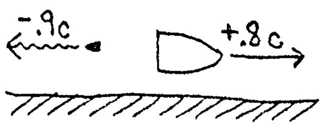
No experiment performed within a closed laboratory can distinguish between the effects of a gravitational field and the effects of an acceleration of the laboratory relative to the stars.

For example, if you are inside a rocket accelerating at 9.80 m/s² in outer space (with no gravity), every experiment you do gives exactly the same results as if you were at rest on the earth with a gravity-caused acceleration of 9.80 m/s². If there are no windows to look out of and see where you are, you cannot tell whether you are on a rocket or on the earth. You can interpret experiments in two equally valid ways: 1) a gravitational force of unknown origin is causing the accelerations I observe, or 2) I am in a non-inertial reference frame and acceleration causes the forces that I observe. (Section 3.1 assumes that "F causes ma", but according to general relativity this cause-effect relationship can also be thought of as "ma causes F"!)

A uniform acceleration and uniform gravitational field are equivalent if *gravitational mass* (in mg or GMm/r^2) and *inertial mass* (in F = ma) are equal. The equivalence of these masses had been verified experimentally (it was thought) but there have been recent attempts to find a "fifth force" that would cause a tiny difference between them. The result of these fifth force experiments is still uncertain; a few scientists claim "yes, there is a fifth force", others say "no, there isn't". ==[get good idea of present physics-community evaluation, edit my --- accordingly]

==[pseudo-forces, as in van-drop (3-##) and centrifugal force (5-#)

General relativity claims that "all physical laws can be formulated in such a way that they are valid for any observer, no matter how complicated his motion". This is not easy to do (it requires complicated mathematics that isn't covered in this book) but it is possible.

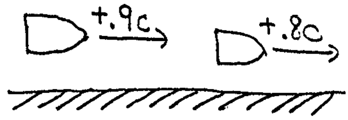


$$V_{bullet} = \frac{V_{rocket} + V'_{bullet}}{1 + \frac{V\,V'}{c^2}}$$

$$= \frac{(+.8c)+(-.9c)}{1 + \frac{(+.8c)(-.9c)}{c^2}}$$

$$= \frac{-.1c}{.28}$$

$$= -.357c$$



$$V_{bullet} = \frac{V_{rocket} + V'_{bullet}}{1 + \frac{V\,V'}{c^2}}$$

$$\frac{+.9c}{1} = \frac{(+.8c) + V'_{SUPER}}{1 + \frac{(+.8c)V'_{SUPER}}{c^2}}$$

$$+.9c + .72V'_{SUPER} = +.8c + V'_{SUPER}$$

$$.1c = .28\,V'_{SUPER}$$

$$+.357c = V'_{SUPER}$$

16.90 Memory-Improving Flash Cards

16.1	Special relativity is used for __ . General relativity is used for __ .	inertial (unaccelerated) reference frames either inertial or non-inertial reference frames
16.1	Special relativity is built on the __ of __ .	constancy, physical laws and light-speed
16.1	Relativity resolved the conflict between __ .	relativity principle and Maxwell's Equations
16.1	The speed of __ depends on relative motion.	everything except light
16.2	Glenn and Rhonda both say __ , but __ . Each sees the other's events go __ but __ .	my clock is OK, your clock runs slowly normal forward direction, in slow motion
16.2	A round-trip rocket twin ages __ because __ .	less, her acceleration destroys "symmetry"
16.2	Observers can disagree about __ if events __ and __ .	simultaneity, occur at different x-positions the observers have relative x-motion
16.2	Relativity makes statements about __ , not __ .	physics, philosophy
16.3	Differences in time, length & mass are a __ .	logical result of the "constancy postulates"
16.3	Properness: For __ , ask __ . But for __ and __ , ask __ . An observer can be __ for one quantity and __ .	time, Does he see i & f at same location? length, mass, Does he see the object at rest? proper, improper for another
16.3	Proper quantities are __ , the __ .	unique, smallest & largest observable
16.3	Improper observers see larger __ , smaller __ . The usual relativistic __ or __ factor is __ , which differs significantly from __ only __ .	time and mass, length dividing, multiplying, $\sqrt{1 - v^2/c^2}$ 1, at extremely high speed (close to c)
16.3	Only length that is __ changes.	parallel to the relative motion
16.4	An object with __ can never __ . Light has __ so it can __ .	mass, reach or exceed the speed of light rest mass = 0, travel at the speed limit "c"
16.4	An object can have energy due to __ or __ . For energy, "total = sum of parts" is __ , so __ , __ and __ are each conserved. At low speeds, __ is much larger than __ , but at very high speeds (above __) __ .	rest mass ($m_o c^2$), mass-in-motion (KE) $E_{total} = mc^2 = m_o c^2 + KE$ energy, mass, rest energy + kinetic energy $m_o c^2$, KE .866 c , KE is larger than $m_o c^2$
16.4	A periodic table lists __ ; m indicates __ .	m_o , an object's "resistance to acceleration"
16.4	Some valid equations are __ .	$F \Delta t = \Delta p$, $F \Delta x = \Delta KE = \Delta E_{total}$
16.4	When v « c, __ and __ equations are __ .	classical, relativistic, almost equivalent
16.5	The relativistic correction factor prevents __ .	predicted speeds from exceeding c
16.5	If speeds "add", the relativistic speed is __ . If speeds "subtract", the relativistic speed is __ .	smaller than classically predicted speed larger than classically predicted speed
16.6	Motion affects light's __ but not its __ . If source and observer are moving apart, __ .	frequency and wavelength, speed frequency decreases & wavelength increases
16.6	Relativity equations contain only __ velocity, because __ .	relative (not absolute) there is no "propagating medium" for light
16.7	Within a __ an observer cannot tell the difference between the effects of __ and __ .	closed laboratory gravity, reference-frame acceleration