Chapter 13

Alternating Current Circuits

Chapter 13 is mostly finished, but is not in camera-ready format.
This file doesn't have any of the diagrams, but it does have
some text-excerpts, with omissions indicated by .....

background: This chapter assumes you have studied Sections 11.10 (RC circuits) & 12.5 (LR circuits).
strategy: Read Sections 13-1 & 13-2 first. Then, if your class studies the time-dependency of voltage and phasor diagrams (or if you're curious and want to learn), read Sections 13.3 and 13.4.

13.1 Alternating Current

Chapter 11 described the behavior of capacitors & resistors in *direct current* (dc) circuits, where negatively-charged electrons always move in one direction, away from the battery's negative terminal toward its positive terminal.

As described in Section 12.4, electric generators at a power plant produce voltage whose magnitude and direction varies as shown by the *sine curve* on the first graph below. This alternating voltage causes *alternating current* (ac). In an *ac circuit,* electrons move one direction and then the opposite direction, as shown in the second graph, oscillating back and forth about relatively fixed positions.

[two graphs will be shown here]

Over one full cycle, ac voltage is positive for $\frac{1}{2}$ cycle and negative for $\frac{1}{2}$ cycle. A sine curve is symmetric, so the + and – voltages cancel each other and $V_{average}$ is zero. But taking the *root mean square average* (a technique shown in Problem 13-#) gives $\mathbf{V_{rms} = V_{max}/\sqrt{2} = .7071\, V_{max}}$. Similarly, $\mathbf{I_{rms} = .7071\, I_{max}}$.

In the United States, most ac current that is delivered to houses and businesses alternates at a rate of 60 cycles per second, with V_{rms} approximately 120 Volts, so $V_{peak} = V_{rms}/.707 = 120/.707 = 170$ Volts.

13.2 Equations for Alternating Current Circuits

The circuit below contains an *ac voltage source* (like an ac generator, or just plugging the circuit into an electrical wall-outlet) with a potential difference of $\Delta \mathbf{V_S}$, an *inductor* " " (I don't have the graphic in this file, so I'll just fill the spaces with INDUCTOR" temporarily) with $\Delta \mathbf{V_L}$ and *inductance* **L** , and a *resistor* "⌐\/\/\- " with $\Delta \mathbf{V_R}$ and *resistance* **R**, and a *capacitor* " ⊣⊢ " with $\Delta \mathbf{V_C}$ and *capacitance* **C**.

[picture of LRC circuit will be here]

The rest of Chapter 13 describes the behavior of this type of *LRC series circuit.*

The current and all four potential differences change throughout the cycle. Graphs of I and ΔV have the "continuous sine wave" shape shown in Section 13.1, with *frequency* **f** and *angular frequency* **ω**. As explained in Section 5.4c, $\omega = 2\pi f$.

At some point during each ac cycle, the current reaches a maximum $\mathbf{I}_{max}$. And at some point in the cycle, the voltage across the INDUCTOR, -/\/\/- and ⊣⊢ reach a maximum; these maximum voltages are $\Delta \mathbf{V}_{\mathbf{L}max}$, $\Delta \mathbf{V}_{\mathbf{R}max}$ and $\Delta \mathbf{V}_{\mathbf{C}max}$.

At any instant of time, current is the same in every part of a simple series circuit. If charge is moving ∅ at the rate of 2 Coulombs/second through the INDUCTOR, at this instant there is 2 Amps ∅ through the -/\/\/-, and also 2 Amps ∅ to and from the ⊣⊢.

But ΔV_{Lmax}, ΔV_{Rmax} and ΔV_{Cmax} always occur at different times, and usually have different magnitudes. The timing of these ΔV-maximums is discussed in Sections 13.3 & 13.4, along with their relationship to ΔV_{Smax}. The magnitudes of the ΔV-maximums are given by the equations below, which each has a format of " ΔV = I ( ) ".

$\Delta \mathbf{V}_{\mathbf{L}max} = \mathbf{I}_{max}\ \mathbf{X_L}$, where $\mathbf{X_L + \omega L}$ is the *inductive reactance* of the INDUCTOR.

$\Delta \mathbf{V}_{\mathbf{R}max} = \mathbf{I}_{max}\ \mathbf{R}$, where R is the *resistance* of the -/\/\/- .

$\Delta \mathbf{V}_{\mathbf{C}max} = \mathbf{I}_{max}\ \mathbf{X_C}$, where $\mathbf{X_C + 1 / \omega C}$ is the *capacitive reactance* of the ⊣⊢ .

$\Delta \mathbf{V}_{\mathbf{S}max} = \mathbf{I}_{max}\ \mathbf{Z}$, where $\mathbf{Z + \sqrt{R^2 + (X_L - X_C)^2}}$ is the *impedance* of the circuit.

In a dc circuit, R is the "resistance" to current flow. Similarly, in an ac circuit Z shows how much the circuit "impedes" the flow of current.

Each element in a circuit contributes to Z, the total impedance of the circuit. The -/\/\/- contributes resistance R, while INDUCTOR and ⊣⊢ contribute "reactances" of X_L and X_C. If you study Sections 13.3 and 13.4, you'll discover why Z is not a simple addition of "$R + X_L + X_C$".

R, X_L, X_C and Z have the same units: ohms, **Ω** .

$\Delta V_L = I_{max}\ \underline{\omega L}$: This equation shows that an INDUCTOR impedes current most when it has a large inductance L (when it is effective at "fighting") and when the frequency ω is large (when current is changing rapidly, thus producing a large ΔI/Δt to "fight against").

The closer a ⊣⊢ comes to holding its full charge, the more it impedes the flow of current. Two factors that help a ⊣⊢ reach full charge, and thus impede current, are small C (capacity to hold charge) and small ω (a long time-per-cycle allows charge to flow in one direction for a long time, thus causing a buildup). This is why ω and C are "on the bottom" in the V-formula: $\Delta V_C = I_{max} / \underline{\omega C}$.

For a dc series circuit with ω=0, after a long time INDUCTOR doesn't impede current at all because there are no changes to fight against, but a ⊣⊢ stops it completely. The X-equations correctly predict these results: substitution of "ω = 0" gives $X_L = \omega L = (0) L = 0$ [showing that INDUCTOR doesn't impede current] and $X_C = 1/\omega C = 1/(0) C = \infty$ [showing that ⊣⊢ impedes the current infinitely].

If an ac circuit is missing any standard elements (L, R or C) the above equations can still be used; just substitute $L = 0$, $R = 0$, or $C = \infty$.

Resonance

$I_{max} = \Delta V_{Lmax} / Z$, so current in an ac circuit is maximum when Z is minimum. The minimum Z (which is equal to $\sqrt{\mathbf{R^2 + (X_L - X_C)^2}}$ occurs at *resonance,* when $X_L - X_C = 0$, when $(\omega L) - (1/\omega C) = 0$, $\omega^2 LC = 1$, and $\omega = \sqrt{1/LC}$:

ac-circuit resonance, with minimum Z and maximum I_{max}, occurs when

$$\omega_{resonance} = \sqrt{\frac{1}{LC}} \qquad \text{and} \qquad f_{resonance} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

Units: ω is in radians/s, f is in cycles/second (also called Hertz, abbreviated Hz).

If a circuit is not resonant, resonance can be achieved by varying ω, L or C.
Problem 13-# shows how resonance is used to "tune" a radio or television.

Power

In an LRC circuit, power is dissipated (turned into thermal energy) only in -/\/\-. The INDUCTOR and ⊣⊢ store and release energy, but don't dissipate it. Because only R (not the entire Z) causes power dissipation, there is a *power factor* of R/Z in ac power formulas. If a circuit could have R = 0, R/Z = 0, and no energy would be dissipated. For a resonant circuit, $Z = \sqrt{R^2 + 0^2} = R$, and $R / Z = 1$. { Optional: some textbooks describe the power factor as cos ø, where cos Ø = (Z/R). The meaning of Ø is discussed in Section 13.4.}

In a dc circuit, P can be calculated in 3 ways: as IV, I^2R, or V^2/R. To calculate the *rms average power* for a full ac cycle, use any dc power formula, replace I & V with I_{rms} & V_{rms}, replace R with Z, and multiply by the power factor of Z/R:

$$P_{rms} = I_{rms} V_{rms} \frac{Z}{R} = (I_{rms})^2 Z \frac{Z}{R} = \frac{(V_{rms})^2}{Z} \frac{Z}{R}$$

P_{rms} can also be found using by using I_{max} & V_{max} (instead of I_{rms} & V_{rms}) and multiplying by ½ . Why? Because $I_{rms}V_{rms} = (.707\ I_{max})(.707\ V_{max}) = \frac{1}{2}\ I_{max}V_{max}$. Similarly, $(I_{rms})^2 = \frac{1}{2}\ (I_{max})^2$ and $(V_{rms})^2 = \frac{1}{2}\ (V_{max})^2$.

Optional: If your class studies *instantaneous power* (the rate of energy dissipation at an instant of time), use the instantaneous values of I & V: $P_{instantaneous} = I_{inst}\ V_{inst}\ (R/Z) = I_{inst}^2\ Z\ (R/Z) = (V_{inst}^2/Z)(R/Z)$. The time-dependence of I & V is discussed in Sections 13.3-13.4.

There are many formulas, but they're just familiar dc formulas with 2 changes: replace R with Z, multiply by R/Z. Use rms-values to get P_{rms}, or use max-values and multiply by ½ . Optional: to find P_{inst}, substitute instantaneous-values.

PROBLEM 13-A

An LRC series circuit with L = 80 mH, R = 20 Ω, and C = 50 μF, is driven by a 60 Hz source with an average voltage of 120 V. Find the circuit's inductive reactance, capacitive reactance, impedance, maximum current, maximum voltage across each circuit element, and average power. How can you change the ⊣⊢'s capacitance to get a "resonant circuit"?

SOLUTION 13-A

Translate words into variable-letters, choose a formula, substitute-and-solve:

inductive reactance $= X_L = \omega L = 2\pi f L = (2\pi)(60)(80 \text{ x } 10^{-3}) = 30.2\ \Omega$,

capacitive reactance $= 1/\omega C = 1/2\pi f C = 1/[(2\pi)(60)(50 \text{ x } 10^{-6})] = 53.1\ \Omega$,

impedance $= \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{20^2 + (30.2 - 53.1)^2} = 30.4\ \Omega$.

From Section 13.1, $\Delta V_{Smax} = \Delta V_{S\,rms}/.707 = (120)/.707 = 170\text{ V}$.

maximum current $= I_{max} = (\Delta V_S)_{max} / Z = 170 / 30.4 = 5.59$ Coulombs/s .

maximum voltages:

$\Delta V_{Lmax} = I_{max}(X_L) = (5.59)(30.2) = 169\text{ V}$,

$\Delta V_{Rmax} = I_{max}(R) = (5.59)(20) = 112\text{ V}$,

$\Delta V_{Cmax} = I_{max}(X_C) = (5.59)(53.1) = 297\text{ V}$.

$P_{rms} = I_{rms} V_{rms} (R/Z) = (.707\, I_{max})(.707\, V_{max})\ (R/Z)$

$= (.707)(5.59)\ (.707)(170)(20/30.4) = 313$ Watts,

$P_{rms} = \frac{1}{2}\ I_{max} V_{max} (R/Z) = \frac{1}{2}\ (5.59)(170)(20/30.4) = 313$ Joules/second

For resonance, $\omega^2 LC = 1$, $([2\pi][60])^2 (80 \text{ x } 10^{-3}) C = 1$, $C = 8.79 \text{ x } 10^{-5}\text{ F} = 879\ \mu\text{F}$.

Most equations in this section are for the "cycle as a whole". If your class studies instantaneous equations (that describe what is happening at an instant of time) and the time-dependence of voltage, read optional Sections 13.3 & 13.4. { No matter what your class does, you may find it helpful to read Section 13.3, which is short, intuitive and non-mathematical. }

13.3 Time-Dependence of Voltage Maximums

Sections 12.5 and 11.10 discuss LR and RC circuits. When the switch is closed in the LR circuit below, INDUCTOR produces ΔV_L to fight the change in current. After a long time, I through -\/\/\- has reached its maximum value and ΔV_R is a maximum; $\Delta I/\Delta t = 0$ so there is no need for INDUCTOR to "fight", and $\Delta V_L = 0$. Notice the order of voltages: first the maximum ΔV_L occurs, then the maximum ΔV_R .

In the rest of Chapter 13, I'll often shorten ΔV to V, but remember that V is a potential **difference**.

When the switch first closes in the RC circuit, I and V_R are maximum. After a long time the fully charged -||- has maximum V_C, but no current flows through the -||- (or -\/\/\-) so $V_R = 0$. First V_R occurs, then V_C.

When the switch closes on the LRC circuit, maximum V_L occurs first (as INDUCTOR reacts to fight the change) but V_R and V_C are zero. Then current increases and reaches a maximum, and so does V_R. Finally the current stops because -||- is fully charged, with maximum V_C; I is constant at zero, so V_L and V_R are zero. The order of voltage maximums is V_L, V_R and V_C: L R C .

[picture of LRC circuit will be here]

If the dc battery is replaced by an ac voltage source, current and voltages will change continuously. But the order of V-maximums is the same as in a dc circuit: first ΔV_{Lmax} occurs, followed $\frac{1}{4}$ cycle later by ΔV_{Rmax}-and-I_{max}, followed $\frac{1}{4}$ cycle later by ΔV_{Cmax}. The relationship between these V-maximums and ΔV_{Smax} is discussed in Section 13.4.

The connection between statements (like "V_L leads I" or "I lags behind V_L") and graphs (like V_L-versus-time) can be confusing. Section 13.4 shows an easy, logical way to understand these graphs.

The order of circuit elements doesn't matter. These circuits all behave the same:
.....[there will be pictures of circuits with elements in different order: LRC, LCR, RLC, RCL, LRC, LCR, or with multiple elements of any one type, as in LRCL, LRCR, and so on].....

13.4 Phasor Diagrams, Graphs, Phase Angles, and Time-Dependent Equations

Voltage relationships can be seen easily on a *phasor diagram.* The length of the L, R and C arrows (labeled on the first diagram below) represent the magnitudes of $\Delta V_{L\,max}$, ΔV_{Rmax} and $\Delta V_{C\,max}$, respectively. The vertical component of the L, R and C arrows (these can be called the *projections* of the arrows onto the vertical axis) show the values of V_L, V_R and V_C at a certain instant of time. The first diagram shows what is happening at the instant we define to be t + 0. Look at .....[picture will be here].....[this section will have more omissions because it's so "visual"]

Look at the tip of the R-arrow in the five diagrams. Do you see it make steady progress around the circle? ..... Do you see the relationship between the height of the R-arrow's tip and height of the V_R curve? { Just as we did in Section 8.2 for *simple harmonic motion,* we can take advantage of the fact that when an object moves around a circle at constant speed, the vertical component of its position imitates a "sine wave". }

You can use this same process to understand the sine curves for V_L and V_C.

A phasor diagram is like 3 cars, spaced $\frac{1}{4}$ cycle apart, moving around circles with different radii. The V_L car peaks first, then V_R, V_C and the empty spot (with no V-arrow). This multiple-cyclic process can be described as: L R C o L R C o L R C o ...

[picture goes here]

note: If your textbook uses phasor diagrams where V is represented by the horizontal (not vertical) component of V_{max}, just substitute "horizontal" for "vertical" in my descriptions.

At any instant, current is the same in every part of a circuit. I and ΔV_R reach their maximum value at the same time; they are "in phase with each other".

The first graph below shows that .....[snip].....

The second graph shows that ..... [pictures] .....

Study the first phasor diagram below and notice .....

Now look at the second phasor diagram, where .....

Kirchoff's law is true for "instantaneous" voltages, but not for "maximum" voltage:
$\Delta V_{L\,max} + \Delta V_{R\,max} + \Delta V_{C\,max} \neq \Delta V_{S\,max}$.

The four main equations of Section 13.2 have the same format: $\Delta V_{max} = I_{max}(\ )$. .....

Don't confuse visual-mathematical symbolism with reality. In Section 8.2 the movement of a race car around a circle is used to describe *simple harmonic motion* in equations, even though the object (a block moving back and forth) is not really moving in a circle. Similarly, we can use phasor diagrams to develop equations for the time-dependence of ΔV, even though the 4 voltages don't really move in a circle like the 4 arrow-tips do. And we add V_{max} arrows (for V_L, V_R and V_C) as vectors to get the correct $V_{S\,max}$, even though voltages aren't vectors.

the Phase Angle "ø"

If we define $t + 0$ when V_L is maximum (as in the earlier phasor diagrams) and $\theta + 0$ as the location of V_R at this time, then Ø gives the angular position of V_S at this $t + 0$ instant, and Ø is a *phase angle.* θ and Ø are defined the same as in Section 8.2 !

In the first diagram below, drawn at $t + 0$ when V_R is maximum, $V_{L\,max}$ is larger than $V_{C\,max}$ so the V_S arrow is ahead of the V_R arrow, and ø is +.

In the second diagram, V_{Cmax} is larger than V_{Lmax}; V_S is behind V_R and ø is –.

ΔV_L fights and delays the buildup of current. It is the deciding factor if ΔV_{Lmax} is larger than ΔV_{Cmax}, as in the first diagram; this causes the maximum of I-and-V_R to occur after the maximum of V_S.

The effect of ΔV_C is opposite that of ΔV_L. In the second diagram ΔV_C is the most important factor, so its "anti-delay" makes I_{max} (and ΔV_{Rmax}) occur before ΔV_{Smax}. Because of the way ø is defined, instead of saying "V_{Rmax} is ahead of V_{Smax}" we can state the same fact as "V_{Smax} is behind V_{Rmax}", which means that ø is –.

[diagrams will be here]

As described earlier, the V_{max}-triangle and XRZ-triangle have the same shape and same ø, so we could draw the analogous XRZ (instead of V_{max}) triangle and use this same "ahead or behind" logic to reach the following useful conclusions.

When X_L is larger than X_C, the circuit's reactance is "inductive" and ø is +.
When X_C is larger than X_L, the circuit's reactance is "capacitive" and ø is –.
When X_L equals X_C, the circuit is "resonant" with no reactance, and ø is zero.

" $ø = \tan^{-1}[(X_L - X_C)/R]$ " gives the correct ø, including the correct ± sign.

Time-Dependent Equations

To understand the equations below, think about this step-by-step derivation:

Each $V_{instantaneous}$ is the vertical component of the V_{max} phasor arrow,
so $V_{instantaneous} = V_{max}\ \sin[\text{angular } \theta\text{-position of } V_{max} \text{ phasor arrow}]$.

(θ-position of arrow) $= \omega t + \theta_i$, where θ_i is the position when $t + 0$.
When $t + 0$, the V_L-arrow is at $+\frac{1}{2}\pi$, V_R-arrow is at 0, V_C-arrow is at $-\frac{1}{2}\pi$,
V_S-arrow is at $+ø$, and I (which is in-phase with the V_R-arrow) is at 0.

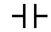
$$\Delta V_L = \Delta \mathbf{V}_{\mathbf{L}\,max} \sin(\omega t + \tfrac{1}{2}\pi) = +\Delta \mathbf{V}_{\mathbf{L}\,max} \cos(\omega t)$$
$$\Delta V_R = \Delta \mathbf{V}_{\mathbf{R}\,max} \sin(\omega t)$$
$$\Delta V_C = \Delta \mathbf{V}_{\mathbf{C}\,max} \sin(\omega t - \tfrac{1}{2}\pi) = -\Delta \mathbf{V}_{\mathbf{C}\,max} \cos(\omega t)$$
$$\Delta V_S = \Delta \mathbf{V}_{\mathbf{S}\,max} \sin(\omega t + \text{ø})$$
$$I = \mathbf{I}_{max} \sin(\omega t)$$

These equations can be "linked" with all equations from Section 13.2 that contain $\Delta V_{L\,max}$, $\Delta V_{R\,max}$, $\Delta V_{C\,max}$, $\Delta V_{S\,max}$ or I_{max}. Also, $\boldsymbol{\omega = 2\pi f}$.

If your textbook uses horizontal (not vertical) components of the V-arrows, or if it defines t + 0 at a different time (for example, when V_S is a maximum), its equations will differ from the equations above. But the basic principles are identical, and when used correctly either set of equations will give correct answers.

13.90 Memory-Improving Flash Cards

13.1	In a dc circuit, electrons __ . In an ac circuit, electrons __ .	always move in the same direction alternate directions (oscillate back & forth)
13.1	The shape of a V(or I)-versus-t graph is __ .	sine wave (is assumed for all of Chapter 13)
13.1	V_{rms} = __ , so V_{max} = __ . I_{rms} = __ .	.707 V_{max}, V_{rms} / .707, .707 I_{max}
13.2	The rest of Chapter 13 describes the __ .	behavior of a series LRC circuit
13.2	Maximum values of V_L, V_R & V_C occur at __ . At every instant, current is __ .	different times equal in every part of a series circuit
13.2	V_{max} equations for L, R, C & S have form __ , where [ ] is __ , which all have __ .	$\Delta V_{max} = I_{max}$ [ ] X_L, R, X_C or Z ; same units (ohms, **Ω**)
	__ and __ both contribute to __ , which indicates __ .	reactance-X's, resistance-R, impedance-Z how much a circuit impedes ac current flow
13.2	An inductor has large ΔV_L when it __ , and impedes most when __ , so __ .	fights against current-changes large L and high f, $X_L = \omega L$
	A capacitor has near-max ΔV_C when it __ , and impedes most when __ , so __ .	is close to holding its full charge small C and low f, $X_C = 1/\omega C$
	At __ frequency, __ is most important. (2)	high, L ; low, C
13.2	If ac circuit is missing LRC elements, __ .	substitute L = 0, R = 0, C = ∞
13.2	Max __ occurs at min __ , which is called __ . This occurs when __ .	I_{max}, Z, resonance $X_L = X_C$, $\omega^2 LC = 1$, $\omega = \sqrt{1/LC}$
	__ is larger than __ . To convert, __ or __ .	ω, f, $\omega = 2\pi f$, $\omega / 2\pi = f$

13.2	In LRC, __ dissipates energy, __ __ energy.	  & INDUCTOR, store & release
13.2	To get P-formulas, use __ , then __ __ , which is the __ .	$P = IV = I^2 R = V^2/R$, replace R by Z multiply by R/Z, power factor
	To get $P_{root\text{-}mean\text{-}square}$, substitute __ or __ .	rms-values, max-values and multiply by $\frac{1}{2}$
	Optional: To get $P_{instantaneous}$, substitute __ .	instantaneous-values
13.3	V_{max} order: in LR __ , in RC __ , in LRC __ .	is L R, is R C, is L R C
	Starting with __ , there is __ between __ .	V_L , $\frac{1}{4}$ cycle ($\frac{1}{4}$ T, 90°, π/2 rads), V_{max}'s
	Circuit behavior doesn't depend on the __ .	order of circuit elements
13.4	A phasor diagram is analogous to __ .	3 race cars ($\frac{1}{4}$ cycle apart, different radii)
	If V_S is also drawn, there are __ that are __ .	4 "cars", located at the tip of each arrow
13.4	At any instant __ is same thru circuit, __ aren't.	current (I), potential differences (ΔV's)
	__ are always in phase. V-cycle is __ .	I and V_R , L R C o L R C o L R C o ...
13.4	$V_{maximum}$ is __ , $V_{instantaneous}$ is __ .	arrow's length, arrow's vertical component (or horizontal component if your class...)
	phasor & graph: same __ for __ & __ .	height, arrow-tip, V-on-graph
13.4	To interpret graph correctly, ask __ not __ .	which peak occurs first? who wins race?
13.4	__ = __ for __ but not for __ .	$V_L + V_R + V_C = V_S$, inst-V, max-V
13.4	right triangle: legs are __ , hypotenuse is __ ,	($V_{Lmax} - V_{Cmax}$) and V_{Rmax} , V_{Smax}
	or legs are __ , hypotenuse is __ .	($X_L - X_C$) and R , Z
	These triangles (__) are __ so they have __ .	V_{max} & XRZ, similar, same ø
13.4	I_{max} always occurs after __ and before __ .	V_{Lmax} , V_{Cmax}
	If __ (so __), V_{Smax} occurs __ I_{max} , ø is __ . { repeat for 3 possibilities }	$\mathbf{X_L} > X_C$, $V_{Lmax} > V_{Cmax}$, before, + $\mathbf{X_C} > X_L$, $V_{Cmax} > V_{Lmax}$, after, − $X_L = X_C$, $V_{Lmax} = V_{Cmax}$, at same t as, 0

Eventually, Chapter 13 will be "finished" in a camera-ready format.