Chapter 4

Work and Kinetic Energy, Impulse and Momentum

Work-Energy and Impulse-Momentum are closely related topics. When your class studies either of them, read Section 4.1, which shows their similarities.

Then use Chapter 4A (starting in Section 4.2) if you study Work-Energy first, or Chapter 4B (starting in Section 4.8) if you're studying Impulse-Momentum.

Section 4.12 is an overview; it shows a "general strategy" for using all of the tools you've learned in Chapters 2 through 4A & 4B.

4.1 Deriving the Work-Energy and Impulse-Momentum Equations

If **F** is constant (so **a** is constant), tvvax equations are valid.
You can use the *a-link,* as shown below, to derive two new equations:

$$v_f^2 - v_i^2 = 2a\,\Delta x$$

Use the a-link; substitute F/m for a.

$$v_f^2 - v_i^2 = 2\left(\frac{F}{m}\right)\Delta x$$

Multiply both sides by $\frac{1}{2}m$, simplify, exchange l & r sides.

$$\tfrac{1}{2}m\left[v_f^2 - v_i^2\right] = \tfrac{1}{2}m\left[2\,\frac{F}{m}\,\Delta x\right]$$

$$F\,\Delta x = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$v_f - v_i = a\,\Delta t$$

Use the a-link; substitute F/m = a.

$$v_f - v_i = \left(\frac{F}{m}\right)\Delta t$$

Multiply both sides by m, simplify, exchange sides.

$$m\left[v_f - v_i\right] = m\left[\frac{F}{m}\,\Delta t\right]$$

$$F\,\Delta t = mv_f - mv_i$$

DEFINITIONS. These math terms are given a name and an abbreviation symbol:

$F\,\Delta x$ is given the name work, abbreviated W.
$\frac{1}{2}mv^2$ is given the name kinetic energy, abbreviated KE or K.
$F\,\Delta t$ is given the name impulse; there is no common abbreviation symbol.
mv is given the name momentum, abbreviated p.

Substituting these definitions, each equation can be written many different ways:

$F\Delta x$	=	$\frac{1}{2}mv_f^2$	–	$\frac{1}{2}mv_i^2$	$F\Delta t$	=	mv_f	–	mv_i
Work	=	KE_f	–	KE_i	Impulse	=	p_f	–	p_i
W	=		ΔKE		Impulse	=		Δp	

Any version of an equation's left side (like $F\Delta x$, Work or W) is equal to any version of its right side ($\frac{1}{2}v_f^2 - \frac{1}{2}v_i^2$, $KE_f - KE_i$ or ΔKE), so any left-side and right-side version can be combined into an equation. This idea is a powerful problem solving tool, as you'll discover in Section 4.7, "Many-Sided Equations".

Notice the difference between KE and ΔKE (or between p and Δp). $\frac{1}{2}mv_f^2$ is the KE at a specific time, the interval's final point. But the equation's entire right side {$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$, or $KE_f - KE_i$} is ΔKE, the change of kinetic energy.

CAUSE ⇒ EFFECT

As with F=ma (in Section 3.1), these new equations are cause-effect relationships:

$F \Rightarrow ma$	$F\Delta x \Rightarrow \Delta(\frac{1}{2}mv^2)$	$F\Delta t \Rightarrow \Delta(mv)$
F causes $m(\Delta v/\Delta t)$.	$F\Delta x$ causes ΔKE.	$F\Delta t$ causes Δp.

F=ma [which is $F = m(\Delta v/\Delta t)$] tells you what is happening at one instant of time, at one point in space. But $F\Delta x=\Delta KE$ and $F\Delta t=\Delta p$ describe the accumulated effects of acceleration that occur during an interval of distance or time.

UNITS: Work and KE have the same units, kg m^2/s^2. This combination is given the name *Joule,* abbreviated "J": 70 kg m^2/s^2 ≡ 70 Joules ≡ 70 J. { As emphasized in Section 3.2, an equation's "=" sign states that every equation-term has the same units. Check the work-equation: do $F\Delta x$, $\frac{1}{2}mv_f^2$ and $\frac{1}{2}mv_i^2$ all have the same units? }

Other energy-related units, Watts and kilowatt-hours, are covered in Section 4.5.

Impulse and momentum, $F\Delta t$ and mv, both have units of kg m/s. There is no SI name or abbreviation for this combination of units.

OPTIONAL CALCULUS: Section 18.31 derives $F\Delta x=\Delta K$ and $F\Delta t=\Delta p$ for situations when F is not constant, and uses the *chain rule* to derive two new relationships.

CHOICES: If your class is studying work-energy, continue on to the next section. If you study impulse-momentum first, skip to Chapter 4B (Sections 4.5 to 4.7.

Chapter 4A: Work ⇒ Δ(Kinetic Energy)

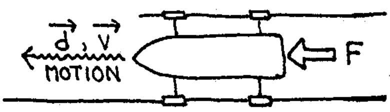
Sections 4.2 to 4.4 are the core of Chapter 4A. The main focus of these sections, the Total Work Equation in 4.3, is a valuable problem-solving tool. When your class studies "power", read Sections 4.5 to 4.7.

4.2 For straight-line motion, Work = F d cosØ.

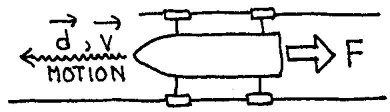
If an object moves in a straight line, and the force acting on it is constant, work can be calculated using "W ≡ F Δx". To let you analyze motion in any direction, Δx is replaced with the object's *displacement vector* "d"; to get d, just draw a straight line between the object's initial & final positions. (Δx and d are analogous. If movement is parallel to the x-axis, displacement can be described with "Δx". But "d" can be used for movement in any direction: x, y, z or anywhere in-between.)

For straight-line motion, an object's displacement and velocity vectors (**d** and **v**) always point in the same direction, because **d** = **v** Δt.

These *bird's-eye pictures* (views from above) show how to find the ± sign of work:



If $\vec{F}$ is in the same direction as $\vec{d}$, the cart's speed increases so $\frac{1}{2}mv^2$ (the KE) increases; ΔKE is +, and W is +.

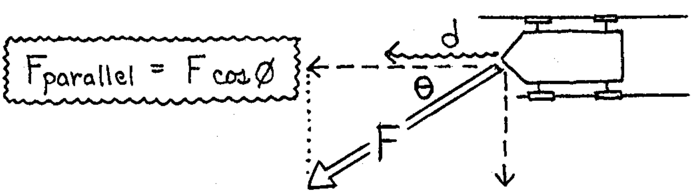


If $\vec{F}$ and $\vec{d}$ are in opposite directions, the cart's speed decreases so $\frac{1}{2}mv^2$ (the KE) decreases; ΔKE is −, and W is −.

In Chapters 2 & 3, a ± sign showed whether a vector (Δx, v, Δv, a or F) pointed in the + or − direction. But W is not a vector. The ± sign of W doesn't show "direction"; instead, a + or − sign shows whether the Work causes ΔKE to be + or −.

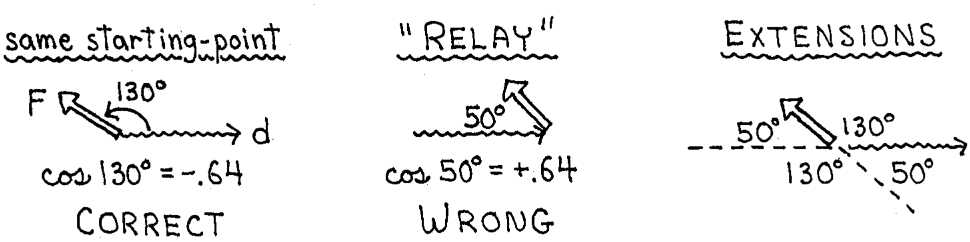
The car is on rails that allow the car to move only in the x-direction. Force in the y-direction is perpendicular to the x-motion; this y-force will not cause KE to change, because only *parallel force* (along the direction of motion) produces Work and ΔKE.

If ø is the angle between the F and d vectors, then Work = $F_{parallel}$ d = F d cos ø:



$$W = F_{parallel}\, d = (F \cos \emptyset)\, d = F\, d \cos \emptyset$$

The drawings below show 3 ways to find the angle between **F** and **d**. In the first picture, where both vectors begin at the same location, "W = F d (cos 130°) = Fd(−.64)" gives the correct magnitude and ± sign for W. In the second picture, where **F** and **d** are drawn head-to-tail using the *relay* method of Section 1.4, "W = F d (cos 50°) = Fd(+.64)" gives the correct W-magnitude, but the ± sign of W is wrong.



The third picture shows that <u>any</u> angle between **F** and **d** (or their extensions) gives the correct magnitude. To get W's ± sign, either be sure you have drawn the vectors "starting from the same location", or use this common sense logic:

If F will cause speed (and thus KE) to increase, the Work done by F is +.
If F will cause speed (and thus KE) to decrease, the Work done by F is –.

ZERO-WORK SITUATIONS: W = 0 if any multiplying factor in "F d cos Ø" is zero. Work = 0 if F= 0, or d = 0 (when no movement occurs), or cos Ø = 0 (when F is ⊥ to d).
{ If you hold a 100 pound box for 10 minutes, will you do any work? Your muscles may get tired, and you are exerting effort; but according to the physics definition, Work = 0 because d = 0. }

TOTAL = SUM OF PARTS: If forces A, B and C act on an object to produce works of W_A, W_B and W_C, respectively, then W_{total} (which causes ΔKE) is $W_A + W_B + W_C$.

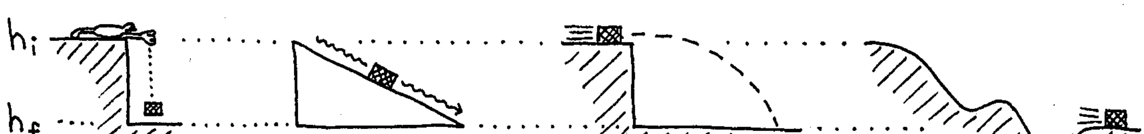
You can use "W = F d cos Ø" only if **F** is constant. If **F** is not constant, you can calculate Work by using *areas* (as discussed in Section 4.6) or *integrals* (an optional way to find area using "calculus", as explained in Sections 18.1, 18.2 & 18.31).

Have you noticed that we're using a new kind of vector equation? In a Δt = Δv, one vector (a) is multiplied by a non-vector (Δt) to produce another vector (Δv). But in "**F** Δ**x** = W", two vectors multiply to produce a non-vector!
OPTIONAL: If your class studies *dot-product multiplication of vectors*, read Section 18.51 now.

4.3 How to Derive and Use **the Total Work Equation**

The Total Work Equation is an easy way to solve most work-energy problems.
To derive it, we'll split total work into 4 categories (the work done by gravity force, by spring force, by friction, and by all other forces), then define "potential energy", and finally recombine everything into one equation.

1) **GRAVITY WORK:** If an object with mass m is near the earth's surface where $F_{gravity}$ = mg, and "up" is defined to be the + direction, and its i & f positions have vertical heights of h_i & h_f, the work done by $F_{gravity}$ is $W_{gravity} = -mg(h_f - h_i)$.
$W_{gravity}$ doesn't depend on how the object gets from h_i to h_f. $W_{gravity}$ is the same for the four journeys below, because each has the same "$h_f - h_i$ = Δh"; it doesn't matter if the path is vertical ↓, diagonal ↘, curved ⤷ or irregular ∿.



2) **SPRING WORK:** { If your class hasn't studied springs yet, skip this and go on to "friction work". } The force caused by a coiled spring is described in Section 3.8. As shown in Problems 4-# (using areas or optional calculus), if a spring's length changes from x_i to x_f, the work it does is $W_{spring} = -[\frac{1}{2}k(x_f - x_e)^2 - \frac{1}{2}k(x_i - x_e)^2]$. Usually, x_e is defined to be zero, and this formula simplifies to "$W_{spring} = -[\frac{1}{2}k x_f^2 - \frac{1}{2}k x_i^2]$", which I'll use in the Total Work Equation.

3) **FRICTION WORK:** Friction force is described in Section 3.7. When friction "produces" motion, as illustrated in Problem 3-#, the work done by friction can be +. But for use in the T.W.Equation, $W_{friction}$ means the work done when *kinetic friction force* (f_k) opposes an object's forward motion.

If an object slides a distance "d", $W_{friction} = -f_k\, d = -\mu_k N\, d$. In many situations (but not all, as emphasized in Section 3.4), $W_{fr} = -\mu_k mgd$ on a horizontal surface and $W_{fr} = -\mu_k\, mg(\cos\theta)\, d$ on an inclined plane. W_{fr} is always –, thus decreasing speed and KE, because f_k always points in the direction opposite to d.

4) **OTHER WORK:** For work done by all other forces, use "$W_{other} = F_{other}\, d \cos \emptyset$".

Some examples of "other forces" are a rope's tension pull, a person pushing an object, and the acceleration (or braking) force that is generated in a car's engine (or brake drums) and is applied through the tires' friction.

POTENTIAL ENERGY, symbolized by either "PE" or "U", is defined only for *conservative forces.* A force is *conservative* if the work done by it is zero during a round-trip cycle. As shown in Problem 4-#, $F_{gravity}$ and F_{spring} are conservative forces, but $F_{friction}$ is not conservative.

For physics, ΔPE (PE change) is used, rather than PE itself. ΔPE is defined for gravity and springs, but not for friction, by changing the ± sign of W: $\Delta PE \equiv -W$.

$$\begin{aligned}\Delta PE_{gravity} &\equiv - W_{gravity} \\ &= -[-mg(h_f - h_i)] \\ &= +mg(h_f - h_i) \\ &= +mg\ \Delta h\end{aligned}$$

$$\begin{aligned}\Delta PE_{spring} &\equiv - W_{spring} \\ &= -[-(\tfrac{1}{2} k x_f^2 - \tfrac{1}{2} k x_i^2)] \\ &= \tfrac{1}{2} k x_f^2 - \tfrac{1}{2} k x_i^2\end{aligned}$$

Some intuitive aspects of Potential Energy are explored in Section 4.4.

It's easy to derive the **TOTAL WORK EQUATION** from "$W_{total} = \Delta KE$".

As you read each of these 4 steps, look at the corresponding math step below.

1) The TOTAL WORK done on an object can be split into 4 categories: the work done by gravity, springs, dissipative friction, and all other forces combined.
2) For conservative forces like $F_{gravity}$ and F_{spring}, $\Delta PE \equiv -W$, so $-\Delta PE = W$; replace $W_{gravity}$ and W_{spring} with their –ΔPE's.
3) Bring 3 terms from the left to right side. $W_{friction}$ is always –; when it is brought to the right side, $W_{friction}$ is always +; this is shown by | | *absolute value signs.*
4) Substitute specific formulas for W_{other}, ΔKE, $\Delta PE_{gravity}$, ΔPE_{spring} and $W_{friction}$.

$$W_{TOTAL} = \Delta KE$$
$$W_{gravity} + W_{spring} + W_{friction} + W_{other} = \Delta KE$$
$$-\Delta PE_{gravity} - \Delta PE_{spring} + W_{friction} + W_{other} = \Delta KE$$
$$W_{other} = \Delta KE + \Delta PE_{gravity} + \Delta PE_{spring} + |W_{friction}|$$

$$F_{other}\, d \cos\emptyset = (\tfrac{1}{2} mv_f^2 - \tfrac{1}{2} mv_i^2) + (mgh_f - mgh_i) + (\tfrac{1}{2} kx_f^2 - \tfrac{1}{2} kx_i^2) + |W_{friction}|$$

When using $mgh_f - mgh_i$, $mg(h_f - h_i)$ or $mg(\Delta h)$, you must define "up" to be +.
$|W_{friction}|$ is always +. $W_{friction} = f_k\, d = \mu_k N d$; it is often $\mu_k mgd$ or $\mu_k mg(\cos\theta)d$.

Another "Total Work Equation" format is shown below. It is the same as above, but all "i" terms are on the left side, and all "f" terms are on the right side.

The following analogy will help you understand what it means. If you begin with \$50 (initial dollars), earn \$70 (dollar input) and lose \$10 (dollar waste), you'll end up with \$110 (final dollars), because 50 +70 −10 = +110. This can be rearranged to get 50 +70 = +110 +10: (initial dollars) + (dollar input) = (final dollars) + (wasted dollars).

Now think "energy" instead of "dollars", and the equation below will make sense.

$$\underbrace{\tfrac{1}{2}mv_i^2 + mgh_i + \tfrac{1}{2}kx_i^2}_{\text{INITIAL ENERGY (KE+PE)}} + \underbrace{W_{OTHER}}_{\text{ENERGY INPUT}} = \underbrace{\tfrac{1}{2}mv_f^2 + mgh_f + \tfrac{1}{2}kx_f^2}_{\text{FINAL ENERGY (KE+PE)}} + \underbrace{|W_{FRICTION}|}_{\text{WASTED ENERGY}}$$

Both forms of the T.W.Equation are identical; the terms are just rearranged. I'll use the bottom form for the rest of this book, but the top form is just as good. If your textbook and teacher prefer one of the forms, it may be convenient for you to also use it. In either format, the T.W.E. is a valuable problem-solving tool; it is easy to use, and adapts to a wide variety of situations. Here are examples.

PROBLEM 4-A: Gravity-Work on an Inclined Plane

Part 1: A block starts from rest at the top of a 5.00 m high 51.2° inclined plane. When it reaches the bottom, its speed is 8.40 m/s. What is μ_k?

Part 2: The diagonal sliding-surface of another 51.2° plane is 25.7 m long; its μ_k is the same as the first plane. If a block has $v_i = 0$ at the top, what is its v at the bottom?

Part 3: The block in Part 1 has a mass of 10.0 kg; find $W_{friction}$, the force of friction, and the coefficient of kinetic friction, without using the μ_k you found in Part 1.

SOLUTION 4-A

Parts 1 & 2) Draw the block's F-diagram, then consider the W done by the 3 forces: N, mg and f_k. N is ⊥ to d, so $W_N = Fd\cos\emptyset = Fd\cos 90° = Fd(0) = 0$. Because N is the only "other force" and $W_N = 0$, W_{other} can be crossed out. There are no springs, so PE_{spring} (the $\frac{1}{2}kx^2$'s) can be crossed out. Substitutions for KE's, mgh's (with h ≡ 0 at the lowest point, the bottom of the ramp) and $W_{friction}$ (= $mg\cos\emptyset\, d$) are shown below. Every term contains "m" so it can be divided out, as shown by the /'s.

Part 1:

$$\tfrac{1}{2}mv_i^2 + mgh_i + \cancel{\tfrac{1}{2}kx_i^2} + \cancel{W_{other}} = \tfrac{1}{2}mv_f^2 + mgh_f + \cancel{\tfrac{1}{2}kx_f^2} + \mu_k mg\cos\theta\, d$$

$$0 + \cancel{m}(9.8)(5.0) = \tfrac{1}{2}\cancel{m}(8.4)^2 + \cancel{m}g(0) + \mu_k \cancel{m}(9.8)\cos 51.2°\left(\frac{5.0}{\sin 51.2°}\right)$$

$$.348 = \mu_k$$

Part 2:

$$\tfrac{1}{2}\cancel{m}v_i^2 + mg\,h_i = \tfrac{1}{2}\cancel{m}v_f^2 + \cancel{m}gh_f + \mu_k \cancel{m} g\, d\cos\theta$$

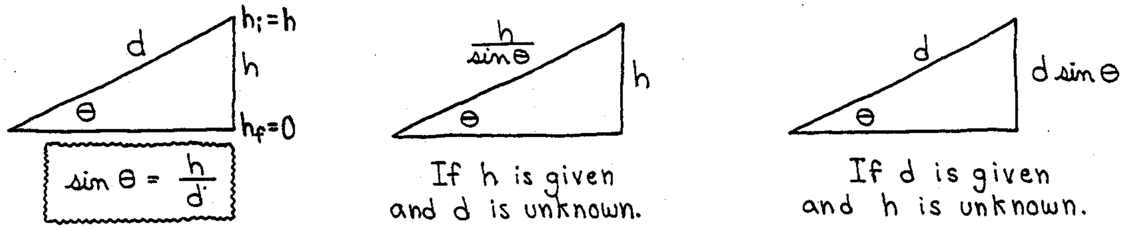
$$\tfrac{1}{2}(0)^2 + (9.8)(25.7\sin 51.2°) = \tfrac{1}{2}v_f^2 + g(0) + .348(9.8)(25.7)\cos 51.2°$$

$$16.4\ \tfrac{m}{s} = v_f$$

Notice the four terms that are underlined with ~~~'s.

The plane-height "h" is used for mgh_i. But the plane-length "d", the distance the block actually slides along the ramp, is used in $W_{friction} = \mu_k mg(\cos\theta)\,d$.

The first triangle below shows the relationship of θ, h and d. h and d can both be defined in terms of one variable, either h [in the second triangle, d = h/sinθ] or d [in the third triangle, h = d sinθ]. These substitutions were used in the algebra above.



Part 2: On the second plane, $mg\Delta h$ is 4 times as large, and $\mu_k mgd(\cos\theta)$ is 4 times as large, but v_f is only doubled. Do you see why? Hint: $W = \Delta KE = \Delta(\frac{1}{2}mv^2)$.

Part 3: This problem emphasizes the difference between friction-work (W_{fr}), friction-force (f_k) and the coefficient of friction (μ_k). Use the same TWE as in Part 1, and substitute for $|W_{friction}| = f_k\, d = \mu_k\, m\, g\, (\cos\theta)\, d$ in three separate steps:

Step 1	Step 2	Step 3
$10(9.8)(5.0) = \frac{1}{2}(10)(8.4)^2 + \lvert W_{fr}\rvert$	$\lvert W_{fr}\rvert = f_k \quad d$	$f_k = \mu_k \quad m \quad g \quad (\cos\theta)$
$+137.2 = \lvert W_{fr}\rvert$	$137.2 = f_k\,(5.0/\sin 51.2°)$	$21.4 = \mu_k\,(10)(9.8)(\cos 51.2°)$
W_{fr} is always –, so $W_{fr} = -137.2$	$21.4 = f_k$	$.348 = \mu_k$

PROBLEM 4-B: Gravity-Work during Free Flight

A 2.0 kg ball is thrown from a 10.0 m high building with initial speed 26.8 m/s. Just before it hits the ground, what is its speed if the ball is thrown straight up? straight down? horizontally? aimed 40° above horizontal? 40° below horizontal?

Can you use the TWE to find the ball's velocity in any of these situations?

SOLUTION 4-B

There is no PE_{spring}, W_{other} or $W_{friction}$. The TWE for all 5 throws is the same:

$$\tfrac{1}{2}\, m\, v_i^2 = \tfrac{1}{2} m v_f^2 + m\, g\, \Delta h$$
$$\tfrac{1}{2}\,(26.8)^2 = \tfrac{1}{2}\, v_f^2 + 9.80\,(-10.0)$$
$$30.2 \text{ m/s} = v_f$$

Is $v_f = 30.2$ m/s the ball's speed or velocity? For each of the 5 throws, visualize the ball-path and just-before-impact velocity of each throw. Are they all the same? No. All 5 throws have the same v_f magnitude of 30.2 m/s, but their v_f directions differ.

The "v" in the Total Work Equation represents only v-magnitude (speed); it doesn't give any information about v-direction!

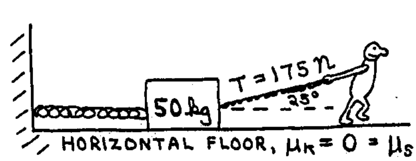
$F\Delta x$, Work and $\frac{1}{2}mv^2$ are *scalars* (non-vectors) that have ± sign but no "direction".

You can't find the direction of v_f with the T.W.E. {although common sense tells you the straight-up and straight-down throws will have a straight-down v_f}, but it's easy to find the magnitude & direction of v_f by using tvvax as in Section 2.8, or with the "3-Dimensional Work Equation" from Section 18.32 (optional, using calculus).

PROBLEM 4-C (If your class hasn't studied springs, skip this and read the Strategy Summary.)

When it is 5.00m from the wall, the block is moving → at 2.0 m/s. The man pulls with constant force. The spring has $k = 40$ N/m, $x_e = 4.20$m. When the block is 8.00m from the wall, what is its speed?

Does the man need "spiked shoes"? Why?



SOLUTION 4-C

Draw the block's F-diagram. As usual, N is ⊥ to d, so $W_N = 0$. Eliminate $PE_{gravity}$ (a "horizontal floor" means that $h_i = h_f$) and $W_{friction}$ (because $\mu_k = 0$). If the floor is really frictionless, the man can't walk without spiked shoes.

The block moves from 5.00 m to 8.00 m, so d = 3.00 m. Form a mental picture of the spring at 4.20, 5.00 and 8.00. If $x_e \equiv 0$, do you see why $x_i = +.80$ and $x_f = +3.80$?

$$\tfrac{1}{2} m v_i^2 + \cancel{mgh_i} + \tfrac{1}{2} k x_i^2 + F_{other}\, d \cos\phi = \tfrac{1}{2} m v_f^2 + \cancel{mgh_f} + \tfrac{1}{2} k x_f^2 + \cancel{|W_{fr}|}$$

$$\tfrac{1}{2}(50)(2)^2 + \tfrac{1}{2}(40)(.8)^2 + 175\,(3.0)\cos 25^\circ = \tfrac{1}{2}(50) v_f^2 + \tfrac{1}{2}(40)(3.8)^2$$

$$+\,3.46\ \tfrac{m}{s} = v_f$$

A Strategy Summary for the Total Work Equation:

Read, think, draw a picture. Choose i & f points for a distance interval.

If a height change or spring-force is involved, define $h \equiv 0$ (usually at i or f, whichever is lower, or at the lowest point in the situation-picture) and $x \equiv 0$ (usually at the spring's equilibrium length "x_e", or at some convenient place like a wall or ...).

Eliminate the unnecessary parts of the TWE (use all clues!). Substitute & solve.

Remember that $|W_{friction}|$ is always +, and to define "up" as + for $mg\Delta h$.

If every equation term contains "m", cancel all m's. $\frac{1}{2}mv^2$, mgh and $\mu_k Nd$ (when N is either mg or $mg\cos\theta$) all have m, so m's often cancel. But $\frac{1}{2}kx^2$ and (usually) W_{other} don't have m, so m-cancelation isn't always possible.}

During substitution and algebra, be sure to "square" the v & x in $\frac{1}{2}mv^2$ & $\frac{1}{2}kx^2$.

The TWE and $F\Delta x = \Delta KE$ are identical {the TWE derivation clearly shows that TWE is just a detailed version of $F\Delta x = \Delta KE$}; you can use either equation to solve problems.

Some books give "semi-total work equations" that omit one or more of the 5 terms in the TWE. Semi-total equations can show useful relationships, but for problem-solving I think it's better to have one equation that always works, then knock off the parts that aren't needed for a particular problem.

If you've read Chapter 4B, you can now use the Section 4.12 overview.

4.4 Transformations of Potential and Kinetic Energy

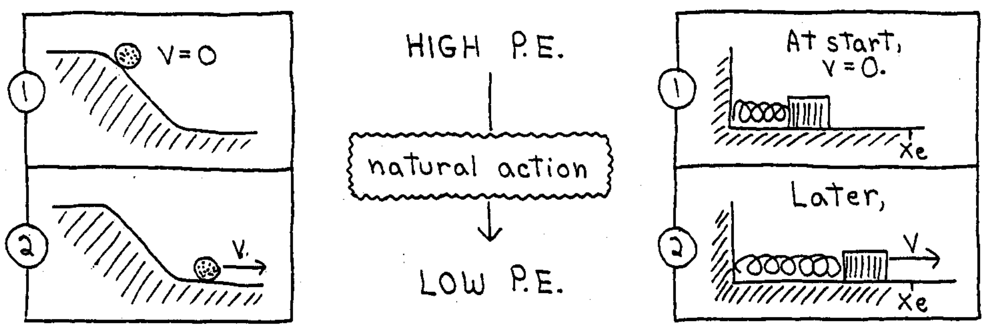
Ball #1 is in *stable equilibrium;* if it is displaced to the left or right, it will move back toward its original *equilibrium position.* #2 is in *unstable equilibrium;* if it is perfectly balanced "on top" it will stay where it is, but if it is displaced a little to the left or right it moves away from its original position. #3 is in *neutral equilibrium;* if it is displaced to the left or right (and released with $v_i = 0$), it stays where it is on the horizontal surface. #4 is *not in equilibrium;* it will move downhill if it is released.

STABLE equilibrium | UNSTABLE equilibrium | NEUTRAL equilibrium | not in equilibrium

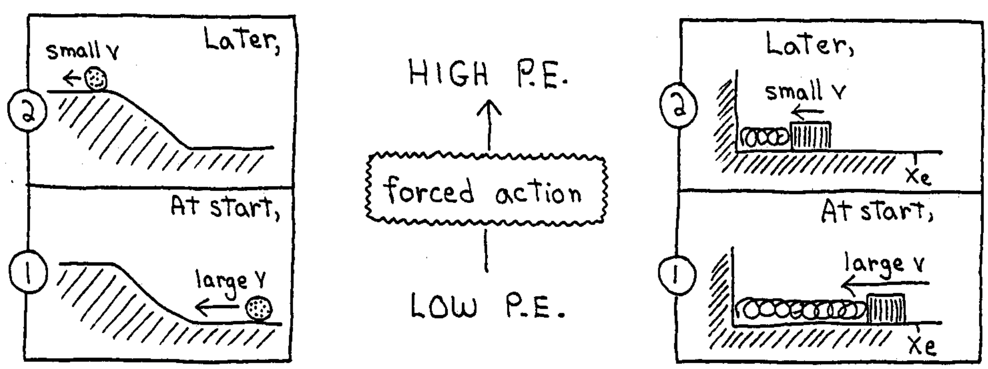
NATURAL ACTION: If an object that is "not in equilibrium" begins with $v = 0$, and you let it "vote with its actions" by doing whatever it wants to do, it will respond in such a way that its potential energy decreases.

In the pictures below, a motionless ball naturally rolls downhill to decrease its gravitational PE (which is equal to "mgh"), not uphill (which would increase its PE). Similarly, a motionless compressed spring will naturally move toward its x_e home-position (this will decrease PE), not away from x_e (which would increase PE).

In the pictures below, a motionless ball naturally rolls downhill to decrease its PE ($PE_{gravity} = mgh$), not uphill (which would increase PE). Similarly, a motionless compressed spring will naturally move toward the x_e home-position to decrease its PE ($PE_{spring} = \frac{1}{2}kx^2$), not away from x_e (which would increase PE).



If $v \neq 0$, PE won't necessarily decrease. In the pictures below, the object "uses up" some of its original KE, transforming it into a PE increase. { notice the 1's and 2's }



The definition $\Delta PE \equiv -W$ shows that "potential energy" isn't really energy. It's just a thinking tool, a convenient intuitive way to describe the work and (because $\Delta PE = -W = -\Delta KE$) corresponding ΔKE that occurs during a certain i-to-f process.

What does "potential" energy mean? The Random House Dictionary definition of *potential* is "capable of coming into actuality...". Potential energy has the potential (capability) of being transformed into kinetic energy or, as discussed in Problem 4-#, into other forms of work or energy.

In the two natural-action examples above, the PE that a system "loses" (when its PE decreases) is transformed into a KE increase. In the two forced-action examples, the "lost KE" is transformed into PE increase.

CONSERVATION OF MECHANICAL ENERGY: If $W_{friction}$ and W_{other} are 0, one TWE format becomes $0 = \Delta KE + \Delta PE$, the other is $(KE + PE)_i = (KE + PE)_f$. Both equations say the same thing: the first equation [$0 = \Delta KE + \Delta PE$] says that what one energy-form loses is canceled by what the other energy-form gains, so [as stated in the second equation] the combination of "KE + PE", which is defined as *total mechanical energy,* is the same at i & f.

ENERGY ACCOUNTABILITY: In Part 3 of Problem 4-A, the PE loss of 490 J is transformed into a 353 J gain in KE, while 137 J is lost (it is actually turned into *heat,* as explained in Section 7.#) because of $W_{friction}$.

Section 7.# shows that *energy conservation* is true for all forms of energy: light, electrical, chemical, nuclear, mass and more.

4.5 Power

Power, abbreviated "P", is defined as WORK DONE divided by TIME ELAPSED.

Notice the '◌'-substitutions below: $W = F d \cos\emptyset$ and $d/\Delta t = v$. By "grouping" $F d \cos\emptyset / \Delta t$ in different ways, you can get either $W/\Delta t$ or $F v \cos\emptyset$.

$$P \equiv \frac{W}{\Delta t} \qquad\qquad F v \cos\emptyset = P$$

$$\Downarrow \qquad\qquad\qquad \Uparrow$$

$$\frac{F d \cos\emptyset}{\Delta t} = F \frac{d}{\Delta t} \cos\emptyset$$

The logic that *TOTAL = SUM OF PARTS* can be used for F_{total}, W_{total} or P_{total}. For example, if F_A, F_B & F_C act on an object to cause W_A, W_B & W_C and P_A, P_B & P_C,

$$F_{total} = F_A + F_B + F_C$$
$$W_{total} = W_A + W_B + W_C$$
$$P_{total} = P_A + P_B + P_C$$

UNITS: The units for $F d \cos\emptyset / \Delta t$, $W/\Delta t$ and $F v \cos\emptyset$ are equivalent (Nm/s = J/s = Nm/s = $kg\,m^2/s^3$. This unit-combination is given the name *Watt,* abbreviated "W": 75 Nm/s = 75 $kg\,m^2/s^3$ = 75 J/s ≡ 75 Watts ≡ 75 W. A common non-SI power unit is the *horsepower,* abbreviated "hp": 1 hp ≡ 746 Watts. {I don't know whose dumb idea this was, but there is also a "metric horsepower" that is 750 Watts.}

When you plan strategy, it is usually best to think of power in terms of **J/s** instead of Watts. For example, (50 J/s)(10 s) = (500 J) clearly shows unit cancellation, but in (50 W)(10 s) = (500 J) the relationship of units is a mystery.

A "kilowatt-hour", abbreviated "kWh", is a unit of energy, not power. 1 kWh is the work done by 1 kW (1000 W) of power in 1 hour: 1 kWh = (1000 J/s)(3600 s) = 3.6×10^6 J.

PROBLEM 4-D

Part 1: A 500 kg elevator carries seven 70 kg passengers; it is pulled upward by a motor with a maximum power of 45 hp. What is the elevator's top speed "v_{max}"? Will the motor exert less force if it pulls the elevator with a speed of $\frac{1}{2} v_{max}$?

Part 2: If the motor's maximum power is used, what is the acceleration when the elevator's speed is v_{max}? when its speed is $\frac{1}{2} v_{max}$? {Hint: Use F=ma and P=Fv.}

SOLUTION 4-D

Part 1: Draw a force diagram with elevator-and-passengers as a *system object*. If the elevator's velocity is constant (whether it is at v_{max} or $\frac{1}{2} v_{max}$), $a = 0$, and $F=ma$ shows that the cable tension T is 9700*N*. If we assume a massless cable, the motor must exert this same T-force at the other end of the cable. **F** and **v** are in the same direction, so $\phi = 0°$. 45 hp must be converted to SI units.

for elevator-passengers "object"

$$F_{total} = m\,a$$
$$+T - 500(9.8) - (7 \times 70)(9.8) = m(0)$$
$$= 9700N$$

$$P_{motor} = F_{motor}\, v_{motor} \cos\phi$$
$$\Downarrow$$
$$(45\,hp)\left(\frac{746\,W}{1\,hp}\right) = (9700)\ v\ \cos 0°$$
$$3.46\ m/s = v$$

T and v point in the same direction, so the W, ΔKE, Δv and P caused by T are +. "mg" and v point in opposite directions, so the W, ΔKE, Δv and P caused by mg are –.

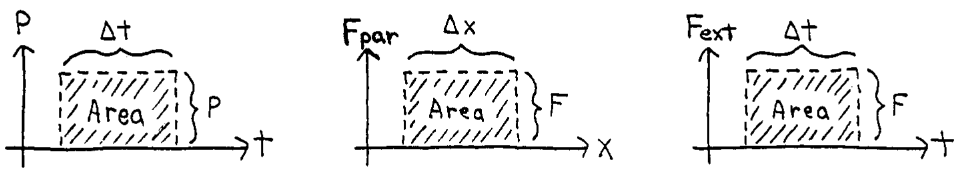
The effects of T and mg cancel each other: v is constant because F_{total} (and a) = 0, W_{total} (and ΔKE) = 0, and $P_{total} = 0$. Notice the difference between partial quantities (like P_{motor} or $W_{gravity}$) and total quantitites (F_{total}, P_{total}, W_{total}).

Part 2: When v = 3.46 m/s, T (= P/v = 33570/3.46 = 9700) is just enough to balance mg, so a = 0. But when v = 1.73 m/s, T (= P/v = 33570/1.73 = 19400), and a = F_{total}/m = [+19400 –(990)(9.8)] / 990 = 9.80 m/s per s upward.

Does it surprise you that constant P does not necessarily mean constant F or a?

4.6 The Meaning of Graph-Areas

Section 2.10 examines the meaning of area for graphs of v-versus-t & a-versus-t. The area-meaning of some Chapter 4A & 4B equations is derived below, by linking geometry [area = height x width] with physics [the substitutions shown by ⇓'s].



Area = $P\,\Delta t$ ⇓ Area = Work

Area = $F_{par}\,\Delta x$ ⇓ Area = Work

Area = $F_{external}\,\Delta t$ ⇓ Area = Impulse

For area of any shape (not just □'s), P-t, F-x and F-t areas give W, W and Impulse.

As emphasized in Problem 2.##, area can be either + or –. Use common sense to interpret the meaning of Work or Impulse that is calculated from graph-areas.

{ In Problem 4-#, F-versus-x area is used to derive the formula "$W_{spring} = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$".}

OPTIONAL: Section 18.# shows how calculus can be used to find work.

4.7 A Many-Sided Equation

In the diagram below, each of the 8 boxes is equal to every other box.

AREA of F ↑ → x	If F_{total} is constant, $F_{total}\ d\ \cos Ø$	$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$
OPTIONAL CALCULUS	W_{total}	$KE_f - KE_i$
AREA of P ↑ → t	If P_{total} is constant, $P_{total}\ \Delta t$	ΔKE

You can equate any two of these boxes to make an equation that fits the needs of a particular problem. Here are some examples.

Problem 4-E: A 40 kg object with v_i = 5 m/s is pulled with constant P_{total} for 10 s (with cos Ø = +1), and has KE_f = 8000 J. Find W_{total} and P_{total}.

Problem 4-F: A 40 kg object pulled with a constant F_{total} of 60*N* (with cos Ø = +1) has v_f = 20 m/s and ΔKE = +7500 J. Find v_i and Δx.

Solutions 4-E and 4-F: In Section 2.4, the "1-out" method for choosing a tvvax equation was described. The same principle is used for a many-sided equation: look for an equation-side with the variable you want to solve for, and another equation-side that contains a lot of the "given information". {If your first equation choice doesn't give a 1-unknown equation, try another option.} Study the 4 equations below. Do you see the reason for choosing each equation-side?

4-E

$W_{TOTAL} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

$W_{TOTAL} = KE_f - \frac{1}{2} m v_i^2$

$W_{TOTAL} = 8000 - .5(40)(5)^2$

$W_{TOTAL} = 7500$ Joules

$P_{TOTAL}\ \Delta t = KE_f - \frac{1}{2} m v_i^2$

$P_{TOTAL}(10) = 8000 - .5(40)(5)^2$

$P_{TOTAL} = 750$ Watts

4-F

$\Delta KE = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

$+7500 = \frac{1}{2}(40)(20)^2 - \frac{1}{2}(40) v_i^2$

$v_i^2 = 25$

$v_i = \pm 5.0$ m/s

$F_{TOTAL}\ \Delta x = \Delta KE$

$(60)\ \Delta x = (+7500)$

$\Delta x = 125$ m

The difference between "partials" and "totals" is important. For example, if forces F_a & F_b act on an object to produce W_a & W_b and P_a & P_b, you must use F_{total}, P_{total} or W_{total} (not F_a, P_a or W_a) to find changes in the object's motion: **F_{total} = ma**, and **W_{total} = ΔKE**. Don't ever use "F_A = ma" or "W_A = ΔKE", because they aren't true!

Some partial-formulas can be used: $F_a\ d \cos Ø_a = P_a\ \Delta t = W_a$, and $P_a = F_a\ v \cos Ø_a$.

If you have read Chapter 4B, you can now use Section 4.12.

Chapter 4B: Impulse ⇒ Δ Momentum

Section 4.8 explains Impulse-Momentum fundamentals; read it first. Then use 4.9-4.11 (2-Dimensional momentum, elastic collisions, center-of-mass calculations) when your class covers these topics. Section 4.12 combines Impulse-Momentum and Work-Energy ideas; read it when you've studied both topics.

4.8 Internal and External Forces

If you haven't read Section 4.1 yet, do it now. 4.1 shows how tvvax and F=ma combine to produce the main equation we'll be using: $F\,\Delta t = m v_f - m v_i$.

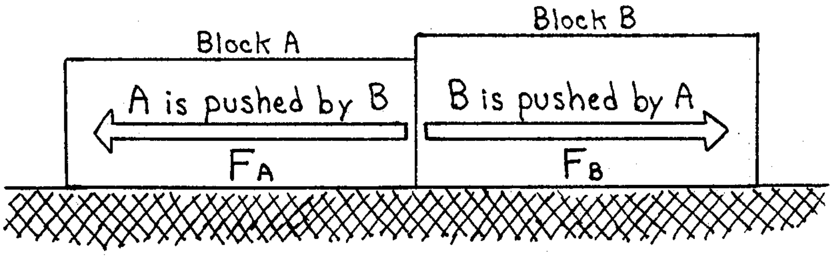
PROBLEM 4-G: A Totally Inelastic Collision

Part 1: Block A, with mass 5.0 kg, moves 8.0 m/s toward the right on a horizontal frictionless surface. Block B, mass 10.0 kg, moves 7.0 m/s toward the left. A and B collide head-on and stick together. Define the rightward direction to be +, and find their final velocity, momentum (mv) and kinetic energy ($\frac{1}{2}mv^2$).

Part 2: Later, the stuck-together blocks slide down a long 12° inclined plane with $\mu_k = .25$. How much time elapses before they stop?

SOLUTION 4-G, Part 1: Internal Forces

At every instant of time during the collision, $\mathbf{F}_A$ (the contact-force felt by A) and $\mathbf{F}_B$ (the contact force felt by B) are equal-and-opposite: $\mathbf{F}_A = -\mathbf{F}_B$. Why? Because, as explained in Section 3.5, they are a *third law mutual interaction force pair.*



$\mathbf{F}_A\,\Delta t = (\Delta \mathbf{mv})_A$, where $(\Delta \mathbf{mv})_A$ is the momentum change of block A. Similarly, $\mathbf{F}_B\,\Delta t = (\Delta \mathbf{mv})_B$. Because $\mathbf{F}_A = -\mathbf{F}_B$, $\mathbf{F}_A\,\Delta t = -\mathbf{F}_B\,\Delta t$, and $(\Delta \mathbf{mv})_A = -(\Delta \mathbf{mv})_B$. The momentum changes of A and B are equal-and-opposite, so they cancel each other and momentum is conserved: $(\Delta \mathbf{mv})_{total} = (\Delta \mathbf{mv})_A + (\Delta \mathbf{mv})_B = 0$.

For Problem 3-B of Section 3.3, we analyzed the "block train" by using a *system* of all-four-blocks-together, and discovered that the *internal forces* exerted by the ropes canceled each other and didn't affect the overall system. Only one force [the *external force*] survived cancellation, to give "3.0(9.8) = 14.7 a".

Combined-object systems can also be used to solve impulse-momentum problems. If we consider A-and-B-together to be a ***system***, then "A is pushed by B" and "B is pushed by A" are just one part of the system being pushed by another part of itself. These ***internal forces*** don't affect the momentum of the overall system. Only ***external force*** causes momentum to change; to emphasize this, a subscript is added to the Impulse-Momentum Equation, showing that "F" is really "external F":

$$F_{external}\,\Delta t = (mv)_f - (mv)_i$$

Now the Impulse-Equation is easier to use, because you can ignore all internal forces; the equation only asks for $F_{external}$! { It would be difficult, maybe impossible, to accurately describe the magnitude of the A-B collision force and how it varies with time. It's nice to be able to ignore it.}

CONSERVATION OF MOMENTUM: The blocks' collision force is internal, and the external vertical forces of N & mg cancel each other, so the system's net $F_{external}$ is zero and "$F_{ext}\ \Delta t = (mv)_f - (mv)_i$" simplifies to "$0 = (mv)_f - (mv)_i$".

When $F_{external} = 0$, momentum is "conserved": $(mv)_i = (mv)_f$

Initially the A-and-B system has two independently moving objects, A & B, so the system's $(mv)_i$ contains two terms: the mv of A, and the mv of B. After the collision, A & B move together as a unit, so $(mv)_f$ has only one term: the mv of A-and-B.

$$\begin{aligned} \overset{0}{\cancel{F_{ext}}}\ \Delta t &= (mv)_f - (mv)_i \\ (mv)_i &= (mv)_f \\ (m_A v_A)_i + (m_B v_B)_i &= (m_{AB})_f (v_{AB})_f \\ 5(+8) + 10(-7) &= (5+10)(v_{AB})_f \\ -2\ \text{m/s} &= (v_{AB})_f \end{aligned}$$

As discussed in Section 2.2, $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$ is a vector equation. The **p** and mv vectors always point in the same direction, with magnitude (and units) differing by a factor of "m". The system's final momentum [$\mathbf{p} \equiv m\mathbf{v}$ = (15 kg)(–2 m/s) = –30 kg m/s] is a vector that points, as shown by the – sign, toward the left.

Block A's mv changes by –50: from (5)(+8) = +40, to (5)(–2) = –10. B's mv changes by +50: from (10)(–7) = –70, to (10)(–2) = –20. As predicted earlier by third-law logic, the mv-changes of A and B cancel each other, so Δ(mv) for the entire system is zero.

The system's final kinetic energy, $\frac{1}{2}mv^2 = \frac{1}{2}(15)(-2)^2 = 30$ J, is much less than its initial KE of $\frac{1}{2}mv^2 = \frac{1}{2}(5)(+8)^2 + \frac{1}{2}(10)(-7)^2 = 160 + 245 = 405$ J. If $F_{ext} = 0$, momentum will be conserved, but this doesn't guarantee that kinetic energy is also conserved. The percentage of KE that is retained is (30/405) x 100 = 7.4%.

Why is momentum conserved during this collision, but not kinetic energy? What happens to the KE that is "lost"? These questions are answered in Problem 4-#.

The following 3 terms can be used to describe how much kinetic energy is retained (preserved) during a collision:

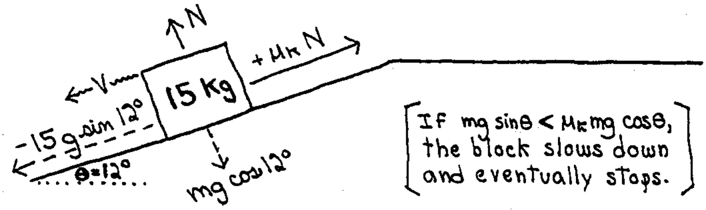
If 100% of a system's KE is retained, a collision is called ***elastic***.

If the colliding objects stick together, minimum retention of KE occurs, and a collision is ***totally inelastic***. In Problem 4-G's totally inelastic collision, 7.4% (not 0%) of the initial KE was retained. "Minimum KE retention" doesn't mean that all KE is lost. Instead, it means that if the blocks don't stick together, KE-retention will be greater than the 7.4% minimum. It might be 7.5%, 50%, even 100%, but it will never be less than 7.4%. {If the m's and v's of the colliding objects are different than in Problem 4-G*, the minimum KE retention can be higher or lower than 7.4%. * like the situations in Problem 4-#, where "elasticity" is explored }

If less than 100% of the KE is conserved, a collision is ***inelastic***. {For the blocks of Problem 4-G, any collision with KE-retention between 7.4% and 99.9% would be "inelastic".}

SOLUTION 4-G, Part 2 : External Forces

When A-and-B slide down the ramp, N and mg(cos 12°) cancel each other, and the net $F_{external}$ is $-mg(\sin 12°) + \mu_k mg(\cos 12°)$. $F_{external} \neq 0$, so $(mv)_i \neq (mv)_f$.



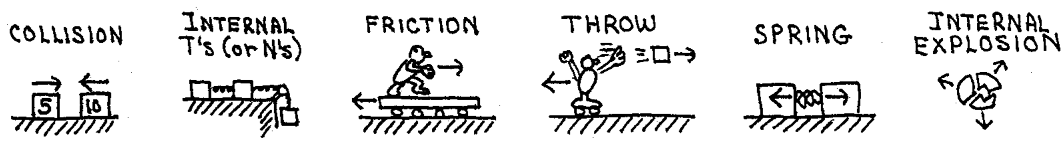
$$F_{external} \quad \Delta t = mv_f - mv_i$$
$$[-15(9.8)\sin 12° + .25(15)(9.8)\cos 12°]\ \Delta t = (15)(0) - (15)(-2)$$
$$\Delta t = 5.6 \text{ seconds}$$

For use with $F_{ext}\,\Delta t = \Delta p$, you can define any combination of objects you want as a "system" (there are no rules!), even if the objects are moving in different directions. { The reason for this, the cancellation of "third-law internal forces", was discussed earlier. }

It's easy to decide if a force is internal or external: Choose the objects you want in your "system" and then, for each F acting on a system-object, ask "Is this F caused by another system-object (which will make it $F_{internal}$) or by something outside the system (which makes it $F_{external}$)?"

Typical external forces are **gravity** [either free-fall mg, or on-a-plane mgsinθ], **air resistance**, **friction** (if one friction-producing surface is a system-object and the other surface is non-system, as in Part 2 of Problem 4-G), and a **T-pull or N-push produced by a non-system source** (like a person pulling or pushing a car).

Typical internal forces are **collision-interactions** (if both colliders are system-objects, as in Problem 4-G), **friction** (if both friction-producing surfaces are part of the system you've chosen; below, the board rests on frictionless rollers, and the system is man + board), "**passive**" **N's or T's** (like the ropes that connect the blocks in Problem 3-B), a "**throw**" (if the thrower and thrown-object are both in the system) or **gunshot**, **spring-force** (if as shown below, the system is block + spring + block), and "**internal explosions**" (like when a firecracker enclosed in a system-object explodes).



The separation of F's into F_{int} and F_{ext} is often the most important step in solving a problem. Stay alert and watch for internal forces! Be ready to ask the internal-external question: "Is the cause of a force within the system or outside the system?"

CONSERVATION OF MOMENTUM: If $F_{external} = 0$, mv is conserved, $(mv)_i = (mv)_f$.

ALMOST-CONSERVATION OF MOMENTUM: During a collision or internal explosion, a system experiences a huge quick F_{int} with large magnitude but small Δt. If the F_{ext}'s acting on the system (like friction or gravity) have relatively small magnitude and are multiplied by a tiny Δt, $F_{ext}\,\Delta t$ [and the corresponding $\Delta(mv)$] will be small and it is often assumed that the initial & final mv's are approximately equal. This is abbreviated $(mv)_i \approx (mv)_f$, where "≈" means "is approximately equal to".

CHOICES: When your class studies 2-Dimensional Momentum, Elastic Collisions, and Center-of-Mass Calculations, read the corresponding section here. If your class skips any of these topics, you can also skip it here.

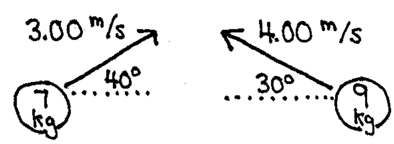
To learn how to use "F-versus-t area" to find Impulse, read Section 4.6.

If you've already studied Chapter 4A, you can use Section 4.12.

4.9 2-Dimensional Collisions

PROBLEM 4-H: A 2-Dimensional Collision

Two objects, moving on a horizontal surface as shown at the right, collide and stick together. Immediately after the collision, what is their velocity and momentum?



SOLUTION 4-H

For each object, N and mg cancel each other. And to make the large collision forces "internal" so they can be ignored, let's define 7-and-9 to be a system. The problem doesn't say the surface is frictionless but it is usually assumed, as explained at the end of Section 4.8, that collision-Δt is extremely small and the "$F_{ext}\,\Delta t$" caused by friction can be ignored: $F_{ext} \neq 0$, but $F_{ext}\,\Delta t \approx 0$, so $(mv)_i \approx (mv)_f$.

$F_{ext}\,\Delta t$, $(mv)_f$ and $(mv)_i$ are vectors, so "$\overrightarrow{F_{ext}\Delta t} = \overrightarrow{(mv)_f - (mv)_i}$" is a *vector equation.* Like other vector equations (4 of the 5 tvvax equations, F=ma, ...), it can be split into components to form an x-equation and a y-equation, as shown below.

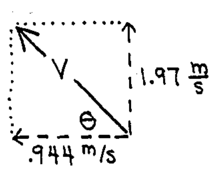
Substitute the x & y components of each vector into the x & y equations, and solve:

For the x-direction,

$$(F_{ext})_x^{\,0}\,\Delta t = (mv_x)_f - (mv_x)_i$$
$$(mv_x)_i = (mv_x)_f$$
$$+7(3)\cos 40° - 9(4)\cos 30° = (7+9)\,v_{xf}$$
$$-.944 \text{ m/s} = v_{xf}$$

For the y-direction,

$$(F_{ext})_y^{\,0}\,\Delta t = (mv_y)_f - (mv_y)_i$$
$$(mv_y)_i = (mv_y)_f$$
$$+7(3)\sin 40° + 9(4)\sin 30° = (7+9)\,v_{yf}$$
$$+1.97 \text{ m/s} = v_{yf}$$



v magnitude

$$v = \sqrt{(.944)^2 + (1.97)^2}$$
$$v = 2.18 \text{ m/s}$$

v direction

$$\theta = \tan^{-1}\frac{1.97}{.944}$$
$$\theta = 64.4°$$

$$\vec{p} = m\vec{v} = (16.0\text{ kg})(2.18\text{ m/s}) = 34.9 \text{ kg m/s}$$

$\vec{p}$ points in the same direction as $\vec{v}$.

When you use $F\Delta t = \Delta p$ for 2-D problems, there are three "separations". They are the same three splits that were used in Chapter 2 for tvvax:

1) The $\overrightarrow{F\Delta t} = \overrightarrow{\Delta(mv)}$ equation is split into components in the x & y directions.
2) mv is calculated at the times you've chosen to be initial & final.
3) To find total momentum, add the mv's for all objects in the system you've chosen.

The following principles will help you do these separations easily, with skill:

1) **Write equation-parts on different parts of the page,** then use this "location separation" to remind you of what each part means.
DIRECTIONS: Write the x & y equations on the left & right sides of the page.
TIMES: If $F_{ext} = 0$, put $(mv)_i$ and $(mv)_f$ on opposite sides of the equation.
OBJECTS: When substituting for $(mv)_i$ and $(mv)_f$, there should be one term for each object at the i & f times. For example, in Problem 4-G there are 2 objects [and 2 terms] at the initial-time, but only 1 object [and 1 term] at the final-time.

2) **Develop good habits, do things one step at a time, be disciplined and alert.** When substituting into an x or y-equation, focus on the x or y direction, respectively. When substituting for $(mv)_i$ or $(mv)_f$, focus on what's happening at the the i or f time; don't mix them together. When you calculate the system's total momentum, systematically consider the contributions of each independently moving object.

4.10 Elastic Collisions

PROBLEM 4-I: An Elastic Collision
If the collision in Problem 4-G is elastic, what is v_f for A and for B?

SOLUTION 4-I
There are two solution methods: use standard algebra, or "shortcut formulas".

STANDARD ALGEBRA: For the same reasons as in Problem 4-G, momentum is conserved. By definition of "elastic", kinetic energy is also conserved. The equations below state these facts in mathematical terms. They can be solved using standard leapfrog substitution; lots of algebra is required, including the Quadratic Formula.

Because mv is conserved,	"elastic" means K.E. is conserved:
$(mv)_i = (mv)_f$	$(KE)_i = (KE)_f$
$5(+8) + 10(-7) = 5\,v_A + 10\,v_B$	$\frac{1}{2}(5)(+8)^2 + \frac{1}{2}(10)(-7)^2 = \frac{1}{2}(5)\,v_A^2 + \frac{1}{2}(10)\,v_B^2$
$(-6 - 2\,v_B = v_A)$ → ①	① ⇓
	$810 = 5(-6 - 2v_B)^2 + 10\,v_B^2$
⇓ ②	$0 = 30\,v_B^2 + 120\,v_B - 630$
$-6 - 2(+3) = v_A$	② ← $v_B = +3$ or (IF NO COLLISION) $v_B = -7$
$-12\ m/s = v_A$	

The SHORTCUT FORMULAS below can only be used for 1-dimensional elastic collisions. They give the same correct answers, with much less time and effort; just substitute-and-solve. The 1 & 2 subscripts represent objects 1 & 2 (in this problem, they are blocks A & B).

For a 1-Dimension collision, elastic (KE conserved), with $F_{ext} = 0$.

$(v_1)_f = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)(v_1)_i + \left(\frac{2\,m_2}{m_1 + m_2}\right)(v_2)_i$	$(v_2)_f = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)(v_2)_i + \left(\frac{2\,m_1}{m_1 + m_2}\right)(v_1)_i$

$$(v_A)_f = \left(\frac{5-10}{5+10}\right)(+8) + \left(\frac{2(10)}{5+10}\right)(-7) \qquad (v_B)_f = \left(\frac{10-5}{5+10}\right)(-7) + \left(\frac{2(5)}{5+10}\right)(+8)$$

$$(v_A)_f = \quad -2.67 \qquad -9.33 \qquad\qquad (v_B)_f = \quad -2.33 \qquad +5.33$$

$$(v_A)_f = \quad -12 \text{ m/s} \qquad\qquad (v_B)_f = \quad +3.00 \text{ m/s}$$

MEMORY TIPS: If your exam is "closed book", if you want to use these formulas during the exam you must memorize them. They look complicated, but they're easy to remember if you practice active recall (flash cards,...) and take advantage of the formulas' "structural consistency": the left sides are v_f's, the right-side multipliers are v_i's (in the left equation, the fraction with $+m_1$ on top is multiplied by v_{1i}, and the fraction with $2\,m_2$ is multiplied by v_{2i}), the fraction-bottoms are all $m_1 + m_2$, and there is symmetry (what 1's do in the first equation, 2's do in the second equation). As soon as you get your exam, write the formulas down on an exam page.

Before an exam, practice these equations a few times, with different "unknowns". For example, use the numbers from Problem 4-6 and pretend you know everything except v_{1i} & v_{1f} (substitute the knowns, and solve for v_{1i} & v_{1f}). Then pretend-and-solve for v_{1i} & v_{2i}, or m_1 & v_{2f}, or m_1 & m_2, or.... { Problem 4-# gives several elastic collision examples.}

Intuitive Principles: Think about what happens when a large-mass bowling ball and a small-mass soccer ball collide. The bowling ball will usually (but not always) continue moving in its original direction, while the soccer ball usually (but not always) changes direction because of the collision.

In a collision between two equal-mass bowling balls, the balls just "exchange" velocities: $(v_1)_f = (v_2)_i$ and $(v_2)_f = (v_1)_i$.

4.11 Center-of-Mass Calculations

To find the center-of-mass for a system that has several objects, use these two strategy-tools.

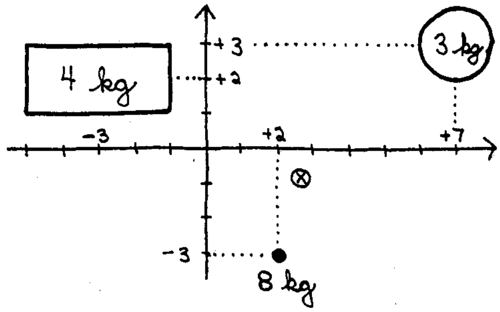
1) Use SYMMETRY: Common sense suggests that the c-of-m for a symmetric uniform-density object is at its center. This is true, and useful.

2) Use the FORMULAS below to find the x & y coordinates of the system's c-of-m.

$m_1, m_2, \ldots$ and $x_1, x_2, \ldots$ are the masses and x-positions of object #1, object #2,.... until all system-objects are included:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \ldots}{m_1 + m_2 + \ldots} \qquad\qquad y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \ldots}{m_1 + m_2 + \ldots}$$

For example, symmetry-and-formulas can find the c-of-m for this 3-object system:



$$x_{c\text{-}of\text{-}m} = \frac{4(-3) + 8(+2) + 3(+7)}{4+8+3} \qquad y_{c\text{-}of\text{-}m} = \frac{4(+2) + 8(-3) + 3(+3)}{4+8+3}$$

You may find it easier to use the $x_{c\text{-}of\text{-}m}$ & $y_{c\text{-}of\text{-}m}$ formulas if you say to yourself, as you consider the contributions of each object, "4 at –3, plus 8 at +2, plus 3 at +7".

Solving the center-of-mass formulas (do the arithmetic for yourself) shows that the system's c-of-m is at $x = +2.78$, $y = -.78$; this is shown by the ⊗ on the diagram.

Optional Calculus: Section 18.33 shows how to use *integrals* to find center-of-mass.

CHOICES: If your class won't be studying v_{cm}, a_{cm}, etc., skip to Problem 4-J. With the formulas below, you can calculate the velocity of a system's c-of-m:

$$(v_{cm})_x = \frac{m_1 (v_1)_x + m_2 (v_2)_x + \ldots}{m_1 + m_2 + \ldots} \qquad (v_{cm})_y = \frac{m_1 (v_1)_y + m_2 (v_2)_y + \ldots}{m_1 + m_2 + \ldots}$$

To calculate the system's a_{cm}, just replace the v's in the above equations with a's.

You can analyze the motion of a many-object system as if all of its mass "m_{total}" is located in **a single object** at x_{cm}, moving with v_{cm} and a_{cm}, acted on by $(F_{ext})_{total}$, where $(F_{ext})_{total}$ is the vector sum of all F_{ext}'s that are acting on the system-objects.

Some equations, like $v_f - v_i = a\,\Delta t$ and $F=ma$ and $F_{ext}\,\Delta t = \Delta mv$, can be adapted for a system of several objects:

$$(v_{cm})_f - (v_{cm})_i = a_{cm}\ \Delta t$$

$$(F_{ext})_{total} = m_{total}\ a_{cm}$$

$$(F_{ext})_{total}\,\Delta t = (m_{total}\,v_{cm})_f - (m_{total}\,v_{cm})_i$$

Two equations that cannot be "adapted for a system" are

$$v_f^2 - v_i^2 = 2\,\mathbf{a}\,\Delta\mathbf{x} \quad \text{and} \quad \mathbf{F}\,\Delta\mathbf{x} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 .$$

For 3-dimensional motion, use x_{cm} & y_{cm} & z_{cm}, $(v_{cm})_x$ & $(v_{cm})_y$ & $(v_{cm})_z$, and so on.

PROBLEM 4-J: Center-of-Mass Calculations

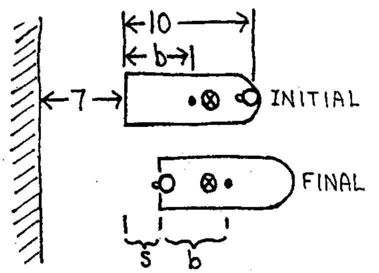
A 150 kg iceboat can slide across horizontal frictionless windless ice. Initially, the back of the motionless boat is 7.0 m from the shore. A 50 kg woman then walks 10.0 m toward the shore, from the front of the boat to the back. As viewed by an observer standing on the ground, what is the displacement (net movement) of the boat and the woman during this walk? { Hint: Draw pictures of the initial & final situations. What is F_{ext} ? }

SOLUTION 4-J

"Horizontal" means that N & mg cancel each other. Wind-force is zero. Friction between the boat and ice is zero. And if boat-and-woman is a system, friction between her feet and the boat is F_{int}. All of this means that $F_{ext} = 0$.

Because $F_{ext} = 0$, the momentum (mv) of the system (boat + woman) is conserved. "mv" and "m" are both constant, so "v" is also constant; $v_i = 0$, so v continues to be 0, and the system's c-of-m (⊗ on the diagram below) doesn't move. This fact is stated in the equation "$(x_{cm})_i = (x_{cm})_f$", which is solved later. All of this logic can be easily summarized: if a system has $(v_{cm})_i = 0$ and $F_{ext} = 0$, $(x_{cm})_i = (x_{cm})_f$.

We don't know whether the boat's weight distribution is symmetric, so we'll let the boat's center-of-mass (•) be an unknown distance "b" from the back of the boat. When the woman walks leftward, the boat moves rightward (why?) a distance I'll call "s". If the back of the boat is $x \equiv 0$, the woman's i & f positions are 10 & s. Do you see why the boat's c-of-m has i & f positions of "b" and "s + b" ?



$$(x_{cm})_i = (x_{cm})_f$$

$$\frac{150(b) + 50(10)}{\cancel{450 + 50}} = \frac{150(s+b) + 50(s)}{\cancel{450 + 50}}$$

$$150b + 500 = 150s + 150b + 50s$$

$$2.5 \text{ meter} = s$$

"s = 2.5" means the boat moves 2.5 m →. The woman has two sources of motion: she walks 10 m ← and is carried 2.5 m → along with the boat. These combine to give her a net movement of 7.5 m ←. { Alternate methods: She moves from $x_i = +10$m to $x_f = +2.5$m, so you can calculate $\Delta x = x_f - x_i = (+2.5) - (+10) = -7.5$ m. Or use visual number-line logic to see that she has moved 7.5m to the left, so $\Delta x = -7.5$m. }

The boat's mass is 3 times as large as the woman's mass (150 kg versus 50 kg), so it moves 1/3 as far with respect to a ground-observer (2.5 m versus 7.5 m).

Extras: A) The 2-unknown equation can be solved because "b" cancels. B) If $x \equiv 0$ at the shore, you still get s = 1.0 m, because "7" cancels; try it for yourself and see. C) OPTIONAL: You can show that $(x_{cm})_i = (x_{cm})_f$ by using the "system equations" that were discussed earlier. Substitute $F_{ext} = 0$ and $v_i = 0$ into "$F_{ext}\,\Delta t = m v_f - m v_i$" and solve for $v_f = 0$, then solve "$x_f - x_i = \frac{1}{2}(v_i + v_f)\,\Delta t$". Or you can solve "$(F_{ext})_{total} = m_{total}\, a_{cm}$" for $a_{cm} = 0$, and then solve "$(v_{cm})_f - (v_{cm})_i = a_{cm}\,\Delta t$" for $(v_{cm})_f = 0$.

4.12 How to Choose a Useful Equation

After you've read Chapters 4A & 4B, many equations are available:
the 5 tvvax's, F=ma, Section 4.7's "many-sided" $F\Delta x = \Delta KE$ & $P \equiv W/\Delta t$, and $F\Delta t = \Delta p$.
How do you know which one(s) to use? Here is a strategy-guiding principle:

As in the 1-out tvvax strategy, choose an equation that contains the goal-variable and the "knowns".

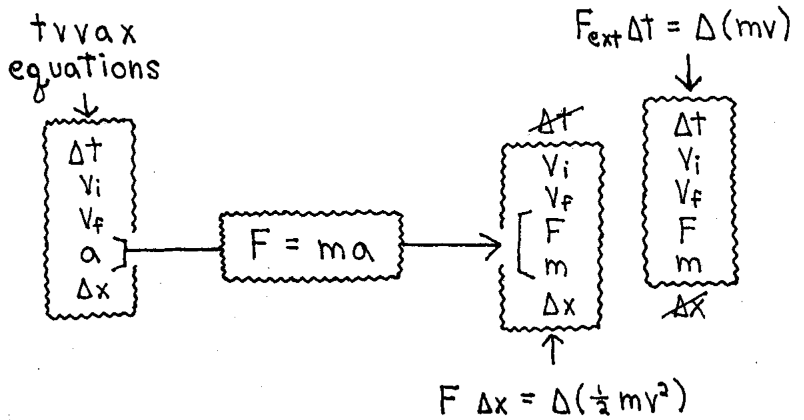
The diagram below will help you do this. It shows the variables in each equation. Notice that tvvax doesn't have F, F=ma has no i or f or Δ, the Work-equation has Δx and the Impulse-equation has Δt.

The diagram below shows the variables in each equation. Notice that tvvax has "a", but the work & impulse equations (after substitution of F=ma) have F & m. tvvax has no F, F=ma has no i or f or Δ, the Work-equation has Δx and the Impulse-equation has Δt. This variable-knowledge will help you choose an equation.

If no force is involved, use one of the 5 tvvax equations.

If there is F but no i-to-f interval, F=ma will help you determine what is happening at a specific instant of time.

If the effects of F accumulate during an interval, use $F\Delta x = \Delta KE$ if Δx is either given or asked for, and use $F\Delta t = \Delta p$ if Δt is given or asked for.



OTHER CLUES: If an object changes height between i & f, think "Aha! This is $mg\Delta h$, so I should use the Total Work Equation form of $F\Delta x = \Delta KE$." If an *internal force* is involved (the end of Section 4.8 suggests ways to recognize them), you can simplify things by using $F_{ext}\Delta t = \Delta p$. If a problem mentions work, power, kinetic or potential energy, impulse or momentum, you have a good clue about what to do.

A GENERAL PRINCIPLE: If you're not sure about what to do, just do something. Read the problem again and look for a "key piece of information" you missed the first time through. Or just choose an equation and substitute; if you can solve it, great! If not, look for a different equation or more information.

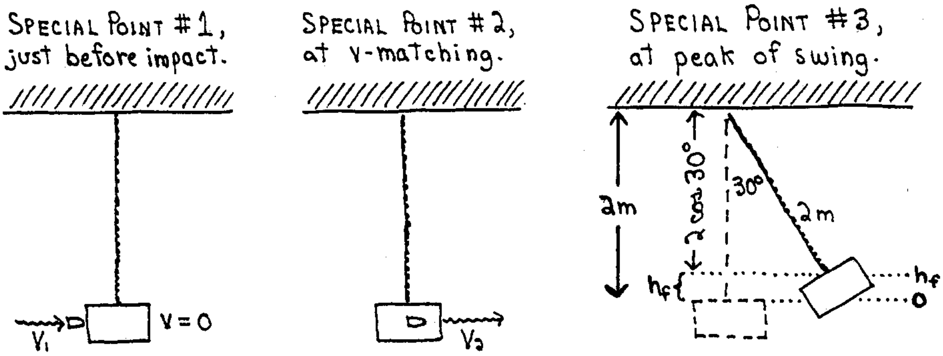
ALTERNATE SOLUTIONS are often possible. For example, F=ma and tvvax can be used to solve many problems in Chapters 4A & 4B. This takes a little more time, because the a=F/m substitution has already been done in deriving $F\Delta x = \Delta KE$ and $F\Delta t = \Delta p$. But if you see the F=ma/tvvax option first, it is quicker to do it immediately and not waste time searching for "the perfect solution".

PROBLEM 4-K: Combining Work-Energy & Impulse-Momentum.
A 3.0 kg block hangs from a 2.0 m string. A 50 gram bullet hits the block but doesn't go all the way through. At the peak of the block's swing, the string makes a 30° angle with vertical. What was the bullet's initial speed?

SOLUTION 4-K

If bullet-and-block are a system, their collision force is internal. Even though $F_{gravity}$ is external, the collision-Δt is so small that momentum is almost-conserved during an interval that ends at the instant the bullet stops moving through the block. At this time, labeled Special Point #2 below, the bullet and block are moving together as a unit and have the same "matching" v_2.

The height change between points 2 & 3 says "Use $F\Delta x = \Delta KE$!". The string is 2.0 m long at Points 2 and 3. The logic of "total = sum of parts" gives $2.0 = 2.0(\cos 30°) + h_f$, so $h_f = .268$ m. From 2 to 3, the string tension is always ⊥ to v, so T doesn't do any work and isn't included in the Total Work Equation. The TWE, stripped of the parts you don't need, can be easily solved.



From 1 to 2

$$(mv)_1 \approx (mv)_2$$
$$.050\, v_1 + 3(0) = (3.05)\, v_2$$
$$.0164\, v_1 = v_2$$

From 2 to 3

$$\tfrac{1}{2} m v_i^2 + mgh_i = \tfrac{1}{2} m v_f^2 + mgh_f$$
$$\Downarrow$$
$$\tfrac{1}{2}(.0164\, v_i)^2 + g(0) = \tfrac{1}{2} m(0)^2 + (9.8)(.268)$$

Chapter 4 Flash-Card Review

Chapter 4A

4.2	Only F that is __ motion produces W, so __ ; **d** (which is __) points in same direction as __ . This formula can be used only if __ .	parallel to, $W = F_{parallel}\, d = F d \cos\theta$ displacement [a line drawn from i to f], v motion is straight-line and F is constant
4.2	W is __ , so the ± sign of W shows __ . If F & **d** point in __ , v & KE __ , ΔKE is __ .	not a vector, ΔKE's ± sign (not "direction") (same direction, ↑, +) (opposite dirctns, ↓, −)
4.2	W = 0 if __ .	F = 0, d = 0 (no motion), cosθ = 0 (F ⊥ d)
4.3	ΔPE can be defined for __ forces, but not __ . For $F_{gravity}$ & F_{spring}, W (and ΔPE) don't __ .	conservative (like $F_{gravity}$ & F_{spring}), $F_{friction}$ depend on the object's i-to-f path
4.3	The TWE splits W_{total} into W done by __ , substitutes __ , then rearranges things.	gravity, springs, friction, other forces $W = -\Delta PE$ and formulas for W_{other} & W_{fr}
4.3	The ⁄ and ∣ sides of a ⊿ are __ or __ . ⁄ is used in __ , and ∣ is used in __ .	d and d sinθ, h / sinθ and h $\mu_k N d$, $mgh_f - mgh_i$ or $mg\Delta h$

4.3	To use TWE, choose __ & __ , eliminate __ , substitute for __ , cancel __ , remember __ .	i & f, $h\equiv0$ and $x\equiv0$, un-needed parts, W_{fr}, m's (if m in <u>all</u> terms), 2 in v^2 and x^2
4.4	If a released object __ , it responds to make __ . But if __ , PE can either __ , because __ .	has $v_i = 0$ and is not in equilibrium, PE ↓ $v_i \neq 0$, ↑ or ↓ , some of KE_i can turn into PE
4.4	ΔPE is a convenient __ way to describe __ . PE has the __ of being transformed into __ .	intuitive, i-to-f work (and the ΔKE it causes) potential (capability), kinetic energy or work
4.4	If __ , __ is conserved (is constant); when PE ↓ , W_{total} is __ because __ , so ΔKE is __ .	W_{other} & W_{fr} are 0, KE + PE, +, ΔPE is – (and $W_{total} = -\Delta PE$), +
4.5	Power formulas are __ , because __ . Joules are used for __ ; __ are P-units, __ isn't.	$P = W/\Delta t$ and $P = Fv\cos\phi$, $d/\Delta t = v$ W & ΔPE & KE, Watts & hp, kWh
4.7	8 boxes in the many-sided W-equation are __ .	(const F) $d\cos\theta$, Fd area; (const P)Δt, Pt area, W_{total}; $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$, $KE_f - KE_i$, ΔKE
4.7	__ are correct F=ma & W-equations, but __ .	$F_{tot} = ma$, $W_{tot} = \Delta KE$; $F_{part} \neq ma$, $W_{part} \neq \Delta KE$

Chapter 4B

4.8	F_{int} doesn't cause __ , because the __ cancel.	Δ(mv), internal third-law FΔt's (impulses)
4.8	For $F_{ext}\Delta t = \Delta mv$, __ can be in a system.	anything! (there are no rules)
	To decide if F is internal or external, ask __ .	Is the F-causer inside or outside the system?
	Five typical external forces are __ .	gravity, air resistance, external T or N or f
	Seven typical internal forces are __ .	internal T or N or f, system-collision, throw or gunshot or explosion "inside system"
4.8	Conservation of momentum occurs if __ .	$F_{ext} = 0$; $\Delta mv \approx 0$ if $\Delta t \approx 0$ causes $F_{ext}\Delta t \approx 0$
4.8	F_{int} doesn't cause __ , but it can cause __ .	Δ momentum, Δ kinetic energy
	If __ of KE is retained, a collision is __ .	(100%, elastic) (< 100%, inelastic)
	A minimum % of KE is retained (but __) when objects __ ; this kind of collision is __ .	this "minimum %" isn't necessarily zero stick together, totally inelastic
4.9	x & y motion is __ , so FΔt =ΔKE can be __ .	independent, split into x & y equations
	3 splits (and simplifying strategies) are __ .	<u>x & y</u> (on left & right page sides); don't mix! <u>i & f</u> (left & right equation sides); don't mix! <u>objects</u>; each system-object at i & f has mv
4.10	The "shortcut formulas" can be used if __ .	motion is 1-dimensional & collision is elastic
4.11	Useful center-of-mass tools are __ .	if uniform density, c-of-m is at object-center $x_{cm} = (m_1x_1 + m_2x_2 +..)/(m_1 + m_2 +..)$, $y_{cm} =$...
4.11	Optional: __ can be adapted for systems.	v ≡, a ≡, 4 (of 5) tvvax, F=ma, FΔt =Δmv
4.11	If __ , system's c-of-m doesn't move, so __ .	$(v_{cm})_i = 0$ and $F_{ext} = 0$, $(x_{cm})_i = (x_{cm})_f$

Choosing Equations

4.1	FΔx=ΔKE & FΔt =Δ(mv) are derived by __ .	combining tvvax equations with F=ma
4.12	A general strategy: choose equation with __ .	the goal-variable and lots of "knowns"
4.12	Use tvvax if __ , F=ma if __ , FΔx=ΔKE if __ , FΔt =Δmv if __ .	interval but no F, no interval (just instant), interval & Δx (no Δt), interval & Δt (no Δx)
4.12	If h changes, __ . If there is internal F, __ .	use TWE (FΔx =ΔKE), use $F_{int}\Delta t = \Delta mv$
4.12	F=ma describes __ . The Work & Impulse equations describe the __ effects of F __ .	what is happening at 1 location at 1 instant accumulated, over an interval of Δx or Δt

Chapter 4A Summary

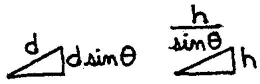
There are two TWE formats: the one shown below, and the one derived in Section 4.3.

	INITIAL ENERGY	+	INPUT	=	FINAL ENERGY	+	WASTE
ENERGY ACCOUNTABILITY	$KE_i + PE_i + PE_i$	+	W_{other}	=	$KE_f + PE_f + PE_f$	+	$\lvert W_{fr}\rvert$
	$\frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2$	+	$F_{other}\, d \cos\phi$	=	$\frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2$	+	$\lvert \mu_k N d\rvert$
CONSERVATION: If $W_{oth} = 0 = W_{fr}$, $(KE+PE)_i = (KE+PE)_f$	$KE_i + PE_i + PE_i$	+	0	=	$KE_f + PE_f + PE_f$	+	0

W_{other} can be a rope pull, person's push or pull, car engine or brakes (thru tire-friction), ...

mgh — For mgh, you must define "up" to be +. h or $d \sin\theta$

$d\,\mu_k N$ — d or $\frac{h}{\sin\theta}$



N is often (but not always) mg or $mg\cos\theta$.

$W_{friction}$ can also be simply "$f_k\, d$".

$\lvert W_{fr}\rvert$ is always +.

Eliminate the TWE parts you don't need, then choose i & f, h≡0 & x≡0.
Substitute for W_{fr}, cancel m's if every term contains m, remember v② & x②.

EVERY BOX EQUALS EVERY OTHER BOX.

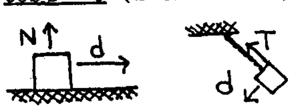
AREA of F-x (graph)	If F_{total} is constant, $F_{total}\; d\; \cos\phi$	$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$
OPTIONAL CALCULUS Section 18.31	W_{total}	$KE_f - KE_i$
AREA of P-t (graph)	If P_{total} is constant, $P_{total}\; \Delta t$	ΔKE

For straight-line motion with constant i-to-f force,

$W = F_{parallel}\; d$

$W = F\; d\; \cos\phi$

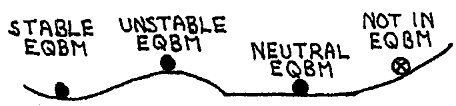
Work = 0 if $F = 0$, or $d = 0$ (no movement), or $\cos\phi = 0$ (F & d are ⊥).



$$P = \frac{W}{\Delta t} = F v \cos\phi$$

$$\frac{F\; d\; \cos\phi}{\Delta t}$$

two different ways to "group" $F d \cos\phi / \Delta t$



If ⊗ (not in equilibrium) is released with $v_i = 0$, it responds so its PE ↓; $F_{parallel}$ & **d** are in same direction, so W is + and KE ↑ (as PE ↓).

If $v_i \neq 0$, object can move "uphill" in PE; W is – so KE ↓ as PE ↑.

If F_{par} & d point same direction, W is +. If F_{par} & d point opposite directions, W is –.

Totals & Partials: $F_{total} = ma$ & $W_{total} = \Delta(\frac{1}{2}mv^2)$, but $F_{part} \neq ma$ & $W_{part} \neq \Delta(\frac{1}{2}mv^2)$.

$F_{total} = F_A + F_B + \ldots$, $W_{total} = W_A + W_B + \ldots$, $P_{total} = P_A + P_B + \ldots$

F causes $m(\Delta v / \Delta t)$, $F\,\Delta x$ causes $\Delta(\frac{1}{2}mv^2)$, $F\,\Delta t$ causes $\Delta(mv)$.

Chapter 4B Summary

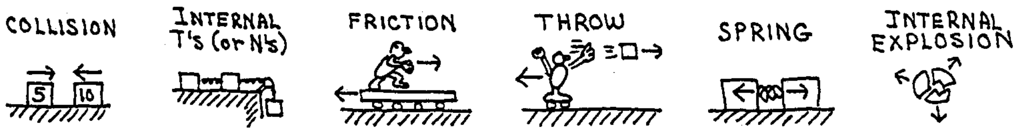
You can define any combination of objects as a "system".
$F_{internal}$ doesn't cause $\Delta(mv)$, because the $F_{int}\,\Delta t$ acting on one part of a system is canceled by an equal-and-opposite $F_{int}\,\Delta t$ (from a "third law mutual-force" partner) that acts on another part of the system; this cancellation makes $(F_{int}\,\Delta t)_{total} = 0$.

$$\mathbf{F}_{external}\ \Delta t = (\mathbf{mv})_f - (\mathbf{mv})_i$$

<u>You can ignore all internal forces</u>; the equation only asks for $F_{external}$!

For each force, ask "Is this F caused by another system-object (making it $F_{internal}$) or by something outside the system (which makes it $F_{external}$)?"

Some external forces are gravity, air resistance, and external friction or T or N.
Some internal forces are internal friction or T or N, a throw or gunshot of a system-object by another system-object, an internal spring-force or internal explosion.



CONSERVATION OF MOMENTUM: If $F_{ext} = 0$, mv is conserved, and $(mv)_i = (mv)_f$.
MOMENTUM IS "ALMOST CONSERVED, $(mv)_i \approx (mv)_f$, if $\Delta t \approx 0$ causes $F_{ext}\,\Delta t \approx 0$.

KE retention in a **collision**: If it is 100 %, **elastic**. If it is less than 100 %, **inelastic**. When the objects "stick", it is the minimum % (not necessarily 0), **totally inelastic**.

For a *1-dimensional elastic collision* of objects 1 & 2, you can use these formulas:

$$(v_1)_f = \frac{m_1 - m_2}{m_1 + m_2}(v_1)_i + \frac{2\,m_2}{m_1 + m_2}(v_2)_i \qquad (v_2)_f = \frac{m_2 - m_1}{m_1 + m_2}(v_2)_i + \frac{2\,m_1}{m_1 + m_2}(v_1)_i$$

- XY independence ⇒ [x & y equations on left & right sides of page]; <u>don't mix x & y</u>!
- If $F_{ext} = 0$, $(mv)_i = (mv)_f$ [i & f on left & right sides of equation]; <u>don't mix i & f</u>!
- Check to be sure you have an mv term for each system-object at the i & f times.

The *center-of-mass* for a symmetric uniform-density object is at its center.

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \ldots}{m_1 + m_2 + \ldots}$$

If a system has $(v_{cm})_i = 0$ and $F_{ext} = 0$, its c-of-m doesn't move, and $(x_{cm})_i = (x_{cm})_f$.
Optional: system-adaptions of some (but not all) motion equations is discussed in Section 4.11.

How to Choose a Useful Equation

two tvvax equations	F = ma "a link"	$F\,\Delta x = \Delta(\frac{1}{2}mv^2)$ $F\,\Delta t = \Delta(mv)$

Choose an equation with the goal-variable and lots of "knowns".

For an interval with no F, use one of the 5 tvvax equations (which don't contain F). F=ma has no i or f or Δ; F=ma shows what is happening at a specific instant of time. If the accelerating effects of F_{total} accumulate during an interval, use $F\Delta x = \Delta KE$ if Δx is either given or asked for, and use $F\Delta t = \Delta p$ if Δt is given or asked for.

If an object changes height between i & f, use "mgΔh" in W=ΔKE.
If "internal force" is involved, simplify things by using $F_{ext}\,\Delta t = \Delta p$.