Chapter 3

Force and Motion (Dynamics)

Read Sections 3.1 to 3.4 first, then use 3.5 to 3.8 in any order you want.

3.1 The Basics of Dynamics: F = ma

Basic Concepts & Equations {your textbook will provide details}:

$$\mathbf{F}_{total} = m\,\mathbf{a}$$

shows how an object's *acceleration* "**a**" depends on its *inertial mass* "m" (which is proportional to the object's weight) and the *net force* "$\mathbf{F}_{total}$" that is pushing or pulling it.

I like to think of F=ma as "cause ⇒ effect" relationship.
The right side tells you WHAT is happening: a mass is being accelerated.
The left side tells you WHY it is happening: a force is acting on the mass.

F = ma.
F causes ma.
Cause ⇒ Effect.

Notice that F does not cause velocity; F causes acceleration (a change of velocity).

F = m**a** is a vector equation. Because m is a non-vector, $\mathbf{F}_{total}$ and **a** always point in the same direction. Their magnitudes (and their units) differ by a factor of "m".

UNITS: As discussed in Section 1.3, a metric prefix can be interpreted two ways. A prefix is usually treated as part of the number: 53 km = 53 x 10^3 m. But for mass, the SI unit (kilogram, kg) differs from the metric unit (gram, g); to be SI-consistent, 53 kg must be interpreted as 53 kg, not 53 kg.

If mass is given in grams, convert to kg with a conversion factor. For example, 40 g = 40 g [1 kg / 1000 g] = .040 kg. MENTAL ARITHMETIC: to divide 40 by 1000, move the decimal point of "040." three places to the left.

F=ma, so F must have the same SI-units as ma: (kg)(m/s^2). This combination of units is called a *Newton,* abbreviated *N*: 5.2 kg m/s^2 ≡ 5.2 Newtons = 5.2*N*.

The **gravity force** acting on an object is called ***weight,*** "w". Near the earth's surface, $F_{gravity}$ has a direction of "straight down" and a magnitude of "w = mg", where m is the object's mass and g is approximately 9.80 m/s^2.

A 1.00 kg mass will, if g = 9.80 m/s^2, produce a weight force of w = mg = (1.0 kg)(9.80 m/s^2) = 9.80*N*; in non-SI units, this force is 2.2046 pounds. { "pounds" is a measure of force, not mass }

3.2 F = ma Problem-Solving Strategies

As in Chapter 2, these Aesop's Problems cover a wide variety of essential problem-solving strategies, so you can learn a lot in a short time.

PROBLEM 3-A: Ratio Logic

When Ron pushes a 1200 kg car on a horizontal road, its acceleration is .8 m/s per second. If Ron and Don (his equally strong twin) both push, what is the car's acceleration? If five passengers, with a total mass of 400 kg, sit in the car, what is the acceleration when Ron and Don push?

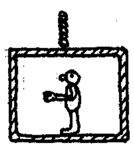
SOLUTION 3-A: Because F=ma, **a** is proportional to F and inversely proportional to m. If F doubles (when Don joins Ron), **a** doubles from .8 m/s^2 to 1.6 m/s^2.

When the car's mass increases from 1200 kg to 1600 kg, it is multiplied by a factor of 1600/1200 = 4/3, and **a** is multiplied by a factor of 3/4, decreasing from 1.6 m/s^2 to (3/4)(1.6 m/s^2) = 1.2 m/s^2.

{ If you haven't read Section 2.9's discussion of ratio logic, I suggest that you do it soon. }

PROBLEM 3-B: An Accelerating Elevator

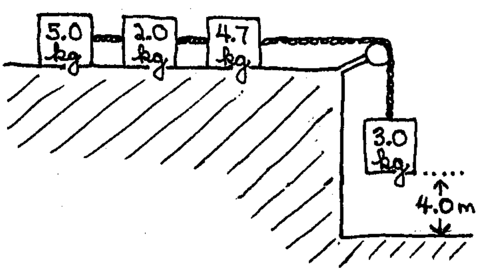
Part 1: A 70.0 kg man rides a 500 kg elevator; both accelerate upward at 2.00 m/s^2. What is the tension in the elevator cable? What force does the man's feet exert on the floor?



Part 2: If the elevator and man move upward with a constant speed of 8.00 m/s, what is the cable tension and the man's force against the floor?

PROBLEM 3-C: A Four-Object Train

What is the tension in each rope, and the acceleration of each block? { Assume that the pulley and ropes are "massless", and there is no friction. }

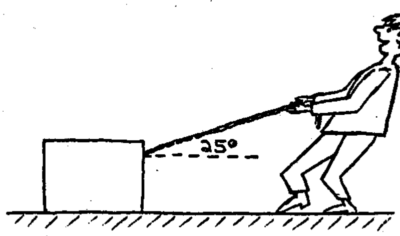


All of the blocks start from rest. When will the 3.0 kg block hit the ground? What is the maximum speed of the 5.0 kg block?

What forces act on the 4.7 kg block? What net (total) force acts on it?

PROBLEM 3-D: 2-Dimensional Force

A man pulls with constant force on a lightweight rope attached to a 2000 *N* block. The block starts from rest; after 5.0 seconds it is sliding across the frictionless horizontal surface with a speed of 2.0 m/s. How much force do the rope and floor exert on the block?



SOLUTION 3-B

Step 1: Read carefully, think, draw a picture of the problem situation.

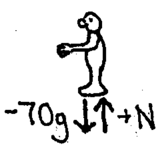
Step 2: Make a Force Diagram. To do this, choose an *object*, look at a drawing of the problem-situation, imagine you are the object and ask "What forces do I feel pushing and pulling on me?", then draw and label these forces.

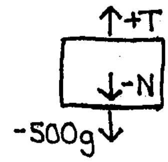
As shown below, this problem requires separate force diagrams for two objects: man and elevator. For the **man's F-diagram and F=ma,** include **only** the forces acting on the man. If you are the man, gravity pulls you downward (–70 g) and the floor pushes you upward (+N). What direction have I chosen to be + ?

Now imagine you're the elevator. You feel gravity pull you downward (–500 g), the cable pull you upward (+T), and the man push you downward (–N). These F's, and **only** these, are included in the **elevator's F-diagram and F=ma equation**.

FORCE-LABELS: If you know the numerical value of a force [like 70 g = 70(9.8)], use it. If not, represent the force with a letter; common labels are "N" for a force caused by surface contact (like man-against-floor) and "T" for rope-tension force.

NEWTON'S THIRD LAW: The contact between man & elevator produces a matched pair of *mutual interaction* forces. "Man is pushed upward by elevator" causes +N on the man's F-diagram, and "elevator is pushed downward by man" causes –N on the elevator's F-diagram. These two forces are equal in size (so their magnitudes are both labeled "N") and opposite in direction (one is +, the other is –).





force-diagram and F = ma <u>for the MAN-object only</u>

$$(F_{TOTAL})_{man} = m_{man}\, a_{man}$$
$$(-70[9.8]) + (+N) = (70)(+2.0)$$
$$N = +140 + 686$$
$$N = +826 \text{ Newtons}$$

F-diagram and F=ma equation <u>for the ELEVATOR-object only</u>

$$(F_{TOTAL})_{elevator} = m_{el}\, a_{el}$$
$$(-500[9.8]) + (-N) + (+T) = (500)(+2)$$
$$-500(9.8) - (+826) + T = (500)(+2)$$
$$T = +6726 \text{ Newtons}$$

Step 3: Substitute <u>vertically</u> into F = ma, and solve. In the equations above, the ⇓'s emphasize *vertical substitution*; put everything you know about F, m & a directly under F, m & a. All m's and a's are given in the problem statement. <u>To substitute for F in an object's F=ma</u>, <u>use the object's F-diagram</u>.

Vertical substitution is important. To see why, consider what happens when F is substituted sideways, turning the man's F=ma into "F = –70(9.8) + N". This equation, although true, is useless for problem-solving because the equation's entire right side [all information about "ma"] has been destroyed.

When you substitute, always decide the ± sign [which indicates the direction] for each F-vector and a-vector. If you don't decide, the sign is automatically +, whether you would have chosen it to be + or not. For the elevator, T and 2 m/s^2 are upward, in the direction I've defined as +, so they are +T and +2. 70g and N are downward, opposite the + direction, so they're –70g and –N.

Use F-symbols (like T, N, g) to represent only magnitude; show F-vector direction with the ± sign in front. For example, g is 9.80, not –9.80. A force of –(500)(9.80) multiplies to give –4900; the – sign shows that gravity force is downward. But if "g = –9.80" is incorrectly used to get –(500)(–9.80), which equals +4900, you are stating that gravity points in the + (upward!) direction.

ALGEBRA: use standard leapfrog-substitution. Solve 1-unknown left equation for N, then substitute N into the right equation (as shown by ⇓) and solve it for T.

Step 4: Answer the questions that were asked. "T = 6730 Newtons" and "N = 826 Newtons" answer the questions about cable tension and man-against-floor force.

This floor-man force is called *apparent weight* because a weighing scale placed under the man's feet would be squeezed with a force of 826 *N*, and would report this as the man's "weight". It is larger than the man's usual weight of mg = (70 kg)(9.80 m/s^2) = 686 *N*. { We might claim that the man's "extra weight-force" (826 – 686 = 140 N) has been caused by his +2 m/s^2 acceleration. This "reversed cause-effect" (with ma causing F) is, in fact, a key principle of the theory of General Relativity. }

A Solution for Part 2: "Constant velocity" means acceleration is now zero, instead of $+2\ m/s^2$. Otherwise, the equations are the same as before: $-70g + N = (70)(0)$ and $-500g - N + T = (500)(0)$. Solving them gives $N = 686N$ and $T = 5586N$.

Velocity was given as 8 m/s. This information is unnecessary (if v is constant at 8 m/s ↑, 20 m/s ↓, or zero, we still substitute "a = 0" into F=ma). "8 m/s" is a ***decoy***, included to lead you down a wrong strategy path if you're not careful. Problem writers sometimes use decoys to give you practice in deciding relevance, so you can learn to recognize what information should be used and what should be ignored.

SOLUTION 3-C

Read-think-draw. Make a force diagram: choose an object, imagine you are this object, draw the forces that push and pull you.

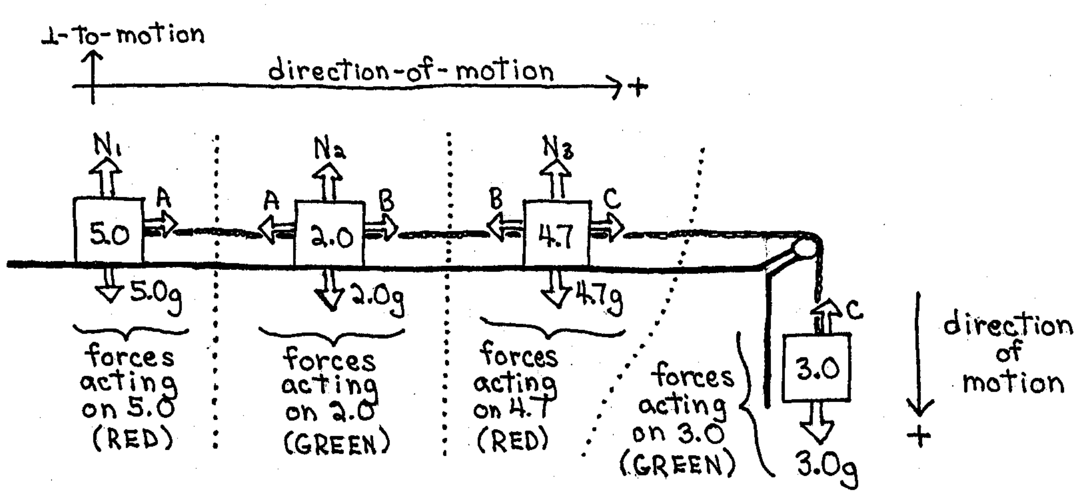
To solve this problem you need the "principle of MASSLESS ROPES": if a rope was truly massless, it would pull with the same tension force at both ends. The first rope pulls the 5.0 & 2.0 blocks with an equal-magnitude force I'll call "A"; the T-force that is felt by each block is shown on the diagram below. Similarly, the second rope pulls 2.0 & 4.7 with force "B". The massless frictionless pulley changes the third rope's direction from ↔ to ↕, but the tension magnitude is still "C" at each end; this T-force pulls 4.7 & 3.0 in whatever direction the rope is pointing.

MULTIPLE-OBJECT FORCE DIAGRAMS: The dividing lines below show which forces act on 5.0, on 2.0, on 4.7, and on 3.0. When you draw an object's F-diagram and substitute these F's into the object's F=ma, be sure to include only the forces that act ON the object, not those exerted BY the object. For example, when you imagine you're the 3.0 block, don't say "I am pulling the rope downward". Instead, say "I am **being** pulled upward by the rope".

I like to use color on force-diagrams. The physical situation (table, blocks, ropes, pulley,...) is drawn in black, and the forces are in color. Use 4 different colors for the 4 objects or, as shown below, just alternate two colors: draw 5.0's forces in red, 2.0's in green, 4.7's in red, and 3.0's in green.

After you've drawn the F's on each individual object, do a quick "mass production check". Ask whether you've drawn all occurences of each kind of force: have you included all weights, all surface-contact forces (N's), all rope forces, all.... ?

Two styles of force-diagrams are "free body" (that show only the object, like the man or elevator in Problem 3-B) and "in context" (showing everything: table, ropes, pulley,...., as in the picture below). Both styles serve the same purpose. The important thing is to choose a specific object and draw only the forces acting on that object.



DIRECTION-OF-MOTION. Each block has a "direction of motion": the 5.0, 2.0 & 4.7 blocks move →, and 3.0 moves ↓. If the ropes don't stretch, the motion of each block is "matched" to the others, and every block has the same direction-of-motion tvvax information: the same Δx, same v, and same a.

XY INDEPENDENCE. As emphasized in Section 2.7, motion in different directions can be analyzed independently. Forces that are ⊥ to motion (like N_1, N_2, N_3, 5.0g, 2.0g and 4.7g) are not included in the "direction of motion" F=ma equations below.

ALGEBRA. "a" is the same for each block, so there are only 4 unknowns [A, B, C, and a] in the objects' 4 equations. These equations can be solved using leapfrog substitution, but there is a better method. Add all of the equations together, as shown below, and solve "29.4 = 14.7 a" for $a = +2.0\ m/s^2$. Then substitute "a = +2.0" into the equations that will, after the a-substitution, contain only 1 unknown. This lets you solve "5.0 only" for $A = 10.0\,N$, and "3.0 only" for $C = 23.4\,N$. Finally, substitute into either "2.0 only" or "4.7 only" to get $B = 14.0\,N$.

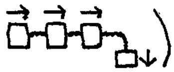
F, m and a for 5.0-only	F, m and a for 2.0-only	F, m and a for 4.7-only	F, m and a for 3.0-only
F = m a	F = m a	F = m a	F = m a
+A = (5.0) a	−A + B = (2.0) a	−B + C = (4.7) a	+3.0(9.8) − C = (3.0) a

Add the equations

$$\begin{aligned} +A &= 5.0\,a \\ -A + B &= 2.0\,a \\ -B + C &= 4.7\,a \\ +29.4 - C &= 3.0\,a \\ \hline +29.4 &= 14.7\,a \end{aligned}$$

SYSTEMS. Here is an alternate approach. All 4 blocks move together as a single unit with the same tvvax info, so you can treat them as one big "system-object". The system-equation below contains F_{total} and m for each individual object. Notice that many force-letters appear twice. For example, the first rope pulls 5.0 forward (this is +A) and pulls 2.0 backward (this is −A). The +A and −A cancel each other, so "A" disappears from F=ma. Similarly, B and C also disappear.

In the context of this 4-block system, A, B & C are *internal forces;* a rope passively transmits equal-and-opposite force from one part of the system to another part of it, but doesn't affect the system as a whole. Only the "3.0g" force of gravity (which is underlined) survives cancellation; it causes the acceleration of 2.0 m/s^2.

for the system () with "bent axis"

$$+A - A + B - B + C - C + \underline{3.0(9.8)} = (5.0 + 2.0 + 4.7 + 3.0)\,a$$

Look at the left side of the four single-object equations (+A; −A, +B;...) and compare these terms with the system-equation's left side. Then compare the equations' right sides. Do you see why the system-equation gives the same result [29.4 = 14.7 a] as adding all 4 single-object equations together?

A "system approach" can be used to solve part of Problem 3-A. If we define elevator-and-man to be an object, and ignore the internal contact forces (the N's) that act between the elevator and man, the elevator-and-man equation is "+T − (500 + 70)g = (500 + 70)(+2.0)", which gives $T = 6726\,N$. This is, of course, the same result we got earlier. But to solve for "N" we need a single-object equation, either "man only" or "elevator only", because N doesn't appear in the elevator-and-man equation.

Now we know 3-of-5 tvvax variables ($v_i = 0$, $\Delta x = +4.0$ m, $a = +2.0$ m/s^2) so the last two problem-questions can be answered. Use the 1-out strategy from Section 2.3 to choose $\Delta x = v_i t + \frac{1}{2} at^2$, and solve it for $\Delta t = 2.0$ seconds. Then solve "$v_f - v_i = at$" for $v_f = +4.0$ m/s. Just before the 3.0-block hits the ground, all four blocks have the same speed of 4.0 m/s. After 3.0's impact, gravity no longer produces acceleration (why?) so the other three blocks continue moving at their maximum speed of 4.0 m/s.

What forces act on the 4.7 kg block? The earlier F-diagram has –B and +C in the direction of motion, $+N_3$ and –4.7g in the direction perpendicular ($\perp$) to motion. If an object slides along a surface and doesn't do bizarre tricks like leaping away from the surface or diving down into it, $v_\perp$ (v in the $\perp$ direction) is zero and remains zero. $v_\perp$ is constant, so $a_\perp = 0$, the $\perp$-to-motion equation is $+N_3 - 4.7(9.8) = (4.7)(0)$, and $N_3 = +46.1\,N$. Or just use common sense: to produce $a_\perp = 0$, N and mg [which are the only $\perp$ forces] must be equal-and-opposite so they'll cancel each other.

The net $\perp$-to-motion force is zero, and the net direction-of-motion force is "$-B + C$". Using earlier results, $-B + C = -14.0 + 23.0 = +9.4$. The 4.7 kg block feels a total vector force of $+9.4N \rightarrow$, which causes its acceleration of 2.0 m/s^2 $\rightarrow$.

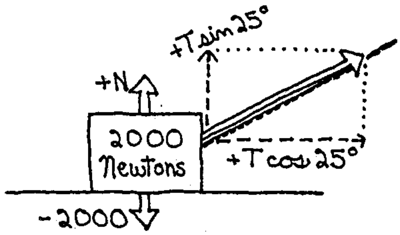
SOLUTION 3-D

Read, think, draw. Imagine you are the block; you feel gravity pull you (2000), the floor push up against you (N), and the rope pull you in the direction it points (T).

Motion in the x & y directions can be analyzed independently, for either tvvax or F=ma. On the diagram below, methods from Section 1.2 are used to split T into x & y components. In the direction of motion, 3 tvvax variables are given (Δt, v_i, v_f) so you can solve for $a_x = .40$ m/s^2. And because the y-direction is $\perp$ to sliding, $a_y = 0$.

Substitute x & y information into the x & y equations. {The block's weight is 2000*N*. To find its mass, use $w = mg$: $2000 = m(9.8)$, so $2000/9.8 = m$.}

The problem asks for the rope & floor forces, T & N. Solve the 1-unknown equation for T, then substitute into the other equation (⇓) and solve for N.



$$F_x = m\,a_x$$
$$+T\cos 25° = \left(\tfrac{2000}{9.8}\right)(+.4)$$
$$T = +90.0$$

$$F_y = m\,a_y$$
$$(+N) + (-2000) + (+T\sin 25°) = \left(\tfrac{2000}{9.8}\right)(0)$$
$$\Downarrow$$
$$+N - 2000 + (+90.0)\sin 25° = 0$$
$$+N - 2000 + 38 = 0$$
$$N = +1962$$

3.3 A Summary of F = ma Strategies

Here is a summary of the Aesop's Fable Strategies that were used in Problems 3-B to 3-D, to gather information and "funnel" it into F=ma:

Make a force-diagram:
Draw a picture, choose one object, imagine you are this object and think: "The forces I feel pushing & pulling me are __." Draw only these forces, and label them.

$w/g = m$

Solve tvvax for a, use it.

If v is constant, $a = 0$.
If object slides, $a_\perp = 0$.

If matched motion, each object has same Δx, v, a.

$$F_{total} = m\,a$$

X & Y motions are independent, so choose axes and then split F's (and a) into x & y components:

$$F_x = m\,a_x \qquad\qquad F_y = m\,a_y$$

When you substitute, always decide whether the ± sign (to show vector direction of **F** or **a**) should be + or –.

F=ma is always written for one object (you choose) and one direction (you choose).
Draw only forces acting on the object (I am being...), not those exerted by the object.

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Review Problems 3-B to 3-D: do you see which strategies were used in each problem?

3-B used two F-diagrams [for the two F=ma's] and "constant v".
3-C used four F-diagrams [and four F=ma's], matched motion, solving tvvax, xy split, $a_\perp = 0$.
3-D used one F-diagram [but two F=ma's because of an xy split], solving tvvax, $a_\perp = 0$, $w/g = m$.

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Here is a summary of the forces that have been used so far in Chapter 3:
mg-direction is a pull "straight down" toward the earth's center,
N-direction is a push ⊥ to the surfaces that are in contact,
T-direction is a pull in the direction the rope points.

$F_{gravity}$ magnitude is "mg". But there is no magnitude formula for N or T; you must find it (as was done in all 3 problems) from context, by solving F = ma.

In Problem 3-C, N = mg because 3 conditions are true: the block is on a horizontal surface, N and mg are the only vertical forces, and the vertical acceleration is zero. In 3-B (Part 1) and 3-D, where $N \neq mg$, which of these 3 conditions is not met? The answer is given in Section 3.6.

When you substitute "mg", use g = 9.80, not –9.80. For example, use –m(9.80), not –m(–9.80).

Units can provide valuable clues; "10 kg block" or "98 *N* block" give all of the mg = w information you need, even though there is no mention of mass (m) or weight (w).

○ ○ ○ ○ ○ ○ ○

When the same variable is in two equations, you can solve for it in one equation and substitute it into the other. An equation-link is often needed to solve a problem. So far, we have used five kinds of links.

Third Law link: In Problem 3-B, "N" is in the man's F=ma (because man is pushed by elevator) and in the elevator's F=ma (because elevator is pushed by man).

Massless Rope link: In Problem 3-C, objects at each end of a "massless" rope are pulled with equal tension, so the same T-force appears in the F=ma of two objects.

a-link: The tvvax equations and F=ma both use *a*. In Problem 3-C, F=ma is solved first, then *a* is used in tvvax. In Problem 3-D the information flow is reversed; *a* is found in tvvax, then used in F=ma. { Links works in both directions, of course.}

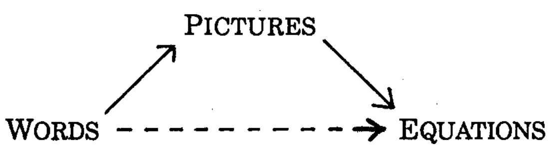
Matched Motion link: In Problem 3-C all four blocks have the same *a* (and Δx, and v) so the same *a* appears in the F=ma of each block.

Split link: In Problem 3-D, "T" is split into x & y components, so T appears in the $F_x = ma_x$ and $F_y = ma_y$ equations.

3.4 Playing with Problems

Modes of Thinking: To be a good problem-solver, you must translate ideas fluently between three modes of thinking: verbal, visual and mathematical.

I like to think of these modes more "concretely", in terms of their most common use in physics problems: words, pictures (on-paper or in-the-mind) and equations:



- - - → shows a translation path often used by beginning students. They notice that equations are used in most solutions, so their immediate goal is to "find an equation". By contrast, the first ——→ shows that **expert solution-finders explore a problem with non-mathematical thought**, "playing with the problem" until words have been translated into a clear, complete mental picture of the physical situation. They get the **big picture first** and fill in the **details later**. In "The Physics Teacher", May 1981, Frederick Reif uses an art analogy to illustrate the process of *successive refinements,* by comparing two ways to paint a woman's portrait:

> ... one might begin by first painting completely the woman's eye, then her eyebrow, then her nose, and so forth until the entire portrait has been completed. The same problem might be approached by a "method of successive refinements" which deals first with the major features of the portrait and then successively with more minor features. Thus one might begin by first drawing a rough sketch of the entire woman, then elaborating this sketch by additional lines, and then successively elaborating with more lines and colors until the entire portrait has been completed. Almost any painter would use the second method. .... In scientific problem solving, such a method of successive refinements can be implemented effectively by using successively more detailed levels of description of the same situation.

The second ——→ shows that a picture-idea often leads to a useful equation. This is why it is important to know the "physical meaning" of equations: if you have a strong, clear idea of the connection between each equation and the physical reality it represents, understanding a physical situation (by exploring the problem with non-mathematical thinking) will naturally lead you to a true-and-useful equation.

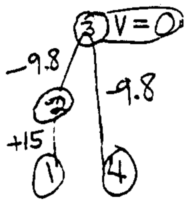
As emphasized in Section 2.3, physics-math is easy. It is the mastery of physics concepts that will make you a problem-solving expert. { The main math we have so far is right-triangle trig (easy), tvvax equations (easy), and F=ma (very easy), but problems can still be challenging if they require that you interpret complex physical situations, or use ideas that are not yet internalized-and-intuitive [for example, if you're still "fighting" the independent analysis of x and y motion, it may be difficult to remember this tool when you need it to solve a problem]. }

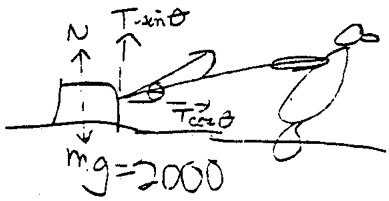
Even though math isn't the problem-solving "key", it is important and shouldn't be ignored. In fact, one goal of this book is to help you be totally comfortable with math so your mind is free to think about words and pictures. If you are confident about coping with math, you can relax and focus your attention on physics!

Drawing to produce Logic (not Art)

When you solve a physics problem, use drawing as a thinking tool, to help you translate words into clear picture-ideas. Aim for logical creative thought, not "art".

Relax and let the ideas flow. Don't worry if the "special points" (for Problem 2-B below) are sloppy, or the man (for Problem 3-D) looks like a penguin.





{ In the context of problem-solving, "pictures" is anything that helps you 1) form a clear idea of the problem situation, and 2) bridge the gap between words and equations. This could be a force diagram [drawn from a word-description, then used to substitute into the F=ma equation] or a tvvax table [whose structure makes it easy to store information so you can see what is "known" and "unknown"] or an actual drawing, a mental image, or ... }

Principles and Practice

Think about how a basketball player learns to play with skill and confidence. First he (or she) learns the fundamentals: how to dribble, pass, shoot, play defense, and cooperate with other players. Then he uses practice to master these skills, to make them natural and instictive. By facing challenges during practice, he learns to respond appropriately in a variety of situations, so he can quickly cope with whatever happens during the game "when it counts".

You can become an expert problem solver the same way: learn the basic tools and "make them your own" through practice. By using principles in different situations you'll become better at improvising, as you discover that your own "variations on a few basic themes" let you cope with a wide variety of problems.

The value of "searching for insight" was emphasized in Sections 2.1 [skier] and 2.5 [welder]. You can learn a lot by "just doing problems" but you'll learn even more if your practice is made effective by insight-principles. Principles-and-Practice are a great team. Working together as cooperative partners, they're more effective than either one by itself, as each increases the effectiveness of the other.

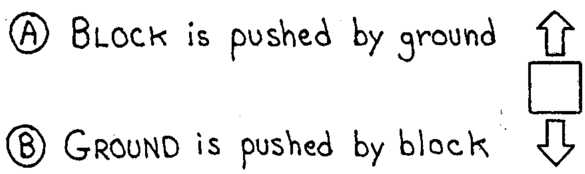
CHOICES: Read the rest of Chapter 3 (Newton's Third Law, inclined planes, friction-force, spring-force) in any order you want.

3.5 Newton's Third Law of Motion

A *Third Law force-pair* is described in Problem 3-B: man is pushed by elevator and (due to their "mutual interaction") elevator is pushed by man. A common summary of such a force-pair is that the forces are "equal and opposite".

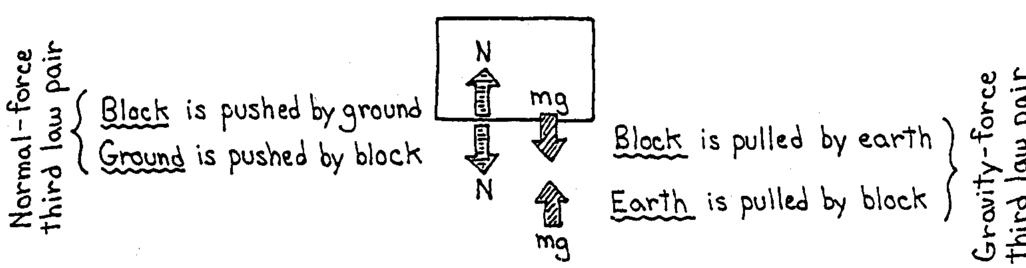
PROBLEM 3-E: When a block slides on level ground with only N & mg forces acting on it, the N & mg forces are equal in size and opposite in direction. Does this equal-and-opposite relationship indicate that N and mg are a third law force-pair?

SOLUTION 3-E: Instead of jumping to conclusions, let's analyze what "equal and opposite" means, by examining the ground-block interaction from two points of view:
A) The BLOCK says "I feel the ground underneath me; it pushes me upward".
B) The GROUND says "I feel the block on top of me; it pushes me downward".



Notice the >< . It shows the "mutual symmetry" produced by the mutual interaction between two members of a third law force-pair:
if block is pushed by ground, then ground is also pushed by block.
{ Another way to state this is "if ground pushes block, then block pushes ground". }

Every type of force (contact, gravity, ...) involves the mutual interaction of two objects. The ground-block mutual interaction forces (labeled "N") are caused by physical contact between the ground and block. The earth-block mutual interaction forces (labeled "mg") are caused by gravitational attraction between the earth and block. Notice that **the members of a third law force-pair are always the** **<u>same kind of force</u>**:



The "affected object" is underlined. The partners in a third law force-pair never act on the same object, so they never appear together on the F-diagram for any object.

The equal-and-opposite N & mg forces are not caused by "mutual interaction with each other". N & mg are not the same kind of force, and they're not a third law pair.

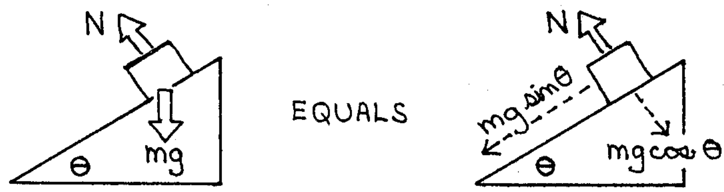
\- - - - - - - - - - -

A third law force-pair involves <u>two</u> EQUAL-AND-OPPOSITE relationships:
1) Third law forces are EQUAL IN SIZE and OPPOSITE IN DIRECTION, and also
2) EQUAL IN "KIND OF FORCE" and OPPOSITE IN "MUTUAL SYMMETRY".

Could you answer a challenging question posed by a lazy horse?

3.6 Inclined Planes

When an object slides along a non-horizontal surface, it is easier to analyze the motion if you choose axes oriented parallel (||) and perpendicular (⊥) to the plane. For example, mg [first picture] can be split into || and ⊥ components [second picture]:



Every time you see this situation (horizontal base, θ is plane's angle-of-elevation) you can immediately split "mg" into these "mg sinθ" and "mg cosθ" components.

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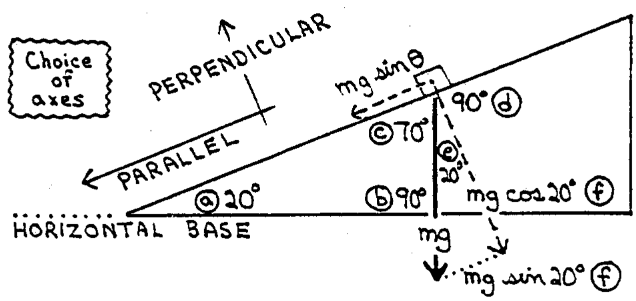
OPTIONAL: If you want to know WHY, here is a derivation.

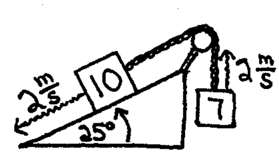


a) The base is chosen to be horizontal. Given: the plane surface is 20° above this.
b) The base is horizontal and the mg force is vertical; the angle between them is 90°.
c) A triangle's three angles add up to 180°. Two angles are 20° and 90°, so this is 70°.
d) The ⊥ component of mg will, by definition of ⊥, make a 90° angle with the plane.
e) A straight line contains 180°. The other angles are 70° and 90°, so this is 20°.
f) Study the right-triangle used for "splitting"; its legs are drawn in the directions the axes point (⊥ and || to the plane surface), and "mg" is the hypotenuse. The side adjacent to 20° is "mg cos 20°", and the side opposite 20° is "mg sin 20°".

- - - - - - - - - - - - - - -

PROBLEM 3-F: A Plane-and-Pulley Problem.

Part 1) A 10.0 kg block slides down the ramp with v_i = 2.0 m/s; the 7.0 kg block moves upward at v_i = 2.0 m/s. Find rope tension, block-acceleration, and force exerted on the 10 kg block by the ramp. Ignore friction, and assume the rope & pulley are massless.

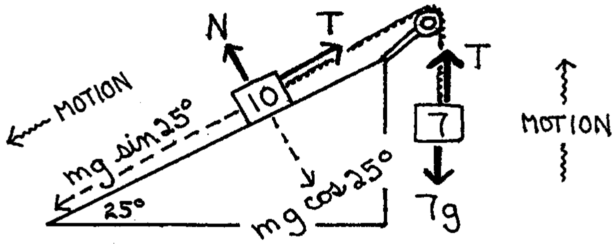


Part 2) Find the acceleration if the "given information" is changed so you know the rope tension is 57.4 Newtons, but don't know the mass of the block that hangs vertically from the rope (it is given as "m", instead of 7.0 kg).

SOLUTION 3-F

Draw F-diagrams so you can substitute for "F" in F=ma: imagine you are the 10 kg block and draw the forces acting on you, imagine you are the 7 kg block and draw the forces acting on you, then write direction-of-motion equations [as shown below] for 10-only, 7-only, and a "system object" of 10-and-7-together. { Problem 3-C discusses a "bent axis" in the direction-of-motion, matched motion, and "system-objects". }

Solve for a, then substitute into either of the other equations and solve for T. (If you want, substitute "T = +57.4" and "a = −1.6" into the 7-only equation as a "check".)



For 10 only	For 7 only	For 10 & 7 together
F = m a	F = ma	F = m a
+10(9.8)sin25° −T = 10 a	+T −7(9.8) = 7a	+10(9.8)sin25° −T +T −7(9.8) = (17)a
⇓		+41.4 −68.6 = 17a
+41.4 −T = 10 (−1.6)		−27.2 = 17a
+57.4 = T		−1.6 = a

The meaning of ± Signs: In the 10-only equation, the "−" sign of "−T" shows that the rope force points in the − direction (up the ramp), while "T" represents only force magnitude. But in "10 a", "a" represents the entire a-vector, both magnitude and direction; acceleration is written as just "a", not "+a" or "−a".

If an algebra solution shows that "a" is −, as in "a = −1.6 m/s²", this tells you that the acceleration vector points in the − direction (up the ramp), opposite the direction that was chosen to be +.

But "T = +57.4*N*" only tells you that force-magnitude (which is all "T" represents) is 57.4*N*; the + sign does not show force-direction*. The direction of the rope force is shown by the "−" sign in the original "−T". * If you try to interpret "T = +57.4" to mean that the rope force points in the + direction (down the ramp), you'll reach the wrong conclusion.

A Summary of ± Sign Logic: If a letter (like T, N, m or g,...) is used to represent only F-magnitude, that letter must have a + value. If the solution for that letter is − (for example, if the algebra had given "T = −57.4") you know that a mistake has been made somewhere in the algebra process: drawing the F-diagram, assigning the ± sign during substitution, or in solving the equation.

Look at the F-diagram and "10 & 7 together" equation, and you'll see a tug-of-war. A 41.4*N* force pulls the blocks in the + direction, the T's cancel, and 68.6*N* pulls in the − direction. The net force of −27.2*N* causes the system to have a = −1.6 m/s².

Initially, v = +2 m/s, but *a* is − because F is −. Δv and **a** always point in the same direction as **F**. But **v** and **a** can, as in this example, point in different directions.

("v = 2.0 m/s" is, as discussed in Problem 3-B, a "decoy". It isn't needed to find a, T, or N.)

In Problems 3-C and 3-D, sliding blocks had $a_\perp = 0$. Similarly, the 10 kg block slides in a straight line along the plane; $v_\perp$ is 0 and remains 0, so $a_\perp = 0$. The ⊥-to-the-plane F=ma is "+N −10(9.8)(cos 25°) = 10 (0)". Solving it gives N = 88.8 Newtons.

Or use intuitive logic. The only ⊥-to-plane forces are +N and −10g(cos 25°). They cancel to cause $a_\perp = 0$, so they must be equal in magnitude: N = 10g(cos 25°).

Often (but not always), N = mg on a horizontal surface, and N = mg cosθ on an inclined plane. As discussed in Section 3.3, N = mg if the surface is horizontal, and N & mg are the only vertical forces, and $a_{vertical} = 0$. In Problem 3-C all three conditions are met, so N = mg. But N ≠ mg in some other problems because a ≠ 0 (in 3-B), N & mg are not the only vertical forces because T has a vertical component (in 3-D), and the surface is not horizontal (in 3-F).

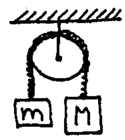
In this Part 1 Solution, the "10 & 7 together" equation was valuable because +T and −T canceled each other, producing a 1-unknown equation that could be solved for *a*.

But in Part 2 the "given information" is changed. T is known, so getting rid of T is not useful. The 10-and-7 equation now has 2 unknowns (m & a), but 10-only has 1 unknown so you can find "a", and then solve 7-only or 10-and-7 for "m":

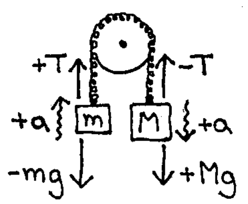
For 10 only	For 7 only	For 10-and-7 together
$+10g\sin 25° - 57.4 = 10a$	$+57.4 - mg = ma$	$+10g\sin 25° - 57.4 + 57.4 - mg = (10 + m)a$

PROBLEM 3-G: two strategies that produce the same result

The acceleration of these blocks (m is smaller than M) can be found in two ways: by defining the direction-of-motion as + for both blocks, or by defining "up" as + for both blocks. Find "a" using each method (it's easy) and show that both methods give the same result.



SOLUTION 3-G: Here are F-diagrams, F=ma's and solutions for each method:



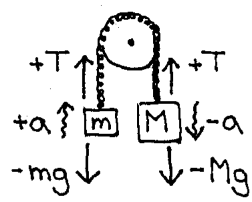
$$+T - mg = m(+a)$$
$$-T + Mg = M(+a)$$

Solving for a-magnitude [which is what "a" represents],

$$\frac{M - m}{M + m}\, g = a.$$

Now look at a-directions above:
$a_{of\ m}$ is +a (in the ↑ direction),
$a_{of\ M}$ is +a (in the ↓ direction).

I prefer this method.



$$+T - mg = m(+a)$$
$$+T - Mg = M(-a)$$

Solving for a-magnitude [which is what "a" represents],

$$\frac{M - m}{M + m}\, g = a\ \text{[magnitude]}.$$

Now look at a-directions above:
$a_{of\ m}$ is +a (in the ↑ direction),
$a_{of\ M}$ is −a (in the ↓ direction).

3.7 Friction-Force

Basic Concepts & Equations {your text will provide details}:

kinetic friction (if friction-producing surfaces slide across each other) is $f_k = \mu_k N$,
static friction (when friction-producing surfaces don't slide) is $f_s \le \mu_s N$,
where f_k and f_s are the forces of kinetic & static friction,
while μ_k and μ_s are the coefficients of kinetic & static friction.

f_k points in the direction opposite to sliding,
f_s points in the direction the object would slide if it wasn't held in place by f_s.

μ_s is always larger than μ_k. μ size depends on the nature of the surfaces in contact (their roughness, chemical composition, lubrication,...) and is usually assumed to be independent of surface area [so f is the same whether a block's orientation is ▭ or ▯ ~~blocks~~] and sliding-speed.

PROBLEM 3-G: Static Friction, and the meaning of "$f_s \le \mu_s N$"

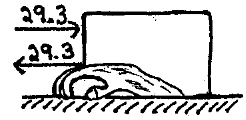
A 10.0 kg block rests on a horizontal surface. The block/surface contact has μ_s = .300, and μ_k = .200. What is the friction force if you push the block in the ⇒ direction with forces of zero? 10.0 Newtons? 29.3*N*? 29.5*N*?

SOLUTION 3-G

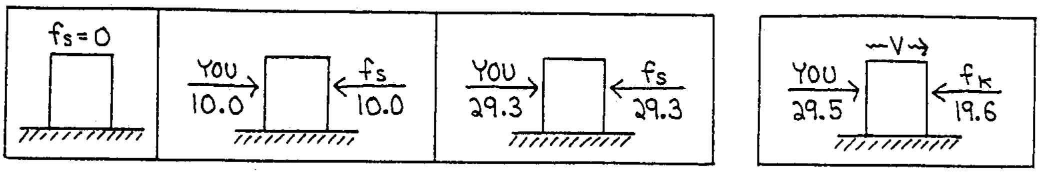
The upper limit of f_s is $\mu_s N = \mu_s mg = .300(10)(9.8) = 29.4N$. If you push the block with a force less than 29.4*N*, f_s will "match" your push [no more, no less] to make

$F_{total} = 0$ and thus keep the block from moving. If you push with 0, 10.0 or 29.3N, static friction pushes back with 0, 10.0 or 29.3N.

f_s is a "passive" force; it doesn't "make something happen", it just tries to prevent sliding. You may find it useful to think of static friction as the block desperately clinging to the surface, "gripping" the surface so the block won't slide.



If your push exceeds the 29.4N limit, f_s cannot prevent movement. In the last picture: when the block starts moving, friction immediately changes to kinetic, and decreases from $f_s = \mu_s N = .300(98.0) = 29.4N$ to $f_k = \mu_k N = .200(98.0) = 19.6N$. The block now accelerates quickly: $a = F_{total}/m = [(+29.5) + (-19.6)]/10.0 = +.99 \text{ m/s}^2$.



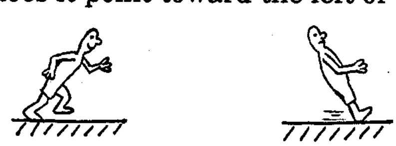
Kinetic friction has a fixed magnitude; $f_k = \mu_k N$, always.

But static friction size can vary from 0 to $\mu_s N$. This is stated by "$f_s \le \mu_s N$", which means "f_s can be less than or equal to $\mu_s N$". If the total non-friction force ($F_{non\text{-}f}$, like your push) increases until it reaches $\mu_s N$, every bit of available static friction must be used to prevent motion. If you push harder, the block will "break away" and start moving. It is only at the "breakaway limit" (often indicated by clue words: least, most, maximum, minimum, smallest,...) that $f_s = \mu_s N$. Below the breakaway limit, $f_s = F_{non\text{-}f}$, because f_s does just enough to cancel $F_{non\text{-}f}$ [no more, no less] so it can prevent movement.

This "breakaway" concept is useful in other situations. For example, if a rope is tied to the 10.0 kg block and you pull upward with gradually increasing T, what are T & N just before the block is lifted off the ground? { To check your answer, do Problem 3-5.}

PROBLEM 3-H: Finding the direction of static friction.

At the start of a race, Fred's legs work hard to make his speed increase quickly. After the race is over, he slows down and stops. If Fred's feet don't slip, do they feel a force due to kinetic friction or static friction? Predict the direction of friction at the start & finish of this race: does it point toward the left or right?



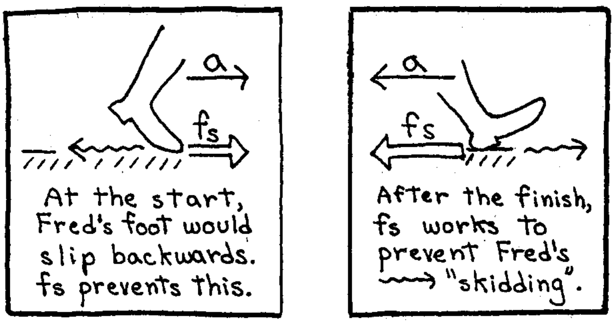
Fred increases speed — Later: slowing down!

SOLUTION 3-H

The names kinetic & static may seem to imply moving & non-moving, but it's better to think in terms of sliding & non-sliding. If you ask "Is Fred moving?", the answer is YES and you may wrongly conclude that friction between his feet and the ground is kinetic. Instead, you should ask "Are his feet sliding?". The answer is NO, and the friction is static.

The direction of f_k is always opposite to actual sliding, but f_s opposes "would-be" sliding. To find the direction of f_s, look at the situation and ask: if there was no friction, what direction would the object slide? Then think: to prevent this would-be motion, f_s must point in the opposite direction. { It may help to imagine what would happen on a low-friction surface like ice. } The pictures below show what the sliding "would be" (⟵〰) if there was no friction, the f_s force (⇒) that tries to prevent this would-be sliding, and Fred's acceleration (→).

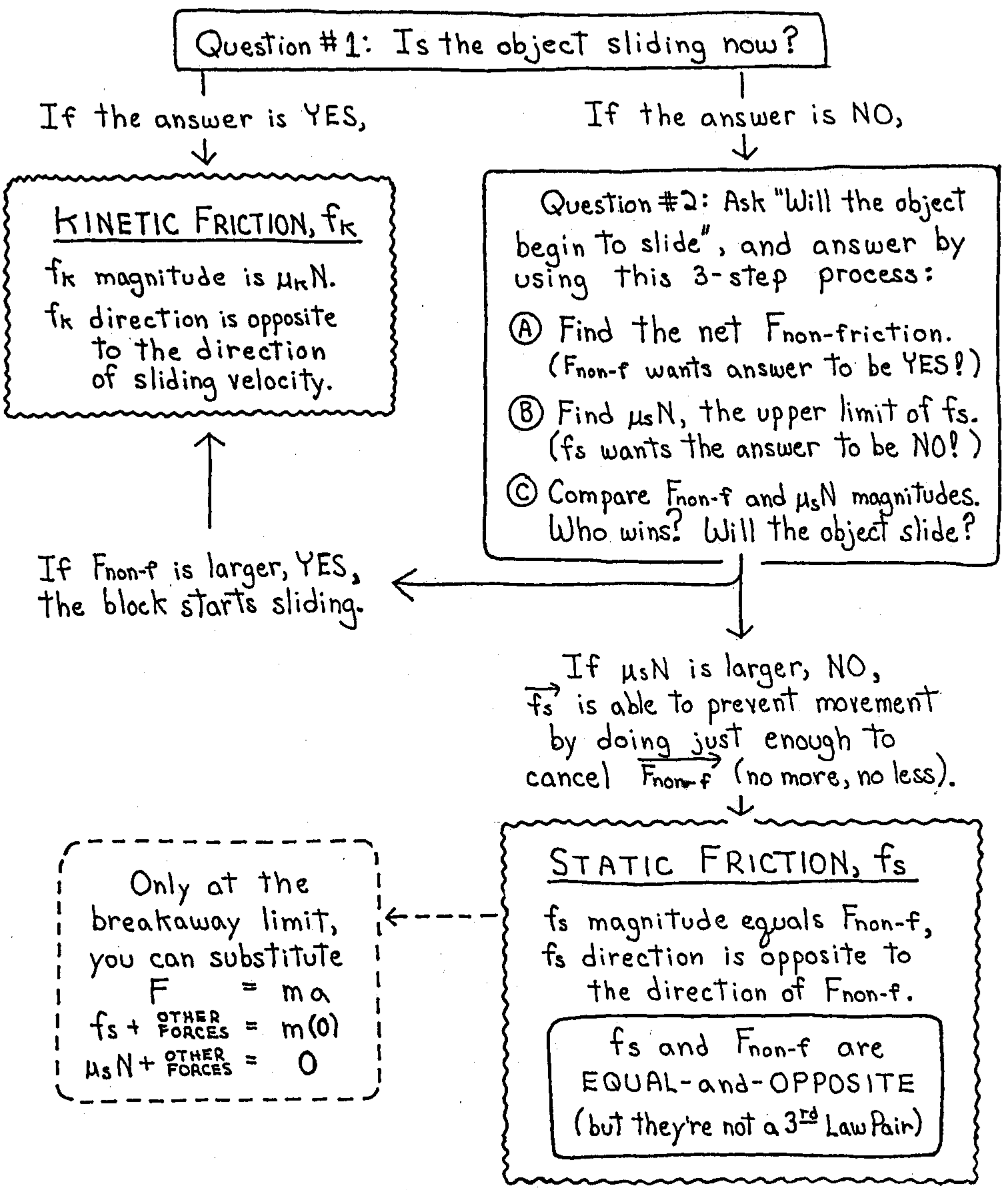
Compare the directions of $\mathbf{f}_s$ and $\mathbf{a}$. Does $\mathbf{f}_s$ oppose $\mathbf{a}$, or cause it?



Friction opposes sliding (either actual or would-be), not "motion". In fact, friction is often the cause of acceleration, as in this example. (As usual, **v** and **a** can point in the same direction or in opposite directions, because **a** is related to Δ**v**, not **v**.)

○ ○ ○ ○ ○ ○ ○

This flowchart organizes the fundamentals of friction into a useful strategy.
(Your textbook won't show you this logic, but they will expect you to use it for problem-solving!)



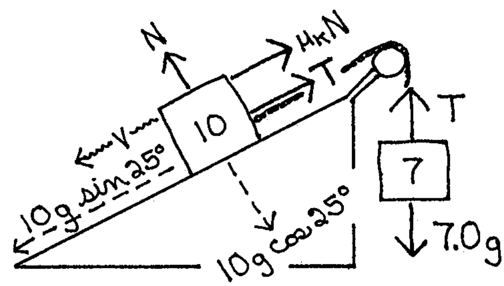
PROBLEM 3-I: Kinetic & Static Friction

Everything is the same as in Problem 3-F, but now there is friction between the block and plane: $\mu_k = .25$ and $\mu_s = .35$. How far will the block move down the ramp? After it stops, will it start moving again? If it moves, what is "a"? If not, what is f_s?

SOLUTION 3-I

Use the Friction Flowchart to guide your strategy. The answer to the flowchart's first question, "Is the object sliding now?", is YES. The block is sliding down the ramp, so f_k is up-the-ramp. f_k magnitude is $\mu_k N$; draw this on the F-diagram.

N appears in both equations below; this solve-and-use link ($\Downarrow$ substitution) gives $a = -2.9 \text{ m/s}^2$. We know 3-of-5 tvvax ($v_i = +2.0$, $v_f = 0$, $a = -2.9$) and want Δx; the t-out equation, $v_f^2 - v_i^2 = 2a\Delta x$, can be solved for $\Delta x = +.69$ meters. (In Problem 3-F, v = 2.0 m/s was a decoy. Now it is useful.)



for 10 & 7 together (in direction-of-motion)

$$F = m\,a$$
$$+10(9.8)(\sin 25°) - .25\,N - T + T - 7g = (10+7)\,a$$
$$\Downarrow$$
$$+41.4 \quad -.25(88.8) \quad -68.6 = 17\,a$$
$$-2.9 \text{ m/s per s} = a$$

for 10 only (in ⊥ direction)

$$F = m\,a$$
$$+N - 10(9.8)(\cos 25°) = (10)(0)$$
$$N = 88.8 \text{ Newtons}$$

The instant the block stops, you answer NO to "Is the object sliding now?". Move on to the next question, "Will the object begin sliding?", and answer it using Steps A, B & C. A) The net $F_{non\text{-}friction}$ in the direction of motion is $+10g \sin 25° - T + T - 7g = -27.2\,N$ up-the-ramp. B) The f_s upper limit is $\mu_s N = .35(88.8) = 31.1\,N$. C) $\mu_s N$ is larger than $F_{non\text{-}f}$ so the block doesn't move; f_s prevents movement by supplying an equal-and-opposite force of $27.2\,N$ down the ramp, just enough to cancel $F_{non\text{-}f}$.

The same surface-to-surface contact produces both N and f. N is ⊥ to the plane-of-contact, while f is parallel to the contact-plane, so N and f are always ⊥ to each other. N is a common "link" because it appears in two equations: in the ⊥-to-motion F=ma as N, and in the direction-of-motion F=ma as $\mu_k N$ (if the object is sliding) or as $\mu_s N$ (if the object is not sliding but is at the "breakaway limit").

CHOICES: Wait until your class studies spring-force, then read this section.

3.8 Spring Force

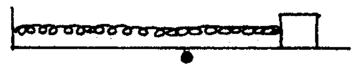
Basic Concepts & Equations

When a coiled-wire spring is at its *equilibrium length* "x_e" it is relaxed and doesn't exert any force. If a spring is compressed or stretched a small amount so its length is "x", $F_{spring} = -k(x - x_e)$, where "k" is the spring's *force constant*. { If x_e is defined to be $x \equiv 0$, F_{spring} is just "$-kx$", because $-k(x - x_e) = -k(x - 0) = -kx$. }

DIRECTION: like a homing pigeon, a spring always tries to get back to its home-position equilibrium length at x_e. If a spring is compressed so it is shorter than x_e, it pushes back toward x_e. If it is stretched, a spring pulls back toward x_e.

PROBLEM 3-J: Find each F_{spring}. Equilibrium length is 40 cm, and k = 800 N/m.

0 .15 .40



0 .40 .55

SOLUTION 3-J:

For the compression,

$$\begin{aligned} F_{spr} &= -k(x - x_e) \\ &= -50(.15 - .40) \\ &= -50(\ -.25\ ) \\ &= +12.5 \text{ Newtons} \end{aligned}$$

F_{spring} is +, ⇒, back toward x_e, like a good homing pigeon.

For the expansion,

$$\begin{aligned} F_{spr} &= -k(x - x_e) \\ &= -50(.55 - .40) \\ &= -50(\ +.15\ ) \\ &= -7.5 \text{ Newtons} \end{aligned}$$

F_{spring} is −, ⇐, back toward x_e, like a good homing pigeon.

3.90 Flash-Card Review for Chapter 3

3.1	Does force cause motion?	Not necessarily; F causes Δv (not v).
3.1	SI units: 5 kg must be interpreted as __ . A 10_ ball has m = 10, a 10_ ball has m = 10/g.	5 kg (not 5 k g) kg (kilogram), N (Newton)

For now, I'm not writing any flash-cards for Sections 3.2 & 3.3.
Study these important sections carefully (re-reading will help) and do your own reviews.

3.4	Expert problem-solvers translate __ , not __ . To find useful equation, combine __ and __ . If you are __ , you can relax and focus on __ .	words → pictures → eqns, words → eqns equation's "physical meaning, clear picture confident about physics-math, physics
	__ (found by __) and __ are a great team.	principles (searching for insight), practice
	Use drawings for __ , not __ .	a thinking tool (words→pictures), "art"
3.5	The F's of a third-law pair are caused by __ . These F's are __ and __ .	mutual interacion, equal (magnitude) & opposite (direction), equal (kind of F) & opposite (mutual symm.)
	Members of a third-law pair never _ because they __ .	"cancel" each other never act on the same object

3.6	On a plane, mg can be split into __ .	mg sinθ "down plane", mg cosθ "into plane"
3.6	F-letters (like T, N, mg, ...) represent __ . In "a" and "–a", <u>a</u> represents __ and __ . If "–T +mg = ma", T = +50 means __ , and a = +3 means __ .	only F-magnitude magn & directn (for a), only magn (for –a) T-magn is 50 [but force (–T) is in – directn] a-magnitude is 3 <u>and</u> a-direction is +
3.6	In a=+5 and a =–5, the + or – shows __ . In F=+5*N* and F =–5*N*, + or – shows __ .	that **a** points in the + or – direction +5*N* is F magnitude, –5*N* is "oops" warning
3.7	The magnitude of f_k and f_s are __ and __ . If __ , $f_k = \mu_k$ N. If __ , $f_s = \mu_s$ N.	μ_k N, range of 0 to μ_s N (matches F_{non-f}) [always] , object is at "breakaway limit"
	The direction of f_k is __ . The direction of f_s is __ .	opposite to sliding (not "motion") opposite F_{non-f} (and thus to would-be sliding)
	Can friction point in the direction of motion?	Yes; friction often causes **a**. [runner, car,...]
3.7	To answer "Will the object start sliding", __ .	find F_{non-f}, find μ_s N, choose the winner
3.7	Six F=ma LINKS are __ .	3rd law (2 objects), massless rope (2 objects), tvvax & F=ma, matched motion (Δx, v, a), XY split, N-link (N in x, μN in y)
3.8	Make a blank force-table (as in the Chapter 3 Summary), then fill in the blanks.	

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from previous editions, 3.3

~~3.2~~	F=ma is always written for a __ and a __ . For a F-diagram, the object says "I am __".	specific object, specific direction <u>being</u> pushed (or <u>being</u> pulled)
~~3.3~~	x & y motion is __ , so F=ma can be __ .	independent, split ($F_x = m\,a_x$, $F_y = m\,a_y$)
~~3.4~~	N and mg are equal if __ . a = 9.8 m/s² downward if __ . a = 0 if __ .	only N & mg and a = 0 and horizontal mg is the only force acting on object constant v, constant v=0, $a_\perp = 0$ if sliding

Chapter 3 Summary

An F-diagram (and its corresponding F=ma) always refer to <u>one specific object</u>; draw a "free body" F-diagram, or use separating-lines (as in Problem 3-B), or colors.

You can define a combination of matched-motion objects as a "system-object" (or just add the F=ma's of individual objects) to make *internal forces* cancel. This canceling can be good (if F_{int} is unknown) or bad (if F_{int} is known).

Make a force-diagram: Draw picture, choose object, imagine you're the object, say "I am <u>being</u> pushed & pulled by __ and __ and ....", then draw and label these forces.

$w/g = m$

If "matched motion", all objects have same v, a, Δx.
If v is constant (whether v is 0 or $\neq 0$), a = 0.
If sliding (or) $a_\perp = 0$.
Solve tvvax for a, use it.

$$F_{total} = m\ a$$

X & Y motion is independent, so choose axes, split F's (and a) into x & y components. For each F-component, decide whether direction should be represented by a + or − sign.

F-letters represent only magnitude: −mg = −m(9.80), not −m(−9.80).

$$F_x = m a_x \qquad \text{and} \qquad F_y = m a_y$$

Force NAME	cause	FORCE MAGNITUDE	FORCE DIRECTION
GRAVITY: weight, w	pull of earth	$mg \approx m(9.80)$ {GMm/r^2; see Chptr 5G}	PULL, "down" toward center of earth
TENSION, T or F_T	string, rope,...	There is no magnitude formula for T or N. (find T or N magnitude by solving F=ma)	PULL, in direction the rope points.
NORMAL, N or F_N or....	surface contact	There is no magnitude formula for T or N. (find T or N magnitude by solving F=ma)	PUSH, ⊥ to surfaces' plane-of-contact
FRICTION f_k, f_s	surface contact	$f_k = \mu_k N$, $f_s \le \mu_s N$ (f_s can vary from 0 to $\mu_s N$) f_s is EQUAL-AND-OPPOSITE to $F_{non\text{-}friction}$	f_k opposes sliding motion; f_s opposes "would-be" motion
SPRING	coiled spring	MAGNITUDE + DIRECTION is $-k(x_f - x_e)$, or (if $x_e = 0$), $-kx$	like "homing pigeon", PUSH or PULL toward x_e
OTHER FORCES include air resistance, fluid pressure and buoyancy (in Ch.6), electrostatic (Ch.11), magnetic (Ch.13), and nuclear (Ch.15).			

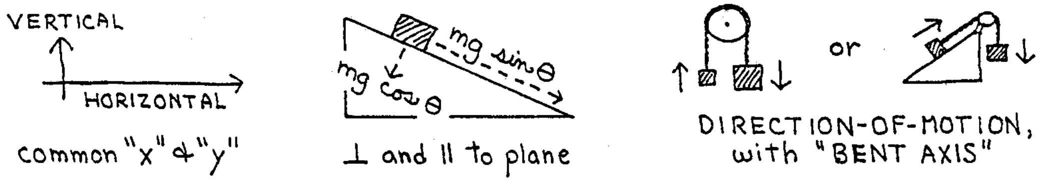
A third law force-pair involves <u>two</u> EQUAL-AND-OPPOSITE relationships:

1) Third law forces are EQUAL IN SIZE and OPPOSITE IN DIRECTION, and also
2) EQUAL IN "KIND OF FORCE" and OPPOSITE IN "MUTUAL SYMMETRY" {for example, "If ground pushes block, then block also pushes ground."}.

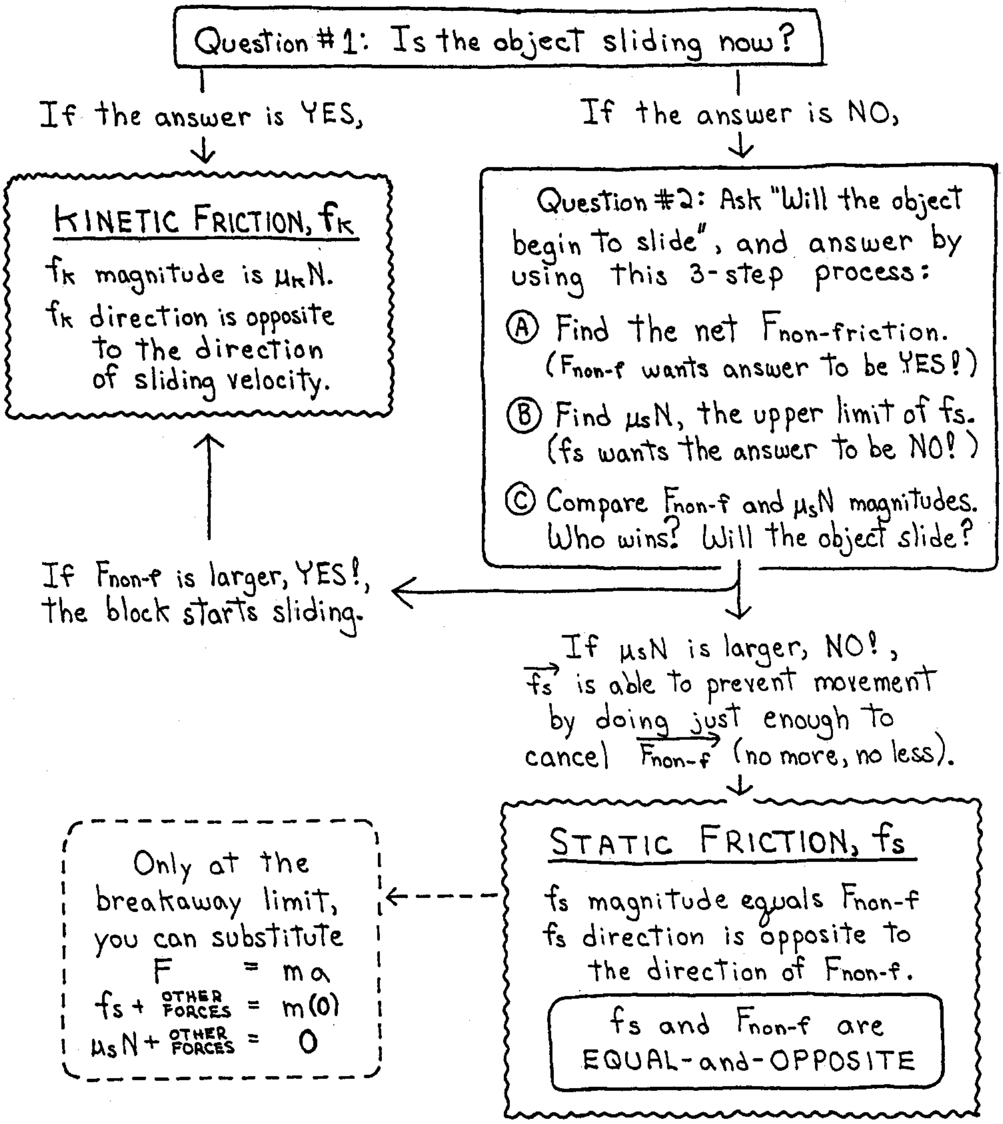
Partners in a third law force-pair don't act on the same object, so they never appear together on the F-diagram for an object. {For example, equal-and-opposite N & mg forces are not a third law pair.}

LINKS:
- third law (mutual-interaction F-partners appear in F=ma for two objects)
- massless rope (pulls object at both ends with equal T, so T is in 2 F=ma's)
- a-link ("a" appears in tvv<u>a</u>x and F=m<u>a</u>; link works in both directions)
- matched motion (if two objects have the same "a")
- split-link (if an F is split into F_x & F_y, that F is in $F_x=ma_x$ & $F_y=ma_y$), N-link (if one F=ma has N, and another F=ma has "friction = μN")

If possible, choose object & axes to get a 1-unknown equation. Some axes-options are:



This flowchart organizes the fundamentals of friction into a useful strategy:



coefficients of friction are μ_k & μ_s, but friction *forces* are f_k & f_s (or μ_kN & μ_sN). μ_k & μ_s are approximately independent of surface-contact area and sliding speed.

{ There are many interesting friction problems in Section 3.91. }