Chapter 2

Motion (Kinematics)

First, read Sections 2.1 to 2.3. At the end of 2.3, there is a "CHOICES" explanation of how to use the rest of Chapter 2.

2.1 Power Tools for Problem Solving

What do you think about these ways to cut a tree?

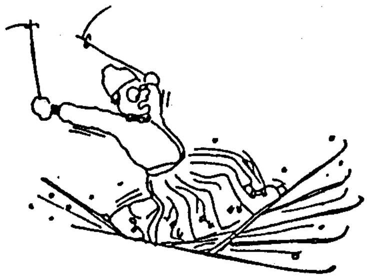
One student is working hard and sweating a lot, but isn't getting much done.
The other student is working smart, using the proper tool, and is getting results!

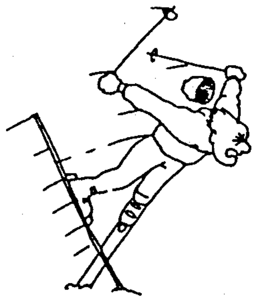
Which kind of student will you be? Without good tools, you are doomed to failure and frustration. With power tools, physics problem solving will bring you success and satisfaction. Your confidence will increase as problem solving becomes a fun game, one you have a chance of "winning", and you will actually look forward to the challenges presented in homework and on exams.

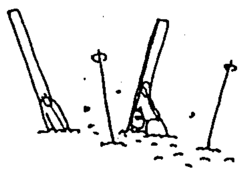
Where do you find these tools? You know many of them already, and you'll learn more from your teacher, from this book and your main textbook, and through your own discoveries. The following true story shows how a *master tool,* the "discovery of insight", leads to effective learning.

Learning from Mistakes (how I didn't learn to ski)

My first day of skiing! I'm excited, but the rental skis worry me. They look much too long, maybe uncontrollable? On the slope, fears come true quickly and I've lost control – totally – roaring down the slope yelling "Get out of my way! I can't stop!". But soon I do stop — flying through the air sideways, a floundering spin, a mighty bellyflop into icy snow. My boot-bindings grip like claws that won't release their captive, and the impact twists my body into a giant, hurting pretzel. Many race-and-crash cycles later I'm dazed, in a motionless heap at the foot of the mountain, wondering what I'm doing, why, and if I dare to try again.







Even the ropetow brings disaster. I fall down and wallow in the snow, pinned in place by my own huge skis; the embarrassing dogpile begins, as skiers coming up the ropetow are, like dominoes in a line, toppled by my sprawling carcass. Gosh, it sure is fun to ski.

With time, some things improve. After the first humorous (for onlookers) and terrifying (for me) trip down the mountain, my bindings are adjusted so I can bellyflop safely. And I develop a strategy of "leap and hit the ground rolling" to minimize ropetow humiliation. But my skiing doesn't get much better so — wet and cold, tired and discouraged — I retreat to the lodge.

The break is wonderful, just what I need for relaxation and recovery.
An hour later, after a nutritious lunch topped off with tasty hot chocolate, I'm sitting near the fireplace in warm dry clothes, feeling happy and adventurous again. A friend tells me about another slope, one that can be reached by chairlift, and I decide to "go for it".

This time the ride up the mountain is exhilirating. Instead of causing a ropetow domino dogpile, the lift carries me high above the earth, like a great soaring bird. Soon, racing down the hill, I dare to experiment — and discover an **insight**! If I press my ski edges against the snow a certain way, they "dig in". This, combined with "unweighting" (a jump-a-little and swing-the-skis-around foot movement) produces a crude parallel turn that lets me zig-zag down the slope under control, and suddenly I can ski!

Continued practice now brings rapidly improving skill, and by day's end I'm feeling great. I still fall down some, but I'm learning from the good and bad things that happen. And I have the confident hope that even better downhill runs await me in the future. Skiing has become fun!

This experience illustrates two important learning principles:

1) INSIGHT and QUALITY PRACTICE: I learned how to ski by doing it correctly, **not** by making mistakes. There was no real improvement until I discovered the edge-unweight turning tool; this "insight" made my practicing effective so I could quickly develop better technique.

2) **PERSEVERANCE**: My morning ski runs weren't fun, and I didn't seem to learn from them, but I kept trying anyway despite the risk of injury to body and pride. Eventually this perseverance paid off — because I refused to quit in response to frustrating morning failures, I experienced the great joys of afternoon success.

Perseverance and insight are also the keys to physics. In Section 2.5, at the start of the *Aesop's Fable Physics Problems* that will help you meet the "ski slope challenge" of physics, there is a simple story about a friend and his search for insight. I hope it will change the way you think about "learning from experience" and will help you turn physics [and everything else you do] into an exciting adventure.

The next 3 sections explain the fundamentals of motion, including "how to choose an equation", so you can build your problem-solving strategies on a solid foundation.

2.2 The Basics of Motion: Velocity & Acceleration

This book assumes that you have already learned the "basics of physics" (concepts and equations) from your textbook or lecture. Sometimes I'll review-and-summarize these basic ideas in a short section like the following.

Basic Concepts & Equations {your textbook will provide details}

The definitions of *velocity* "v" and *acceleration* "a" are:

$$\frac{\text{(change of position)}}{\text{(change of time)}} \equiv \mathbf{v} \qquad\qquad \frac{\text{(change of velocity)}}{\text{(change of time)}} \equiv \mathbf{a}$$

"Δ" means "change of", is calculated by taking "final value – initial value", and is pronounced "delta". For example, if a car-velocity changes from –15 m/s to +20 m/s, change of v ≡ Δv (delta v) = $v_f - v_i$ = (+20 m/s) – (–15 m/s) = +35 m/s. Similarly, change of position is $\Delta x = x_f - x_i$, and change of time is $\Delta t = t_f - t_i$.

The definitions of velocity & acceleration can be written with Δ's, and rearranged:

$$\frac{\Delta x}{\Delta t} = v \qquad\qquad \frac{\Delta v}{\Delta t} = a$$

$$\Delta x = v\,\Delta t \qquad\qquad \Delta v = a\,\Delta t$$

AVERAGE VELOCITY: If constant acceleration makes a car's velocity change from v_i = +20 m/s to v_f = +30 m/s, the *average velocity* is (just as you would expect) +25 m/s, halfway between v_i and v_f: $v_{average} = \frac{1}{2}(v_i + v_f)$.

The defining-equations for v & a can have v replaced by $\frac{1}{2}(v_i + v_f)$, Δx by $x_f - x_i$, and Δv by $v_f - v_i$. {And to make equations more compact, textbooks usually replace Δt with t.}

$$\Delta x = \tfrac{1}{2}\ v\ \Delta t \qquad\qquad \Delta v = a\ \Delta t$$

$$(x_f - x_i) = \tfrac{1}{2}(v_i + v_f)\ t \qquad\qquad v_f - v_i = a\ t$$

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If the same variable appears in two equations, forming a link between them, you can solve for that variable in one equation and use it (substitute it) in the other. Can you find the 3 variables that link "$x_f - x_i = \frac{1}{2}(v_i + v_f)t$" and "$v_f - v_i = a\,t$"? These *solve-and-use links* are the key to the following process for deriving new equations.

Step 1: **Solve** "$v_f - v_i = a t$" for v_f, v_i and t; $v_f = v_i + at$, $v_f - at = v_i$, $(v_f - v_i)/a = t$.
Step 2: **Substitute** v_f, v_i or t, one at a time (as shown by the ⇓'s) into $\Delta x = \frac{1}{2}(v_i + v_f)t$.
Step 3: **Rearrange** and simplify, to get three new equations. In each derivation, notice that the "linking variable" does not appear in the new equation.

substituting for v_f	substituting for v_i	substituting for t
$\Delta x = \frac{1}{2}(v_i + v_f)\ t$	$\Delta x = \frac{1}{2}(v_i + v_f)t$	$\Delta x = \frac{1}{2}(v_i + v_f)\ t$
⇓	⇓	⇓
$\Delta x = \frac{1}{2}(v_i + [v_i + a\,t])\,t$	$\Delta x = \frac{1}{2}([v_f - a\,t] + v_f)\,t$	$\Delta x = \frac{1}{2}(v_i + v_f)\,[(v_f - v_i)/a]$
{after rearranging}	{after rearranging}	{after rearranging}
$\Delta x = v_i t + \frac{1}{2} a t^2$	$\Delta x = v_f t - \frac{1}{2} a t^2$	$v_f^2 - v_i^2 = 2 a \Delta x$

There are now 5 equations (the 2 originals, plus these 3 new equations) that describe the motion of an object whose acceleration is constant:

$$\mathbf{v_f - v_i = a\ t} \qquad (\Delta x \text{ is missing})$$

$$\mathbf{(x_f - x_i) = \tfrac{1}{2}(v_i + v_f)\,t} \qquad (a \text{ is missing})$$

$$\mathbf{(x_f - x_i) = v_i\,t + \tfrac{1}{2} a\,t^2} \qquad (v_f \text{ is missing})$$

$$\mathbf{(x_f - x_i) = v_f\,t - \tfrac{1}{2} a\,t^2} \qquad (v_i \text{ is missing})$$

$$\mathbf{v_f^2 - v_i^2 = 2\,a\,(x_f - x_i)} \qquad (t \text{ is missing})$$

These 5 equations have 5 variables: Δt, v_i, v_f, a, and "$x_f - x_i$" (which can be written as "Δx" and considered to be one variable). Notice that each 4-variable equation is missing one of the variables. As you'll soon discover, this "missing variable" is the key to a simple yet powerful equation-choosing strategy.

{ Your textbook will have equations that are equivalent to these, even though they may look slightly different. For example, many books rearrange the second equation to "$x_f = x_i + \frac{1}{2}(v_i + v_f)t$", and omit the fourth equation. }

2.3 How to Choose a Useful Equation

The following problem shows how easy it is to choose a motion-equation:

Two seconds after Becky's race car crosses the finish line, her velocity is +80 m/s. She hits the brakes and slows down at a constant rate of $-4.0\ m/s^2$. How far does her car travel while the brakes are applied?

Step 1: Read carefully, think, draw pictures. Do whatever is needed to **form a clear idea of the problem situation**, so you can **choose initial & final points for a useful constant-acceleration interval**.

The problem asks for distance "while the brakes are applied" so we ignore the first 2 seconds, and choose the instant when Becky's foot hits the brake as "initial" (with t_i, v_i, a_i, x_i) and the instant her car comes to rest as "final" (with t_f, v_f, a_f, x_f).

Step 2: Make a tvvax table (with the 5 variables: t v_i v_f a Δx) **and look for a 3-of-5 subgoal**. As discussed in Step 3, <u>if you know any 3 of the 5 variables</u>, <u>you can find the other 2</u>. This table shows what you know and, by circling it, what you want to find.

Δt =
v_i = +80 m/s
v_f = 0
a = $-4\ m/s^2$
(Δx) =

By asking 5 specific questions (What is Δt? What is v_i? ...) an unfilled tvvax table will push you into action. If you don't know 3-of-5, you must do something! Re-read the problem and dig deeper; have you missed information that is stated or implied? Do you have a clear idea of what is known (and unknown) at the initial & final points? Would it help if you redefined either i or f? Keep asking and answering questions, exploring and making progress until you've solved the problem.

Step 3: Choose an equation using "1-out Strategy", then Substitute & Solve. If you want to solve an equation for Δx (the goal-variable), that equation must contain Δx. And to make best use of the 3 knowns, the equation should also have them. Of the 5 tvvax variables, only 1 is unwanted: Δt is the "1-out". It's easy to look at the 5 tvvax equations and find the one that doesn't have Δt in it. (If you search for the equation that has all of the 4 "wanted" variables, it is more difficult; try it and see!) Then substitute and solve:

$$
\begin{aligned}
v_f^2 - v_i^2 &= 2\ a\ \Delta x \\
(0)^2 - (+80)^2 &= 2\,(-4)\ \Delta x \\
-6400 &= -8\ \Delta x \\
+800 \text{ meters} &= \Delta x
\end{aligned}
$$

Step 4: Answer the question that was asked. (Yes, "800 meters" is the answer.)

With a little practice, you'll find that Steps 2 & 3 of this "tvvax system" make it easy to organize information and choose an equation. When you're confident about the math, you can focus your attention on the creative, productive "Step 1": translating the words of a problem-statement into an accurate mental picture of what is happening, and choosing the initial & final points for a useful interval.

This illustrates a very important principle: <u>the key to solving physics problems is understanding physics!</u> <u>The math is usually easy after you form a clear idea of what is happening</u>.

To develop a true understanding of motion (or any other part of physics) you must learn to <u>unite intuitive and mathematical thinking</u>. To do this, think about your own experience with moving objects, learn all you can from your textbook, pay attention to classroom lectures & demonstrations, be curious, and ask questions.

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Here is an easy way to handle UNITS correctly: be careful, relaxed, and careful.

<u>Be careful at the start</u>. In the tvvax table and during substitution, use a consistent system of units. Most books, including this one, use SI (System Internationale) units: Δx in **m**, Δt in **s**, v in **m/s**, a in **m/s²**. (For example, if Becky's speed was given as 288 km/hr, to be SI-consistent you would have to convert into m/s by using standard conversion factors or [this is even easier] Section 1.4's "substitution method": $(288 \times 10^3 \text{ m})/(3600 \text{ s})$.

<u>Be relaxed in the middle</u>. While solving the algebra, you can safely ignore units. If you've substituted properly, every equation term has the same units (for example, Δx, $v_i t$ and $\frac{1}{2} at^2$ all have units of "m") and everything will work out smoothly.

<u>Be careful at the end</u>. A correct answer includes units; the stopping distance is "800 meters", not just "800". If you have substituted SI units properly, the Δt, v, a or Δx you solve for will have units of s, m/s, m/s^2 or m, respectively.

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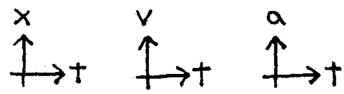
CHOICES: The 4-step "tvvax strategy" will help you solve most motion problems. The next five sections examine objects flying through the air (in 2.4), and situations where information must be "split" into two or more tvvax tables for different time intervals (2.5), objects (2.6), or directions (2.7-2.8). Ratio logic [a very useful skill] is explored in 2.9, and 2.10 looks at the "relative motion" of trains, boats and planes.

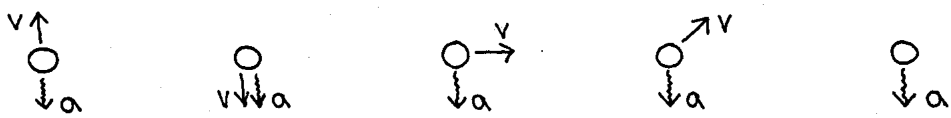
Sections 2.4-2.8 should be read in order, but you can use 2.9 and 2.10 at any time, whenever your class studies these subjects.

Four characteristics of motion graphs (like these) are explored in Section 19.1.
OPTIONAL: Sections 19.2 and 19.3 use "calculus" to analyze motion.



2.4 Free Flight: the motion of falling objects. Acceleration is not the same as Velocity!

If an object moves freely through the air near the surface of the earth, with no force acting on it except air resistance (which we'll ignore, considering it negligible) and gravity, the object is in *free flight*. The object's acceleration is 9.8 m/s per second downward, no matter what direction it is moving. Each of these free-flight objects has the same acceleration, even though each has a different velocity:

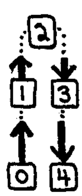


upward v	downward v	sideways v	diagonal v	v = 0, at peak
a = ↓ 9.8	a = ↓ 9.8	a = ↓ 9.8	a = ↓ 9.8	a = ↓ 9.8

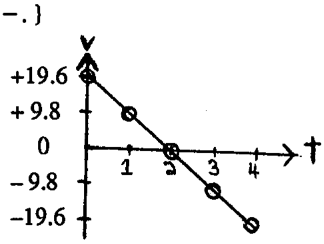
Notice that <u>there is no connection between the magnitudes or directions of v and a</u>. As shown above, a-direction does not depend on v-direction. And a-magnitude is the same whether a ball is moving (as in the first 4 pictures) or motionless at the peak with v = 0 (in the last picture). { Or consider a car cruising at a constant 120 miles/hr; it has large v, but a = 0. At the start of a dragrace when the driver hits the gas, the car has large a but v = 0.}

But there is a close relationship between acceleration and <u>change</u> of velocity. For example, if a ball moves upward with speed +19.6 m/s at t = 0, its velocity at t = 1s, 2s, 3s and 4s are shown below: in a simple picture, with words, and on a "v vs. t" graph.

{ I have arbitrarily defined "up" to be the + direction, so upward velocity is +, and velocity in the opposite "down" direction is −. }



At t = 0, v = +19.6 (fast upward)
At t = 1s, v = + 9.8 (slow upward)
At t = 2s, v = 0 (only for an instant)
At t = 3s, v = − 9.8 (slow downward)
At t = 4s, v = −19.6 (fast downward)



Study the middle column and notice that during each 1-second interval the ball's v changes by −9.8 m/s, whether it changes from +19.6 to +9.8, +9.8 to 0, 0 to −9.8, or −9.8 to −19.6. Do you see what is meant by an acceleration of "9.8 m/s per second"?

UNITS: −9.8 m/s per second = (−9.8 m/s)/s. When m/s is divided by s, this becomes −9.8 m/s^2. Although the SI unit of acceleration is usually expressed as m/s^2, you should <u>think</u> of acceleration as "<u>m/s per second</u>".

The graph above shows the steady decrease of number-line velocity from +19.8 m/s to −19.8 m/s, as v changes by −9.8 m/s each second.

Important: decrease of number-line velocity doesn't necessarily mean decrease of speed. For example, during its downward flight the ball's *velocity* (magnitude-and-direction) decreases from 0 to –19.6 m/s, while its *speed* (magnitude) is increasing from 0 to 19.6 m/s.

While the ball is moving ↑, **v** and **a** point in opposite directions (**v** is ↑, **a** is ↓) and speed is decreasing. When it moves ↓, **v** and **a** point the same direction (both are ↓) and speed is increasing. These relationships can be stated as rules that are always true for 1-dimensional "straight-line" motion:

When speed is increasing (getting faster), **v** and **a** point in the same direction.
When speed is decreasing (getting slower), **v** and **a** point in opposite directions.

Because Δ**v** and **a** are both vectors (as indicated by bold-face type) with magnitude and direction, "Δ**v** = **a** Δt" is a *vector equation.* Multiplying **a** by Δt (which is a non-vector) doesn't produce a change of direction, so **a** points in the same direction as Δ**v**. In the free-flight example above, **a** and Δ**v** both point downward, in the direction I've chosen to represent with a "–" sign.

The magnitudes of **a** and Δ**v** differ by a factor of Δt. For example, in the interval from 0 s to 3 s, v changes from +19.6 m/s to –9.8 m/s, with $\Delta v = v_f - v_i$ = (–9.8 m/s) – (+19.6 m/s) = –29.4 m/s. Expressed in a formula, **(–29.4 m/s) = (–9.8 m/s²)** (3 s). The magnitude [and units] of **a** and Δ**v** differ by a factor of "Δt = 3 s".

This section contains many important concepts. Some of these ideas may be new and it will take awhile for you to "internalize" them. Because of this, I think you'll find it useful to review this section several times (right now, and also later) until these ideas about acceleration and velocity-change become natural and intuitive.

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Throw a ball straight up and watch the "peak" where the ball stops for an instant between going-up and coming-down. At this *peak point,* vertical velocity is zero. It is often useful to choose the peak as initial or final point for an interval, because this gives you an extra piece of information and brings you closer to the 3-of-5 goal.

When you read a problem, watch for ***zero-velocity words*** (peak, stop, from rest, is dropped, maximum height,...) that tell you "v = 0" at some special point.

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If you drop a heavy bowling ball and light baseball at the same time from the same height, which one hits the ground first? To find out, just do a "reality check"; drop the balls and watch what happens. The result may surprise you, because both balls hit the ground at almost exactly the same time!

2.6 Insight, Progress, and Aesop's Problems. Problems with more than one <u>Time Interval</u>.

Learn from Experience by Searching for Insight

A friend who quickly became an expert welder got this wise advice from his first teacher: every time you do a welding job, do it better than the time before. This is also a good way to learn thinking skills: every time you finish a problem, ask yourself "<u>What can I learn from this problem that will help me do better in the future?</u>".

Don't define your amount-of-studying by the time you spend or the number of problems you do, but by **how much you learn**. If you do a problem in 10 minutes and use 2 minutes to analyze what you've done, you'll probably learn more (that will help you do future problems) in the 2 minutes of analysis than in the first 10 minutes. Does that sound like a good deal?

You can make Rapid Progress

There are two main strategies for learning quickly: **work hard** and **work smart**.

Usually, the more you study, the more you'll learn. Working hard pays off. And so does working smart; using "power tools" will make your study time more efficient and more fun.

Much of your skill improvement will come one step at a time. Each step you take prepares you for the next step as you make slow, steady progress.

But you can also travel in leaps. This is possible because many physics skills are interdependent, which is bad news (if you're missing an important tool, everything you do suffers from this weakness) and good news (key insights can, as described in the skiing story, let you make rapid progress).

If you are persistent in searching for insights, your steps and leaps will produce a wonderful transformation. You will find, increasingly often, that goals which earlier seemed impossible are becoming things you can now do with ease.

Aesop's Problems

In Aesop's Fables, each story is designed to teach a useful moral lesson.

Similarly, problems in this book are carefully designed to teach specific problem-solving strategies. Aesop's Problems are an efficient, fun way to learn, because they quickly give you experience with the most common problems (and the strategies that let you solve them), using examples that are easy to understand.

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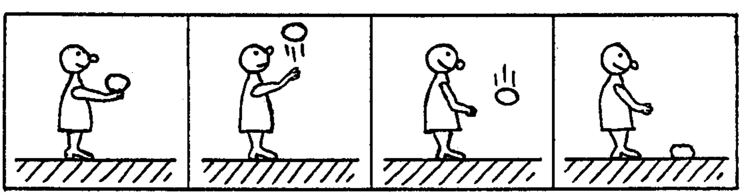
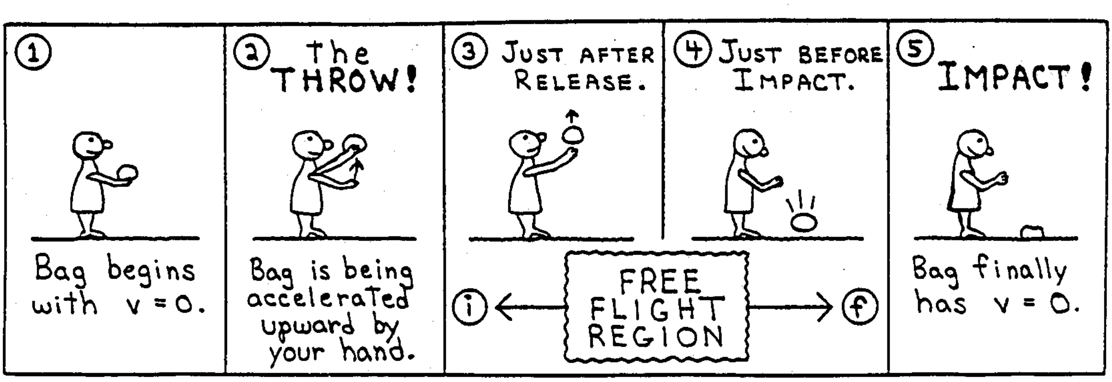
PROBLEM 2-A: For the up-and-down throw shown below, what are good choices of initial & final points for a free flight interval? (Hint: Remember that tvvax equations can be used only for a constant-aceleration interval.)



PROBLEM 2-B: A Rocket Ship

Starting from rest, a rocket accelerates upward at 16.0 m/s^2 for 5.00 seconds, then shuts off its engines. How long does the entire trip take, from liftoff to impact?

SOLUTION 2-A

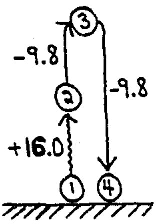


It may seem like pictures 1 & 5 should be i & f, since they're the start & finish of the entire action sequence. But acceleration isn't constant between 1 and 5, so 1-to-5 cannot be a tvvax interval. The "free flight" 3-to-4 interval, from just-after-release to just-before-impact, cannot be mixed with the *throw* (an interval when the bag is accelerated upward by your hand) or *impact* (when the bag crashes into the ground and is quickly brought to rest). Choose 3 & 4 as initial & final free-flight points.

SOLUTION 2-B

As shown below, the trip can be split into intervals separated by ***special points:***

1) liftoff, 2) engine shutoff, 3) peak, 4) just-before-impact.



The 5 tvvax equations should only be used when a is constant, so the 1-to-2 interval (with a = +16.0) cannot be combined with 2-to-4 (free flight, a = –9.8). The total 1-to-4 time cannot be calculated in one step. Instead, make a separate tvvax table for each constant-acceleration time interval.

From 1 to 2			From 2 to 4		
Δt	=	5.00 s	Δt	=	
v_1	=	0 ("from rest")	v_2	=	
v_2	=		v_4	=	
a	=	16.0 m/s^2	a	=	–9.8 m/s^2
Δy	=		Δy	=	

You know 3-of-5 for the first table. Use 1-out strategy to choose useful 1-unknown equations: it is easy to solve "$v_f - v_i = a\,t$" for v_2 = +80.0 m/s, and "$\Delta y = v_i t + \frac{1}{2} a\, t^2$ " for Δy = +200 m. {If you want, do the algebra for yourself now.}

Because v_i & v_f have been replaced with the more informative v_1 & v_2, v_2 & v_4, it is easy to see that v_2 is the end of 1-to-2 and the start of 2-to-4, just like midnight on New Year's Eve is the end of one year and start of the next. Take advantage of a ***solve-and-use link:*** solve for v_2 = 80.0 m/s in the first table, then use it in the second table.

From 1 to 2				From 2 to 4		
Δt	=	5.00 s		Δt	=	
v_1	=	0 ("from rest")		v_2	=	[+80.0 m/s]
v_2	=	+80.0 m/s	········· a link ········	v_4	=	
a	=	16.0 m/s^2		a	=	–9.80 m/s^2
Δy	=	+200 m	········ another link ······	Δy	=	[–200 m]

Another Link: Use the fact that Point #2 is 200 m above the ground [which is Point #4] to conclude that the rocket falls 200 m (so Δy = –200 m) during the 2-to-4 interval.

IMPORTANT: Δy depends only on the position of the initial & final points, #2 & #4. It doesn't matter how high above #2 the rocket rises while reaching its peak.

Now you know 3-of-5 for the second part of the trip. To find Δt, you can either solve one quadratic equation, or take a "Quadratic Detour" with two regular equations.

the QUADRATIC FORMULA

{ If you understand the Q-formula, this algebra is easy. If not, read Sec. 18.7 }

$$\Delta y = v_i\, t + \tfrac{1}{2}\, a\, t^2$$

$$(-200) = (+80)t + .5\,(-9.8)\,t^2$$

$$+4.9\,t^2 - 80\,t - 200 = 0$$

$a = +4.9 \quad b = -80 \quad c = -200$

$$t = \frac{-(-80) \pm \sqrt{(-80)^2 - 4(+4.9)(-200)}}{2(+4.9)}$$

$t = +18.5\,s$ or $t = -4.4\,s$

a QUADRATIC DETOUR

Step 1: Solve for unwanted variable,

$$v_f^2 - v_i^2 = 2\, a\, \Delta y$$

$$v_f^2 - (+80)^2 = 2(-9.8)(-200)$$

$$v_f^2 = 10320$$

$$v_f = \pm 101.6$$

Does $v_f = +101.6$, or -101.6 ?

Step 2: 4 equations can be solved for Δt. The easiest is

$$v_f - v_i = a\, t$$

$$(-101.6) - (+80) = (-9.8)\,t$$

$$+18.5\,s = t$$

Do you see why, during the Q-Detour, you must choose $v_f = -101.6$? (Hint : At Point #4, is the rocket moving upward or downward? Does this make the velocity + or – ?)

Quadratic Bonanza: In Section 18.7, there is 1) a "hot tip" for quickly solving the Quadratic Formula, 2) four ways to avoid the Q-Formula, and 3) a discussion of what the two solutions from above {+18.5 and –4.4} really mean.

To answer the problem-question, use the logic that ***total = sum of parts.***
Time for entire trip = (time for 1-to-2) + (time for 2-to-4) = (5.0 s) + (18.5 s) = 23.5 s.

Two extra comments: A) If the problem had asked "What is the rocket's maximum height?", you would also need a table for the 2-to-3 interval.

B) It doesn't matter what direction you choose to be +. If you choose ↓ (instead of ↑) to be +, the equations will look different [for example, the Q-Detour equation is now "$v_f^2 - (-80)^2 = 2(+9.8)(+200)$", which gives $v_f = \pm 101.6$ and (because downward is +) v_f is +101.6 m/s], but you'll get the same result of $\Delta t_{total} = 23.5$ s.

2.7 Problems with more than one <u>Object</u>

PROBLEM 2-C: A Hare and Tortoise Drag Race

Cars H and T run an "almost quarter-mile" 400 m drag race. Car H accelerates at 5 m/s^2, Car T at only 2 m/s^2, but T gets a 7 second headstart. Who wins the race?

How far from the finish line (and on which side of it) is Hare when it passes Tortoise? Hint : Form a clear picture-idea of the instant when H passes T, then describe this situation in terms of tvvax variables.

SOLUTION 2-C

Who wins? If the entire race is chosen as i-to-f, you know 3-of-5 for each car [$v_i = 0$, $a = +2$ or $+5$, $\Delta x = +400$] so you can calculate the times required to reach the finish line: 20.00 s for T, and 19.65 s (= 7.00 + 12.65) for H. So who wins?

Now we'll use an "idea picture" to help answer the second question. Both cars start at the same point, which we'll define as $x_i \equiv 0$. When H passes T (and only at this point) they have the same x-value, which we'll represent by "$x_f \equiv P$".

Because of T's headstart the Hare will, once it begins moving, always have a driving time that is 7 seconds less than T's time. If T's time is represented by "J", then H's time is "J – 7". (Another option: If you let "J" to represent H's time, T's time is J + 7. This correctly shows that T drives for 7 seconds more than H.)

Make a separate tvvax table for each moving object:

for the TORTOISE		
Δt	=	J
v_i	=	0
v_f	=	
a	=	+2
$x_f - x_i$	=	P – 0

for the HARE		
Δt	=	J – 7
v_i	=	0
v_f	=	
a	=	+5
$x_f - x_i$	=	P – 0

Do we know 3-of-5? For each table, v_i and a are ***knowns*** because we know their numerical value, Δt and Δx are ***semi-knowns*** because we know something about them (that J & P in the T-table are the same as J & P in the H-table) but not their number value, and v_f is ***unknown*** because we know nothing about it.

Each table has only 2 knowns, so there is no 3-of-5. But we can take full advantage of the available information by choosing equations that don't contain v_f, which is the "total unknown". This gives a T-equation with J & P as unknowns, and an H-equation that also has J & P as unknowns.

Below, these 2 *simultaneous equations* are solved for the 2 unknowns, using a strategy of ***solve-and-use links.*** As you follow the numbered steps (1, 2, 3, 4 and 5), notice the pattern of back-and-forth "leapfrog substitution": solve one equation, substitute the results into the other equation and solve it, substitute and solve, ...
(Simultaneous equations are discussed more fully in Section 18.8.)

for the Tortoise

$\Delta x = v_i t + \frac{1}{2} a t^2$

$(P-0) = (0)J + .5(+2)(J)^2$ (1)

$P = 1.0\, J^2$

(4)

$P = 1.0(19.05)^2$ (5)

$P = 363$ meters

for the Hare

$\Delta x = v_i t + \frac{1}{2} a t^2$

$(P-0) = (0)t + \frac{1}{2}(+5)(J-7)^2$

(2)

(3)

$(1.0 J^2) = 2.5(J-7)^2$

Use Q-formula or $\sqrt{\ }$-trick.

$J = +4.29 s$, or $J = +19.05 s$

QUADRATIC OPTIONS: There are two ways to solve $1.0J^2 = 2.5(J-7)^2$. You can multiply $2.5(J-7)(J-7)$ and then solve the Q-Formula with $a = +1.5$, $b = -35$ and $c = +122.5$. Or use this easy $\sqrt{\ }$-Trick: take the $\sqrt{\ }$ of both sides and solve the two resulting equations, $1.0J = +1.581(J-7)$ and $1.0J = -1.581(J-7)$. If you do the algebra, you'll find that each option gives the same solutions: $J = +4.29$ (this is impossible; it is before H begins) and $J = +19.05$ (this is the correct time).

"The passing point is 363 meters from the starting line" is a true statement. But does it answer the question that was asked? The problem asked "How far from the finish line... ?", so the correct answer is "37 meters before the finish line".

A strategy summary for Two-Object Problems: **Use all available information** (no more, no less) to mathematically describe each variable as fully as possible, whether this is known, semi-known [using letters] or unknown. Use logic, imagination, and maybe a quick drawing to form a clear picture of the problem situation, then translate this idea into tvvax information like "J, J–7, and P–0". If you can't get the 3-of-5 knowns that let you solve a 1-unknown equation, try for 2 unknowns in 2 equations.

PROBLEM 2-D: Answers with Symbols. If a car starts from rest with constant acceleration "a" and completes a race in "M" seconds, how long is the race track?

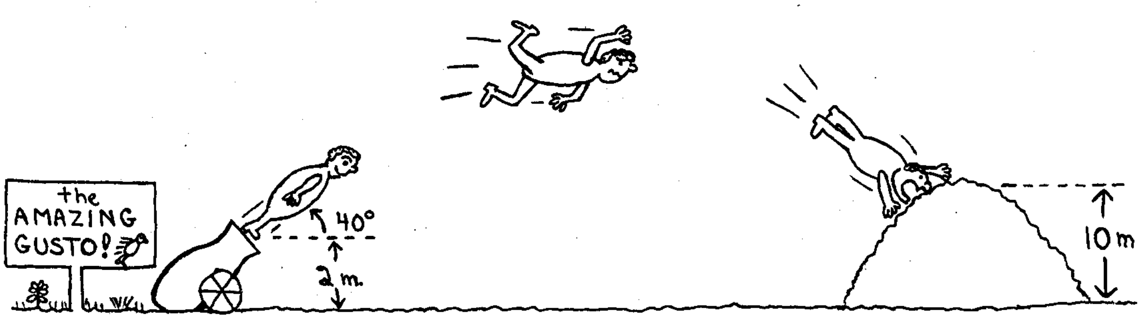
SOLUTION 2-D: This question expects you to treat a & M as if they were knowns, and express your answer in terms of a & M. You know 3-of-5 (Δt = M, $v_i = 0$, a = a) so "$\Delta x = v_i t + \frac{1}{2} a t^2$" can be solved for $\Delta x = \frac{1}{2} a M^2$ = length of track.

CHOICES: Wait until your class studies 2-Dimensional motion, then use this section.

2.8 Problems with more than one Direction: 2-Dimensional Motion

PROBLEM 2-E: A Bullet Race. A man drops a bullet and at the same time, from the same height, shoots another one horizontally from a gun. Which bullet hits the ground first? (Consider the earth to be flat; ignore its "curvature".)

PROBLEM 2-F: A Human Cannonball
Mile-a-Minute Gus will be shot from a cannon. The circus hires you to make a huge pile of whipped cream for him to land in, for a sweet & spectacular end to his journey. a) How far away should you place the creampile? b) How long will the flight take? c) What peak height above the ground does Gus reach? d) What is his velocity at the peak? e) What is his velocity just before impact? (v_i = 1 mile/minute = 26.8 m/s. Ignore air resistance.)



SOLUTION 2-E: Both bullets have the same facts in the vertical direction [Δy = –2 m, a = –9.8 m/s^2, and (do you see why "horizontally" is a zero-word?) $v_i = 0$] so they have the same Δt. The bullets hit the ground at the same time! Why? Because the gun-bullet's horizontal movement doesn't affect its vertical fall; its → and ↓ motions occur independently.

For many students, this "XY independence" is difficult to believe. But it is true. Maybe it makes sense to you now. Maybe a logical argument or lecture-demonstration will convince you of its truth. In any case, please accept it and use it for motion analysis and problem-solving.

- - - - - - - - - - - - - -

When an object has zero acceleration, its motion is simplified; here is an example.

As discussed in Section 2.4, a "free flight" object has a vertical (y) acceleration of a_y = –9.80 m/s^2, and a horizontal (x) acceleration of $a_x = 0$. (I've chosen "up" to be +.)

When $a_x = 0$, two of the five tvvax equations become "$v_i = v_f$" (this is logical; if $a = 0$, v is constant) and the other three become "$\Delta x = v_x \Delta t$". For free-flight motion in the horizontal direction, the 5-variable/5-equation tvvax system simplifies to 3 variables in 1 equation; instead of trying for a 3-of-5 subgoal, look for 2-of-3.

"xy independence" lets you make separate tvvax tables for an object's x & y motion:

For x motion when $a_x = 0$, 2-of-3 subgoal.

$\Delta x = v_x \; \Delta t$

( ) = ( )( )

For y-motion in free flight, 3-of-5 subgoal.

Δt =
$(v_y)_i$ =
$(v_y)_f$ =
a = -9.8 m/s^2
Δy =

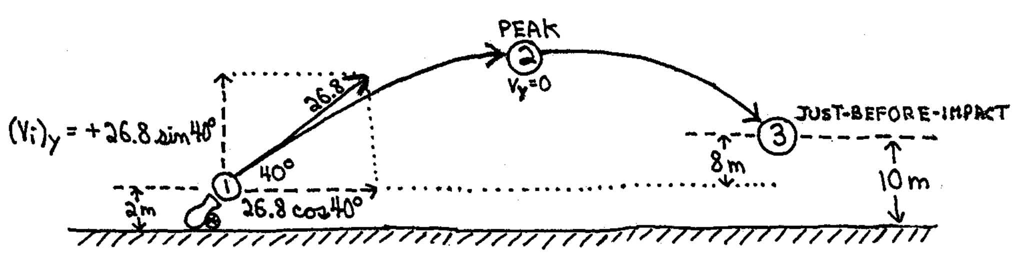
When you fill in the x-table, focus on the sideways ↔ direction; ignore ↕ things.
When you fill in the y-table, focus on the up/down ↕ direction; ignore ↔ things.

Don't mix x-facts and y-facts together! **Put each piece of information in its proper place; always make a decision** by asking "Is this fact relevant for the ↔ direction (if it is, put it in the x table) or the ↕ direction (then it belongs in the y table)?"

SOLUTION 2-F

Choose "special points" for intervals. We need special points 1, 2 & 3 as shown below, because the problem asks about the total 1-to-3 flight (distance & time), the peak point (height & velocity) and just-before-impact point (velocity). Your problem-solving skill will improve if you practice forming a clear mental picture of **points** [like 1( ), 2( ), 3( )] and **intervals** [like 1-to-2( ), 2-to-3( ), and 1-to-3( )]. A clear picture-idea will also help you follow my explanations.

Study this picture closely. It is a good visual summary of the problem information. Notice the "splitting" of v_1 into x & y components, why the peak-point has $v_y = 0$, and how comparison of the i & f heights gives a 1-to-3 Δy of +8 m.



Useful tvvax tables are given below, for two directions (x & y) and two time intervals (1-to-3 & 1-to-2). If you ask "How can I know which intervals to choose?", some answers are A) Use the problem-statement as a guide; if a question is asked [or if information is given] about a point or interval, you'll probably want a table for it. B) With experience, you'll develop a feeling for what is needed. C) Use trial-and-error; do something to get started and then improvise.

x, from 1 to 3

Δx = v Δt
Δx = $(+26.8 \cos 40°)($ $)$ ······ TIME LINK ······ (to Δt of y table)
↓
Δx = $(+20.5)(2.96)$

y, from 1 to 3

Δt =
V_1 = $(+26.8 \sin 40°)$ m/s
V_3 =
a = -9.8 m/s^2
Δy = $+8$ m

y, from 1 to 2

Δt =
V_1 = $(+26.8 \sin 40°)$ m/s
V_2 = 0
a = -9.8 m/s^2
Δy =

The problem asks for the distance from cannon to creampile; this is Δx for the 1-to-3 interval. To find it you need Δt for the 1-to-3 x-motion (do you see why?), which is the same as Δt for the 1-to-3 y-motion. As shown above, Δt is a "link" between the x & y motion; if you solve for Δt in either table, you can use it in the other. (But be careful! The Δt intervals must be "matched". For example, a 1-to-2 Δt cannot equal a 1-to-3 Δt.)

You must find v_3 eventually [to answer "What is his velocity just before impact?"] so you might as well solve for v_3 first and then find the 1-to-3 Δt. This order of solving lets you avoid the Quadratic Formula. After finding that $v_3 = -11.8$ m/s, you can find $\Delta t_{1\text{-to-}3} = 2.96$ s. Then substitute this Δt into the x-equation, as shown by ⇓, and solve for $\Delta x_{1\text{-to-}3} = 60.7$ meters. Answers for questions a & b: 60.7 m, 2.96 s.

If the reason that "x-time = y-time" isn't clear to you, consider two ways to define the time interval between points 1 & 3. Laurie measures Δt for the x-table; she clicks her stopwatch on when Gus leaves the cannon barrel at Point 1, and clicks it off when he reaches x = 60.7 m (impact at Point 3). Ellen, who measures Δt for the y-table, clicks her watch on when Gus leaves the barrel at Point 1, and off when he descends to y = 10.0 m (impact at Point 3). Laurie [who looks at the x motion] and Ellen [who looks at the y motion] click their watches on at the same time, and off at the same time, so they both measure the same Δt for the 1-to-3 interval.

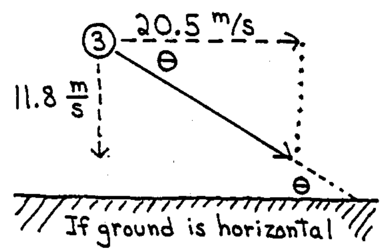
For 1-to-2, $\Delta y = 15.1$ m. Read the c-question carefully, study the diagram and use visual logic. After you have the c-answer, read the next paragraph.

15.1 m is the highest distance Gus rises above the cannon barrel. The cannon barrel is 2.0 m above the ground, so the question's answer is "17.1 meters".

To find Gus's velocity at the peak or just-before-impact, **add v_x and v_y as vectors.**

Question d: At the peak, $v_y = 0$, but $v_x = +20.5$ m/s. Gus is not motionless at the peak; he is moving straight sideways at 20.5 m/s.

Question e: The velocity just-before-impact is v_3. v_x is constant, so $(v_x)_3 = +20.5$ m/s. Earlier, $(v_y)_3$ was found to be -11.8 m/s. The *vector reconstruction* method of Section 1.2 is used below to find the v_3 vector, which has a magnitude of 23.7 m/s and a direction that is 29.9° below horizontal:



V_3 MAGNITUDE	V_3 DIRECTION
$V_3 = \sqrt{11.8^2 + 20.5^2}$	$\theta = \tan^{-1}\left(\frac{11.8}{20.5}\right)$
$V_3 = 23.7$ m/s	$\theta = 29.9°$

2.8 More Strategies for 2-Dimensional Motion

SYMMETRY: An object's flight path is symmetric if 1) the object is in "free flight", and 2) the initial and final points of an interval are at the same height.

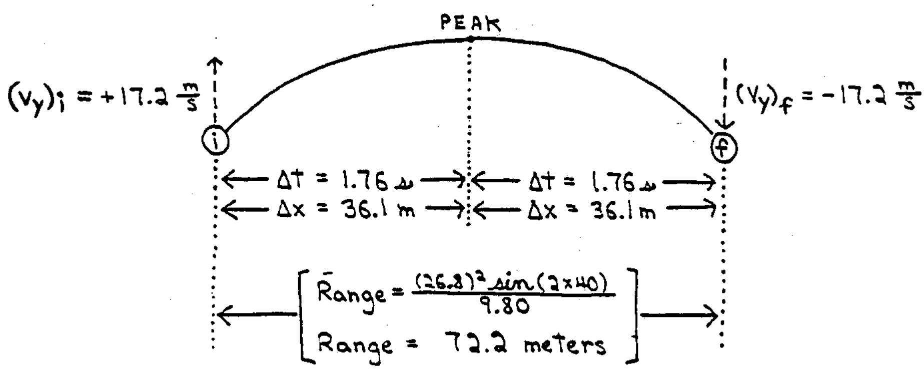
Under these conditions,

A) The time interval before the peak equals the time interval after the peak. Distance traveled before the peak equals distance traveled after the peak.

B) The vertical v_i and v_f are equal in magnitude but opposite in direction. (For example, if v_y is 17.2 m/s upward at "i", v_y will be 17.2 m/s downward at "f".)

C) The Δx between i & f can be calculated using a formula: $\Delta x = v_i^2(\sin 2\theta_i)/g$.

In 3-F, do you see why the 1-to-3 flight of Gus is not symmetric? This diagram shows what would happen if his takeoff and landing spots were at the same height:



3-Dimensional free flight is analyzed like 2-D free flight. In the vertical direction, $a = 9.8$ m/s^2 downward; look for 3-of-5. In any horizontal direction (like east-west or north-south), $a = 0$; use only "distance = v Δt" and look for 2-of-3.

CHOICES: When your class studies circular motion (which is a special category of 2-dimensional motion) and centripetal acceleration, read Sections 5.1 & 5.2.

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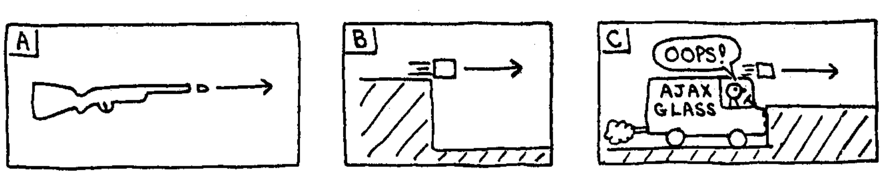
PROBLEM 2-G: A Moving Drop. While a car is moving 25 m/s, a passenger holds a bowling ball and ping-pong ball out the window, and drops them. An instant before the balls hit the ground you take a photograph. If the effect of air resistance is negligible and the car's velocity is constant, will the balls hit ahead of the passenger, alongside him, or behind him? { Answer this question now, read about "release" and decide if you want to change your answer, then look at the solution. }

Here are 3 ways to give an object an initial velocity with $v_x = 10$ m/s and $v_y = 0$:

A) A bullet is fired horizontally from a 10 m/s gun {*horizontally* means $(v_y)_i = 0$}.

B) A block is moving at 10 m/s on a horizontal surface; then it slides off the edge.

C) A van that is moving 10 m/s suddenly stops; a block on the roof keeps going.

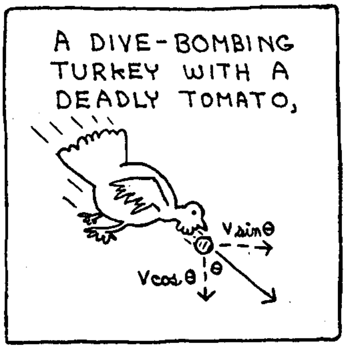


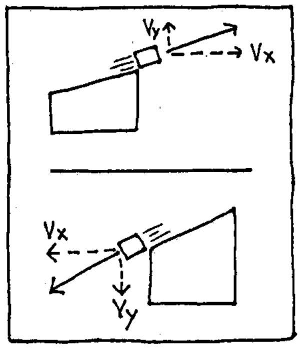
In each example, initially the object must move with a certain velocity because of the gunbarrel, surface or vanroof. If you choose the initial point of an interval to be an instant after the object is "released" from whatever was controlling its movement, the object still has the same velocity (magnitude & direction) it had just before the release.

$$\mathbf{v}_{\text{just-before-release}} = \mathbf{v}_{\text{just-after-release}}$$

Think about Problem 2-G. Is your answer consistent with this release-principle?

This principle can be used for motion in any direction, not just horizontal. Here are examples of 2-dimensional release, with v split into x & y components.





SOLUTION 2-G: If the balls aren't slowed by air resistance, they continue to move forward (as do the car & passenger) at 25 m/s until they hit the ground alongside the passenger.

Do you think air resistance will be negligible for the bowling ball? for the ping pong ball?

To greatly reduce air resistance, drop a ping pong ball inside a car. Will it fall straight down? Will an observer on the curb say that it falls straight down? What happens if the driver hits the brakes? If the car accelerates? If it turns a corner? These questions are explored in Problem 2-12.

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Ideas for Problem-Solving!

If you're working on a problem and can't reach a 3-of-5 subgoal, Sections 2.4-2.8 give ideas about what to do. Look for links: New Year's Eve {as in 2.5}, semi-known letters {2.6}, or an x-y time link {2.7}. Maybe a 1-unknown equation isn't possible, and you should look for a 2-unknown/2-equation situation {2.6}.

Read the problem again, to be sure you understand it. Decide whether you want to redefine the i & f points for intervals. Check for missed zero-words {2.4} or between-the-lines implications, or situations where the release principle {2.8} must be used.

Chapter 2's Flash Cards & Summary are good reviews of problem-solving strategies.

2.9 Ratio Logic

If your money-amount increases from \$200 to \$1000, by how much does it change?

There are two ways to answer this question. You can subtract, to find that money increases by $\$_f - \$_i$ = \$1000 – \$200 = \$800. Or divide, to find that money increases by a *multiplying factor* of $\$_f / \$_i$ = \$1000/\$200 = 5.

Ratio logic uses multiplying factors to predict how a change in one variable affects other variables. This is a practical skill that will help you both inside and outside the classroom. In fact, for overall usefulness, I think that ratio logic is more valuable than any other mathematical-thinking skill.

To answer Problems 2-H and 2-I, use "$y = v_i\, t + \frac{1}{2}\, a\, t^2$".

PROBLEM 2-H: The constant velocity of Car F (Fast) is 3 times that of Car S (Slow). If S goes 100 miles in a certain time, how far will F travel in the same time?
If S travels 120 miles in 6 hours, how long will it take F to drive this same distance?

PROBLEM 2-I: Car F's constant acceleration is 4 times that of Car M; the cars race from rest, and leave the starting line at the same time.

When F has gone 100 meters, how far has S traveled?
If S travels 100 meters in a certain time, how far will it go if it drives 3 times as long?
If S reaches the finish line in 16 seconds, how long does it take F to finish the race?
If F travels 100 meters in "T" seconds, how far will S travel in "3T" seconds?

SOLUTION 2-H

Velocity is constant, so $a = 0$, $y = v_i\,t + \frac{1}{2}(0)\,t^2$, and the equation is "$y = v\,t$".

If you have a good intuitive sense about ratios, you'll think "F goes 3 times as fast, so (in the same time) it will go 3 times as far, which is 300 miles", and "F is 3 times as fast, so (to go the same distance) it will take 1/3 as much time, which is 2 hours".

This intuitive logic is a great way to solve problems, and I suggest that you use it. But you should also know the following "equation-based tool", which is especially useful for ratio problems that are more complicated.

The first question involves **two kinds of information**: **absolute** (S goes 100 miles) and **relative** (F's speed is 3 times as fast, and both cars travel for the same time).

At the left below, relative F-info (the size of F-variables compared with S-variables) is listed above "y = vt". Because F's v is 3 times as large as S's v, "x 3" is written above the v. Because F-time is the same as S-time, "x1" is written above the t. These two multiplying factors (x3 and x1) make the equation's right side 3 times larger, so to keep the equation balanced its left side must also be multiplied by a factor of 3. This conclusion is shown by a "x 3" inside the circle. (It will be easier for you understand this process if you <u>do it</u>. Write "y = v t" on a separate page, then re-read this paragraph; as the multiplying factors are described, write them above your own equation.)

$$\begin{matrix} \times 3 & = & \times 3 & \times 1 \\ y & = & v & t \end{matrix} \qquad\qquad \begin{matrix} \times 1 & = & \times 3 & \times\frac{1}{3} \\ y & = & v & t \end{matrix}$$

To answer the first question, you must combine relative and absolute knowledge. F's y is 3 times as large as S's y (which is 100): F's y is 3 x 100 = 300 miles.

The second question is answered on the right side. The multiplying factors for y and v are "x1" and "x3". To counterbalance v's "x3" factor, t's factor must be "x1/3". Then use common sense; F's t is 1/3 of S's t (which is 6), so F's t = 1/3 x 6 = 2 hours.

The equal-signs on the F-information lines above state that "left-side multiplying factors = right-side multiplying factors": (x3) = (x3)(x1) and (x1) = (x3)(x1/3).

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Proportionality

v and y are **directly proportional**: when v increases (↑) by a factor of 3, so does y. In the three equation-versions below, y and v are either on opposite sides [with both on top] or on the same side [with one on top, the other on the bottom].

v and t are **inversely proportional**: when v ↑ by a factor of 3, t ↓ (it is multiplied by a factor of 1/3). Notice that v and t are either on the same side [with both on top] or on opposite sides [with one on top, the other on the bottom].

<u>DIRECT PROPORTIONALITY</u>

$$\overset{\uparrow}{y} = \overset{\uparrow}{v}\,t \qquad \frac{\overset{\uparrow}{y}}{t} = \overset{\uparrow}{v} \qquad \frac{\uparrow y}{\uparrow v} = t$$

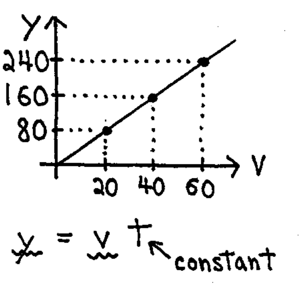
<u>INVERSE PROPORTIONALITY</u>

$$y = \overset{\uparrow}{v}\,\overset{\downarrow}{t} \qquad \frac{y}{\downarrow t} = \overset{\uparrow}{v} \qquad \frac{y}{\uparrow v} = \overset{\downarrow}{t}$$

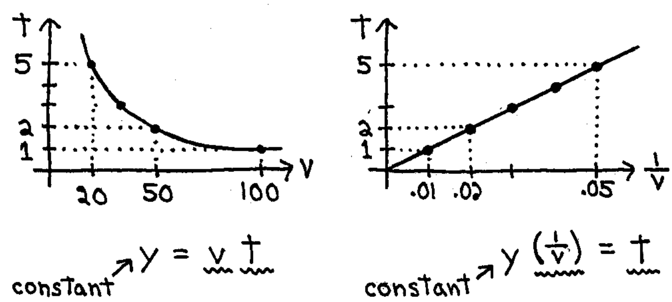
Inverse proportionality is illustrated by numbers that multiply to give a constant, like 4 x 6 = 24. If the multiplication result is to stay constant at "24" and one of the multipliers is doubled, the other must be cut in half. These two changes (multiplying by 2 and by 1/2) cancel each other, so the numbers still multiply to give 24. For example, 4(x 2) x 6(x 1/2) = 8 x 3 = 24. Similarly, 4(x 1/2) x 6(x 2) = 2 x 12 = 24.

The first graph shows a direct-proportion relationship. The graph is a straight line that slopes upward (/): if t is constant (at 4 hours), when v increases (from 20 to 40 to 60 miles/hour), y also increases (from 80 to 160 to 240 miles). { The middle •-point has y = 160, v = 40, t = 4. Substitute these numbers into y = vt ; do they "fit" the equation? Does the v-t-y information for the other two •-points also fit? }

DIRECT PROPORTION



INVERSE PROPORTION



The second graph shows that inverse proportionality gives a non-linear "curving" graph that slopes downward (\ ): if y is constant (at 100), a v-increase (20 to 50 to 100) causes t-decrease (5 to 2 to 1). { Does the y-v-t information for each • fit y = vt? }

As shown on the third graph, y = vt can be rearranged to get a direct-proportion equation: $y \frac{1}{v} = t$. Because 1/v and t are proportional, a graph of t-versus-1/v is a straight line. { Do you see that v's of 20, 50 and 100 give 1/v's of .05, .02 and .01? }

- - - - - - - - - - -

SOLUTION 2-I: "from rest" means $v_i = 0$, so $y = (0)t + \frac{1}{2} a t^2$ becomes $y = \frac{1}{2} a t^2$. "t" is used as a multiplying factor twice, so this equation can be written as $y = \frac{1}{2} a t t$.

F's acceleration is 4 times that of S, so the multiplying factor for S (compared with F) is 1/4. Because the first question asks about S, you write S's multiplying factor above the variables, as shown below. Then use common sense logic. S [slow] travels less distance by a factor of 1/4; instead of 100 m, it travels (1/4)100 = 100 m.

The 2nd question compares the same car for two time intervals. t's multiplying factor of "x 3" is used twice, so the distance is 9 times as much: (3)(3)100 = 900 m.

Question #1

($x\frac{1}{4}$)	=		$x\frac{1}{4}$	x1	x1
y	=	$\frac{1}{2}$	a	t	t

Question #2

(9)	=		same	3	3
y	=	$\frac{1}{2}$	a	t	t

Instead of writing "x1" above an unchanged variable and "x3" above a tripled variable, I usually write "same" and "3". This type of notation is used for Question #2 above. { As long as it helps you organize the ratio-changes that occur in a problem, you can use any notation you want. }

3rd question: Do you see why two t-multiplying factors of 1/2 are needed?
4th question: Two changes combine to give a multiplying factor of 9/4.

Question #3

same	=		4	($\frac{1}{2}$	$\frac{1}{2}$)
y	=	$\frac{1}{2}$	a	t	t

$t_{Fast} = (\frac{1}{2})(16\ s) = 8\ s$

Question #4

($x\frac{9}{4}$)	=		$x\frac{1}{4}$	$(x3)^2$
y	=	$\frac{1}{2}$	a	t^2

$y = \frac{9}{4}(100\ m) = 225\ m$

PROPORTIONALY: If $y = \frac{1}{2} a t^2$, y and a are proportional, y and t^2 are proportional, but a and t^2 are inversely proportional. { By taking the $\sqrt{\ }$ of both equation-sides, the last two relationships become "$\sqrt{y}$ and t are proportional" and "$\sqrt{a}$ and t are inversely proportional". }

- - - - - - - - - - -

Thinking Intuitively about Proportionality

For Problem 2-H, car speeds differ by a factor of 3, and $y = vt$ (there are no "squared terms") so multiplying factors will be either 3 or 1/3. Form a clear mental image of the fast F-car, then use "physics common sense" to decide which multiplying factor to use. For F-distance, multiply by 3 because fast speed causes larger distance. For F-time, multiply by 1/3 because fast speed causes smaller time.

For $y = \frac{1}{2} at^2$, you must be careful because of the t^2. Because t is multiplied twice, a t-change will cause a big change in y or a. { If t triples and a is constant, the y-multiplier is $3^2 = 9$. And if t triples while y is constant, the a-multiplier is $1/3^2 = 1/9$. } But changing y or a has a much smaller effect on t. { For example, if y triples and a is constant, t increases by a factor of only $\sqrt{3} = 1.73$. And if a triples while y is constant, the t-decreasing factor is just 1/1.73. }

Here are some tips to improve your intuitive feeling for proportionality. When you see a physics equation, think about the "physical reality" that is represented by each letter. { For example, if you form a clear mental picture of the equation-variables y, v, t & a, you'll find it easier to solve ratio problems using $y = vt$ and $y = \frac{1}{2} at^2$. } Then study ratio-relationships: look at a variable and ask "If this triples, what happens to each of the other variables, and [use your mental pictures] why do these changes occur?". { For example, if t triples and y is constant, v must be 1/3 as large. If t triples while v is constant, y is 3 times as large. Think about why these changes make sense. Then consider what happens if y triples, or if v triples. }

How to Solve Ratio Problems by DIVIDING EQUATIONS

In Problem 2-I, $v_i = 0$, so $y = \frac{1}{2} a t^2$. For the S-car, $y_S = \frac{1}{2} a_S t_S^2$. And for the F-car, $y_F = \frac{1}{2} a_F t_F^2$. As shown below, we can divide the left & right sides of the S-equation by the left & right sides of the F-equation. Do you see why this "does the same thing" to both sides of the S-equation, and is thus an acceptable algebra operation?

To answer Question #4, we substitute for y_F, a_F, t_F and t_S, then solve for y_S:

$$\frac{y_S}{y_F} = \frac{\cancel{\frac{1}{2}}\, a_S \; t_S^2}{\cancel{\frac{1}{2}}\, a_F \; t_F^2}$$

$$\frac{y_S}{100} = \frac{a_S}{(4\, a_S)} \frac{(3T)(3T)}{(T)\,(T)}$$

$$y_S = 100\left(\tfrac{1}{4}\right)(3)(3)$$

Which equations correctly state that "F's acceleration is 4 times as large as S's"?

$4 \; a_F = a_S$	$a_F = 4 \; a_S$	$\frac{1}{4} \; a_F = a_S$
4 (large) = (small)	(large) = 4 (small)	$\frac{1}{4}$ (large) = (small)

Many students think the first equation is correct, probably by thinking "a_F is larger, so it **deserves** to have the 4 on its side". But this is wrong. Instead, use this logic: because a_S is smaller it **needs** the 4 to make it equal (as promised by the "=" sign) to the larger a_F. The last two equations are correct. The middle-equation is used to substitute for a_F above; or you could substitute for a_S using the right-equation.

Another option: Instead of dividing equations, you can substitute-and-solve them:

$$y_s = \frac{1}{2}\, a_s\, t_s^2 \qquad\qquad y_F = \frac{1}{2}\, a_F\, t_F^2$$

$$\Downarrow \qquad\qquad 100 = \frac{1}{2}(4a_s)(T)^2$$

$$y_s = \frac{1}{2}\left(\frac{100}{2T^2}\right)(3T)^2 \qquad\qquad \frac{100}{2T^2} = a_s$$

$$y_s = 100\left(\frac{9}{4}\right)$$

We've solved Question #4 using three different methods: with multiplying factors, by dividing equations, and substitution. The *multiplying-factors* method gives an intuitive feeling for ratio logic, and is good for quick "mental arithmetic". *Dividing equations* may work better for complicated problems. *Substitute-and-solve* is usually slower, but it is correct. I suggest that you learn and practice all three methods. This gives you more problem solving flexibility, so you can think quickly-and-correctly in a wider variety of situations.

PROBLEM 2-J:
If x + y = z, and x is doubled while y is tripled, by what factor is z multiplied?

SOLUTION 2-J: This question cannot be answered. There are 3 terms (not 2, as in previous problems), so z is affected independently by changes in x and changes in y.

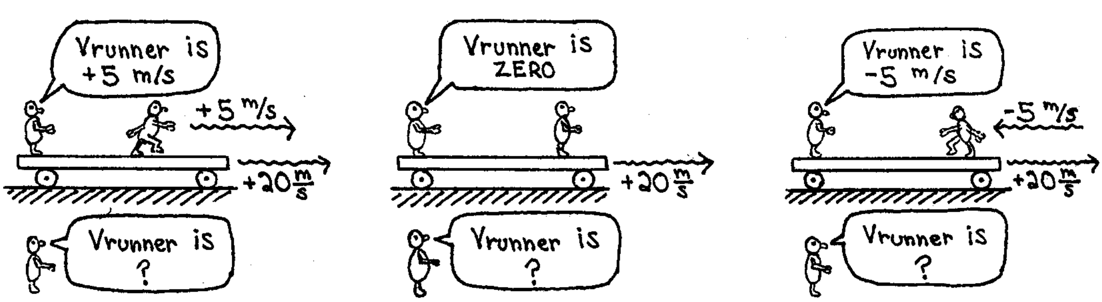
If you study the examples below, you'll see that information about single variables (x doubles, y triples) is not enough to determine z's multiplying factor. We must also know the relative sizes of x and y. { Do you see that when x is large (compared with y) the x-multiplying factor has a large effect on z's multiplying factor? But when y is larger than x, the y-multiplier is more important in determining z's multiplier. }

x + y = z	x + y = z	x + y = z	x + y = z	x + y = z
10 + 0 = 10	9 + 1 = 10	5 + 5 = 10	1 + 9 = 10	0 + 10 = 10
↓×2 ↓×3 ↓(×2.0)	↓ ↓ ↓(×2.1)	↓ ↓ ↓(×2.5)	↓ ↓ ↓(×2.9)	↓ ↓ ↓(×3.0)
20 + 0 = 20	18 + 3 = 21	10 + 15 = 25	2 + 27 = 29	0 + 30 = 30

This example shows that "**multiplying factors" ratio logic can be used only for equations that have 2 terms (one on each side of the equation).**

2.10 Relative Motion (trains, boats and planes)

PROBLEM 2-K: Common-Sense Relative Motion
An observer on the 20 m/s train reports the runner-velocities shown. What runner velocities will you observe if you are standing on the ground?



SOLUTION 2-K

In the first picture you'll see two "sources of motion". The train moves rightward at +20 m/s, and the runner moves rightward (with respect to the train) at +5 m/s, so you observe (+20 m/s) + (+5 m/s) = +25 m/s. Similarly, in the last two pictures you'll see (+20) + (0) = +20 m/s, and (+20) + (–5) = +15 m/s.

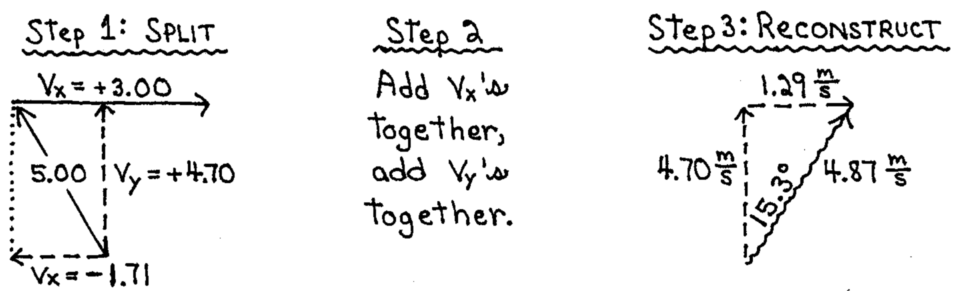
These results are just what "physical common sense" would lead you to expect.

PROBLEM 2-L: A Boat on a River

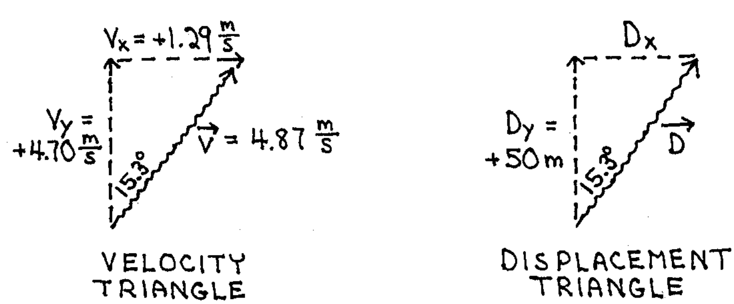
A motorboat travels 5.0 m/s in still water. A river that is 50 m wide flows eastward at 3.0 m/s. The boat starts on the south shore and aims 20° upstream (20° west of north). How long does it take to cross the river? Where does the boat land, with respect to a point that is straight across from its starting point?

SOLUTION 2-L

While crossing the river, the boat has two "velocity sources": its 5 m/s motor (this v must be split into x & y components) and the 3 m/s current. To find the velocity that is seen from the ground, add these vectors using the "split, add, reconstruct" method:



Using logic from Section 2.4: Δt is a non-vector and "**v** Δt = Δ**x**", so velocity and displacement vectors always point in the same direction. Because of this, the triangle formed by boat-velocity (**v**) and its components (v_x, v_y) is *similar to* the triangle formed by boat-displacement (**D**) and its components (D_x, D_y). These triangles have the same shape [the same angles], but different magnitudes and different units:



8 math tools can be used to analyze these velocity & displacement triangles:

1 similar-triangles ratio equation,

4 trigonometry equations: the definitions of sin, cos & tan, and "$ADJ^2 + OPP^2 = HYP^2$",

3 motion equations (in the x, y & "total" directions): $v_x \Delta t = D_x$, $v_y \Delta t = D_y$, $\mathbf{v} \Delta t = \mathbf{D}$.

Some of these tools are used below, to calculate D_x in three different ways:

SIMILAR TRIANGLES HAVE THE SAME SIDE/SIDE RATIOS.

$$\frac{1.29\ m/s}{4.70\ m/s} = \frac{D_x}{50\ m}$$

$$13.7\ m = D_x$$

USE STANDARD TRIGONOMETRY (SECTION 1.4).

$$\frac{D_x}{50\ m} = \tan 15.3°$$

$$D_x = 13.7\ m$$

Solve equations for x-direction and y-direction

Step 2

$$D_x = V_x\ t$$

$$D_x = (1.29\ \tfrac{m}{s})(10.6\ s)$$

$$D_x = 13.7\ m$$

Step 1

$$D_y = V_y\ t$$

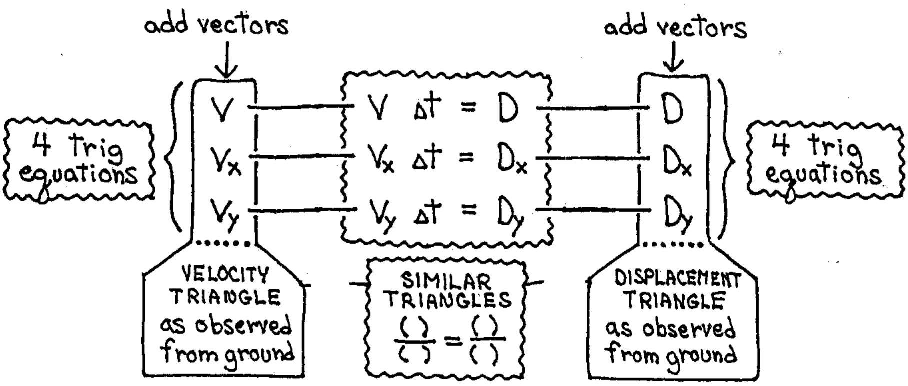
$$(50\ m) = (4.70\ \tfrac{m}{s})(t\ s)$$

$$10.6\ s = t$$

After we know that Δy = 13.7 m, **D**-magnitude can be calculated six different ways:

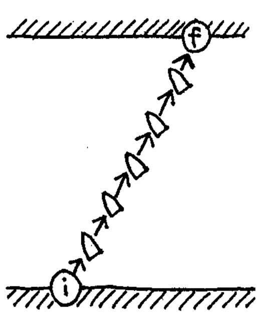
x + D ratios	y + D ratios	PYTHAGOREAN	≡ of sin	≡ of cos	MOTION EQUATION
$\frac{1.29}{4.87} = \frac{13.7}{D}$	$\frac{4.70}{4.87} = \frac{50}{D}$	$D = \sqrt{13.7^2 + 50^2}$	$\sin 15.3° = \frac{13.7}{D}$	$\cos 15.3° = \frac{50}{D}$	$(4.87\ \tfrac{m}{s})(10.6\ s) = D$

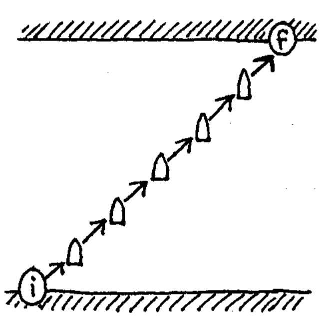
Here is a summary of the many tools (inside the ▭'s) that are available:



The left picture below shows the boat at 5 times during its trip. Even though the boat "aims" 20° upstream, it actually travels 15.3° downstream with respect to a fixed ground-point, and lands 13.7 m downstream [to the east] of a straight-across point.

The right picture shows what happens if the boat aims straight across.





(2.10) 19.1 Motion Graphs

This section will help you master an information-gathering, problem solving, visual thinking skill that is useful in many situations, both in and out of physics class.

It has 4 parts, because there are 4 main things to look for on a motion graph: POINT, SLOPE, SHAPE, AREA.

POINT

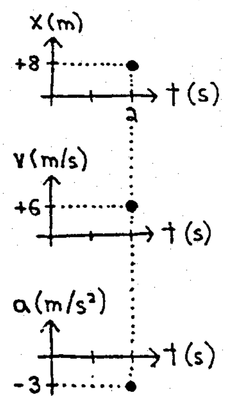
On the three graphs below, the *axis labels* show what is represented on each axis.

All three horizontal axes show time, measured in seconds. The vertical axis on the top, middle and bottom graphs show the object's x-position (in meters), velocity (in m/s) and acceleration (in m/s²). I'll call these the x-t, v-t and a-t graphs.

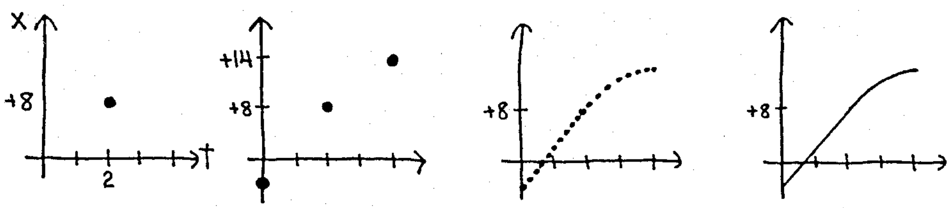
You are probably familiar with the y-x graphs (y ↕ → x) used in math courses. Motion graphs (x-t, v-t, a-t) use the same principles as y-x graphs.

On the graphs below, information about the x-position, velocity and acceleration of a car is communicated by the position of points. Each graph point states that two "matched pair" things are true simultaneously. For example, the •-point on the x-t graph has horizontal and vertical positions (emphasized by the ···· lines) that tell you "t = 2s and x = +8m", or "when t = 2s, x = +8m". The v-t and a-t graphs say "when t = 2s, v = +6 m/s", and "when t = 2s, a = −3 m/s² ".

An object's x-t, v-t and a-t graphs are usually drawn with identical t-axes, stacked on top of each other and aligned so a vertical line represents the same time (in this case, 2s) on each graph; when t = 2s, x = +8m and v = +6 m/s and a = −3 m/s².



As shown below, if you keep increasing the number of graph points they will eventually form a continuous graph line that shows the x-position (or v, or a) at every instant of time between 0 s and 4 s:



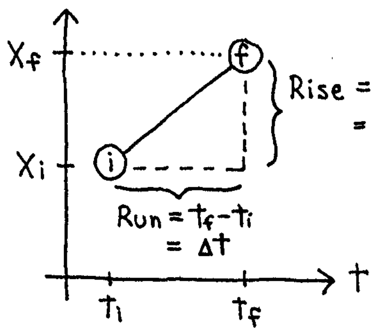
Important: <u>A graph doesn't necessarily look like the event it describes</u>. The object in the graph above moves in a straight line for awhile, then slows down and stops. A time-lapse photograph of its motion looks like —, but its x-t graph is /⌒ .

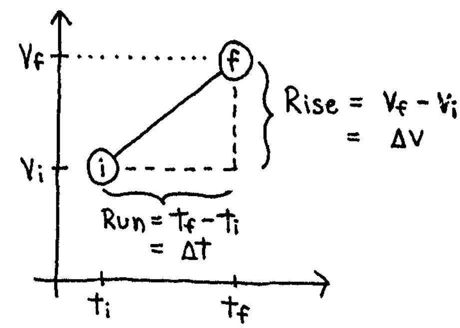
SLOPE

The slope of a line is defined as its RISE divided by its RUN, or RISE/RUN, where RISE is the initial-to-final change of the variable that is plotted on the vertical axis, and RUN is the i-to-f change of the variable that is plotted on the horizontal axis.

The slope of an x-t or v-t graph has a "physical meaning" that is discovered by comparing the geometry & physics interpretations of RISE/RUN and using this logic:

If A = B and A = C, then B = C.





Geometry : $\frac{\Delta x}{\Delta t} \equiv$ SLOPE

Physics : $\frac{\Delta x}{\Delta t} \equiv$ VELOCITY

Logic ⇒ SLOPE = VELOCITY

> The slope of x-t at a certain time gives the velocity at that time.

Geometry : $\frac{\Delta v}{\Delta t} \equiv$ SLOPE

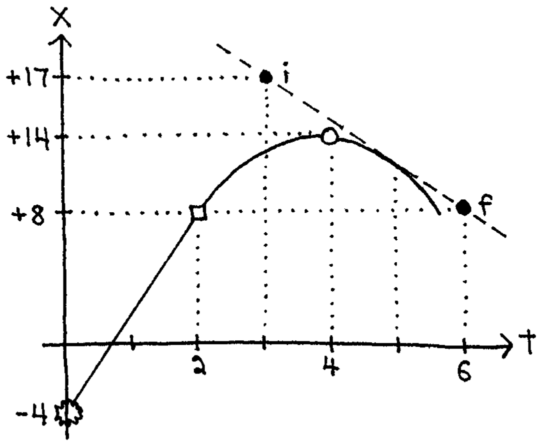
Physics : $\frac{\Delta v}{\Delta t} \equiv$ ACCELERATION

Logic ⇒ SLOPE = ACCELERATION

> The slope of v-t at a certain time gives the acceleration at that time.

Here are four examples of how to calculate a slope, using the graph below.

- In the straight-line region from 0 to 2 s, the slope is always the same. You can use any two points on the line (like ○ and □) as i & f points to calculate "x-t slope $\equiv \Delta x/\Delta t = (x_f - x_i)/(t_f - t_i)$". As calculated below, in this region the slope is +6 m/s.
- After 2 s, the x-t points form a curve, and the x-t slope keeps changing. To find the *instantaneous x-t slope* at 5 s, which is the *instantaneous velocity* [that the car's speedometer reads] at 5 s, use this 3-step process: a) Draw the *tangent line,* shown by - - - -, that matches the direction of the graph-line at t = 5 s (imagine that the graph is a highway viewed from above; the headlights & taillights of a car driving on it will point along the tangential line). b) Arbitrarily choose i & f points (like •i and •f) on the tangent line, and c) use these points to calculate "x-slope = $(x_f - x_i)/(t_f - t_i)$".
- It is easy to find the instantaneous slope at 4 s. At the peak of the curve, the tangent line is horizontal so the slope [which is the car's $v_{instantaneous}$] is zero.
- To find the *average x-t slope* (which is the *average velocity*) for the interval from 0 to 4 s, use the actual x-values at t = 0 & t = 4 s (shown by ○ and o) as the i & f points for calculating "x-slope = $(x_f - x_i)/(t_f - t_i)$".



From 0 to 2s: use ✿ and □.

$$\text{Slope} \equiv \frac{x_f - x_i}{t_f - t_i}$$

$$\text{Slope} \equiv \frac{(+8)-(-4)}{(2)-(0)}$$

Slope = +6 m/s

Instantaneous slope at t = 5s: use •i and •f.

$$\text{Slope} \equiv \frac{x_f - x_i}{t_f - t_i}$$

$$\text{Slope} \equiv \frac{(+8)-(+17)}{(6)-(3)}$$

Slope = −3 m/s

Average slope from 0 to 4s: use ✿ and o.

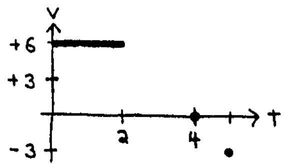
$$\text{Slope} \equiv \frac{x_f - x_i}{t_f - t_i}$$

$$\text{Slope} \equiv \frac{(+14)-(-4)}{(4)-(0)}$$

Slope = +4.5 m/s

Between 0 s and 2 s, the x-slope is +6 m/s, so the location of every v-point between 0 s and 2 s is +6 m/s. This is shown on the v-t graph below. Similarly, the x-slopes of 0 (at 4s) and −3 m/s (at 5 s) mean that the v-points are 0 (at 4 s) and −3 m/s (at 5 s).

Every instantaneous slope on the x-t graph is a point on the v-t graph.



SHAPE

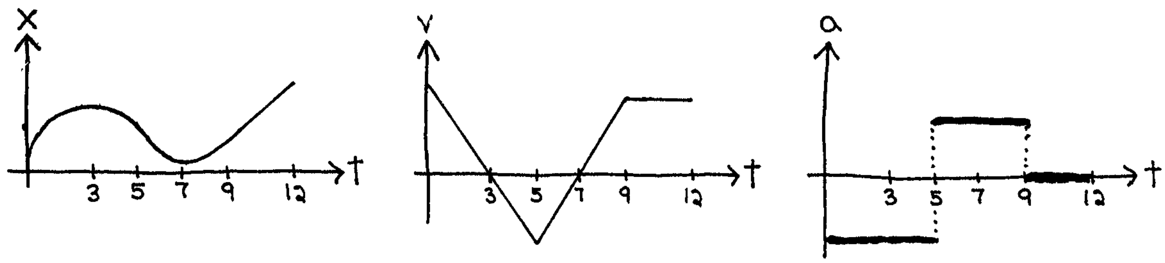
Imagine that you are walking toward the right along a graph-line. If you walk uphill , the [instantaneous] slope is +. If you walk downhill , the slope is −. If you walk horizontally , the slope is zero.

If the slope is steep (either or) it has large magnitude. If the slope is shallow (or) its magnitude is small. If the graph-line is horizontal (either —— , or the peak-spot of ∩ or bottom of ∪) the slope magnitude is zero.

Look at the x-t graph below. At t = 0, the slope is + with large magnitude (steep). As you move to the right, slope magnitude gradually decreases (gets shallower) until it is zero at 3 s. From 3 s to 5 s, the slope is − with increasing magnitude. From 5 s to 7 s, the slope is −; its magnitude decreases until it is 0 at 7 s. From 7 s to 9 s, the slope is + with increasing magnitude; from 9 s to 12 s, it is + with constant magnitude.

Now read the x-t description above, but replace "slope" with "velocity". Do you see why this modified paragraph [At t = 0, the velocity is + with large magnitude. As you move to the right, velocity magnitude ...] now describes the location of points on the v-t graph?

Compare the v-t and a-t graphs. From 0 s to 5 s the v-slope is – and constant, so the a-points (which equal the v-slopes) are – and constant. From 5 s to 9 s, the v-slope and a-points are + and constant. From 9 s to 12 s, the v-slope and a-points are zero.



SHAPES: From 0s to 5s, the x-t shape is a "Mountain", and **v** is decreasing so **a** is Minus. From 5s to 9s, the x-t shape is a "Pit", and **v** is increasing so **a** is Positive. From 9s to 12s, the x-t graph is Straight, and **v** is constant so **a** is Zero. It is easy to remember the connection between x-t's shape and **a**'s sign: MM, PP, SZ.

Concavity: In math courses, mountain & pit shapes are called "concave down" and "concave up".

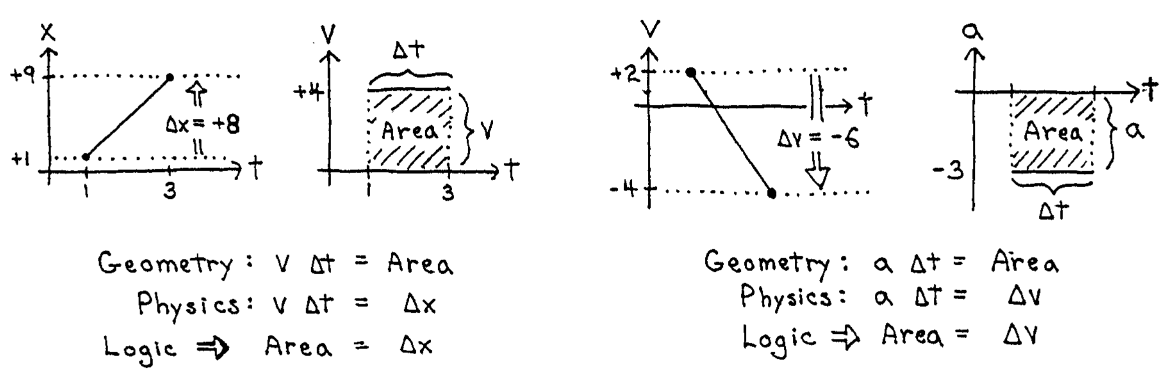
The independence of velocity and acceleration: As stated in Section 2.4, free-flight acceleration is 9.8 m/s^2 downward, whether velocity is ↑, ↓, →, ↗, or zero.

The graphs above also show that **v** & **a** can point in different directions. While **a** is – (from 0 s to 5 s), **v** is + (0 s to 3 s) and 0 (at 3 s) and – (3 s to 5 s). While **a** is + (5 s to 9 s), **v** is – (5 s to 7 s) and 0 (at 7 s) and + (7 s to 9 s). And while **a** = 0 (9s to 12s), **v** is +.

From 0 s to 5 s (and 5 s to 9 s), v-magnitude changes while a-magnitude is constant.

AREA

The "physical meaning" of v-t area and a-t area can be discovered by comparing geometry and physics definitions, and using this logic: if A = B and A = C, then B = C.

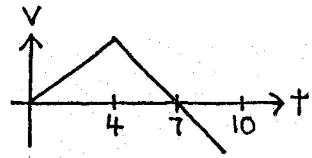


These area-principles (v-area = Δx and a-area = Δv) are true for area of any shape, not just rectangles. (Problem 2-27 shows how to estimate irregular areas.)

+ and – AREA: On the left graph above, the i-to-f v-area is + [above the t-axis] because v is +, so the corresponding i-to-f Δx is +. But on the right graph, the i-to-f a-area is – [below the t-axis] because a = –3; this causes the i-to-f Δv to be –.

The v-area between i & f tells you Δx between i & f, but says nothing about x_i or x_f. If x_i or x_f wasn't given on the left graph above, you would know that Δx = +8 because the v-area is +8 m, but you wouldn't know whether x goes from 0 to +8, from 1000 to 1008, from –1000 to –992, or .... (Similarly, a-area gives Δv, but not v_i or v_f.)

PROBLEM 19-A: When is this object furthest from its starting point?



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The **Chapter 2 Summary** gives a good overview of motion-graph strategies. If you study it closely, you'll learn a lot. Here is a very brief summary:

x-concavity	x-slope	Δx
↓↑	↓↑	↑
	v-point	v-area
	v-slope	Δv
	↓↑	↑
a-sign	a-point	a-area

Problem 2-26 shows the relationships between a graph's points, slopes, shapes and areas.

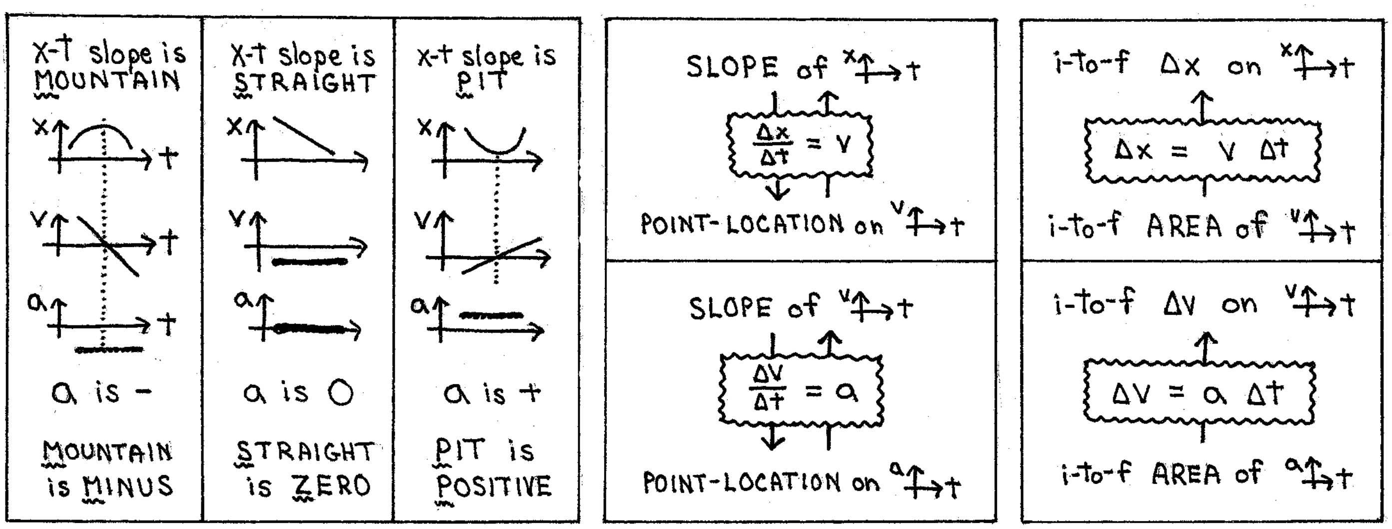
OPTIONAL: If you understand this section you already know quite a bit about three main topics of *calculus: derivatives* are just slopes, a *second derivative* is the "slope of a slope" (this was discussed in "Shape"), and *integrals* are an easy way to find areas. If your class uses calculus to analyze physics, or if you're curious and want to learn it on your own, read Sections 19.2 and 19.3.

SOLUTION 19-A: If you look at the slope-shape of v-t and think "maximum x occurs at the graph's peak, at 4s", or if you try to interpret the graph as if it was a "photograph" of the object's motion, you'll get the wrong answer.

But if you focus on point-location and think "as long as v is +, the object is moving in the + direction and x will keep increasing", you'll get the correct answer of 7s.

(Or you can try to maximize the + v-area, which maximizes the +Δx ; this occurs at 7s.)

Each type of graph (x-t, v-t, a-t) and characteristic (point, slope, shape, area) gives a certain kind of information, but says absolutely nothing about other things. To interpret graphs properly, you must know what to look for **and** what to ignore.



Average Slope : Use actual graph-points for i + f, calculate RISE/RUN.
Instantaneous Slope : draw tangent line, choose i + f, calculate RISE/RUN.

2.90 Flash-Card Review for Chapter 2

I strongly recommend that you use these "flash card" sections.
As discussed in Section 1.90, these reviews will help you improve your problem-solving skill.

You may want to mark the question-answer pairs that you find especially useful, so you can review these pairs more often.

2.1	With __ , physics problem-solving is __ . __ helps you "learn more from experience".	power tools, satisfying and fun searching for insight
2.2	Δ means __ , is calculated by __ .	"change of", final value – initial value
2.2	___ can be used to derive new equations.	solve-and-use links
2.3	The 5 motion equations are true if __ . Each of these equations has __ , but __ . The strategy for using tvvax tables is __ .	a is constant between the i & f you've chosen 4 variables, is missing 1 variable make, look for 3-of-5, choose 1-out equation
2.3	The key to problem-solving is __ because __ . To truly understand physics, unite __ .	understanding physics, math is usually easy intuitive and mathematical thinking
2.3	To do units correctly-and-easily, be __ .	careful (start), relaxed (middle), careful (end)
2.4	An object is in "free flight" if __ . A free-flight object has **a** = __ if **v** is __ . a = –9.8 m/s² means that __ changes by __ .	gravity is only force & air resistance is ignored 9.8 m/s² ⇓ , any magnitude or direction number-line v, –9.8 m/s during each 1 second
2.4	**a** and __ always point in the same direction; their __ and __ differ by a factor of __ because __ [equation] and Δt is a __ with __ .	Δ**v** (but not **v**) magnitude, units, Δt Δ**v** = **a** Δt , non-vector, no direction
2.4	Straight-line motion: **v** and **a** __ if speed ↑. but while speed is ↓ ing, **v** and **a** __ .	point in same direction (have same ± sign) have opposite directions and ± signs
2.4	"rest" is a __ , and so are __ .	zero-v word; stop, drop, peak, max height,...
2.5	After every problem, ask yourself __ . Aesop's Problems teach __ .	What can I learn from this problem? specific problem-solving strategies
2.5	To learn quickly, __ and __ . __ is possible because many physics tools __ .	work hard, work smart rapid progress, are "interdependent"
2.5	A free-flight interval lasts from __ to __ .	just after "throw release", just before "impact"
2.5	For special points 1, 2, 3 & 4, several possible ways to find $\Delta t_{2\text{-to-}4}$ are __ . Instead of v_i & v_f, use __ to clarify __ . Δy (= __) depends only on __ , not on __ .	direct ($\Delta t_{2\text{-}4}$), adding ($\Delta t_{2\text{-}3} + \Delta t_{3\text{-}4}$), subtracting ($\Delta t_{1\text{-}4} - \Delta t_{1\text{-}2}$) v_1 & v_2 & v_3 ... , "New Year's Eve" t-links $y_f - y_i$, y_i & y_f , what occurs between i & f
2.6	Use __ information to describe variables as __ If you can't get a __ , try for __ .	all available; known, semi-known, unknown 1-unknown eqn, 2 eqns with 2 unknowns
2.6	Quadratic equations can be solved by __ .	2-step detour, √‾ trick, Quadratic Formula

2.7	Make separate tvvax tables for different __ .	time intervals, objects, directions
2.7	x & y motion can be __ . If $a_x = 0$, for tvvax you need __ of __ . [x & y] When filling in x & y tables, always __ .	analyzed independently 2-of-3 (1 eqn) for x, 3-of-5 (5 eqns) for y decide [each fact must be put in correct table]
	Δt for x & y motions are equal if __ .	intervals are "matched" ($\Delta t_{1\text{-to-}2} \neq \Delta t_{1\text{-to-}3}$)
	If you know v_x and v_y, find v by __ .	vector reconstruction (basic trig, Section 1.2)
	Form a clear mental picture of __ and __ .	special points, intervals
2.8	If __ , an object's path is symmetric, so __ , __ , and __ .	an object is in free-flight and $y_i = y_f$ Δt & Δx before peak = Δt & Δx after peak, $(v_y)_i = -(v_y)_f$, $\Delta x = v_i^2 (\sin 2\theta)/g$
2.8	The "release principle" says that __ . __ is a zero-word that often tells you __ .	$v_{\text{just before release}} = v_{\text{just after release}}$ "horizontal", $v_i = 0$
2.8	If you can't solve a problem yet, __ .	Re-read and search for missed implications [or zero words or "object releases" or ...]. Re-draw your pictures, re-think i & f choices, look for links or "2 unknowns in 2 equations".
2.9	"How much change" is found by __ or __ . If changes are made, an equation's __ = __ . If $k = m + n$, __ .	subtraction $(x_f - x_i)$, division (x_f / x_i) left-side multiplying factors, r-side multipliers "multiplying factors" ratio logic can't be used
2.9	If $x = yz$ [$x/y = z$], y & z are __ , x & y are __ ; graphs of __ are straight, __ is curved.	inversely proportional, directly proportional x-versus-y or 1/y-versus-z, y-versus-z
2.9	If $a = ce^2$ [a = cee], __ change → big __ change, but changing __ → smaller change in __ .	e, a or c a or c, e
2.9	If b is 3 times larger than c, the equation is __ . If d is 1/3 as large as e, their equation is __ .	$b = 3c$ [large = 3(small)] , or $\frac{1}{3}b = c$ $d = \frac{1}{3}e$ [turn words into equation], $3d = e$
2.9	Dividing equations is acceptable because __ . Ratio problems can be solved by using __ .	it "does the same thing" to both equation-sides multiplying factors (mentally or on-paper), dividing equations, standard substitution
2.10	v-vectors and **D**-vectors can never __ .	be "mixed" in same vector-triangle
2.10	Eight useful tools are __ . { Hint: 1, 4, 3 }	v-Δ and **D**-Δ are *similar* (same side/side ratios) trigonometry (≡ of sin, cos, tan ; $a^2 + b^2 = c^2$) motion ($D_x = v_x \Delta t$, $D_y = v_y \Delta t$, $\mathbf{D} = v \Delta t$)
19.1	x-t point, area & instantaneous slope give __ . v-t point , area & instantaneous slope give __ . a-t point , area & instantaneous slope give __ .	x at t , ? , v at t ($\Delta x / \Delta t = v$) v at t, i-to-f Δx ($\Delta x = v \Delta t$), a at t ($\Delta v / \Delta t = a$) a at t, i-to-f Δv ($\Delta v = a \Delta t$), rate of a-change
19.1	x-t shapes [∩ , ∪ , /] show __ .	**a** is – (m is m), **a** is + (p is p), **a** = 0 (s is z)

Chapter 2 Summary

the tvvax system

5 variables, 5 equations (each is missing 1 variable)

$v_f - v_i$	=	$a\ t$	Δx is missing
$(x_f - x_i)$	=	$\frac{1}{2}(v_i + v_f)\ t$	a is missing
$(x_f - x_i)$	=	$v_i\ t + \frac{1}{2} a\ t^2$	v_f is missing
$(x_f - x_i)$	=	$v_f\ t - \frac{1}{2} a\ t^2$	v_i is missing
$v_f^2 - v_i^2$	=	$2a\ (x_f - x_i)$	Δt is missing

These equations are true only if a is constant between i & f.

Step 1: Read carefully [for words, sentence structure, implications], think [creative and logical], draw [form a clear picture-idea, on paper and/or mentally]. Choose initial (i) and final (f) points for a useful constant-a interval.

Step 2: Make a "tvvax table", to show what you know about the 5 tvvax variables.
Look for zero-v words: from rest, stop, is dropped, peak, maximum height,...
"t" means Δt. You can substitute "$x_f - x_i$" for Δx whenever it's helpful.
Δx depends only on x_i and x_f positions, not on what happens between i & f.

Step 3: Look for a 3-of-5 subgoal: if you know any 3 variables, you can find the other 2. If you can't get 3-of-5, re-read the problem-statement more carefully, look for "links" where the same variable occurs in two tvvax tables [like New Year's Eve {Section 2.6}, semi-known symbols {2.7}, x-time = y-time {2.8},...].

Step 4: Use "1-out strategy" to choose the equation with 3 knowns & the goal-variable, substitute-and-solve. If necessary, use simultaneous equations {2.7}, or a quadratic option {19.7} like the Q-formula {2.6} or 2-step Q-Detour {2.6} or $\sqrt{\ }$ Q-trick {2.7}; for each Q-option you must choose between + and – solutions.
UNITS: Use SI (s, m/s, m/s^2, m). Be careful at start (during substitution), relaxed in middle (algebra solution), careful at end (answering the question).

Step 5: Answer the question that was asked.

FREE FLIGHT {2.5}: If only gravity affects an object and air resistance is ignored, it has $a_x = 0$; a_y = 9.80 m/s per second downward, which is –9.80 m/s^2 if "up" is +.
Don't mix a free flight interval with a "throw" or "impact".

SPLITS: Make a tvvax table for each time interval and/or object and/or direction.

TIME SPLIT {2.6}: Split the action into useful constant-a intervals, separated by special points. Be specific about v-labels; use v_1, v_2, v_3,... (not v_i & v_f), look for New Year's Eve links. Use "total = sum-of-parts" logic, like $\Delta t_{1\text{-to-}2} + \Delta t_{2\text{-to-}4} = \Delta t_{1\text{-to-}4}$.

OBJECT SPLIT {2.7}: Translate the words of a problem into a clear picture that helps you define tvvax knowns and unknowns and semi-knowns (where the same variable-letter is in different tvvax tables). Use "$\Delta x = x_f - x_i$" for each object; at a certain special time (like a passing point) two objects may have the same x-position.

DIRECTION SPLIT {2.8 & 2.9}: Analyze x & y motion independently, make separate x & y tvvax tables for each time interval and object. For free-flight motion,

X: 2-of-3 subgoal.

$\Delta x = v_x\ \Delta t$ .... t-LINK (if same i & f) .... Δt =

(   ) = (   )(   )

Y: 3-of-5 subgoal.

Δt =
v_i =
v_f =
a = –9.8 m/s^2
Δy =

Use Section 1.3 methods to split v_i into v_x & v_y, and reconstruct the $(v_{total})_f$ vector.

SYMMETRY: For free-fall motion when $y_i = y_f$, $\Delta t_{\text{before peak}} = \Delta t_{\text{after peak}}$, $\Delta x_{\text{before peak}} = \Delta x_{\text{after peak}}$, $\Delta x_{\text{i-to-f}} = v_i^2 (\sin 2\theta_i)/g$, and $(v_y)_i = -(v_y)_f$.

RELEASE PRINCIPLE: $v_{\text{just-before-release}} = v_{\text{just-after-release}}$ (magnitude & direction).

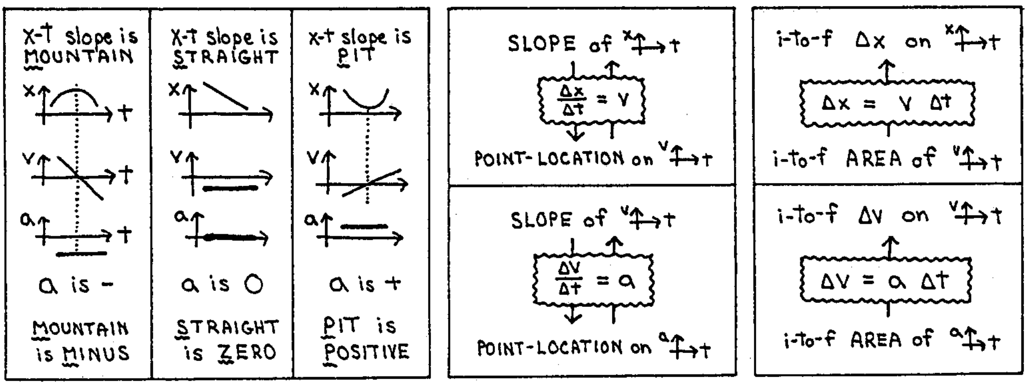
A vector's x (or y) component is + if the vector points in the x (or y) direction you've chosen to be +. The x-component is – if it points in the opposite direction.

$\Delta \mathbf{x} = \mathbf{v}_{\text{average}}\ \Delta t$, so $\Delta \mathbf{x}$ and $\mathbf{v}_{\text{average}}$ always point in the same direction.
$\Delta \mathbf{v} = \mathbf{a}_{\text{average}}\ \Delta t$, so $\Delta \mathbf{v}$ and $\mathbf{a}_{\text{average}}$ always point in the same direction.

v and **a** can have different directions; examples occur in Sections 2.2 & 2.10-Shapes.

While number-line **v** is increasing, a is +; while number-line **v** is decreasing, a is –.
While speed ↑, **v** & **a** have the same ± sign; while speed ↓, **v** & **a** have opposite ± signs.

MOTION GRAPHS {2.0}: Point, Slope, Shape (Concavity), Area.



Average Slope: Use actual graph-points for i+f, calculate RISE/RUN.
Instantaneous Slope: draw tangent line, choose i+f, calculate RISE/RUN.

RELATIVE MOTION (2.11)

$v^{\text{train}}_{\text{ground}}$ {v of tr, obs from gr} + $v^{\text{runner}}_{\text{train}}$ {v of ru, obs from tr} = $v^{\text{runner}}_{\text{ground}}$ {v of ru, obs from gr}

$v^{\text{train}}_{\text{ground}} = -v^{\text{ground}}_{\text{train}}$, vectors can be added in any order: $v^{\text{train}}_{\text{ground}} + v^{\text{runner}}_{\text{train}} = v^{\text{runner}}_{\text{train}} + v^{\text{train}}_{\text{ground}}$.

Use consistent reference frames: $\Delta x^{\text{runner}}_{\text{ground}} = v^{\text{runner}}_{\text{ground}}\ \Delta t^{\text{runner}}_{\text{ground}}$, but $\Delta x^{\text{runner}}_{\text{train}} \neq v^{\text{runner}}_{\text{ground}}\ \Delta t^{\text{runner}}_{\text{ground}}$.

Here is a TOOL SUMMARY for 2-dimensional "boat & plane problems":