Chapter 1

Math for Physics:
Geometry, Trigonometry,
Metric Units, Conversion Factors

Chapter 1 is not really about "physics". It is a collection of math tools you'll need for physics: reviews of Geometry & Trigonometry (in Sections 1.1 & 1.2), and discussions of Metric System Prefixes & Conversion Factors (in 1.3 & 1.4). You can read these sections [one at a time, or all at once] whenever you think it will be helpful.

In addition, Algebra for Physics is summarized in Chapter 18, and Calculus for Physics (optional) is in Chapter 19.

1.1 Geometry for Physics

Here are the main geometry tools you'll need for physics:

An easy way to remember these tools is \( \Delta XYZ \): \( \Delta \) is a triangle, \( X \) is crossed lines, \( Y \) represents the \( 90^\circ \) (\( \vee \)) and \( 180^\circ \) (\( \bar{1} \)) rules, and \( Z \) shows parallel lines.

Many diagrams can be analyzed only if you DRAW EXTRA LINES. How do you know what lines to draw? Often, useful extra lines are extensions of existing lines, parallel to existing lines, perpendicular to existing lines, or are needed to form a right triangle that can be analyzed using the trigonometry methods of Section 1.3. Be creative and experiment with extra lines. If you're not sure whether you should draw a line, go ahead and draw it; if it isn't needed, you can erase it or ignore it.
Trigonometry and **SIMILAR TRIANGLES** are discussed in Section 1.2.

**TOTAL = SUM OF PARTS** logic can be used to find "x = 3" for this diagram:

```
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;-x&lt;-[4]</td>
<td>&lt;-7&lt;-[7]</td>
</tr>
</tbody>
</table>
```

## 1.2 Trigonometry for Physics

This section shows you how to master essential problem solving tools. You'll use these tools often, so it's worth investing a few extra minutes to learn them well.

In the right triangle below, the longest side (15.0) is opposite the 90° angle; this side is the *hypotenuse*, abbreviated HYP. One of the two shorter sides (8.6, OPP) is opposite the 35° angle, while the other (12.3, ADJ) is adjacent to 35°.

The 3 basic trigonometry functions (sine, cosine, tangent) are side/side *ratios*, the length of one side divided by the length of another side:

```
\[
\sin 35° = \frac{\text{OPP}}{\text{HYP}} = \frac{8.6}{15.0} = 0.57 \\
\cos 35° = \frac{\text{ADJ}}{\text{HYP}} = \frac{12.3}{15.0} = 0.82 \\
\tan 35° = \frac{\text{OPP}}{\text{ADJ}} = \frac{8.6}{12.3} = 0.70
\]
```

These side/side ratios are stored in the memory of your calculator. If you know the angle is 35° and want to find the ratio of OPP/HYP, use the "sin" button: punch "35 sin". But if you know this ratio and want to find the angle, use the "sin⁻¹" button: punch "8.6 ÷ 15.0 = sin⁻¹". Practice with your calculator until you know how to use all 6 trig buttons; the diagram below will help you check your answers.

Your calculator has a button (it is often labeled "DRG") that determines whether an angle has units of Degrees, Radians or Grads. If it's in "degree mode", which probably occurs automatically when the calculator is turned on, and you punch "35 sin", it interprets "35" to be 35 degrees (not 35 radians or 35 grads) and gives "0.574..." as the answer. (Radian units, which are useful for understanding rotational motion, won't be discussed until they're needed — in Section 5.4.)
Either non-90° angle can be used for trig functions, if adjacent & opposite are defined correctly. The triangle we used earlier has angles of 90°, 35° and (because its angles add to 180°) 55°. Do you see why \( \cos 35° = \sin 55° \)? Look at the 12.3-side:

12.3 is adjacent to 35°, so \(12.3/15.0 = \text{ADJ/HYP} = \cos 35° = 0.82\),
but 12.3 is opposite 55°, so \(12.3/15.0 = \text{OPP/HYP} = \sin 55° = 0.82\).

There are 4 right-triangle variables: \(\text{ADJ}, \text{OPP}, \text{HYP}\), and \(\theta, \sin \theta, \cos \theta, \tan \theta\).
(\(\sin \theta, \cos \theta\) and \(\tan \theta\) are really 1 variable, not 4, because if you know one of them you can find the others.) As shown below, there are also 4 equations: 3 trigonometry definitions plus the Pythagorean Theorem, and each equation contains 3 of the 4 variables.

\[
\sin \theta = \frac{\text{OPP}}{\text{HYP}} \quad \cos \theta = \frac{\text{ADJ}}{\text{HYP}} \\
\tan \theta = \frac{\text{OPP}}{\text{ADJ}} \quad \text{ADJ}^2 + \text{OPP}^2 = \text{HYP}^2
\]

This useful principle will help you plan "the next step" in your problem solution:
If you know any 2 of the 4 right-triangle variables, you can easily find the other 2.

For example, if you know that \(\theta = 35°\) and \(\text{ADJ} = 12.3\), the equations with \(\theta\) and \(\text{ADJ}\) \(\cos \theta = \text{ADJ/HYP}, \tan \theta = \text{OPP/ADJ}\) can be solved for \(\text{HYP}\) and \(\text{OPP}\), respectively.

**SPLITTING A VECTOR INTO COMPONENTS**
(I'm assuming your textbook has discussed vectors, which are quantities that have both magnitude and direction.)

If a plane flies 150 miles/hour in a direction 55° to the eastward side of due north (abbreviated 55° E of N), find its speed in the eastward and northward directions.

Known: HYP and \(\theta\)
Unknown: ADJ and OPP

In the drawing above, the velocity vector is a solid arrow-line, and the components are dashed arrow-lines. The "extra" dotted lines are parallel to (and the same length as) the components. These --- lines help form right triangles that can be analyzed.

Each equation that contains both "knowns" can be solved for an unknown:

\[
\cos \theta = \frac{\text{ADJ}}{\text{HYP}} \quad \sin \theta = \frac{\text{OPP}}{\text{HYP}} \\
\text{HYP} \cos \theta = \text{ADJ} \quad \text{HYP} \sin \theta = \text{OPP} \\
(150) \cos 55° = \text{ADJ} \quad (150) \sin 55° = \text{OPP} \\
86 = \text{ADJ} \quad 123 = \text{OPP}
\]
The HYP is always the longest side of a right-triangle; ADJ and OPP are always smaller. The \( \cos \theta \) and \( \sin \theta \) ratios are "cut-down multiplying factors" (like \( \cos 55^\circ = 0.574 \), and \( \sin 55^\circ = 0.819 \)) that tell you how much smaller the ADJ and OPP sides are:

\[
\text{large (cut-down factor)} = \text{small} \quad \frac{\text{HYP}}{\cos \theta} = \text{ADJ} \quad \frac{\text{HYP}}{\sin \theta} = \text{OPP}
\]

A memory trick: the first letters of ADJ & \( \cos(A,c) \) are early in the alphabet, while OPP & \( \sin(O,s) \) come later.

Some textbooks connect \( \cos \) & \( \sin \) with \( x \) & \( y \), respectively, implying that a vector's x-component is \( \text{HYP}(\cos \theta) \), and its y-component is \( \text{HYP}(\sin \theta) \). Sometimes, as in the example above, these formulas give the wrong answer. Is this something you need? I strongly recommend that you connect \( \cos \) & \( \sin \) with ADJ & OPP (not \( x \) & \( y \)), so you will always get the correct answer.

**RECONSTRUCTING A VECTOR FROM ITS COMPONENTS**

If you know an airplane has a northward speed of 86 mi/hr and an eastward speed of 123 mi/hr, can you find the magnitude and direction of its velocity vector?

![Diagram of vector components](image)

Known: ADJ and OPP
Unknown: HYP and \( \theta \)

As with "splitting", the 2 knowns appear in equations that can be solved.

\[
\tan \theta = \frac{\text{OPP}}{\text{ADJ}} \\
\theta = \tan^{-1} \frac{\text{OPP}}{\text{ADJ}} \\
\theta = \frac{\text{123}}{86} \\
\theta = 55^\circ
\]

Another memory trick: to remember "\( \tan \theta = \frac{\text{OPP}}{\text{ADJ}} \)" think TOAD.

Here is a visual summary of vector splitting and vector reconstruction:

![Diagram of vector components](image)

**SIMILAR TRIANGLES**: If two triangles have the same 3 angles, they have the same "shape" and the same side/side ratios. One of these triangles will be a "scaled up" version of the other triangle. For example, each side in the airplane-velocity triangle (86, 123, 150) is 10 times larger than the corresponding side of the earlier triangle (8.6, 12.3, 15.0) because both triangles have a 35°-55°-90° shape. Do you see why each triangle will have the same side/side ratios? For example, the tan 55° ratios (they are 123/86 and 12.3/8.6, respectively) are both 1.43.
SIMILAR TRIANGLES: If two triangles have the same 3 angles, they have the same "shape" and the same side/side ratios. (One of these triangles will be a "scaled up" version of the other triangle. For example, each side in the airplane-velocity triangle (86, 123, 150) is 10 times larger than the corresponding side of the earlier triangle (8.6, 12.3, 15.0) because both triangles have a 35°-55°-90° shape. Do you see why side/side ratios (like the tan 55° ratios of 123/86 and 12.3/8.6, respectively) are the same for both triangles?)

Here are two trigonometry relationships you can use, and one you should avoid.

Two correct formulas are: \( \frac{\sin \theta}{\cos \theta} = \tan \theta \), \( (\sin \theta)^2 + (\cos \theta)^2 = 1 \).

The relationship below looks like it might be true, using the logic that "if \( 3(a + b) = 3a + 3b \), then we can do the same thing with trig functions like "cos". But with trig functions, this math-technique is wrong, as shown by the ≠ sign:

\( \cos(a + b) ≠ \cos a + \cos b \)

In case you ever need them, the correct angle-addition formulas are summarized in Section 18.9#.

ADDING VELOCITIES (using "graphical" and "splitting into components" methods of analysis) is discussed in Problem 1-7.

RELATIVE MOTION: A common example of vector addition is the behavior of a boat in river current, or an airplane in wind. Such problems are studied in Section 2.10.

VECTOR MULTIPLICATION: Dot-Products and Cross-Products (optional techniques that are used in most calculus-based physics courses) are discussed in Section 19.11.

1.3 Two Interpretations of Metric Prefixes

The metric system, which is used in most countries, measures length in meters, mass in grams, and time in seconds. These units are abbreviated m, g, and s.

PREFIXES: To make smaller or larger units, the metric system uses a combination of prefix & base-unit, as shown in the table below. Any prefix (M, k, c,...) can be used with any base-unit (m, g, s,...).

[Scientific notation, like \( 3.4 \times 10^3 \) being used to represent 3400, is discussed in Section 18.6.]

<table>
<thead>
<tr>
<th>PREFIX &amp; MEANING</th>
<th>EXAMPLES</th>
<th>PREFIX NAMES</th>
</tr>
</thead>
<tbody>
<tr>
<td>M means ( x \times 10^6 )</td>
<td>( 5.7 , \text{Ms} = 5.7 \times 10^6 , \text{s} )</td>
<td>Ms is megasecond</td>
</tr>
<tr>
<td>k means ( x \times 10^3 )</td>
<td>( 5.7 , \text{kg} = 5.7 \times 10^3 , \text{g} )</td>
<td>kg is kilogram</td>
</tr>
<tr>
<td>c means ( x \times 10^{-2} )</td>
<td>( 5.7 , \text{cm} = 5.7 \times 10^{-2} , \text{m} )</td>
<td>cm is centimeter</td>
</tr>
<tr>
<td>m means ( x \times 10^{-3} )</td>
<td>( 5.7 , \text{mg} = 5.7 \times 10^{-3} , \text{g} )</td>
<td>mg is milligram</td>
</tr>
<tr>
<td>µ means ( x \times 10^{-6} )</td>
<td>( 5.7 , \text{µm} = 5.7 \times 10^{-6} , \text{m} )</td>
<td>µm is micrometer</td>
</tr>
<tr>
<td>n means ( x \times 10^{-9} )</td>
<td>( 5.7 , \text{nm} = 5.7 \times 10^{-9} , \text{m} )</td>
<td>nm is nanometer</td>
</tr>
<tr>
<td>p means ( x \times 10^{-12} )</td>
<td>( 5.7 , \text{ps} = 5.7 \times 10^{-12} , \text{s} )</td>
<td>ps is picosecond</td>
</tr>
</tbody>
</table>

There are two useful ways to interpret combined prefix/base-unit combinations.

In the table above, the prefix is considered to be part of the number, as illustrated with this underlining: \( 5.7 \, \text{cm} = 5.7 \times 10^{-2} \, \text{m} \). For physics problems this is usually the best way to handle prefixes.
But you can also keep the c & m together to form a "cm" unit that is 1/100 as large as a meter: in 5.7 cm, there are 5.7 of these cm-size length units.

1.4 Conversion Factors

A Conversion Factor (CF) converts a thing from one unit to another. For example,

\[
\frac{48 \text{ inches}}{12 \text{ inches}} = \frac{4 \text{ feet}}{1 \text{ foot}}
\]

\begin{align*}
\uparrow & \quad \uparrow \\
\text{old unit} & \quad \text{ONE} & \quad \text{new unit}
\end{align*}

1 foot/12 inches = 1 because what is on top (1 foot) is the same as what’s on the bottom (12 inches). (This is the same reason that 4/4 = 1, or 12 inches/12 inches = 1.)

When we multiply 48 inches by [1 foot/12 inches], we multiply it by 1. This keeps the 48 inch length the same size; its length-units just change from inches to feet.

A correct CF-Equation is needed, to show the proper relationship between units. This equation must use the correct number ratio ("14 inches = 1 foot" is wrong), and it must match the numbers and units correctly ("12 feet = 1 inch" is wrong).

Proper matching uses one of these types of logic: "1 small-size unit = a fraction of a large-size unit", or "many small-size units = 1 large-size unit". Two examples are:

\[
1 \text{ cm} = 1 \times 10^{-2} \text{ m} \quad 100 \text{ cm} = 1 \text{ m}
\]

(1 small unit = fraction of large unit) \quad (many small units = 1 large unit)

The first CF-equation is simple; just use the prefix-meaning of "c" [multiply by \(10^{-2}\)] that is described in Section 1.3. The second CF-equation, after both sides of the first CF-equation are multiplied by 100, is more intuitive because it uses whole numbers; if you look at a meter-stick, it is easy to see that 1 meter contains 100 cm.

Correct Use of CF-Fractions: If a CF-fraction is flipped upside down it still equals 1, because the fraction's top and bottom are still equal. Therefore, one CF-equation produces two CF-fractions. Each CF-fraction is useful in a different situation:

To convert 48 inches to __ feet,

\[
48 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 4 \text{ ft}
\]

To convert 4 feet to __ inches,

\[
4 \text{ ft} \cdot \frac{12 \text{ in}}{1 \text{ ft}} = 48 \text{ in}
\]

If you choose the wrong CF, units won't cancel and you'll get a mess:

\[
48 \text{ in} \cdot \frac{12 \text{ in}}{1 \text{ ft}} = 576 \text{ in}^2 \text{ ft}^{-1}
\]

square inches per foot ??

WRONG!

\[
4 \text{ ft} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 33.3 \text{ ft}^2 \text{ in}^{-1}
\]

square feet per inch ??

WRONG!

To do a conversion, just multiply by the CF-fractions that are needed to transform the "old units" into the "new units" you want. When 100 km/hour is converted to m/s units, notice that two conversions ( km ⇒ m, hour ⇒ s ) are needed:

\[
\frac{100 \text{ km}}{1 \text{ hour}} \cdot \frac{10^3 \text{ m}}{1 \text{ km}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} \cdot \frac{1 \text{ minute}}{60 \text{ s}} = 27.8 \text{ m/s}
\]
For some conversions you can use a "substitution shortcut". In this example, "k" and "1 hour" can be replaced by something that is equivalent to them:

\[
\frac{100 \text{ km}}{1 \text{ hour}} = "k" \quad \text{and} \quad 1 \text{ hour} = 3600 \text{ s} \Rightarrow \frac{100 \times 10^3 \text{ m}}{3600 \text{ s}} = 27.8 \frac{\text{m}}{\text{s}}
\]

To practice conversions, use 1 minute = 60 s, 1 hour = 60 minutes, 1 ft = 12 in, 1 m = 39.37 in and 1 mile = 5280 ft to convert 60 miles/hour to other speed units:

60 mi/hr = 1 mi/minute = 88 ft/s = 26.82 m/s = 96.56 km/hr

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1.90 Flash-Card Review for Chapter 1

To use a physics tool, you must remember it. Two good ways to improve your memory are organization and active practice.

The Chapter Summaries, starting after Chapter 20, provide logical organization. And these end-of-chapter "flash card reviews" are a quick way to do active practice: just cover the answer-column with a card, look at the question and answer it, then slide the card down to check. (Don't worry if you can't get the answer the first time you see a question. This is not an "exam"; it is a way to make many quick reviews.) Repeat the list until you can easily remember all of the tools; it won't take long! If necessary, go back to the appropriate section (listed in the far-left column) for review. If you want, add question-answer pairs of your own.

Section 20.1, "Improving Your Memory", discusses the principles of organization and practice, and also specific techniques: summary notes, flash cards, and more.

QUESTIONS

1.1 What are 8 useful geometry tools?

1.2 A right-triangle's ___ variables are ___.
   If you know ___, you can ___.

1.2 A vector has ___.
   ___ can be ___.
   The SPLITTING formulas are ___.
   The RECONSTRUCTING formulas are ___.

1.2 The ( )'s are ___.

\[
\begin{align*}
\cos 70^\circ &= \sin \underline{\quad}, \text{ so } \\
20^\circ, \text{ either non-90}^\circ \text{ angle can be } "\theta"
\end{align*}
\]

1.2 Two "extra" trig-equations are ___ and ___.

1.3 A unit-prefix is a ___.
   4 km is ___ or ___.

1.4 A CF-equation must have correct ___ and ___.
   A CF-equation gives ___.
   A CF-fraction = ___.

1.4 Using a ___ shortcut, 90 km/hour is ___.

ANSWERS

\[
\begin{align*}
\Delta X Y Z: \Delta, X, 90^\circ, 180^\circ, Z. \quad \text{Draw extra lines, similar-}\Delta \text{ ratios, total = sum of parts.} \\
4; \quad \text{HYP, ADJ, OPP, } \theta/\sin \theta/\cos \theta/\tan \theta \quad \text{any 2 of these, find the other 2} \\
\text{magnitude and direction} \\
\text{split into } \rightarrow \text{ and } \uparrow \text{ components} \\
\text{ADJ = HYP } \cos \theta, \quad \text{OPP = HYP } \sin \theta \\
\text{HYP = } \sqrt{\text{ADJ}^2 + \text{OPP}^2}, \quad \theta = \tan^{-1}(\text{OPP/ADJ})
\end{align*}
\]

\[
\begin{align*}
V \sin \beta \\
V \cos \beta
\end{align*}
\]

20°, either non-90° angle can be "θ"

\[
\begin{align*}
\cos^2 \theta + \sin^2 \theta &= 1, \quad \sin \theta/\cos \theta = \tan \theta \\
\text{multiplier; } 4 \text{ km } (4 \times 10^3 \text{ m}), 4 \text{ km} \\
\text{number ratio, number/unit matching} \\
\text{two CF-fractions, 1} \\
\text{substitution, } (90 \times 10^3 \text{ m})/(3600 \text{ s})
\end{align*}
\]
Chapter 1  Summary

GEOMETRY FOR PHYSICS: ΔXYZ.

DRAW "EXTRA LINES": extensions of existing lines, parallel to existing lines, perpendicular to existing lines, or to form right-azines.

TOTAL = SUM OF PARTS: If \[ \begin{align*} x & \longrightarrow & 3 & \longrightarrow, \text{ then } x = 4. \\
& & 7 & \end{align*} \]

SIMILAR TRIANGLES: If two triangles have the same angles (and thus the same shape), they have the same side/side ratios.

VECTORS have both magnitude and direction.

If you know 1 of [θ, sinθ, cosθ, tanθ], you can find the others. Calculator: \[ \theta \leftarrow \text{"cos"} \rightarrow \text{ADJ/HYP ratio}, \]

and \[ \text{ADJ/HYP ratio} \leftarrow \text{"cos}^{-1}\text{"} \rightarrow \theta \text{ angle}. \]

\[ \cos \theta = \sin(90^\circ - \theta), \] and \[ \sin \theta = \cos(90^\circ - \theta) \]

If you know 2 of the 4 right-triangle variables [ADJ, OPP, HYP, \(\theta/\sin\theta/\cos\theta/\tan\theta\)] you can find the other 2, by solving the equations that contain the knowns.

\[ \begin{align*} \sin \theta & = \frac{\text{OPP}}{\text{HYP}} \\
\cos \theta & = \frac{\text{ADJ}}{\text{HYP}} \\
\tan \theta & = \frac{\text{OPP}}{\text{ADJ}} \\
\text{ADJ}^2 + \text{OPP}^2 & = \text{HYP}^2 \end{align*} \]

TRUE: \[ \sin \theta / \cos \theta = \tan \theta \]

\[ \sin^2 \theta + \cos^2 \theta = 1 \]

FALSE: \[ \cos (a + b) \neq \cos a + \cos b \]

\[ \cos (a, b) \neq (\cos a)(\cos b) \]

To add vectors, A) Draw the vectors head-to-tail, like a "relay race", B) choose axes and split each vector into x & y components, C) add x-components to get \(x_{total}\), add y-components to get \(y_{total}\), D) use these x & y components to "reconstruct" the resultant vector.

There is a difference between components (which are always \(\perp\)) and originals.

OPTIONAL: Vector multiplication (dot product & cross product) is explained in Sections 18.51-18.53.