asks about higher dimensional figures and introduces the quaternions. The chapter does not go deeply into the material but intends to leave the reader curious and intrigued. The concluding chapter describes occurrences of fractals as physical objects in nature (shorelines, clouds, trees, etc.), returning to the topic found in Mandelbrot's introductory book.

Chapter 8, "Fractals and the Christian Worldview," is an interlude to the mathematics, returning to the claim that of the two suppositions, a Christian or a non-Christian worldview, only the Christian worldview truly explains fractals. Yes, the infinite complexity of the Mandelbrot set is beautiful. Many mathematicians agree that beautiful objects like this are independent of human thought, a form of mathematical *platonism*. But the leap from mathematical *platonism* to belief in a creator and then to belief in the biblical God is not well supported by Lisle. He ignores the difficulties involved in these steps: first from mathematical *platonism* to deism, and then from deism to belief in the God that Christians worship.

In the final (twelfth) chapter, Lisle returns to his argument that mathematics points to the God of the Bible. He quotes physicist Eugene Wigner's article, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," which discusses the "miracle" of mathematics in explaining the modern world.[2] Lisle then quickly dismisses other religious views and claims that only the Bible makes sense of our universe. The book ends with a gospel presentation.

One can argue (Rom. 1:20) that God's divine nature is visible in the beauty of mathematics, but Lisle quickly dismisses the beliefs of atheists and non-Christian religions and leaps to claiming (as implied by the book's subtitle) that the *only* legitimate reaction to fractals is to believe in the Christian God. While most of my mathematical colleagues identify with mathematical *platonism*, their beliefs vary across a spectrum from atheism/agnosticism through Judaism, Islam, and Christianity. The jarring leap from "the beauty of fractals comes not from people" (p. 125) to the Christian worldview, will leave a thoughtful skeptic with whiplash. At no place is the "secret code" to creation explained explicitly.

Lisle's approach to apologetics is that of presuppositionalism. He assumes that only a Christian worldview is reasonable. However, presuppositional apologetics has several significant flaws. It can quickly become a circular argument: if one assumes the truth and accuracy of the Bible as an axiom then the Christian worldview is a foregone conclusion. This approach receives quick approval from people who already believe the scriptures but is readily dismissed by the sceptic. Even when the circular argument is avoided, the best one can argue is that the universe—and mathematics—appears to be beautiful, appears to have design. The appearance of design is roughly equivalent to mathematical *platonism* and parallels the argument of Romans 1. But the sceptic who accepts this argument will immediately point out that there are many worldviews that begin with this assumption. The leap to the Christian worldview is not proven by this approach; it requires the additional confirmation of special revelation.

In other publications, Lisle rejects both the big bang theory and evolution. Ironically, this beautiful book on fractals makes it clear that elegant and complex structures do indeed arise from quite simple processes. This is a concept that underlies the theory of evolution, which Lisle opposes.

Would I put this book on my coffee table? No, because ultimately this book is an attempt at apologetics. The flaw in the apologetics will be apparent to the thoughtful sceptic. And the author's attempt to establish the Christian worldview includes simplistic claims that are dismissive of people with other beliefs.

Notes

[1]Benoit B. Mandelbrot, *The Fractal Geometry of Nature* (New York: W. H. Freeman, 1982).

[2]E. P. Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," *Communications on Pure and Applied Mathematics* 13 (1960): 1–14.

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History of Science

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GENERATIONS OF REASON: A Family's Search for Meaning in Post-Newtonian England by Joan L. Richards. New Haven, CT: Yale University Press, 2021. 456 pages, with 21 b/w illustrations, 1,218 endnotes, and a 35-page index. Hardcover; $45.00. ISBN: 9780300255492.

The title gives no clue who this book is about. Nor does the publisher's description on its website, the abbreviated blurb inside the book jacket, the four endorsements posted on the jacket's back ("beautifully written," "epic masterpiece," "magnificent study," "compelling and wide-ranging"), or even the chapter titles. The reader first learns whom the book is about and how it came into focus in the author's Acknowledgments. In studying the divergent interests of Augustus De Morgan and his wife, Sophia, the importance of De Morgan's father-in-law William Frend's thinking became apparent. This is turn led Richards to delve into the lives and beliefs of two ancestors from the previous generation, Francis Blackburne and Theophilus Lindsey, who felt compelled by their commitment to "reasoned conclusions about matters of faith" (p. x) to move away from orthodox Anglicanism and establish the first Unitarian church in England. Thus the book eventually evolved into chronicling the lives of three generations over a century and a half during (roughly) the Enlightenment era.

A central motif running through the experiences, beliefs, and work of these families was their steadfast commitment to a form of enlightened rationality that provided coherence and foundational meaning for their lives. Reason informed their ecclesiastical commitment to Unitarianism, their views of science and mathematics, and their public activity favoring social and educational reforms. But also, paradoxically, their search for reason led to the beliefs and practices (of some family members) that today would be considered pseudo-scientific—mesmerism, phrenology, and spiritism, among others.

As Richards notes in the book's opening sentence, for her, *Generations of Reason* is "the culmination of a life devoted to understanding the place of mathematics in modern European cultural and intellectual history." The mathematics and logic of early- to mid-nineteenth-century Britain has been an ongoing research interest for Richards during her forty-year tenure as a historian of mathematics at Brown University. It is this that largely drew me to the book and which I will focus on here: it climaxes in a substantive treatment of the progressive mathematics of De Morgan, whose work contributed to transforming British algebra and logic. This is in stark contrast with the radical ideas of Frend, who refused to admit negative numbers into mathematics.

A central figure behind the developments under investigation is John Locke, whose *Essay Concerning Human Understanding* (1689) and *The Reasonableness of Christianity, as Delivered in the Scriptures* (1695) exercised a tremendous influence over and challenge for eighteenth- and nineteenth-century British thinkers. Locke's ideas defined and emphasized rationality in relation to knowledge generally and to scientific and religious knowledge in particular, providing dissenters with a rationale for combatting traditional theology and conformist science and philosophy. For Locke, however, a literal reading of Scripture was still authoritative for religious beliefs. This was true for Frend and De Morgan also, even though they held tolerant attitudes toward a wide latitude of thinkers.

Locke's view of reason also affected period reflections on mathematics. Like others in the early modern and Enlightenment eras, Locke had held up mathematics as a model of absolutely certain knowledge because of the clarity of its ideas and the supposed self-evidence of its axiomatic truths. Of course, this characterization applied more to Euclidean geometry than to the burgeoning domains of analytic mathematics, such as calculus, which, as Berkeley charged, still lacked a sound theoretical basis. As for logic, Locke had an acute antipathy toward traditional argument forms and proposed that one should reason with ideas rather than words, assessing their agreement or disagreement in less convoluted ways than in a syllogism. In expressing such relations with language, though, one should use meaningful and unambiguous terms. This was somewhat problematic in algebra and calculus, where symbolic expressions were manipulated to produce useful and important results, even when their meaning was less than clear.

Around the turn of the nineteenth century, Frend campaigned to bring algebra in line with Lockean reasoning: algebra was conceptualized at that time as universal arithmetic, containing such laws as the transposition rule *if* $a+b=c$ *then* $a=c-b$. Thus, no expression should be employed if its meaning was unintelligible. In the above equations, one must assume the condition $b<c$ to rule out negative values, since numbers, which represent quantities of discrete things, cannot be less than 0. Excising negative quantities from mathematics was extreme but necessary in order to adhere to a literalistic view of rationality.

British mathematicians largely resisted following Frend down this path of purity, though they were unsure how to rationally justify their use of negative and imaginary quantities without going outside mathematics and appealing to things like debts. Robert Woodhouse, in an 1803 work, was one of the first Cambridge mathematicians to propose a more formalistic algebraic approach in calculus. This agenda was furthered a decade later by members of Cambridge's Analytical Society, one of whom was George Peacock. His and others' attempts to convert Cambridge analysis from Newtonian to Leibnizian calculus were waged through translating a French textbook and making notational changes in Cambridge's mathematical examinations.

In 1830 Peacock's *Treatise on Algebra* introduced a more formalistic approach in algebra. Richards argues, drawing upon some fairly recent research, that Peacock's position was grounded in a progressivist view of history: arithmetic developed naturally out of fluency with counting, and algebra out of familiarity with arithmetic. Arithmetic suggests equivalent forms (equations, or symbolic assertions like the above rule) that can also be accepted as equivalent/valid in algebra without being constrained by restrictions appropriate to arithmetic. Such transitions, he thought, constitute genuine historical progress. Algebra thus splits into two parts for Peacock, arithmetical algebra and symbolical algebra, the latter based upon his principle of the permanence of equivalent forms, as found in his 1830 *A Treatise on Algebra*.

Peacock's approach to algebra set the stage for later British mathematicians such as De Morgan (Peacock's student), Boole, and others. Initially inclined to follow his future father-in-law's restrictive approach in algebra, De Morgan was soon won over to Peacock's point of view, even going beyond it in his own work. In a series of articles around 1840, De Morgan identified the basic rules governing ordinary calculations, but he also began entertaining the notion of a symbolical algebra less tightly tied to arithmetical algebra. By more completely separating the interpretation of algebra's operations and symbols from its axioms, symbolical algebra gained further independence from arithmetic. This gave algebra more flexibility, making room for subsequent developments such as the quaternion algebra of William Rowan Hamilton (1843) and Boole's algebra of logic (1847).

After exploring the foundations of algebra, De Morgan turned his attention to analyzing forms of reasoning, a topic made popular by the resurgence of syllogistic logic instigated at Oxford around 1825 by Richard Whately. Traditional Aristotelian logic parsed valid arguments into syllogisms containing categorical statements such as *every X is Y*. De Morgan treated such sentences extensionally, using parentheses to indicate total or partial inclusion between classes *X* and *Y*. Thus, every *X* is *Y* was symbolized by *X*)*Y* since the parenthesis opens toward *X;* to be more precise, one should indicate whether *X* and *Y* are coextensive or *X* is only a part of *Y*. By thus quantifying the predicate, as it was called, De Morgan allowed for these two possibilities to be symbolized respectively by *X*)(*Y* and *X*))*Y*, in compact symbolic form as ')(' and '))'. Combining the two premises of a syllogistic argument using this notation, one could then apply an erasure rule to draw its conclusion. De Morgan enthusiastically elaborated his symbolic logic by adopting an abstract version of algebra that paved the way for operating with formal symbols in logic. De Morgan's symbolism is not as inaccessible as Frege's later two-dimensional concept-writing (though the full version of De Morgan's notation is more complex than indicated here), but it is still rather forbidding and failed to find adherents.

In addition to expanding Aristotelian forms by quantifying the predicate, yielding eight basic categorical forms instead of the standard four, by 1860 De Morgan was generalizing the copula "is" in such sentences to other relations, such as "is a brother of" or "is greater than." He began to systematically investigate the formal properties of such relations and the ways in which relations might be compounded. Though intended as a way to generalize categorical statements and expand syllogistic logic, his treatment of relations was later recognized as an important contribution that could be incorporated into predicate logic. Richards's treatment gives the reader a fair sense of what De Morgan's logic was like, and while a detailed comparison is not developed, the reader can begin to see how De Morgan's system compares to Aristotelian logic, Boole's algebra of logic, and contemporary mathematical logic.

However, as indicated at the outset, exploring De Morgan's algebraic and logical work is only a subplot of Richards's story. Her book is principally a brief for how reason grounded the work and lives of several significant thinkers in an extended family over three generations. As she ties various threads together, the reader occasionally senses that the presentation may be too tidy, drawing parallels between vastly different developments to make them seem of a piece, all motivated by the same driving force of reason. Nevertheless, Richards's account forces the reader to continually keep the bigger picture in mind and to connect various facets of the actors' lives and work to their deeper commitment to reason. Her book thus offers a commendable case study for how technical trends in mathematics might be tied to broader cultural and philosophical concerns.

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OF POPES & UNICORNS: Science, Christianity, and How the Conflict Thesis Fooled the World by David Hutchings and James C. Ungureanu. New York: Oxford University Press, 2022. 263 pages. Hardcover; $39.95. ISBN: 9780190053093.

Readers of *PSCF* are familiar with the "warfare thesis" for the history of science and religion. This interpretation, framed as a historical analysis that stretches from the ancient Greeks to the modern period, explains the way in which science and religion have always been in conflict with each other. At the center of this interpretation are John William Draper's *History of the Conflict between Religion and Science* (1874), and Andrew Dickson White's *A History of the Warfare of Science with Theology in Christendom* (1896). Since the publication of these books, numerous professional historians as well as the general public have accepted and perpetuated many of the claims made within them. The problem with this line of interpretation, however, is that Draper and White were often wrong. For instance, Christopher Columbus (and people in the medieval period) did not think the earth was flat. Christians did not oppose anesthesia. There was no Dark Ages. Christians did not believe in unicorns. Premodern medical diagnosis did not merely appeal to supernatural causation. And the list could continue.