Danilo R. Diedrichs

# Mathematics Reveals Patterns That Reflect the Orderly Character of God

*Danilo R. Diedrichs*

*As the "queen and servant of the sciences," mathematics plays a complex role vis-à-vis the other fields of science. This primarily non-empirical method of organizing arguments and deriving truths deductively also proves to be remarkably effective in describing the physical world. Increasingly, the natural and social sciences are becoming "mathematized" and turning to mathematical models to describe patterns in their observations of the world. Although most models fall short of expressing an absolute, universal law of nature, they remain effective tools to reveal structures of order where none are apparent. This article presents the unique place of mathematical descriptions of the physical world in the Judeo-Christian tradition, and their contribution to our understanding of God. Mathematical models reveal God's character of order as well as his reliability, faithfulness, and uniqueness. They also provide a lens through which God's roles as creator and sustainer of the world become visible. As pranalogical instruments of worship, mathematical models help shape a proper biblical worldview and a better understanding of God's creation in order to improve the quality of life on this earth.*

As one of the earliest and oldest applications of the ability of the human mind to engage in abstract thoughts, mathematics has a special place in the history of human beings' understanding of the world. Every civilization has developed a system of abstracting numbers into a system that lends itself to visualize relationships between quantities and abstract patterns. Our ability to reason with abstract thoughts is at the foundation of mathematics, just as it lies at the core of our understanding of God.

Mathematics stands alone among other fields of knowledge: it is a science, systematically arranged and subject to general laws, but unlike the other sciences, its content is not primarily empirical. The practice of mathematics does not require any application to the physical world and can be done entirely within the bounds of the human mind. However, mathematics can also be used, and with great effectiveness, to describe elements of the physical world.

Complex concerns in the material world often drive us to reach out to our ability for abstraction to provide order and understanding. Once we discover the mathematical rules that describe patterns revealed by this abstract process, we can dissociate ourselves from the material world completely and continue the study of mathematics in complete abstraction, without any need or concern for the material world. Yet the mathematical discoveries made in deepest abstraction often come full circle, proving themselves useful in providing answers to open questions about the physical world. For example, the concepts of geometry, originally developed out of practical needs for engineers and merchants to measure

**Danilo R. Diedrichs** *is an associate professor of mathematics at Wheaton College, Illinois, with research interests in the mathematical modeling of dynamical systems in biology, ecology, and epidemiology. Raised and educated in Switzerland, he holds a PhD in applied mathematical and computational sciences from the University of Iowa, as well as a master's degree in civil engineering from the Swiss Federal Institute of Technology in Lausanne, Switzerland.*

lengths, angles, areas, and volumes, eventually led to the development of an entire system of abstract thought patterns based on postulates and axioms no longer connected to physical concerns. Geometry progressed further into abstraction, extending in the nineteenth century to non-Euclidean geometries and topology. Then Einstein used a non-Euclidean geometry as the framework for his theory of relativity; in addition, the abstract concepts of knot theory, a subarea of topology, provided useful tools for studying the properties of enzymes that unknot the complex knotted structure of DNA molecules, thereby facilitating progress in molecular biology and modern medicine.

The increasing mathematization of the natural and social sciences in the twentieth century led to the widespread use of *mathematical modeling*, a concept that originated in the late seventeenth century with Newton's *Principia Mathematica*, but remained confined to physics and chemistry until the twentieth century. This approach brought together the strengths of experimental science and mathematics into *mathematical models*, the "tendons connecting the muscles of mathematics to the bones of science."[1] The twentieth century saw the emergence of another type of mathematical model called *empirical models*, or *data-driven models*, based on statistical approaches that are common in the life and social sciences. Their primary purpose is less about finding an equation that describes a law of nature, and more about finding an equation that fits a particular data set.

Mathematics is a uniquely effective tool for studying patterns and orderly structures. Mathematician Keith Devlin defines mathematics as the science of patterns that "can be either real or imagined, visual or mental, arising from the natural world or from within the human mind, the quintessential tool for searching structures of order in life, mind and universe."[2]

Philip Davis and Reuben Hersh go even further, using language that mirrors the ordering described in Genesis 1:

> To some extent the whole object of mathematics is to create order where previously chaos seemed to reign, to extract structure and invariance from the midst of disarray and turmoil … To create order—particularly intellectual order—is one of the major human talents, and it has been suggested that mathematics is the science of total intellectual order.[3]

The Christian faith is unique in claiming that, although God is invisible, he delights in the material world and uses it to reveal himself and to reach out to us. The scriptures also emphasize order as being one of the primary attributes of the Christian God. The creation account in the first chapter of Genesis puts emphasis on God's action to create order out of disorder, to give a purpose to the material world.[4] Throughout scripture, whenever God speaks, order appears, and God declares it to be good.

The process of building and defining a mathematical model has been identified by several Christian mathematicians as a unique means for understanding the world God created, and the role he has given us as we interact with his creation. The beauty of patterns revealed by models resonates with our soul, pointing to God's role as creator of order and to the *imago Dei*.[5] Furthermore, the modeling process integrates and shapes our faith by reflecting our motives and attitudes; indeed, models can be used either for destructive purposes or to improve the quality of life on this earth.[6]

This article highlights the role of mathematical models to reveal, study, and explain the patterns observed in the physical world and discovered empirically by experimental science. Some of these patterns are so complex and hidden that experiments do not reveal them; the order within them becomes apparent only through a mathematical model, in that the very definitions of these patterns are based not on the behavior of the systems themselves, but on the mathematical properties of the models used to describe them.

This article begins with an outline of the history and philosophy of the relationship between mathematics and the physical world, leading to the modern concept of mathematical modeling. Then it highlights the effectiveness of mathematics to uncover and study hidden patterns of order in the physical world. Finally, it presents examples of how mathematical models of seemingly "chaotic" systems reveal patterns where none were suspected, thereby providing a means to reflect the underlying order imbued in all of God's creation.

## History and Philosophy of the Relationship between Mathematics and the Physical World

The nature of the relationship between mathematics and the physical world has been under debate since the era of pre-Socratic philosophers. One school of thought, formalized by Plato, held that mathematics has its own existence in the "Platonic realm," independent of human beings, and that we humans are merely discovering what has been there from eternity. According to Plato, the most fundamental kind of reality is composed of nonmaterial, abstract Forms, which our senses allow us to perceive only as shadows on the wall of a dark cave with light shining behind us. Mathematics frees us from the cave of our perceptions so that we can directly perceive the Forms through reason alone. Thus, mathematics is *discovered* and is independent of experience. In contrast to this point of view, the realist or empiricist view rejects this idea and instead claims that mathematical forms are *invented* by the human mind; they are artificial constructs that we imagine, and then use to describe the physical world we observe.[7] This debate, which began 2,500 years ago, is far from being settled today. Physicist and engineer Derek Abbott estimates that most pure mathematicians today lean to a Platonist view, whereas most physicists and engineers are non-Platonists, with applied mathematicians falling somewhere in between.[8]

Throughout the Christian era, many Christian philosophers have adapted a Platonist view of mathematics to a Christian worldview. According to St. Augustine, God created the world, both visible and invisible, including the eternal truths of mathematics, which originated in the eternal mind of God and with which God created the patterns of the world. Being created in God's image, our minds possess the ability to apprehend the basic mathematical truths. Johannes Kepler believed that God had embodied some of his essential mathematical nature in creation, and that we humans can think his thoughts after him. True knowledge of natural phenomena can be attained when the geometric schemes in our mind correspond to those prototypes in the Divine mind that have been copied into the world.[9]

The strongest counter-current to the Platonist view in the Christian world is the Aristotelian natural philosophy, systematized in accordance with Christian theology by Thomas Aquinas in the thirteenth century. Aquinas adopted Aristotle's idea that matter is the basis of all that exists, and that the true form of an object, being contained within the object itself, can be perceived using one's senses. His analyses of physical objects, place, time, and motion were especially influential among the Dominicans and Jesuits.

The earliest expressions of the natural philosophies of Plato and Aristotle held that the physical world was too changeable and imperfect to be explained by mathematics, but the fourteenth and fifteenth centuries saw increasing attempts to apply mathematics to the physical world under Aristotle's growing influence, thanks to Aquinas. The gradual development of experimental practices beginning in the thirteenth century saw mathematics as a tool to organize and analyze experimental data. Eventually, mathematics rose to occupy an important role as an ancillary field of knowledge endorsed by the church to help understand the higher disciplines of theology (the "queen of the sciences"), philosophy, law, and medicine.

In the sixteenth century, the Reformation threatened the stability of the Roman Catholic Church's teachings, prompting widespread theological and philosophical disputes. Under the impetus of the Jesuit mathematician Clavius, Catholic theologians turned to mathematics and to the geometrical proofs in Euclid's *Elements* as a model to derive eternal truths deductively and to prove them decisively and irrefutably.[10] At the same time, scientific practice was being formalized into a philosophy of experiment by Mersenne and Gassendi, among others. Christian natural philosophers discovered the ability afforded to them by mathematics coupled with experimental practice to describe, understand, predict, and, ultimately, to control the natural world around them. They put forward the new doctrine that God had structured the universe according to mathematical laws, in which case it was not only possible, but also God's will, that efforts be made to understand those laws,[11] justifying their efforts as a response to God's creation mandate (Gen. 1:28).

One of the chief natural philosophers of that era, Galileo, championed this view of mathematics, but was not satisfied with it, going further and taking the bold step of equating mathematics with God's native tongue.[12] Following Augustine, he claimed that God had written two books of equal importance and significance: the Holy Scriptures, God's word, to be interpreted by the study of theology; and the Book of Nature, to be interpreted by the study of mathematics. Around the same time, Johannes Kepler was successful in fitting planetary data points to orbits, and Descartes developed significant applications of geometry to the physical world. Eventually, the locus of truth concerning the natural world shifted from theology to mathematics and natural philosophy, causing theology to be supplanted by mathematics as the new "queen of the sciences."[13]

The discovery of the calculus by Newton and Leibniz in the seventeenth century led to increasing mathematization in natural philosophy. Over the next two hundred years, natural philosophers used mathematical equations to write fundamental laws that described the mechanics of the universe, gradually removing much of the mystery from phenomena that had puzzled them for centuries (planetary motion, mechanics, electromagnetism, light, optics, diffusion, heat transfer), and accelerating the scientific revolution.

A shift occurred in the early twentieth century, when many of the laws of nature assumed to be immutable and universal were found not to be as universal as once thought. The laws of Newtonian mechanics were revised to include the theory of relativity, for which Einstein used a formulation based on a geometry inconsistent with Euclidean geometry, also previously assumed to immutably reflect the nature of the known physical world. Furthermore, many fields in the life and social sciences remained resistant to mathematization by models based on theory. These difficulties led to the emergence of a different form of mathematical modeling, now based on experimental data without necessarily having any explicit physical causes, a fundamental shift from the Platonist view held by St. Augustine, Galileo, and their successors. These models are mere tools designed by the human mind to study a specific situation. Under this guise, mathematics has become the "servant of the sciences," providing useful tools to study certain parts of the world, but not all of them, and certainly not with infinite accuracy. As Davis and Hersh state in *The Mathematical Experience,*

> The realization that physical theories may change or may be modified (Newtonian mechanics vs. Einsteinian mechanics, for example), that there may be competing theories, that the available mathematics may be inadequate to deal with a theory in the fullest sense, all this has led to a pragmatic acceptance of a model as a "sometime thing," a convenient approximation to a state of affairs rather than an expression of eternal truth. A model may be considered good or bad, simplistic or sophisticated, aesthetic or ugly, useful or useless, but one is less inclined to label it as "true" or "false."[14]

## Laws of Nature, Scientific Laws, and Mathematical Models

The terms *laws of nature*, *laws of science*, and *laws of physics* were coined in the seventeenth century to describe the laws formulated under the experimental methods of that century. These laws emerge after a large number of repeated scientific experiments reveal an underlying regularity, or pattern, in nature.

However, the word "laws" also reflects the prevalent Christian notion that they were ordered by a divine lawgiver. Thus a "law of nature" was more than just a summary of observable features of the world; it reflected the divine decision of the way the world was intended to behave. According to physicist Paul Davies, laws of nature are universal, absolute, omnipotent, and eternal.[15] Although the general belief in a divine lawgiver has been eroded in the secular scientific community, the properties assigned to the laws of nature coincide with those assigned to the Christian God.[16]

All known fundamental laws of nature are mathematical in form, and the earliest laws were also characterized by their simplicity, encapsulated in a single, often linear, mathematical equation. Although linear equations include only a small subset of all equations, until the twentieth century the practice of attempting to write laws of nature as mathematical equations focused almost exclusively on them. It is understandable that scientists would first turn to the simplest and most tractable form of equations, spurred by the widespread belief that laws of nature could not be anything but linear. The orderly and predictable nature of solutions of linear equations was consistent with the character of God who was assumed to have written them.

Although Newton believed that God was continually at work sustaining the order of the universe, his discoveries opened the way to deism, which grew rapidly during the eighteenth century. The laws of nature were considered to be expressions of the secondary causes with which God had empowered his creation and through which he orchestrated the world's order. With this point of view, God plays the role of a hands-off Master Engineer who rules his creation through deterministic laws, never needing to intervene, as he would have the infinite wisdom and power to make his laws perfect. Deists deny that God plays a direct role in continually sustaining the order of the universe: this function is shifted to nature itself.[17]

Until the end of the nineteenth century, nonlinear models were largely ignored because of the difficulty in solving them. Since the majority of nonlinear models cannot be solved analytically, it is only in the last few decades that computers have facilitated the implementation of numerical methods to approximate their solutions. By the early twentieth century, natural scientists had discovered that most of the principles that describe the world are nonlinear; thus the simple, early formulations of the laws of nature were progressively supplanted by more complicated, nonlinear ones. But these early formulations are not forgotten, as they remain good approximations of reality, accurate enough for practical purposes, and easier to work with than the more universal forms. For example, the Newtonian laws of classical mechanics remain a good approximation of reality in many cases, and are still called "laws." But the fundamental shift is in the knowledge that they are no longer a perfect expression of the reality of the physical world, but merely a conveniently simple approximation for particular applications in certain instances. However, these models are still connected to first principles, albeit imperfectly, so they are distinct from the data-driven models that are based on observations alone.

The fact that the models which approximate reality are still almost universally called "laws" in the twenty-first century is a source of great consternation among modern natural philosophers. Michael Scriven and Nancy Cartwright have attempted to clarify, expand, and redefine the terminology. According to them, the great majority of laws that were once thought to be laws of nature are, in fact, what they define as "scientific laws," approximations of the truth that apply only to idealized models of reality, always subject to the possibility—and often the actuality—of refutation, abandonment, and replacement.[18]

## Mathematics: The Tool That Makes the Invisible Visible

### *Hidden Patterns Revealed by Mathematics*

In the physical world, a distinctive sign that something has order is the presence of a pattern, a regular structure in space and/or time that seems to have been deliberately designed and placed there. As creatures of order created by a God of order, we are naturally drawn to notice these patterns; they resonate with our soul, and we find them intriguing, mysterious, and beautiful. The creeds of the Christian church begin with the affirmation that God created everything visible and invisible. He delights in his creation, and reveals himself to us through his created world. As the "science that makes the invisible visible,"[19] mathematics is the language of choice to describe anything in the world that obeys a certain order or pattern, but its true power is revealed in its ability to describe and study abstract structures and hidden patterns.[20] Humankind was created to perceive mathematical beauty, and the world was intentionally created with the abstract-concrete "fit" to benefit humankind.[21]

Although God remains hidden from human eyes, he provides enough light to reveal himself to those who search for him. Thus, he respects our freedom to either accept or reject him. In his *Pensées*, Pascal writes about the tension God maintains between his revelation and hiddenness, so that those who desire him may find him and those who do not want him would not be forced by bludgeoning evidence into believing against their will. He writes:

> Instead of complaining that God has hidden Himself, you must thank Him for revealing so much of Himself … It would not have been right for Him to appear in a way that is plainly divine and absolutely bound to convince all mankind; but it was not right either that He should come in a manner so hidden that He could not be recognized by those who sought Him sincerely. He chose to make Himself perfectly knowable to them; and thus, wishing to appear openly to those who seek Him with all their heart, and hidden from all who flee Him with all their heart, he tempered the knowledge of Himself, with the result that He had given signs of Himself which are visible to those who seek Him, and not to those who do not seek Him.[22]

Theologians distinguish between special and general revelation as the two ways by which God has chosen to reveal himself to humanity. Special revelation refers to the miraculous means God employs to make himself known to us, including most importantly his physical, human form in the person of Jesus Christ, and his written Word recorded in the scriptures by the inspiration of his Holy Spirit. Through general revelation, God reveals his existence, power, intelligence, and transcendent nature to all humanity, at all times and in all places through nature. Psalm 19:1–4 (NIV) declares,

> The heavens declare the glory of God;
> the skies proclaim the work of his hands.
> Day after day they pour forth speech;
> night after night they display knowledge.
> There is no speech or language
> where their voice is not heard.
> Their voice goes out into all the earth,
> their words to the ends of the world.

In Romans 1:20 (NIV), Paul declares,

> For since the creation of the world God's invisible qualities—his eternal power and divine nature—have been clearly seen, being understood from what has been made, so that men are without excuse.

According to these verses, we can know something of God by observing the universe. The knowledge of God is embedded in his creation; although God himself remains invisible,[23] we can perceive his handiwork in creation.[24] We see in this duality the idea put forth by Augustine and Galileo, among others, that the Bible and the Book of Nature were both written by God, books of equal importance, but to be studied and interpreted by different means.

Echoing Pascal's description of the balance between God's hiddenness and revelation, Davies, although a non-Christian, marvels at our ability as humans to discover the hidden laws of nature when we earnestly search for them.

> What is remarkable is that human beings are actually able to carry out this code-breaking operation, that the human mind has the necessary intellectual equipment for us to "unlock the secrets of nature" and make a passable attempt at completing nature's "cryptic crossword." It would be easy to imagine a world in which the regularities of nature were transparent and obvious to all at a glance. We can also imagine another world in which either there were no regularities, or the regularities were so well hidden, so subtle, that the cosmic code would require vastly more brainpower than humans possess. But instead we find a situation in which the difficulty of the cosmic code seems almost to be attuned to human capabilities … No feature of this uncanny "tuning" of the human mind to the workings of nature is more striking than mathematics, the product of the human mind that is somehow linked to the secrets of the universe.[25]

In his awe at our ability to use mathematics to decode the secrets of nature, Davies echoes the amazement of physicist Eugene Wigner, Nobel Laureate, who, in his paper "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," notes that all of nature seems to follow persistent, unchanging patterns that have been observed since ancient times, but that when these patterns are described in the form of mathematical equations, a miracle occurs.[26] Although Wigner does not mention God or draw any religious conclusions, he uses the word "miracle" twelve times and states that the effectiveness of mathematics is "something bordering on the mysterious … a wonderful gift which we neither understand nor deserve."[27]

Furthermore, writing the laws of nature as mathematical equations uncovers relationships between the laws themselves. Thus, mathematics has the property of making the invisible laws of nature visible, as well as making visible certain relationships between seemingly disparate natural phenomena. Mathematics unifies theories and amplifies our perception; in our attempts to study the behavior of a certain natural system, often we succeed not only in uncovering the law that describes it, but also in discovering several other laws that we had not expected to discover.

In his *Principia*, Newton showed that falling bodies on Earth's surface, the orbits of the Moon around Earth, and the satellites orbiting Saturn and Jupiter, as well as the orbits of the planets around the Sun, all behave like falling masses according to the law of gravity. Later he tied the phenomenon of the tides to the Moon's orbit, thereby demonstrating that celestial and terrestrial phenomena both obey the same physical principles. Similarly, Maxwell's equations were successful in explaining the relationship between electricity and magnetism—two invisible forces which had been observed for centuries, but required a set of mathematical equations to clearly understand the interplay between them. Maxwell's electromagnetic theory unlocked even further mysteries, explaining the nature of light and predicting the existence of radio waves. The Navier-Stokes equations describe the complex motion of fluids, as well as diverse phenomena such as weather patterns, ocean currents, water flow in a pipe, and air flow around the wing of a bird. Fourier's *heat equation*, originally written to explain the diffusion of heat in a solid, is now commonly referred to as the *diffusion equation*, because it has been found to apply equally to many other diffusion phenomena, such as the diffusion of a pollutant in water or a population in an ecosystem.

The discoveries—whether made accidentally or intentionally—of the many relationships between seemingly unrelated phenomena have encouraged scientists to search more intensively than ever for unifying theories, and even to consider the possibility of a Theory of Everything, a unique, all-encompassing theoretical framework that fully explains and links together all physical aspects of the universe. For Christians, this search is consistent with the desire to know God through his creation and with the belief that all facets of our universe were created by the same, unique God. Christianity is founded on the belief that no part of the universe was created at random, that every element of God's creation has a purpose and a function that is connected to every other in an orderly framework, and that it is God's will for humans to perceive him as creator through the orderly patterns of his creation.

## Order and Chaos

Christians have always believed that God created the natural world in such a way as to follow a prescribed order. As Newton studied the motion of planets in the late seventeenth century, he assumed that they followed an orderly pattern in space and time, as does much else in nature. Central to Newton's approach was the belief that the motions of the planets were "imprest" in them at some stage by an intelligent, calculating God who was adept at mathematics and engineering. Newton thought that no natural cause by itself could have produced the harmonious arrangement by which each planet along with its satellites was endowed with precise locations, masses, and velocities that it now had, nor could it have given rise to the mathematically precise laws that described their interaction. Like Kepler before him, he believed that he was a privileged expert—a mathematically adept "priest" authorized to decipher the mathematical texts used by God.[28] Although Laplace, Kant, and many others offered alternative explanations for the order of the natural world, historically, Christians have sought theological reasons for the apparent order of creation.

Newton and Leibniz's development of calculus and differential equations proved to be extremely effective in teasing out the regularities of complex patterns in phenomena that would appear, at first sight, not to have any, such as the motion of planets—a pattern which had baffled astronomers for centuries. With Kepler's and Newton's laws, all of these mysteries are now elucidated. A mathematician need only solve the equations to be able to predict the exact position, velocity, and acceleration of any planet in any direction at any time.[29]

By the end of the nineteenth century, many natural patterns of the world were successfully explained and represented as mathematical laws: motion, hydrodynamics, electricity, magnetism, light waves, and so on. However, there remained some systems, especially in the life and social sciences, that resisted all attempts to explain and predict their behavior by mathematics, displaying a seemingly total absence of any kind of perceivable pattern, earning them the label of *chaotic*. In vain, scientists turned to more and more complex mathematical formulations in their attempts to write the laws and thereby predict and control these systems, until they discovered that the unpredictable nature of these systems was not caused by an incorrect or oversimplified mathematical formulation of the laws but, rather, by the mathematical formulation itself.

The earliest attempt at defining chaos dates back to 1887, as Poincaré studied the disorderly orbits that arise in the dynamics of three attracting bodies, despite the relative simplicity of the underlying equations. This led him to the qualitative definition of *chaotic systems* as those whose behavior appears to be disorderly and random, even though their behavior can be modeled by well-defined, deterministic equations. In addition, Poincaré discovered that the solutions to these equations also have the intrinsic property of being extremely sensitive to initial conditions, making predictions quasi-impossible. In the 1960s, Edward Lorenz discovered these same distinctive features in the nonlinear equations he was using to model weather patterns.

Poincaré suspected that what appeared to be disorderly was, in fact, a pattern so complex that its orderly features could not be seen by the human eye. His suspicions were confirmed with the advent of computing tools, which made it possible to visualize the solutions of equations that model chaotic systems and reveal deep structures of order hidden beneath their seemingly random and unpredictable behaviors. In 1975, mathematician James A. Yorke coined the term *chaos theory* to describe the systematic study of chaotic systems from a mathematical point of view. Since then, chaos theory has been applied to the study of systems as diverse as meteorology, thermodynamics, cryptography, ecology, physiology, and epidemiology.[30]

Currently, the most common method of quantifying the amount of chaos in a dynamical system is by calculating the global Lyapunov exponent (GLE) of the system, a dimensionless number that characterizes the rate of exponential separation of infinitesimally close orbits, with a positive GLE indicating that the system is chaotic.[31] The most recent definition of chaos (2015) introduces the related concept of *expansion entropy* of a system, in an attempt to provide a definition that is quantitative, general, simple, and computable.[32] It is essential to note that all the definitions and descriptors of chaos in a system, whether qualitative or quantitative, refer to properties of the mathematical models used to describe them.

The recent development of chaos theory and nonlinear dynamics has brought the recognition that nonlinear systems are all around us. Although a finite-dimensional dynamical system must be nonlinear to exhibit chaos, it need not be "complex"; indeed, some models can exhibit chaos despite being deceivingly simple, and the same model may exhibit chaotic and nonchaotic solutions under different conditions.

Now that scientists are actively looking for chaos, they find it everywhere—in economics, biology, epidemiology, and other disciplines. As is the case with all mathematical modeling, the strange attractors found in such widely different areas all exhibit the same characteristics; thus, learning about them in one area of interest suddenly unlocks mysteries in others. Freed from the constraining requirements of linear models, it is surprising that we actually find any systems in the world that can be adequately modeled by linear equations at all.

The definitions of chaos outlined above all attempt to describe the same thing, either by describing a qualitative behavior or by quantifiable measurements. For over a century, mathematicians have continued to come up with new definitions, each one shedding just a bit more light on what constitutes this elusive concept. There is no satisfactory single definition of chaos, because of its many different manifestations in different situations. Trying to lock the definition in a box is futile; the concept is just too big for one single definition. When we study a chaotic system, we must make ourselves blind to the entirety of the concept and focus on only a limited number of aspects of the system's chaotic nature.[33]

Christians are familiar with the challenge of defining an unperceivable concept by means of a set of images and partial definitions, each one shedding light from a different angle on an elusive concept, each one increasing our understanding and clarifying our vision of that which we cannot see. Christ's teachings on the nature of the kingdom of God (or kingdom of heaven) employ a series of parables to describe projected images of the kingdom that we can perceive in our world. In his parables, Jesus describes the kingdom of heaven as a hidden treasure, a fine pearl, leaven in bread dough, a grain of mustard seed, and a fishing net. Through these images, we catch different glimpses of the kingdom, all of which will come together in the last days when "the kingdom of the world has become the kingdom of our Lord and of his Christ …" (Rev. 11:15b, ESV). In this sense, the complex nature of chaos provides a pranalogical example of faith integration,[34] that is, a practical analogy that informs our Christian view of heavenly reality.

The parallel between the kingdom of God and chaos moves beyond their definitions alone to the order hidden within them. Just as the chaotic behavior of a system points to structures of hidden order embedded within it, the parables of Jesus that describe the kingdom of God all point to a prescribed order in his kingdom, a way of how things function in God's economy and according to which we Christians should order our lives in this world. Research has revealed that the hidden order of chaos often plays a role in healthy, life-sustaining systems. Recent advances in mathematical physiology have discovered that the occasional chaotic patterns exhibited in a human heart rate are not only innocuous, but actually necessary for its functioning.[35] Similarly, mathematical neuroscientists who study epilepsy have discovered that they can predict the onset of an epileptic seizure by detecting the exact time when the electrical activity in the brain moves away from its naturally chaotic state.[36] Thus, just as Jesus's parables reveal that his kingdom is at work accomplishing the Father's purposes, the orderly patterns in chaos prove to be purposeful in sustaining life and in promoting human well-being in this world.

## Nonlinearity in the World and Order in God's Creation

Although the world is fundamentally nonlinear, described by mathematical equations that contain within them the potential for chaos, the prevalence of chaos is surprisingly rare. A large number of natural phenomena lend themselves to be studied extensively, and often with great precision, using nonchaotic mathematical models that are described by patterns which are unmistakably orderly—reminiscent of the orderly character of the God who created them and sustains them. The rarity of chaos despite the ubiquity of nonlinearity presents a mystery: the world seems to be fine-tuned to maintain the delicate balance between chaos and order that is necessary for human flourishing.

In Isaiah 45:12 (ESV), God says,

> I made the earth and created man on it; it was my hands that stretched out the heavens, and I commanded all their host,

and verse 18 reads,

> For thus says the LORD, who created the heavens (he is God!), who formed the earth and made it (he established it; he did not create it empty, he formed it to be inhabited!): "I am the LORD, and there is no other."

These verses, along with many others, reveal that God has a purpose for his creation. He created humans to inhabit the earth and interact with his creation, just as he desires humans to have a relationship with him. Our interaction with a world that displays evidence of a sustained order leads us to consider the author of this order, a God who reaches out to us in love and reveals his character of order and goodness to those who search for him.

In *Divine and Contingent Order*, Thomas Torrance argues that the development of empirical science rests on the Judeo-Christian doctrine of God as Creator of the orderly universe, who brought it into existence out of nothing and continuously preserves it from lapsing back into chaos and nothingness. Thus, the cosmos is *contingent*, freely created by God, having an existence, freedom, and rational order of its own, while still dependent on him. This claim of contingency, once obscured by Newtonian physics, is now once again drawing attention to itself with modern discoveries in relativity and quantum theories. The universe can be found to be consistently rational only if it is dependent on a creative rationality behind it. The very fact that we derive our understanding of the world from experiments, theories, and mathematical models, implies that we assume the world to be contingent upon God and his character of order. Torrance says,

> The contingency of the creation as it derives from God is inseparably bound up with its orderliness, for it is the product not merely of his almighty will but of his eternal reason. It is not only the matter of the universe, therefore, but its form that comes into being out of nothing, for under the rational creativity of God, matter and form are fused indivisibly together from the very beginning. There is no contingency without order and no order without contingency, for contingency is inherently orderly and order is essentially contingent.[37]

In *The Lost World of Adam and Eve*, John Walton echoes Torrance's claim that there is a connection between the order of nature and the Christian doctrine of creation, which, consistent with Jewish teachings, claims that God is not only the author of life, but also the sustainer of life, and, indeed as Jesus claimed, life itself (John 14:6). According to this doctrine, when God rested on the seventh day, he rested only from his role as *creator* of the world, but not from his role as *sustainer* of life within it. God is always at work in sustaining life, even while resting from creation, as can be seen when Jesus continues to perform life-sustaining miracles, even on the Sabbath.

Torrance and Walton also draw a connection between creation declared as "good" and the order that God formed in the midst of non-order. Walton refers to the Garden of Eden as a "sacred space" of order, and to Adam as a "priest" of that space, with the mandate to extend the order of that sacred space throughout the world.[38] On the other hand, the absence of order can be related to consequences of the Fall—to evil, decay, destruction, and death. Christian doctrine and the scriptures claim that God is essentially good, so he does not create anything for the purpose of being evil. But he has the power to redeem, restore, and re-order, for good purposes, that which has been corrupted by evil. Torrance says,

> The objective divine order of the good and rational does not merely negate evil, but lays hold of it in a re-creative and re-ordering movement with a view to mastering it, repairing that which is disordered, and making it serve a fuller dimension of order than might have been possible otherwise.[39]

The miraculous ways in which God not only replaces something evil with good, but also restores its goodness to a fuller measure than before its corruption by evil, can be seen in the scriptures from Genesis to Revelation. In Genesis, after being sold into slavery by his brothers, Joseph is elevated to a greater position in Egypt than might have been possible otherwise. God not only restores the relationship between Joseph and his brothers, but also brings about the salvation of both Egypt and Israel in the process. In the New Testament, the risen Lord not only restores his relationship with Peter, the disciple who had denied him, but also entrusts him with the leadership of his church. Ultimately, Jesus claims victory over evil and death itself and reveals his purpose of restoring the world to good by his resurrection, which would not have been manifested without his death caused by the evil of the cross.

Just as Walton views Adam as a priest over the sacred space of the Garden of Eden, Torrance views humans as mediators through which God not only redeems his creation, but also brings it to a greater level of good and order. He says,

> It is his [man's] task to save the natural order through remedial and integrative activity, bringing back order where there is disorder and restoring peace where there is disharmony. Since it is through interaction with man, the culminating point of rational order, that nature unfolds and develops its possibilities, it should not only be "pacified" through man, … but in a significant sense also be "humanized," that is, through human cultivation and development, nature should bring forth forms of order and beauty of which it would not be capable otherwise.[40]

But humans, being inclined toward evil by their fallen nature, are also capable of infecting nature with their own disorder, even as they perform their priestly functions. Conversely, nature itself is capable of exhibiting structures of order independently of humans, and thereby it brings about good to human life. Torrance says,

> In and through the profound interconnection of order and disorder in which man and nature share together, nature constantly reveals surprising new possibilities in spite of man, which can have a healing and rectifying effect on him, for after all it is much more in man himself than in nature that evil has lodged itself.[41]

This point of view is consistent with the stability we observe in nature's life-sustaining structures of order, such as the surprising life-giving effects attributed to seemingly disorderly chaotic patterns exhibited in a human heart rate or electrical brain activity. In this sense, the tension between good and evil lodged in the human soul is mirrored in the mysterious tension between order and disorder exhibited by chaotic systems in nature. As created beings, we can look to nature and creation as a mirror into our own human nature, while simultaneously living out our role as priests over it and obeying our God-given mandate of extending the order of the original created sacred space throughout the world.

## Conclusion

The historical narrative of this article demonstrates that mathematics, "queen and servant of the sciences," has always played a central role in humanity's attempts to understand the world. Just as God brought order out of the primordial chaos and created a world for us to live in and care for, we are drawn to order and compelled to bring order and understanding to our observations of the world we live in. So great is the effectiveness of mathematics to represent the laws of nature and reveal patterns in places where none were apparent, that many otherwise-nonreligious scientists and philosophers throughout history have assigned a divine influence to, or at least spoken in religious terms of, its ability to enhance our knowledge of the world. This mystery is nowhere more apparent than in the process of mathematical modeling, by which a human-built model is often surprisingly effective to replicate observable patterns and to reveal concealed structures of order and unforeseen relationships with other seemingly unrelated systems. Just as the power of mathematics lies in its ability to reach beyond the material world into the imperceptible realm of abstraction, so we Christians know that the material world is a reflection of the God who created it. In particular, the patterns of order revealed by mathematical models are a visible manifestation of the orderly character of the Creator, a powerful witness to one of his most fundamental attributes.

Mathematical models teach us about God and enhance our worship as we strive to behold him and put ourselves in a state of awe and adoration of him. But because of God's hidden nature, we cannot yet behold him exactly as he is, although as children of God, we are being conformed to his image, and we have the promise though scripture that the day will come when we shall behold him face to face. Beholding God requires not only perceiving him through our five senses, but also perceiving his character, his attributes, and his invisible qualities. Mathematics, the science that makes the invisible visible, can be enlisted as a tool to enhance our beholding of God.

One element about God's character that transpires from the exercise of mathematical modeling is his reliability and faithfulness. We humans intuitively feel comforted by the reliability of mathematics and by the fact that the Creator designed his creation to follow certain laws—laws that can be described and studied by mathematics—revealing much of God's faithful and reliable character and his desire that the order of the universe would reflect his character and point to him. Under the old covenant, God guides his children by the Law and the Prophets, reaching out to us and establishing our relationship to him through promises, most of which are stated in the same cause-and-effect ("if …, then …") syntax in which mathematical theorems are stated. Just as we come to trust in the reliability of the laws of nature by beholding the repeating patterns of the physical elements of this world, we can trust in the fulfillment of God's promises as laid out in his laws and covenants. This idea is summarized by Gary De Young in his following statement:

> The consistency reflected in laws and patterns is a reflection of God's upholding hand in creation. As we seek to understand the world around us, we see the resulting consistency in creation and transfer this property to our reasoning. This property, in turn, leads to the general belief in the reliability of mathematical knowledge. Thus mathematical knowledge is ultimately based on God's providential and sustaining hand in creation.[42]

Mathematical modeling also points to the uniqueness of the Creator, as mathematical models reveal how different phenomena found in creation can be described by the same model. For example, light and sound waves are modeled by the same equation, so from a mathematical point of view, the phenomena of sight and sound transmission are exactly the same. The consistent and regular discoveries of mathematical similarities in different areas, which have enabled the unification of many physical theories throughout the centuries, point to a common and unique source and author of all.

When we use mathematical modeling to study, understand, predict, and ultimately control the outcome of a physical system for the good of humanity, we are effectively responding to the cultural mandate expressed in Genesis 1:26–28 and Psalm 8, by which God commands his children to subdue and replenish the earth, and, as Walton explains, to extend the order of the sacred space throughout the world.[43] Mathematical models are used in environmental science to understand and control the harmful effects of groundwater and atmospheric pollution; in meteorology, to predict storms and typhoons, allowing for timely mitigation or evacuation measures; in epidemiology, to understand and control the spread of diseases; and in cell biology, to understand cellular mechanisms and thereby design medical treatments. Through mathematical modeling, we Christians have an opportunity to follow God's command to protect and uphold life and to care for our world, as well as the responsibility to use this tool wisely.

Finally, the language of mathematics lends itself naturally to extensions beyond the physical world of our perceptions. Although many advances in mathematics were (and continue to be) motivated by the desire to find models that accurately reflect the reality of this world, many theories of mathematics have been developed by extending, generalizing, and abstracting the mathematical tools originally developed for the use of mathematical modeling. Analysis, topology, and non-Euclidean geometry are examples of rich mathematical theories that have freed themselves from their ties to the physical world to explore abstract worlds beyond our limited perceptions. Thus, mathematics equips us humans to look beyond the visible, while maintaining a mental anchor in the visible world. For example, we use the word *hypersphere* to describe an object that we humans will never be able to behold in this world, but since it has the same mathematical properties of a familiar three-dimensional sphere, we can attempt to accurately imagine a hypersphere in our mind's eye. God calls us to the same mental exercise every time he uses familiar words and images of our world to describe the yet unperceivable realities of the world to come and in which we are called to live with him. ⌂

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## Notes

1Glenn Ledder, *Mathematics for the Life Sciences: Calculus, Modeling, Probability, and Dynamical Systems* (New York: Springer, 2013), 2.

2Keith Devlin, *The Language of Mathematics: Making the Invisible Visible* (New York: W. H. Freeman, 1998), 3.

3Philip J. Davis and Reuben Hersh, *The Mathematical Experience* (Boston, MA: Birkäuser, 1981), 172–73.

4John H. Walton, *The Lost World of Genesis One: Ancient Cosmology and the Origins Debate* (Downers Grove: IL: InterVarsity Press, 2010).

5Jason Wilson, "Integration of Faith and Mathematics from the Perspectives of Truth, Beauty, and Goodness," *Perspectives on Science and Christian Faith* 67, no. 2 (2015): 100–110.

6Douglas C. Phillippy, "A Pranalogical Approach to Faith-Integration with Students," *Perspectives on Science and Christian Faith* 67, no. 2 (2015): 89–99.

7Russell W. Howell and W. James Bradley, eds., *Mathematics in a Postmodern Age: A Christian Perspective* (Grand Rapids, MI: Wm. B. Eerdmans, 2001).

8Derek Abbott, "The Reasonable Ineffectiveness of Mathematics," *Proceedings of the IEEE* 101, no. 10 (2013): 2147–53, https://doi.org/10.1109/JPROC.2013.2274907.

9Howell and Bradley, *Mathematics in a Postmodern Age.*

10Amir Alexander, *Infinitesimal: How a Dangerous Mathematical Theory Shaped the Modern World* (New York: Scientific American, 2014).

11Devlin, *The Language of Mathematics.*

12Mario Livio, *Is God a Mathematician?* (New York: Simon & Schuster, 2009).

13Howell and Bradley, *Mathematics in a Postmodern Age,* 65.

14Davis and Hersh, *The Mathematical Experience,* 86–87.

15Paul Davies, *Mind of God: The Scientific Basis for a Rational World* (New York: Simon & Schuster, 1992), 73.

16However, this definition is controversial in today's scientific community, as many scientists disagree that a law must be eternal or immutable. The definition is also problematic among many Christians who believe that only God is eternal, so that the laws, being created things, are not eternal. See Colin E. Gunton, *The Triune Creator: A Historical and Systematic Study* (Grand Rapids, MI: Eerdmans, 1998).

17Colin E. Gunton, *The One, The Three, and The Many: God, Creation and Culture of Modernity* (New York: Cambridge University Press, 1993).

18Nancy Cartwright, *The Dappled World: A Study of the Boundaries of Science* (New York: Cambridge University Press, 1999).

19Devlin, *The Language of Mathematics,* 10.

20Ibid.; and Max Tegmark, *Our Mathematical Universe: My Quest for the Ultimate Nature of Reality* (New York: Alfred A. Knopf, 2014).

21Wilson, "Integration of Faith and Mathematics from the Perspectives of Truth, Beauty, and Goodness," 105.

22Charles Sherrard MacKenzie, *Pascal's Anguish and Joy* (New York: Philosophical Library, 1973), 70.

23The issue of the hiddenness of God is somewhat complicated: many have argued, based on Romans 1, that humans repress the truth of God that is revealed through creation.

24R. Albert Mohler Jr., "The Way the World Thinks: Meeting the Natural Mind in the Mirror and in the Marketplace," in *Thinking. Loving. Doing.: A Call to Glorify God with Heart and Mind*, ed. John Piper and David Mathis (Wheaton, IL: Crossway, 2011), 47–66.

25Davies, *Mind of God,* 148–49.

26Eugene P. Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," *Communications on Pure and Applied Mathematics* 13, no. 1 (1960): 1–14.

27Ibid., 14.

28Robert C. Bishop, "God and Methodological Naturalism in the Scientific Revolution and Beyond," *Perspectives on Science and Christian Faith* 65, no. 1 (2013): 10–23; and Rob Iliffe, "Newton, God, and the Mathematics of the Two Books," in *Mathematicians and Their Gods: Interactions between Mathematics and Religious Beliefs*, ed. Snezana Lawrence and Mark McCartney (New York: Oxford University Press, 2015), 121–44.

29In reality, our solar system may actually be a chaotic system, which implies that there may be a temporal horizon beyond which the predictability breaks down, as will be explained later. See Gerald Jay Sussman and Jack Wisdom, "Chaotic Evolution of the Solar System," *Science* 257, no. 5066 (1992): 56–62.

30N. Katherine Hayles, "Introduction: Complex Dynamics in Literature and Science," in *Chaos and Order: Complex Dynamics in Literature and Science*, ed. N. Katherine Hayles (Chicago, IL: University of Chicago Press, 1991), 1–36.

31Evelyn Sander and James A. Yorke, "Chaos and Its Manifestations," *SIAM News* 48, no. 7 (September 2015): 8.

32Brian R. Hunt and Edward Ott, "Defining Chaos," *Chaos: An Interdisciplinary Journal of Nonlinear Science* 25, no. 9 (2015): 097618, https://doi.org/10.1063/1.4922973.

33Sander and Yorke, "Chaos and Its Manifestations."

34Russell W. Howell, "The Matter of Mathematics," *Perspectives on Science and Christian Faith* 67, no. 2 (2015): 74–88.

35Leon Glass, "Introduction to Controversial Topics in Nonlinear Science: Is the Normal Heart Rate Chaotic?," *Chaos* 19, no. 2 (2009): 028501, https://aip.scitation.org/doi/10.1063/1.3156832; and Christopher Lampton, *Science of Chaos: Complexity in the Natural World* (Collingdale, PA: Diane Publishing, 1992).

36Tahar Haddad et al., "Temporal Epilepsy Seizures Monitoring and Prediction Using Cross-Correlation and Chaos Theory," *Healthcare Technology Letters* 1, no. 1 (2014): 45–50, https://doi.org/10.1049/htl.2013.0010.

37Thomas F. Torrance, *Divine and Contingent Order* (New York: T&T Clark, 2005), 109.

38John H. Walton, *The Lost World of Adam and Eve: Genesis 2–3 and the Human Origins Debate* (Downers Grove, IL: InterVarsity Press, 2015), 104.

39Torrance, *Divine and Contingent Order,* 110.

40Ibid., 112.

41Ibid., 113.

42Gary De Young, "Perspective on Mathematical Modeling," *Pro Rege* 38, no. 4 (2010): 3–4.

43Walton, *The Lost World of Genesis One;* ____, *The Lost World of Adam and Eve*.