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Cultivating Mathematical Affections: The Influence of Christian Faith on Mathematics Pedagogy

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The goal of this article is to make the case that Christian faith has an opportunity to impact the discussion on best practices in mathematics, not primarily through the cognitive discussion on objectives and standards, but through the affective discussion on the formation of values, the cultivation of mathematical affections – not merely knowing, but also loving, and practicing the truth, beauty, and goodness inherent in mathematics. First, I will outline the work being done on affect in mathematics education, examining what values are actually endorsed by the community of mathematics educators. After summarizing this work on affect, it will be clear that, even in the words of leading researchers, the field is lacking any cohesive, formal approach to analyzing and assessing the affective domain of learning. Secondly, I will argue the thesis that Christian faith offers solutions to the frustrations and shortcomings admitted by researchers on affect in mathematics education. Christian faith offers insight into how mathematical affections might actually be shaped. Here I will draw heavily on the work of philosopher James K.A. Smith and make explicit connection between his work and the mathematics classroom. Finally, I will conclude with a call to action discussing how we as Christian educators might begin to have fruitful contributions to and dialogue with the current research being done in mathematics education.

"When am I ever going to use this?" is a statement that is often on the ears of every mathematics teacher. Please notice that I referred to this as a statement and not as a question. It has been my experience as an educator (and validated through many conversations with fellow colleagues in the profession) that the true nature of "When am I ever going to use this?" is typically not a legitimate inquiry as to the appropriate timeframe in which the student will eventually apply the material at hand in a "real-life" scenario. Rather, the phrase more often arises as a statement. It is a statement of frustration. It is the culmination of confusion and stress, and it usually serves as an exclamation of their withdrawal from the mental activity at hand. In other words, the answer to the question, "When am I ever going to use this?" has already formed in the student's mind as "I will never use this, so learning it is a waste of time."

The real issue being raised by students is not one of application, but rather one of *values*. I have found that the best response to such a statement/question is to translate it into what I believe the student truly meant to express: "Why should I *value* this?" I believe that this is the question of ultimate concern in the mathematics classroom, and this is the question upon which the Christian faith exerts the greatest influence on the pedagogy of mathematics.

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In his introductory essay, Russell Howell notes the recent explosion of pedagogical practices in mathematics being driven by the technological revolution. Howell calls for Christian perspectives in evaluating these current trends in education. This article is meant to answer that call by suggesting that a Christian perspective can steer the analysis of pedagogical practices from a technological/application lens to a lens focused on the values inherent in mathematics education.

Let us begin by considering why students might phrase their value judgment in terms of the practicality of mathematics? Utility is the language in which the students' culture—including their teachers—has conditioned them to speak. Now, to be sure, application is certainly important to consider in the teaching of mathematics as a powerful pedagogical tool. Application should not be ignored. The goal of this article is simply to call our attention to the deeper issue: our students' desperate desire to find something of value in this world and specifically in the mathematics classroom.

As educators, we play a central role in the formation of students' value systems. As Christian educators, the framework of inculcating values in students and the pedagogical steps we take to achieve this goal are motivated and guided by the transforming grace of the gospel and the historical tenets of the Christian faith. I would argue that as Christian mathematics educators we are afforded a unique venue to act missionaly in contributing answers to a very real need in mathematics education research and practice. I will argue that the question most in need of addressing in the mathematics classroom today is not on the level of cognition—it is not a question of what information (be it in the form of national standards or daily class learning objectives) needs to be passed on to our students. Rather, the question most in need of addressing in the mathematics classroom today it is on the level of the affections—it is a question of formation, of what type of people we desire our students to be, of how we answer, "Why should I *value* this?"

From a Christian perspective, learning has little meaning unless it produces a sustained and substantial influence not only on the way people think, but also on how they act, feel, and ultimately worship. There is ample opportunity now, perhaps more so than ever, for Christian mathematics educators to influence the development of what I will term mathematical affections: not merely knowing, but also loving, and practicing the truth, beauty, and goodness inherent in mathematics.

Values in Mathematics Education: Neglecting Mathematical Affections

Education is inherently value laden.[2] There might be some educators who feel that discussion of values and virtues has no place in an academic setting, especially a public/secular one. The mathematics classroom even more so has a tendency to be seen as values neutral. If we as Christian educators are going to be in a dialogue with secular mathematics educators in any meaningful way, it is important to first make clear that education, and specifically mathematics education, is inherently value laden. It is not a question of "*Are* you teaching values?" but, rather, "*Which* values are you teaching?" Even the statement "We should not be focusing on values in the classroom" is itself a value-based statement.[3] The good news is that the door is open, so to speak, for this values-in-mathematics conversation to begin in a substantive manner.

Noted philosopher of mathematics education Paul Ernest dedicates an entire chapter of his book *The Philosophy of Mathematics Education* to demonstrating the value-laden nature of mathematics, noting that "within mathematics there are implicit values."[3] Now, *where* exactly those values derive from may be up for debate, but that is beyond the scope of this article.[4] For our purposes, the simple recognition that values exist in mathematics (and by extension in the mathematics classroom) is a foundational starting point.

Beyond Ernest, value language is scattered throughout national policy documents on the teaching of mathematics.[5] We see this language in national standards such as the NCTM (National Council of Teachers of Mathematics) *Professional Standards for Teaching Mathematics* (1991): "Being mathematically literate includes having an appreciation of the *value* and beauty of mathematics as well as being able and inclined to appraise and use quantitative information" (emphasis added).[6] Mathematical literacy, according to the NCTM, involves not merely

using quantitative information, but also giving the discipline of mathematics its proper *value*. Another national policy document, *Adding It Up: Helping Children Learn Mathematics*, a report published by the National Research Council, argues that mathematical proficiency has five strands, one of which is termed "productive disposition." Productive disposition is defined as "the habitual inclination to see mathematics as sensible, useful, and worthwhile."[7] The current Common Core State Standards Initiative grounds its standards for mathematical practice in part upon the same five proficiency strands proposed by the National Research Council.[8] To be mathematically *proficient* (not just literate), the valuation of mathematics must lead to a habit of seeing mathematics as worthwhile—that is, valuable to justify time or effort spent. Mathematics education is inherently value laden.

So the conversation now moves from addressing the existence of values to the questions, *which* values? *where* do they come from? and *how* do educators instill them into students? It is this last question, how to instill values into students (or, in other words, how to cultivate mathematical affections), which this article focuses on.

In examining the current perspectives on affect in mathematics education, I will construct my argument as follows: (1) research on affect in mathematics education tends to misrepresent what affect actually is; (2) this misrepresentation leads to a body of research that largely attempts to address affect in terms of cognition; and (3) the confusion that exists in 1 and 2 results in a shaky foundation (if any at all) for building a discussion as to how to go about cultivating mathematical affections in students. This will set the stage for discussing the impact of Christian faith upon this issue later in this article.

As a first step, consider a foundational document for composing the learning objectives and outcomes of an academic course: Bloom's Taxonomy (figure 1).[9]

Behaviors from simple to complex	Cognitive Domain (Mental Activity)	Affective Domain (Character and Conscience)	Psychomotor Domain (Physical Activity)
	Creating (compose, originate, design, invent)	Characterizing (revise, require, rate, avoid, resist, manage, resolve)	Originating (arrange, build, construct, initiate)
	Evaluating (judge, criticize, evaluate, appraise, recommend)	Organizing (discuss, theorize, formulate, balance, prioritize)	Adapting (alter, rearrange, vary, revise)
	Analyzing (compare, classify, rank, infer, extrapolate)	Valuing (measure proficiency, subsidize, support, debate)	Mechanizing (assemble, calibrate, fasten, measure, mend)
	Applying (organize, solve, generalize, produce)	Responding (comply, follow, commend, volunteer, acclaim, engage in)	Guided Responding (copy, trace, reproduce, react)
	Understanding (explain, infer, interpret, summarize, paraphrase)	Receiving (differentiate, accept, listen for, respond to)	Setting (begin, move, show, state)
	Remembering (recite, quote, list, define)		Perceiving (choose, identify, relate, select)

Figure 1. Bloom's Taxonomy.

The title "Bloom's Taxonomy" is typically used only in reference to the cognitive (mental/knowledge) domain of learning,[10] while the affective (heart/feeling) domain of learning is more specifically referred to as "Krathwohl's Taxonomy," due to the work of David Krathwohl.[11] The affective domain is not simply based on subjective emotions (though emotion may play a small part in affective learning); rather, it is about demonstrated behavior, attitude, and characteristics of the learner[12]—all of which are deeply rooted to success in the mathematics classroom, and all of which are largely misunderstood in mathematics education research.

A quick glance at this chart will reveal that "application" falls under the cognitive domain of learning while "valuing" falls under the affective domain of learning. So when a student asks, "When am I ever going to use this?" (but really means, "Why should I value this?") and a teacher responds to the surface level application question without digging any deeper, the student receives a cognitive response to an affective question. Such a reply also implicitly reinforces in the student's mind that value stems from utility. No wonder students are confused as to why they should value mathematics: their teachers, by and large, are confused as well. Why? Because, even though affective language permeates national published standards on the teaching of mathematics as an ideal that we should strive to inculcate into students, there is little discussion on *how* to go about accomplishing this task.

Affective learning tends to be seen as subjective and emotional; therefore it does not fit well with the objective mindset we have about mathematics teaching and learning. In a special issue of *Educational Studies in Mathematics* devoted entirely to affect in mathematics education, Rosetta Zan states:

> Affect has been a focus of increasing interest in mathematics education research. However, affect has generally been seen as "other" than mathematical thinking, as just not part of it. Indeed, throughout modern history, reasoning has normally seemed to require the suppression, or the control of, emotion.[13]

This quote reveals the tendency in mathematics education to see affect as equivalent with emotions. If affect is indeed synonymous with emotions (or at least viewed that way by the teacher), then it is a very subjective domain and much trickier to navigate than the (at least seemingly) objective cognitive domain. Application of mathematical concepts is much more objective, and something educators are much more familiar with, in the context of mathematics teaching as compared with values. So why do students not, by and large, value mathematics for its own sake, for the beauty, truth, and goodness it reveals? Why do students not look beyond utility to find value? Because their teachers, following the lead of their own teacher preparatory programs and mathematics education research, have taught them otherwise.

The misconception of what affect actually is, and has been historically defined as, has led to a body of research that approaches affect primarily through the lens of cognition—an area that can be analyzed and assessed much more tangibly and objectively. I have organized my summary of this research to follow the levels of Krathwohl's affective domain of learning as illustrated in figure 1: receiving, responding, valuing, organizing, and characterizing. As Christian educators, I believe that it may be more appropriate to view Krathwohl's levels as being grouped into two strands: instilling values and practicing virtues.

In a foundational article on affective learning in mathematics in the *Handbook of Research on Mathematics Teaching and Learning*, Douglas McLeod states:

> Affective issues play a central role in mathematics learning and instruction. When teachers talk about their mathematics classes, they seem just as likely to mention their students' enthusiasm or hostility toward mathematics as to report their cognitive achievements. Similarly, inquiries of students are just as likely to produce affective as cognitive responses, comments about liking (or hating) mathematics are as common as reports of instructional activities. These informal observations support the view that affect plays a significant role in mathematics learning and instruction. Although affect is a central concern of students and teachers, research on affect in mathematics education continues to reside on the periphery of the field … All research in mathematics education can be strengthened if researchers will integrate affective issues into studies of cognition and instruction.[14]

This 1992 article is still applicable today. McLeod goes on to cite efforts to reform mathematics curriculum and those reform efforts' emphasis on the role of affect. The specific documents he cites are the

NCTM *Professional Standards for School Mathematics* (1989) and the National Research Council's report on mathematics education titled *Everybody Counts* (1989). A shift forward in time to statements made in the NCTM's *Professional Standards for Teaching Mathematics* (1991 and 2000) and the National Research Council's report *Adding It Up: Helping Children Learn Mathematics* (2001), reveals that a strikingly similar argument to McLeod's can be made today, with noticeably unchanging language of national published standards, and the similar situations of finding research on affect "on the periphery." It can be argued that McLeod's work has yielded few results and is in need of an adjustment. You will also notice the concluding remark on integrating the study of affect into "studies of cognition." As we will see below, this is the dominant approach taken by researchers in the field and the primary reason that McLeod's work has yielded little by way of results.

The strand of "values" that I propose for organizing our thoughts on affect covers Krathwohl's categories of receiving (the student's willingness to attend to particular phenomena of stimuli), responding (active participation on the part of the student), and valuing (the worth or value a student attaches to a particular object, phenomenon, or behavior). The term Values is essentially referring to developing an attitude toward a particular subject (in this case mathematics). Support for offering this classification of values stems from the NCTM *Professional Standards for Teaching Mathematics* (1991) quoted above. To see how much of the work being done under the strand of instilling values is motivated primarily by cognitive issues, we can turn to another quote from McLeod:

> The emphasis on affective issues (in the U.S. reform movement in mathematics education) is related to the importance that the reform movement attaches to higher-order thinking. If students are going to be active learners of mathematics who willingly attack non-routine problems, their affective responses to mathematics are going to be much more intense than if they are merely expected to achieve satisfactory levels of performance in low-level computations' skills.[15]

This quote as well as numerous examples from research being done on affect[16] seem to indicate a trend that much of the research on developing values[17] in the mathematics classroom is largely driven by increased attention to higher-order cognitive thinking and its impact on the affections of students, rather than vice versa. This ordering of the cognitive as primary and the affective as subservient to the cognitive tends to lead to discrepancies in actually defining what we are talking about (namely, "beliefs" language is classified under affective research, though in actuality it can be argued that beliefs are much more cognitive in nature).[18] In light of this body of research, Anna Sfard writes:

> Finally, the self-sustained "essences" implied in reifying terms such as knowledge, beliefs, and attitudes constitute rather shaky ground for either empirical research or pedagogical practices—a factor of which neither research nor teachers seem fully aware.[19]

It is difficult to develop a robust body of research on affect when it is unclear what exactly affect is and what terminology should be used.

The next strand of affective learning that I proposed, "virtues," is sadly not on any stronger footing in current research than that of "values." The proposed strand of "virtues" covers Krathwohl's categories of organization and characterization. "Virtues" simply refers to allowing values to inform practices—to form habits based on proper values. We can find this language present in "the habitual inclination to see mathematics as … worthwhile" from *Adding It Up: Helping Children Learn Mathematics*.[20] In discussing practicing virtues in the mathematics classroom, I am most interested in exploring research that takes seriously the last two stages of Krathwohl's taxonomy of the affective domain of learning: organizing (bringing together different values, resolving conflicts between them, and beginning the building of an internally consistent value system), and characterizing by value or value set (individual has a value system that has controlled his or her behavior for a sufficiently long time for him or her to develop a characteristic "life style"—thus the behavior is pervasive, consistent, and predictable).

There seems to be very little, if any, research in mathematics education that is focused on the practicing of virtues (the actual demonstration of values through actions).There are several reasons for the dearth of material in this area; I would like to mention two of them. First, as the quote offered by Zan mentioned above indicates, there has been a sepa-

ration of affect from mathematics research. Second, due to the variability in the research on instilling values, very little groundwork has been laid to take the research a step further: to analyze the practice of those values on a consistent basis. One fruitful point of research on addressing these two issues is offered by Marja-Liisa Malmivuori,[21] who builds off the work of McLeod. While there may be some underlying issues in McLeod's approach as discussed above, we have here an example of continuing work on the level of organization and characterization of student affections in mathematics classes. Malmivuori presents affect as an essential aspect of a student's self-reflection and self-regulation (which fits well with Krathwohl's "organization" category in the affective domain[22]). Students are viewed as agents who constantly interpret and evaluate their experiences and regulate their behavior, in interaction with their mathematics learning environment. Not only are students organizing a value system in mathematics, but they are also evaluating it and allowing it to inform their behavior and habits. However, this research is still being driven largely through the lens of cognition—in this case, focusing on affect in the context of mathematical problem solving.

In summary, there is very little research available with regard to developing the organization and characterization levels of the affective domain in mathematics apart from viewing affect as secondary to cognition. By seeing cognition as the primary goal of the mathematics classroom, there is confusion in defining what exactly we mean when we speak of affect: are we discussing beliefs, or emotions, or attitudes, or values? If space allowed for further study, we would find that work on affect in mathematics can largely be classified as trying to reconcile these various models for understanding what affections actually are, and attempting to explain the complex interaction between affect (whether that is termed as attitudes, or beliefs, or values, or something else) and cognition. Without a solid base of understanding affections, little has been done to analyze at a practical level *how* we as educators might go about cultivating mathematical affections. Removing cognition as the primary lens through which affect is analyzed in mathematics education is no easy task. As Gerald Goldin notes:

> Mathematics educators who set out to modify existing, strongly held belief structures of their students are not likely to be successful addressing only the content of their students' beliefs … it will be important to provide experiences that are sufficiently rich, varied, and powerful in their emotional content to foster students' construction of new meta-affect. This is a difficult challenge indeed.[23]

By "meta-affect" Goldin is referencing affect about affect—or, in other words, how one feels about feeling. For instance, one might experience the feeling of fear when attending a horror movie, but find it enjoyable to do so. This meta-affect level at which students determine what emotions, attitudes, and beliefs are preferable to others is akin to our discussion of value formation, and hence the aptness of Goldin's quote. Values are not going to be modified simply by focusing on content and cognition. The experience of the student needs to change in order to see growth in this area. As we will now see in the next section, a Christian perspective on the teaching of mathematics is up to Goldin's challenge.

Values in Christian Faith: Cultivating Mathematical Affections

What we are really talking about when discussing the affective domain of learning are the habits of our students, how they are instilled, how they are encouraged (or discouraged), and how they are evaluated. For believers, Christian faith will have an obvious impact on this discussion; however, the purpose of this section is to examine how Christians might influence the work being done on affect in mathematics education in a way that would be accepted by all practicing researchers, regardless of their faith commitments. I will begin by briefly summarizing some of the key work that has focused on a Christian approach to mathematics pedagogy, and clarify how what is being proposed here differs from the work that has already been done and how it contributes to this much-needed body of research and resources. Then I will make use of James K. A. Smith's work in *Desiring the Kingdom* to demonstrate how the specific frustrations of researchers in the field of affect in mathematics education can be addressed from a Christian perspective, by ultimately viewing human beings as primarily affective (and secondarily cognitive) creatures. Finally, I will conclude with some practical suggestions for cultivating mathematical

affections in the classroom and offer a call to action for developing more resources along these lines.

Let me take a moment to define more clearly what I mean by mathematical affections. The title of this article is in homage to Jonathan Edwards's *Treatise Concerning Religious Affections*.[24] Edwards's goal was to discern the true nature of religion, and in so doing, dissuade his congregation from merely participating in a Christian culture (a mimicked outward expression) and motivate them to long for true Christian conversion (an inward reality of authentic Christian character). The purpose of this article is to engage us as educators in discerning the true nature of mathematical pedagogy and in determining how we as Christian educators can approach the teaching and learning of mathematics: does it simply mimic the modern culture of utility by requiring outward demonstrations of knowledge retention and application, or does it aim deeper at analyzing true inward character formation?

For Edwards, affections were not synonymous with emotions as they tend to be in today's culture (or in today's mathematics education research as noted by Zan). Edwards understood affections as aesthetics—a way of orienting your life via a mechanism that determines what is beautiful and worthwhile. Affections are character producing and habit forming. It is Edwards's definition of affections (orientation of life, determining worth) that actually appears in policy documents that we have cited.

Consider once more that being mathematically literate involves having an appreciation of the value and beauty of mathematics, and being mathematically proficient involves a habitual inclination to see mathematics as worthwhile. Foundational documents in the area of mathematics education plainly portray mathematics as beautiful, of value, and affecting the habits of the learner to see mathematics as worthwhile. However, as we have seen, none of these documents develops *how* we as teachers are to go about accomplishing this task. It is almost as if these phrases are included in these documents as a courtesy—as a way of saying, "This is how we teachers feel about mathematics, and it would be nice for our students to feel this way too. But feeling is subjective, so there is no real way for us to instruct objectively, or to assess students in this regard." This is a point of connection that we as Christian educators can make with the educational system as a whole—we can answer the questions of *how*. We have much to contribute here, and we do not have to be overtly religious in the presentation.

Now let us return to our initial question, "Why should I value this?" and consider how we might respond from a Christian perspective. Michael Veatch notes,

> There is a prevalent attitude that one learns what is good mathematics by seeing and doing it, not by discussing values. The knowledge needed by the person entering the field will rub off on her. The classroom clearly reflects this attitude.[25]

As it stands, our current methods of teaching mathematics are producing untold numbers of students who see mathematics as more a function of natural ability rather than effort, who are willing to accept poor performance in mathematics, who often openly proclaim their ignorance of mathematics without embarrassment, and who treat their lack of accomplishment in mathematics as a permanent state over which they have little control.[26] The reason for this is that we have given values (affections) a backseat in the mathematics classroom.

In *The Abolition of Man*, C.S. Lewis writes, "Education without values, as useful as it is, tends to make man a more clever devil."[27] This is a fairly accurate statement of the modern-day system of mathematics education. If we do not focus on values, if we do not focus on the affective learning of our students, then their education will still be useful in the sense that they will increase in cognitive ability and learn to apply their thinking. But is that outcome really valuable in and of itself? Without a proper sense of values to guide their application, are we not really just making students "more clever devils"?

As we have already noted, education is inherently value laden, so values cannot actually be removed from education. Lewis's point is that the value we instill in education should be affective—loving learning for its own sake and valuing wisdom. If you do not focus on affections, then you still have usefulness, but is that really beneficial? In the words of the Bishop in Victor Hugo's *Les Misérables*: "The beautiful is as useful as the useful … Perhaps more so."[28] Aesthetics can be more useful than utility. I have defined mathematical affections not simply as knowing, but also as loving, and as practicing the truth,

beauty, and goodness inherent in mathematics. A Christian perspective on the pedagogy of mathematics has much to offer in this regard.

While there are many resources that examine a Christian perspective of mathematics pedagogy (that is, the teaching of mathematics from a Christian perspective, not just an understanding of mathematics from a Christian perspective), there are three that I would like to briefly mention.[29] David Klanderman addresses a Christian response to the constructivism espoused by Ernest above. The goal is to analyze constructivism as a philosophy of mathematics and offer it as an example of how Christians might form their own thinking and offer their own justifications for teaching decisions within the mathematics classroom. Klanderman focuses on the formation of a broader philosophical base from which to approach the teaching and learning of mathematics, rather than addressing specific pedagogical practices and their outcomes, though he does address many of the national policy documents and published standards. He concludes,

> In the areas of teaching and learning of mathematics (Christian) perspectives may result in policies that are similar to those espoused by people with differing views, but for very different reasons.[30]

Although Christians have no right to expect explicitly Christian standards to be proposed by a publicly funded and supported organization such as NCTM, we nonetheless need to have these conversations in the context of Christian community. Where this article differs from Klanderman is that I believe, if argued appropriately, new standards on affect in mathematics that are rooted in an explicitly Christian worldview could indeed be drafted by organizations such as NCTM and implemented across a variety of classrooms, not only Christian ones.

Harold Heie describes the Christian motivation behind the pedagogical strategy of posing integrative questions. By integrative question, he means a question that cannot be addressed without formulating coherent relationships between academic disciplinary knowledge and biblical/theological knowledge.[31] While certainly a valuable tool, and a highly recommended teaching strategy, integrative questions still only target cognition in students. Heie argues for a Christian pedagogy based on shaping beliefs and worldview.[32]

James Nickel notes the need to move beyond "thought" in developing objectives for a biblical Christian mathematics curriculum, noting that mathematical thought, from a Christian perspective, is meant to further God's purposes of redemption and dominion, and thus move us to action.[33] While Nickel does encourage moving beyond thought (or cognition) in determining our teaching practices, his focus tends to be more along the lines of the utility discussed in the introduction, motivated simply from a Christian worldview—or cognitive perspective. One could argue that there is still an underlying assumption that affections are formed primarily through a cognitive understanding of the Christian faith. If this is the fullest approach we take to teaching mathematics from a Christian perspective, we as Christian educators will face the same dilemmas encountered by secular researchers in attempting to examine how to cultivate mathematical affections in students.

The preceding works by Klanderman, Heie, and Nickel contribute greatly to a Christian understanding of what it means to teach mathematics well. However, as beneficial as those resources are for those teaching in explicitly Christian contexts, they lose their value in secular contexts that are extremely unlikely to adopt their underlying faith commitments.[34] It is my contention that integrating the work of James K. A. Smith into mathematics education has the potential to produce research on affect in mathematics that can be accepted broadly by all mathematics educators. Smith urges Christian educators to move beyond worldview and belief language, as such language tends to result in pedagogies that still operate on the level of disseminating information.[35] While space may not allow for a complete analysis of Smith's work,[36] I want to highlight some of the main themes. Then I believe it will be apparent how his distinctly Christian perspective to what human beings are and how they learn, provides some answers that researchers on affect in mathematics education are searching for.

"Behind every pedagogy is a philosophical anthropology."[37] Before you can teach a human being you must first have a notion of what a human being is. Smith notes that a pedagogy that focuses on cognition, that sees education as primarily disseminating information, tends to assume human beings are primarily "thinking things" and cognitive machines.[38]

Smith's thesis is that human beings are primarily affective beings before they are cognitive beings, and this anthropology bears itself out in our current educational system regardless of whether we recognize it. As Smith describes education:

> Education is not primarily a heady project concerned with providing information; rather, education is most fundamentally a matter of formation, a task of shaping and creating a certain kind of people. What makes them a distinctive kind of people is what they love or desire—what they envision as "the good life" or the ideal picture of human flourishing. An education, then, is a constellation of practices, rituals, and routines that inculcates a particular vision of the good life by inscribing or infusing that vision into the heart (the gut) by means of material, embodied practices. And this will be true even of the most instrumentalist, pragmatic programs of education (such as those that now tend to dominate public schools and universities bent on churning out "skilled workers") that see their task primarily as providing information, because behind this is a vision of the good life that understands human flourishing primarily in terms of production and consumption. Behind the veneer of a "value-free" education concerned with providing skills, knowledge, and information is an educational vision that remains formative. There is no neutral, nonformative education; in short, there is no such thing as a "secular" education.[39]

For Smith we are first and foremost creatures of desire before we are creatures of thought or even creatures of belief. Our affections pull us through life toward our vision of "the good life" rather than our cognitions rationally pacing out our steps. We are creatures of love, and love requires practice.[40] In other words, our affections are shaped by the practices/habits/rituals that we are immersed in. Smith refers to these as liturgies—rituals of ultimate concern: rituals that are formative for identity, that inculcate particular visions of the good life, and do so in a way that means to trump other ritual formations.[41] While Smith offers much to unpack for Christian educators, for our purposes of examining affect in mathematics education, the following points are significant to note: (1) the argument that human beings are primarily affective rather than cognitive beings, and (2) the argument that our affections are shaped by practices (liturgies).

What if human beings are primarily affective learners and only secondarily cognitive learners? All of the research cited above treats the affective domain of learning as needing to be interconnected with the cognitive domain (a position which Smith would agree with), but none of the research (with the possible exception of Goldin's work—though this needs to be explore in greater depth) argues for the primacy of the affective domain. Smith would argue that, as Christian educators, we should advance this point further in the research of our respective academic fields. What is refreshing is that Smith notes how this ancient Christian understanding of human beings as creatures of love is finding support in contemporary philosophy and psychology. Therefore there is a base from which to further research on affect in mathematics education (and really in all education) that does not require explicit Christian faith commitments in order to be accepted.

Smith notes that much work has been done in the last century to suggest shifting the center of gravity of the human person from the cognitive to the noncognitive—from the cerebral head to the affective region of the body.[42] The reference "affective region of the body" is a significant one. Often the affective dimension of the human person is associated with the heart and emotion (as we saw in our analysis on affect above). However, Smith's work seems to support the notion that it is the actions/habits of the body that work to form and portray our affections.

This philosophical notion seems to be confirmed by contemporary work in cognitive science as well. It is bodily practices that train the body (including the brain) to develop habits or dispositions to respond automatically in certain situations and environments. Claims regarding material, bodily formation of our noncognitive dispositions are as old as Aristotle, but now they receive support and evidence from contemporary neuroscience and cognitive science.[43] Christian Smith, in his methodological manifesto for the social sciences, noted that the dominant paradigms of social sciences reflect human beings as rational machines, and he calls for a more holistic understanding of humans as believing (affective) or what he terms "narratological" animals, that is, creatures driven by story at an affective level rather than by logic and rationality at a cognitive level.[44]

Charles Taylor notes that what we as humans think about is just the tip of the iceberg and cognition cannot fully or adequately account for how or why we make our way in the world. For Taylor, there is something beneath the cognitive, what he terms "the imaginary"—defined as the way ordinary people imagine their social surroundings that is not expressed in theoretical terms but is carried in images, stories, and legends.[45] Here Taylor uses "imaginary," not in the romantic sense of invention, but, rather, in reference to a precognitive framework or lens through which we view and interact with the world. All of this research is summarized here to note the potential for Christian mathematics educators to build an argument for the primacy of affect in education from a foundation that does not necessarily attach itself to Christian faith commitments and thereby does not lack transference into secular research.

While much of the above work in philosophy and cognitive science needs to be developed in more explicit detail as it pertains to mathematics education, it nonetheless establishes the groundwork that such academic work on the primacy of affections is out there and is, in fact, growing. The key question then seems to be, "What if human beings are primarily affective learners and only secondarily cognitive learners?" If this work is indeed true, and it changes the way we see human beings, then it necessarily must change the way we teach human beings. The majority of research on affect proceeds with an (often unstated) assumption that we are primarily cognitive beings, and the results of that research bear this point out as we have seen—framing arguments that focus on cognition, confusing terminology and learning objectives, and so forth. As Christian mathematics educators, we have the opportunity to contribute the following analysis to work on affect: if human beings are primarily affective learners, *how* then do we develop the affections? As we have seen, James K. A. Smith argues that this occurs through the liturgies of the classroom. Before moving to this last point to discuss some possible ways in which we might cultivate mathematical affections in students, allow me to make note of several other studies on affect in mathematics education in light of the preceding discussion on philosophy and psychology.

Some work being done in the research of mathematics education takes these ideas into account. Such work aims to produce a new unit of analysis for the study of mathematical activity, integrating affectivity and cognition.[46] While this is certainly a step in the right direction, integrating the affective and cognitive, it does not go the extra step to suggest the primacy of the affective.

A stronger statement with regard to the primacy of affective learning is made by Markku Hannula.[47] In examining motivation in the mathematics classroom, Hannula notes that, in order to understand student behavior in classrooms, we need to increase our understanding of what motivation is and how it is regulated. The first relevant issue that he discusses is the importance of the unconscious (or preconscious) in motivation. He also goes on to note that, as a potential, motivation cannot be directly observed, but rather it is only observable as it manifests itself in affect and cognition (for example as beliefs, values, and emotional reactions). Goldin discusses a research-based theoretical framework based on affect as an internal representational system.[48] Key ideas include the concepts of meta-affect and affective structures, and the constructs of mathematical intimacy and mathematical integrity. Goldin understands these as fundamental to powerful mathematical problem solving, and deserving of closer attention by educators. We see in Hannula a recognition of the pre-conscious (and hence precognitive) place of motivation that then influences students' affective actions. In Goldin we find an approach that sees affect as an internalized organization structure which is necessary for students to succeed in the cognitive task of mathematical problem solving.[49]

Finally, let us consider *how* one goes about cultivating mathematical affections. I will offer a few ideas, focused from Smith's notion of liturgies, and drawn specifically from the mathematics classroom. However, this is the area in which we as Christian mathematics educators need to do more work. This article is meant to serve largely as a call to action—a realization of the opportunity we have before us to contribute to a much-needed body of research on affect. There are three brief examples I wish to discuss in light of everything that has been discussed thus far.

1. More consideration needs to be given to assessment. The NCTM *Assessment Standards for School Mathematics* (1995) states, "It is through assess-

ment that we communicate to students what mathematics are valued." If our goal is to cultivate mathematical affections (values) in students, assessment is the primary means by which we do so. We need to consider what liturgies of assessment we participate in at both the formative and summative levels. For instance, is the emphasis on correctness of a student response? Perhaps a teacher poses a question to the class and a student answers incorrectly. The teacher responds with a simple "no" and moves on to call upon another student who they know will provide the right answer and move the lesson along. If we fall into this pattern (liturgy) of formative assessment, we are instilling into students the notion that mathematics is only about getting to a correct answer, and we are ignoring the productive struggle that it takes to get there. At a summative level, as long as high-stakes standardized exams exist in which the main goal is to achieve a certain percentage of correct responses, we will always be fighting an uphill battle in getting students to value mathematics for its creative processes.

2. More consideration needs to be given to technology. We need to be careful not to implement the newest technological accessories in our classroom just because students are used to having technology in their lives outside of school. If we are trying to offer up mathematics as being the technologically savvy discipline and, therefore, worth the interest of students, I would argue that we are largely going to lose that battle. We are offering mathematics as a competing interest against the newest apps, games, and electronic devices that students are inundated with on a daily basis. As much as I love mathematics, I know that this is a competition it will not win. What if instead we focused on technological liturgies in the classroom that utilized mathematics as a way of examining and critiquing technological advancements rather than simply using those advancements to try to make mathematics more fun? What if these liturgies could instill in students a sense of mathematics (and education as a whole) as something other than just a competing product for their attention and, rather, a foundation for their life that informs the product choices and decisions they make? What if we stopped feeding the culture of immediacy that technology has engrained in us and purposefully use the classroom as a time to step back and reflect? Perhaps then students would not automatically jump to the calculator when faced with a difficult problem and proceed to give up if the answer is not achieved in under a minute.
3. More consideration needs to be given to service. There is much that can be contributed to service-learning in mathematics. Personally, after I began implementing service-learning projects in all of my classes, I was amazed at the impact it had on students on both a cognitive and affective level. Service to the community turns the focus away from individualistic goals of education (such as what grade the student receives) to the more altruistic aims of education. In their reflection from a recent project, one of my students wrote "The service-based aspect of the project made it more engaging because we met new people and we had the mindset that we could actually help someone by completing this project." By comparison, Matthew 20:26–28 states, "Whoever wishes to become great among you shall be your servant, and whoever wishes to be first among you shall be your slave; just as the Son of Man did not come to be served, but to serve, and to give his life a ransom for many." If the goal of education is the formation of a certain type of person, then the more that we can get students to express sentiments rooted in scripture as the result of their experience in the math classroom, then the more likely it is that we are heading in the right direction. More resources need to be produced in this regard.[50]

In summary, I believe that there is a need for more work to be done on developing values in students apart from a primarily cognitive approach, and I am convinced that Christian faith has much to offer in this regard. Though cognition and affection are certainly interrelated, more research needs to be done on the assumption of the affections as primary to the students' learning process. There is ample opportunity now, perhaps more so than ever, for Christian mathematics educators to have a major influence on the cultivation of mathematical affections: not merely knowing, but also loving, and practicing the truth, beauty, and goodness inherent in mathematics. ◈

Notes

[1]For an excellent discussion of this point, see C. S. Lewis, *The Abolition of Man* (New York: HarperOne, 2009).

[2]As noted by Russell Howell, "The Matter of Mathematics," *Perspectives on Science and Christian Faith* 67, no. 2

(2015): 84, citing Vern S. Poythress, "A Biblical View of Mathematics," in *Foundations of Christian Scholarship: Essays in the Van Til Perspective*, ed. Gary North (Vallecito, CA: Ross House Books, 1976), 158–88.

[3]P. Ernest, *The Philosophy of Mathematics Education* (New York: Falmer Press, 1991), 259.

[4]Ernest, in presenting his philosophy of mathematics education known as social constructivism, roots these values in the cultural contexts of the human creators of mathematics (ibid., 261). For a thorough discussion of a distinctly Christian perspective on values in mathematics, I highly recommend M. Veatch, "Mathematics and Values," in *Mathematics in a Postmodern Age: A Christian Perspective,* ed. R. Howell and J. Bradley (Grand Rapids, MI: Eerdmans, 2001), 223–49. For a distinctly Christian perspective on Ernest's social constructivism and its impact on the math classroom, I recommend D. Klanderman, "Teaching and Learning Mathematics: The Influence of Constructivism," in *Mathematics in a Postmodern Age*, ed. Howell and Bradley, 338–59. Both Veatch's and Klanderman's work will be referenced throughout this article.

[5]National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics* (Reston, VA: NCTM, 1989); National Council of Teachers of Mathematics, *Professional Standards for Teaching Mathematics* (Reston, VA: NCTM, 1991); National Council of Teachers of Mathematics, *Assessment Standards for School Mathematics* (Reston, VA: NCTM, 1995); National Research Council, *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (Washington, DC: National Academies Press, 1989); and Mathematics Learning Study Committee, *Adding It Up: Helping Children Learn Mathematics* (Washington, DC: National Academies Press, 2001).

[6]National Council of Teachers of Mathematics, *Professional Standards for Teaching Mathematics* (1991), http://www.fayar.net/east/teacher.web/math/standards/previous/ProfStds/TeachMath.htm.

[7]Mathematics Learning Study Committee, *Adding It Up*, 116.

[8]Common Core State Standards Initiative, *Standards for Mathematical Practice*, http://www.corestandards.org/Math/Practice/.

[9]Composed based on B. S. Bloom, M. D. Englehart, E. J. Furst, W. H. Hill, and D. R. Krathwohl, *Taxonomy of Educational Objectives: Handbook I. Cognitive Domain* (New York: McKay, 1956); and D. R. Krathwohl, B. S. Bloom, and B. B. Masia, *Taxonomy of Educational Objectives: Handbook II. Affective Domain* (New York: Longman, 1964). Underlying graphic credit goes to C. L. Jorgensen, http://cljorgensen.com/tag/blooms-taxonomy/.

[10]Bloom et al., *Taxonomy of Educational Objectives: Handbook I. Cognitive Domain;* and L. W. Anderson and D. R. Krathwohl, *A Taxonomy for Learning, Teaching, and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives* (New York: Pearson, 2000).

[11]Krathwohl et al., *Taxonomy of Educational Objectives: Handbook II. Affective Domain*.

[12]Ibid.

[13]R. Zan, L. Brown, J. Evans, and M. S. Hannula, "Affect in Mathematics Education: An Introduction," *Educational Studies in Mathematics* 63, no. 2 (2006): 113.

[14]D. B. McLeod, "Research on Affect in Mathematics Education: A Reconceptualization," in *Handbook of Research on Mathematics Teaching and Learning*, ed. D. A. Grouws (New York: Macmillan, 1992), 575.

[15]Ibid.

[16]See, for example, T. Haladyna, J. Shaughnessy, and J. M. Shaughnessy, "A Causal Analysis of Attitude toward Mathematics," *Journal for Research in Mathematics Education* 14 (1983): 19–29; D. B. McLeod, "Affective Issues in Mathematical Problem Solving: Some Theoretical Considerations," *Journal for Research in Mathematics Education* 19 (1988): 134–41; D. B. McLeod, "The Role of Affect in Mathematical Problem Solving," in *Affect and Mathematical Problem Solving: A New Perspective*, ed. McLeod and Adams, 20–36; D. B. McLeod, "Beliefs, Attitudes, and Emotions: New Views of Affect in Mathematics Education" in Ibid., 245–58; D. B. McLeod, "Research on Affect and Mathematics Learning," *Journal for Research in Mathematics Education* 25 (1994): 637–47; R. S. Prawat and A. L. H. Anderson, "The Affective Experiences of Children during Mathematics," *Journal of Mathematical Behavior* 13, no. 2 (1994): 201–21; I. M. Gomez-Chacon, "Affective Influences in the Knowledge of Mathematics," *Educational Studies in Mathematics* 43 (2000): 149–68; M. L. Malmivuori, "The Dynamics of Affect, Cognition, and Social Environment in the Regulation of Personal Learning Processes: The Case of Mathematics" (PhD diss., University of Helsinki, 2001); G. C. Leder, E. Pehkonen, and G. Törner, eds., *Beliefs: A Hidden Variable in Mathematics Education?* (The Netherlands: Kluwer Academic Publishers, 2002); X. Ma, "Cognitive and Affective Changes as Determinants for Taking Advanced Mathematics Courses in High School," *American Journal of Education* 113 (2006): 123–49; M. L. Malmivuori, "Affect and Self-regulation," *Educational Studies in Mathematics* 63 (2006): 149–64; J. Maaß and W. Schölglmann, *Beliefs and Attitudes in Mathematics Education: New Research Results* (Rotterdam: Sense Publishers, 2009); M. Cotic and M. V. Zuljan, "Problem-Based Instruction in Mathematics and Its Impact on the Cognitive Results of the Students and on Affective-Motivational Aspects," *Educational Studies* 35, no. 3 (2009): 297–310; M. Lebens, M. Graff, and P. Mayer, "The Affective Dimensions of Mathematical Difficulties in Schoolchildren," *Education Research International* (2011): 1–13; and J. Molera-Botella, "Is There a Relationship in Primary Education between Affective Factors in Mathematics and Academic Performance?," *Estudios sobre educación* 23 (2012): 141–55.

[17]To be clear, the term "values" is rarely used explicitly in research on affect. D. B. McLeod in "Beliefs, Attitudes, and Emotions: New Views of Affect in Mathematics Education," in *Affect and Mathematical Problem Solving: A New Perspective*, ed. McLeod and Adams, 245–58; "Research on Affect in Mathematics Education: A Reconceptualization," in *Handbook of Research on Mathematics Teaching and Learning*, ed. Grouws; and "Research on Affect and Mathematics Learning," *Journal for Research in Mathematics Education* 25 (1994): 637–47, divides the affective domain into emotions, attitudes, and beliefs—listed in increasing order with a connection to cognition. V. A. DeBellis and G. A. Goldin in "Affect and Meta-affect in Mathematical Problem Solving: A Representational Perspective," *Educational Studies in Mathematics* 63, no. 2 (2006): 131–47, add a fourth domain: "values, ethics, and morals." It is my position that the "value" language from policy standards has all of these domains in mind, and so I have placed research that addresses any one of these domains under the larger umbrella of "values."

[18]J. K. A. Smith, *Desiring the Kingdom: Worship, Worldview, and Cultural Formation* (Grand Rapids, MI: Baker Academic, 2009).
[19]A. Sfard, *Thinking as Communicating: Human Development, the Growth of Discourses, and Mathematizing* (Cambridge: Cambridge University Press, 2008), 156.
[20]Mathematics Learning Study Committee, *Adding It Up*, 116.
[21]M. Malmivuori, "The Dynamics of Affect, Cognition, and Social Environment in the Regulation of Personal Learning Processes: The Case of Mathematics" (PhD diss., University of Helsinki, 2001); and M. Malmivuori, "Affect and Self-regulation," *Educational Studies in Mathematics* 63 (2006): 149–64.
[22]Krathwohl et al., *Taxonomy of Educational Objectives: Handbook II. Affective Domain*.
[23]G. A. Goldin, "Affect, Meta-affect, and Mathematical Belief Structures," in *Beliefs: A Hidden Variable in Mathematics Education?*, ed. Leder, Pehkonen, and Törner, 71. Emphasis added.
[24]Jonathan Edwards, *A Treatise concerning Religious Affections* (New York: Cosimo Classics, 2007).
[25]Veatch, "Mathematics and Values," in *Mathematics in a Postmodern Age: A Christian Perspective,* ed. Howell and Bradley, 243.
[26]McLeod, "Research on Affect in Mathematics Education: A Reconceptualization"—And still just as true today.
[27]Quote attributed to C. S. Lewis though there is no direct evidence of him having written it. The quote does aptly summarize a major point of Lewis's treatise on education, *The Abolition of Man,* http://lanternhollowpress.com/2012/07/15/meditations-with -c-s-lewis-a-more-clever-devil/.
[28]Victor Hugo, *Les Misérables* (New York: Penguin Group, 1987), 23.
[29]My focus will be on the pedagogy of mathematics and not the content of specific mathematics courses. There are numerous resources that address how Christianity impacts our understanding of mathematics as a discipline and how we might modify the content of courses to underscore Christian faith commitments, many of which have appeared in this journal. Recommended in this regard are Howell and Bradley, eds., *Mathematics in a Postmodern Age*; J. Bradley and R. Howell, eds., *Mathematics through the Eyes of Faith* (New York: HarperOne, 2011); J. Mays, "Why Math Works: Answering Wigner et al.," *Association of Classical and Christian Schools* 19, no. 4 (2012): 14–16; D. F. M. Strauss, "A Historical Analysis of the Role of Beliefs in the Three Foundational Crises in Mathematics," *Facets of Faith and Science*, vol. 2, ed. J. M. van der Meer (New York: University Press of America, 1996); "Mathematics: A Christian Perspective," a curriculum developed by the Kuyers Institute at Calvin College.
[30]Klanderman, "Teaching and Learning Mathematics: The Influence of Constructivism," 359.
[31]H. Heie, "Developing a Christian Perspective on the Nature of Mathematics," in *Teaching as an Act of Faith*, ed. A. C. Migliazzo (Bronx, NY: Fordham University Press, 2002), 101–102.
[32]Ibid., 99.
[33]J. Nickel, *Mathematics: Is God Silent?* (Vallecito, CA: Ross House Books, 2001), 235.
[34]See also J. Barrett and D. Klanderman, "A Christian Constructivist? The Impact of Worldview on Learning Theories and the Mathematics Education Research Community," *Journal of the Association of Christians in the Mathematical Sciences* (2006), http://www.acmsonline.org/journal/2006/KlandermanBarrett.pdf, for a discussion of attempts to reconcile competing theories of learning mathematics within a Christian context falling on the deaf ears of the secular research community.
[35]Smith, *Desiring the Kingdom*, 18.
[36]Smith himself notes that in analyzing how our affections are shaped by something like teaching, the form of the teaching is inextricably linked to the message behind the teaching. So for me to summarize his work is doing it a great injustice, and I therefore strongly encourage a full reading of *Desiring the Kingdom*. My apologies to Dr. Smith.
[37]Smith, *Desiring the Kingdom*, 27.
[38]Ibid.
[39]Ibid., 26–27.
[40]Ibid., 76ff.
[41]Ibid., 86.
[42]See M. Heidegger, *Being and Time,* trans. John Macquarrie and Edward Robinson (New York: Harper and Row, 1966); and E. Brann, "Are Humans Ultimately Affective?," *Expositions: Interdisciplinary Studies in the Humanities* 1 (2007): 53–70.
[43]See J. A. Bargh and T. L. Chartrand, "The Unbearable Automaticity of Being," *American Psychologist* 54 (1999): 462–79; and T. D. Wilson, *Strangers to Ourselves: Discovering the Adaptive Unconscious* (Boston, MA: Harvard University Press, 2004).
[44]C. Smith, *Moral, Believing Animals* (Oxford: Oxford University Press, 2003).
[45]C. Taylor, *Modern Social Imaginaries* (Durham, NC: Duke University Press, 2004).
[46]See C. R. Arujo, F. Andrade, I. Hazin, J. T. R. Falcao, J. C. do Nascimento, and M. M. L. Lessa, "Affective Aspects on Mathematics Conceptualization: From Dichotomies to an Integrated Approach" (paper presented at the 27th International Group for the Psychology of Mathematics Education Conference held jointly with the 25th PME-NA Conference, Honolulu, HI, July 13–18, 2003), v2, 269–76.
[47]M. S. Hannula, "Motivation in Mathematics: Goals Reflected in Emotions," *Educational Studies in Mathematics* 63 (2006), 165–78.
[48]DeBellis and Goldin, "Affect and Meta-affect in Mathematical Problem Solving."
[49]Another area of emerging research that I believe is fruitful in regard to cultivating affections and instilling values that is not discussed at great length here is the area of aesthetics. See N. Sinclair, *Mathematics and Beauty: Aesthetic Approaches to Teaching Children* (New York: Teachers College Press, 2006).
[50]Some resources that currently exist are C. R. Hadlock, ed., *Mathematics in Service to the Community: Concepts and Models for Service-Learning in the Mathematical Sciences*, no. 66 (Washington, DC: Mathematical Association of America, 2005); K. Black, K. Crisman, and D. Jardine, "Introductory Editorial: Special Issue on Service-Learning," *PRIMUS* 23, no. 6 (2013): 497–99; and Service-Learning and Mathematics webpage, http://www.math-cs.gordon.edu/~kcrisman/.