PERSPECTIVES on Science and Christian Faith

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In This Mathematics Theme Issue …

The Matter of Mathematics

A Pranalogical Approach to Faith-Integration with Students

Integration of Faith and Mathematics from the Perspectives of Truth, Beauty, and Goodness

Cultivating Mathematical Affections: The Influence of Christian Faith on Mathematics Pedagogy

Practical Applications of an Integrally Christian Approach to Teaching Mathematics

"The fear of the Lord is the beginning of Wisdom."
Psalm 111:10

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Perspectives on Science and Christian Faith

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Conflict and Collaboration

James C. Peterson

Collaboration can be enriched by conflict between the parties, if each is to bring a contribution to the table. Granted sometimes the conflict can be so complete that each has nothing to offer the other, but more often there is something in each differing perspective that can add to a better resolution going forward. That is not to say that the truth is always in the middle, nor that the compromise that is essential to a democratic process of living together is always the highest goal for academia. In academic study, we often have the luxury of seeking the most accurate description whether it supports practical cooperation or not. But even in academia, the hope is for eventually recognized agreement that knowledge has been advanced in a particular way. In pursuit of that goal, highlighting persistent conflict can still be a form of constructive collaboration, as it helps to map out approaches along with their strengths and weaknesses. We see such conflict, and collaboration, in this issue.

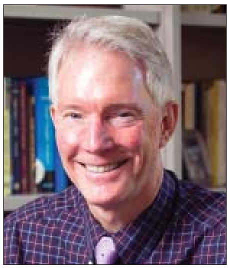
We begin with an invitation article by Russell Howell. One might find it confounding that God is left out of some current theologies, but not as surprising to study mathematics without reference to God. Yet to the contrary, Howell finds many points of interaction between mathematics and Christian faith, especially at the metalevel. In the following four articles that were spurred by Howell's essay, each author has their own perspective on recognizing and building upon a Christian connection with mathematics, both theoretically and practically. With this varied case for Christian perspectives shaping, in particular, the teaching of mathematics, how much more insight might there be here for teaching physics, chemistry, and biology? This is an opportunity to collaborate, not only in regard to mathematics, but also across the sciences.

Our next two essays show considerable conflict. They pick up where the discussion left off last December on human-triggered climate change. In this second round, they can clarify more exactly where they do agree, why their starting points and conclusions add up so differently, and how we might find more resources to hone our interpretation and response. Our book review section follows, along with a vigorous exchange of letters to the editor.

Conflict and collaboration. When authors send the journal their best effort, the first review is whether the essay will be considered by full peer review. This protects expert peer-reviewer time. If the essay has enough potential to warrant that next step, the best response the author can hope for is not rousing applause and cries of "Perfection!" Such just does not happen. Reviewers always have questions, corrections, challenges ... The best response to hope for from the journal is a request for a rewrite that takes into account the reviewers' responses. This does not mean that the author is expected to capitulate, but rather, that the author has now further input to strengthen the argument and the communication of it. When an article is eventually published, the collaboration continues as yet more fellow scholars consider and respond to the piece in conversation, citations, and, in the journal specifically, by dialogue in letters to the editor and in later articles that take the discussion into yet newer territory.

So, welcome to the process of both conflict and collaboration here at *PSCF*. May we appreciate each other in both, and be better for it. ◊

James C. Peterson, *editor*



Russell W. Howell

Article

The Matter of Mathematics

Russell W. Howell

This issue of PSCF *is dedicated to mathematics. The general public would likely scoff at the idea that the Christian faith could possibly have any bearing on the subject. Yet for the past thirty-plus years, the Association of Christians in the Mathematical Sciences (ACMS) has devoted much of its energy focusing on precisely that issue. The following lead article begins by asking whether such an effort makes sense, concludes that it does, and highlights several broad categories (with examples) that will hopefully stimulate further conversation. The articles following draw from these categories (or propose new ones) with a special focus on the teaching of mathematics.*

Does faith matter in mathematics? Not according to the Swiss theologian Emil Brunner. In 1937 he suggested a way to view the relationship between various disciplines and the Christian faith. Calling it the "Law of Closeness of Relation," he commented,

> The nearer anything lies to the center of existence where we are concerned with the whole, that is, with man's relation to God and the being of the person, the greater is the disturbance of rational knowledge by sin; the further anything lies from the center, the less the disturbance is felt, and the less difference there is between knowing as a believer or as an unbeliever. This disturbance reaches its maximum in theology and its minimum in the exact sciences and zero in the sphere of the formal. Hence it is meaningless to speak of a "Christian mathematics."[1]

Thus, Brunner holds a nuanced version of the doctrine of noetic depravity: sin affects the reasoning ability of humans, but does so in varying degrees depending on how "close" the object of reasoning is to their relationship with God. Mathematics, being a purely formal discipline, is beyond the reach of any adverse noetic effects. Christians and non-Christians will therefore come to the same mathematical conclusions, so that, for Brunner, the phrase *Christian mathematics* is an oxymoron.

Of course, on one level Brunner is correct. If one agrees to play the game of mathematics, then one implicitly agrees to follow the rules of the game. Different people following these rules will—Christian or not—agree with the conclusions obtained in the same way that different people will agree that, at a particular stage in a game of chess, white can force checkmate in two moves. In this sense mathematical practice is "world-viewishly" neutral. Moreover, the paradigm for mathematical practice has remained relatively unchanged since Euclid published his masterpiece, *The Elements*, in 300 BC. That paradigm is to derive results in the context of an axiomatic system.[2]

It would be a mistake, however, to apply Brunner's dictum to all areas of mathematical inquiry. One can be committed to the mathematical game, but also participate in analyzing it (and even criticizing it) from a metalevel. In doing so, faith perspectives will surely influence the conclusions one comes to on important questions about mathematics.[3] But is the investigation of such questions really a legitimate part of the

Russell Howell *is Kathleen Smith Professor of Mathematics at Westmont College. He specializes in complex analysis and has recently received an NSF grant to pursue curricular reform in that subject. A co-author of the textbook* Complex Analysis for Mathematics and Engineering, *he is also a contributor to and co-editor of the books* Mathematics through the Eyes of Faith *(HarperOne) and* Mathematics in a Postmodern Age: A Christian Perspective *(Eerdmans).*

mathematical enterprise? At least two reasons can be given for an affirmative answer: (1) such questions are actually taken up at every annual joint meeting of the American Mathematical Society and the Mathematical Association of America; (2) historically, such questions have always been investigated by the mathematical community. Indeed, David Hilbert, one of the greatest mathematicians of the twentieth century, chose two topics for discussion in conjunction with the oral defense of his doctoral degree. The first related to electromagnetic resistance. The second was to defend an intriguing proposition: "That the objections to Kant's a priori nature of arithmetical judgments are unfounded."[4] Hilbert is credited as being a founder of the school of formalism, which insists that axiomatic procedures in mathematics be followed to the letter. It is thus interesting that even those who held a strict view of mathematical practice and meaning saw the investigation of important metaquestions relating to mathematics as a legitimate undertaking by mathematicians.

Is there a helpful classification for metalevel questions that Christian mathematicians might pursue as they attempt to explore the interaction between their discipline and faith? Arthur Holmes suggests four categories of faith-integration in his well-known book *The Idea of a Christian College*: the foundational, worldview, ethical, and attitudinal.[5] The remainder of this article will look at some developments in mathematics that lead naturally to questions in those categories. It will also suggest (and define) a fifth category for consideration: the *pranalogical*. The ideas presented throughout are by no means meant to be exhaustive, or even representative. It is hoped, though, that they will serve as sufficient triggers for further comment, and for thinking about a wide range of additional metaquestions worthy of investigation.

1. Foundational Issues

Holmes states that curricular studies reveal history and philosophy to be common disciplinary areas considered as foundational in higher education.[6] Within the scope of such an education, each discipline has historical and philosophical components that have shaped its practices, procedures, and paradigms. Mathematics has a particularly rich tradition. This section delineates a sampling of perspectives that lead to important interactions with the Christian faith.

1.1 Logic

Gottlob Frege thought that all of mathematics is reducible to logic. In 1903 he was about to take a big step in pushing through his program. He had just completed his seminal work, *Grundgesetze der Arithmetik* (The Basic Laws of Arithmetic), volume 2. It contained five axioms that, Frege hoped, would lay the necessary groundwork for all of arithmetic. The axioms were supposedly clear logical statements describing universal truths. If this work succeeded, his goal of producing an unshakable logical foundation for mathematics would be realized. Unfortunately, just before the book was to be published, Frege received a disturbing letter from Bertrand Russell, who pointed out that Frege's fifth axiom was in conflict with the other four. In other words, Frege's system was inconsistent. It was too late to stop production, so Frege desperately tried to patch things up and inserted a last-minute appendix in which he modified his fifth axiom. He also openly explained the situation:

> Hardly anything more unfortunate can befall a scientific writer than to have one of the foundations of his edifice shaken after the work is finished. This was the position I was placed in by a letter of Mr. Bertrand Russell, just when the printing of this volume was nearing its completion.[7]

It was subsequently shown that Frege's fix did not work, but the effort to ground mathematics on a rock-solid foundation went on. In 1922 the logicians Ernst Zermelo and Abraham Fraenkel produced a collection of axioms that, together with another axiom called the axiom of choice, serves as the basis for a large portion of mathematics (the theory of sets, which can model what one normally thinks of as arithmetic). This axiom set is still in use today, and is referred to as ZFC. Depending on its formulation, ZFC amounts to about ten axioms.[8]

Why use *these* axioms? As we will see in a moment, there is not absolute agreement that ZFC is appropriate, but most mathematicians will give at least two reasons for adopting them: (1) the axioms ring true (i.e., they seem worthy of belief);[9] (2) they produce the desired results. The second condition is important. An axiom set that yields unsatisfactory results is not worth much. But this situation raises an interesting question: what renders these results as "desired"? Is it that they conform to commonly shared empirical experiences, or are they independent ontological entities that mathematicians nevertheless somehow sense? If the former, do different people possibly mean slightly different things when they refer to, say, the number five? If the latter, where are these entities located? In God's mind? Section 1.2, Ontology, briefly explores some of these questions.

Logical Disagreements

One of the disputes regarding the axioms of ZFC arises over the "C" in the acronym, which refers to the axiom of choice. Loosely speaking, this axiom stipulates that, given any collection of non-empty bins, it is possible to select one item from each of them. There is no disagreement among mathematicians over the use of this axiom unless the collection of bins is infinite. Even then, there would not be disagreement if, in a specific instance, there were a specified rule or procedure for the selection. For example, if it were known that the bins consisted of positive integers, one could stipulate—even if some bins had infinitely many positive integers—that the smallest integer is to be chosen from each. If, on the other hand, the only knowledge about the bins were that they contained real numbers (positive or negative), then no constructive procedure could be stipulated ahead of time that would yield a selection. Those accepting the axiom of choice could nevertheless use it to produce a hypothetical selection; those rejecting it would not be able to do so.

Logic and God's Nature

Those who insist that constructive procedures be available in the setting just described likely belong to a school of mathematics known as *Intuitionism*. In general, intuitionists deny that there is any external reality to mathematical objects. Rather, mathematical results are only established by human mental constructions. For them, a mathematical result cannot be established by refuting the claim that the result is false; it must be positively proven within the framework of acceptable intuitionistic assumptions. Thus, intuitionists do not subscribe to the law of excluded middle, which states that, for any proposition P, either P is true or not-P is true.[10] Intuitionists do subscribe to the law of noncontradiction, which states that, for any proposition P, it cannot be the case that P and not-P both hold.[11]

Intuitionism grew out of objections to results that arose in part from the axiom of choice. Its chief proponent was Luitzen Brouwer (1881–1966), who strongly objected to the seeming paradoxes of Georg Cantor's theory of infinite sets. Section 5, Pranalogical Issues, discusses some of these paradoxes. For now we ask a faith-based logical question: Can logical laws be biblically grounded? For example, might 2 Timothy 2:13,[12] "If we are faithless, he remains faithful, for he cannot deny himself," support the law of noncontradiction?[13] What about other laws of logic? The answer to these questions depends on whom you ask.

On the one hand, Sir Michael Dummett (1925–2011), an advocate for intuitionism and a staunch Roman Catholic, rejected classical logic for purely philosophical reasons. He further claimed that his philosophical stance was not influenced in any way by his religious convictions.[14] On the other hand, John Byl, who opts for mathematical realism, attempts to ground a portion of mathematics—including the law of noncontradiction, the axiom of choice, and notions of a completed infinity—on attributes of God found in the scriptures.[15] More generally, Vern Poythress argues that the entire metaphysics of mathematics only holds together coherently because it is part of God's being.[16]

Logic and Gödel

Mathematicians, of course, want coherence, especially in the axioms that help form the building blocks of their edifice. Unfortunately, the theorems that Kurt Gödel produced in 1931 demonstrate that coherence cannot be guaranteed.[17] To explain in full detail the scope of these theorems would go beyond the purpose of this article. Even lengthy treatises by well-known scholars have come under attack by Gödel himself for inaccuracies or misrepresentations.[18] With that caveat out of the way, however, it will be helpful to supply a very brief sketch of Gödel results, as they have important spin-offs for integrative issues. The results apply to any formal axiomatic system that generates an arithmetic capable of addition and multiplication, such as ZFC.[19] In what follows, the phrase *the system* will refer to such an axiomatic system.

Painting with very broad strokes, Gödel created a mechanism for associating a unique number with every well-formed proposition.[20] Thus, if P is a particular proposition of the system it will have a number p associated with it, known as its Gödel number. Gödel then created a proposition G that says, loosely, "The proposition whose Gödel number is g cannot be proved using the results of the system." The remarkable feature about G is that its Gödel number actually is g. Thus, Gödel found a way to have a self-referential statement without the use of potentially ambiguous indexical terms such as the word *this*. In other words, Gödel created an unambiguous way to formulate a proposition that says, roughly,

> G: "This proposition cannot be proved within the system."

Gödel then proved two spectacular results:

Theorem 1: *Within the system, G can be proved if and only if not-G can be proved*.

There are two important implications of Theorem 1:

a. If the system is consistent, then neither G nor not-G can be proved within it.
b. If the law of excluded middle is allowed, then one of the propositions must be true because they are negations of each other. Thus, if the system is consistent, it contains at least one proposition (either G or not-G) that is true, but cannot be proved.

Corollary: *If the system is consistent, then G is true.*

This corollary can be made plausible via metareasoning. The proposition G says, of itself, that it cannot be proved. But if the system is consistent, then, indeed, G cannot be proved, so that G asserts the truth (i.e., G is true).

Theorem 2: *The system cannot be proved to be consistent using the rules of the system.*

The proof of this theorem proceeds as follows: suppose the system could be proved to be consistent. Then, by the above corollary, we would know that G is true, so we would have effectively proven G. But then by Theorem 1, we would also have proven not-G. Thus, G and not-G could *both* be proved, which means that the system is not consistent, a contradiction to our assumption. In other words, the assumption that the system can be proved to be consistent leads to an inconsistency. Recall that *the system* refers to any axiomatic system powerful enough to produce an arithmetic capable of addition and multiplication.

Gödel's results have generated a plethora of specious pronouncements. Following is a sample, whose references are not worth reproducing: "Gödel's theorem tells us that nothing can be known for sure"; "Gödel's incompleteness theorem shows that it is not possible to prove that an objective reality exists"; "By equating existence and consciousness, we can apply Gödel's incompleteness theorem to evolution."

Regardless of what these comments actually mean, it is worth noting the apparent common misunderstanding, that Gödel produced *one* theorem. Perhaps an articulation of that misconception is a red flag to consider when evaluating various pontifications.

Are there any lessons that can be legitimately drawn from Gödel's work? Minimally, his results undercut anyone who might subscribe to a "hyper-foundationalist" program, that is, a program that sets out to prove (in a Descartes-like manner) *everything* that is true by starting with a finite set of indisputable truths or axioms. Gödel demonstrated that not even all mathematical truths can be so established with such a program.

Logic and Mechanism

The Oxford philosopher John Lucas has generated much discussion as a result of his claim that Gödel's theorems refute mechanism.[21] Briefly, Lucas points to the corollary of Gödel's Theorem 1, given earlier: *if the system is consistent, then G is true*. Now, Gödel demonstrated that the truth of G cannot be established within the formal system that generated it, and any computer (and computer program) is an instantiation of a formal system (presumably, of course, capable of addition and multiplication): it operates according to the rules governed by its hardware-software configuration. Thus, no computer can "know" the truth of G. Lucas claims, however, that humans can see that G is true.

This is the point at which the argument gets interesting. According to the corollary, humans can see that G is true, but only if they know that "the system" is consistent. Yet Gödel's second theorem stipulates that a proof of this consistency is impossible. So, then, how is it that humans can know this fact? Lucas, of course, has responded to this critique. In addition, the well-known physicist Roger Penrose agrees with the main conclusion that Lucas draws about mechanism, though perhaps for slightly different reasons.[22] Most people, however, disagree with the reasoning Lucas employs—even those who agree with his conclusion that mechanism is false.

Logic and God

In September 2013, the scholars Christoph Benzmüller and Bruno Woltzenlogel Paleo drew renewed attention to Gödel's ontological proof of God's existence, which he first gave about ten years after his famous incompleteness theorems.[23] Public interest was also captivated by headlines such as "Computer Scientists 'Prove' God Exists."[24]

Gödel's work is a variation of Anselm's ontological argument, which Anselm introduced in chapter two of his famous *Proslogium*:

> Hence, even the fool is convinced that something exists in the understanding, at least, than which nothing greater can be conceived. For, when he hears of this, he understands it. And whatever is understood, exists in the understanding. And assuredly that, than which nothing greater can be conceived, cannot exist in the understanding alone. For, suppose it exists in the understanding alone: then it can be conceived to exist in reality; which is greater.[25]

Benzmüller and Paleo formulated a version of Gödel's argument into a formal system containing five axioms, three definitions, three theorems, and one corollary. The main conclusion is expressed by Theorem 3: *Necessarily, God exists* (in symbols, $\Box\exists x G(x)$). The axioms can be debated, of course, but the system was verified with the help of mechanical theorem provers.[26]

Logic and Computers

Using computerized theorem provers, or using computers in the assistance of a mathematical proof, remains a controversial issue among mathematicians. The controversy came to a head in 1976, when, at a conference in Toronto, Kenneth Appel and Wolfgang Haken announced that they had, with the help of a computer, produced a proof of the "Four Color Theorem." The theorem states that, given any map, it is possible to color it in such a way that no two *adjacent* regions (such as countries or states) have the same color. The term "adjacent" means that the regions in question share a measurable linear distance, and not that they meet only at a point (as do Arizona and Colorado in a map of the United States). The proof involves a branch of mathematics known as graph theory, and it was the computer-assisted bit that caused the stir.

For starters, the program did something that no human could possibly do: it verified the theorem to be true for hundreds of thousands of possible cases. A proof requiring a human to do something like that would at least violate the criterion of surveyability that Ludwig Wittgenstein popularized.[27]

At a press conference, Appel and Haken were asked several questions about the proof:

> Q: How do you know that the computer itself works properly?
>
> A: We've run the program on different machines and gotten the same results.
>
> Q: How do you know that you've considered all the cases?
>
> A: Actually, the other day someone sent us a letter pointing out that we had missed several cases. But we entered those missing cases into the computer program, and it still came out correct.[28]

The first question can actually be broken down into three parts: How do you know the computer hardware behaves as advertised? How do you know the program you created is correct? How do you know the compiler that translates your program into machine language is correct? There are formal methods for verifying the correctness of computer programs, but hardware and compiler verification have been of very limited scope.

The answer given to the second question is a bit disconcerting, but the two original questions give rise to interesting additional queries: Is there a Christian perspective on the role of computers and mathematical proof? Would such a perspective involve giving up a certain standard of certainty, a standard normally associated with traditional (and surveyable!) proofs?

1.2 Ontology

Many people have an intuition that mathematical truths are independent of humans. In the words of Martin Gardner,

> If two dinosaurs met two others in a forest clearing, there would have been four dinosaurs there—even though the beasts were too stupid to count and there were no humans around to watch.[29]

Additionally, mathematical results seem to remain constant across cultures. The mathematical historian Glen Van Brummelen comments that even pre-modern China, which, for all practical purposes, was mathematically isolated from the rest of the world, exhibits an impressive array of results shared by other cultures, such as the binomial theorem, the solution of polynomial equations via Horner's method, and Gaussian elimination for the solution of systems of linear equations.[30]

Ontological Options

What accounts for this intuition, an intuition that is seemingly reinforced by the apparent similarity of shared conclusions? A common belief is that mathematical objects have some type of objectively real status that we can access in some way. An alternate approach is to suggest that our common brain structure generates both the intuition and shared conclusions.

Supporters for both views can be found among thinkers from within and outside the Christian tradition. The physicist Sir Roger Penrose posits the existence of three separate worlds with complex interactions: the physical world, the mental world, and the (Platonic) mathematical world.[31] His proposal has generated a series of objections and responses.[32] Likewise, the mathematician Alain Connes, who argues for an objective, independent existence of mathematical objects, has debated the biologist Jean-Pierre Changeux, who argues that mathematics is merely a product of neural interactions in the human brain.[33] Problems arise in defending each of these positions. The one reducing mathematics to neural brain interactions has to account for the commonsense notion depicted by the intuition of Gardner, mentioned above. For people with views similar to Penrose and Connes, there is the problem of determining where the mathematical world is located, and coming up with a way to explain how humans have access to this world.

Ontological Realism

The earliest Christian perspective supporting an objectively real mathematics that is independent of human thinking is probably due to Augustine, who locates propositions such as "5 + 7 = 12" in God's mind.[34] With such a view, the ontological question relating to the location of mathematical objects dissolves. Further, the means by which we access these ideas can be explained by our having been created in God's image. In other words, it makes sense that God would create humans whose minds reflect, in some very limited sense, his own rationality.

As attractive as it sounds, there are difficulties with Augustine's view that demand sorting out. Mathematical truths seem to be necessarily true. If so, is God's freedom impaired by the requirement that he must conceive these mathematical thoughts? Christopher Menzel has written in detail on issues like this one.[35] An answer to this question, Menzel states, rests on an appeal to God's nature. To say that God necessarily thinks logical thoughts is only to say that God is rational. He cannot refrain from generating them in the same way that he cannot positively commit a sinful act. He cannot do the latter because he is perfectly good. Likewise, being perfectly rational, he cannot do otherwise than conceive all possible well-formed logical thoughts.

That appears to be a nice solution, but some Christians take issue with it. Roy Clouser, for instance, puts God's thoughts on a different plane from that of humans: "Whereas creatures can't break the law of noncontradiction because they're subject to it, God's transcendent being can't break that law because it doesn't apply to God's being at all."[36]

Those who are comfortable with the idea of logic as part of God's nature, however, have a more serious issue to address. It relates to the contradiction identified by Russell that was mentioned in Section 1.1, Logic. Basically, Russell showed that a set being a member of itself is an incoherent notion. But if God knows all mathematical truths, then he presumably can conceive of all possible sets. This conception is tantamount to a set of all sets, which would mean that such a set has itself as a member. Menzel gets around this difficulty by appealing to what philosophers call an impredicative definition, which is a definition that generalizes over a totality to which the entity being defined belongs. The upshot is that if *S* is a collection (i.e., a set) of sets, then the sets in that collection must have been well formed "before" (in a logical sense) they can be aggregated into the set *S*. Thus, there can be no "set of all sets." To account for God's seeming omniscience of logical constructs, Menzel's model has God collecting these logical entities in a hierarchical type-scheme. This model has been formalized in a theory that includes ZFC, and it is provably consistent relative to ZF. Nevertheless, certain difficulties remain,[37] so more work can profitably be done in this area.

Ontological Nominalism

Problems with mathematical realism have led some thinkers to the view that there are no universals or abstract objects.[38] People belonging to this school are dubbed *Nominalists*, coming from the Latin word *nomen*, meaning name. Thus, for Nominalists, mathematical objects have no objectively real status. Sets, numbers, and propositions are simply convenient naming devices humans have devised to describe common experiences or thoughts.

Historically, many important philosophers have held this view, for example, William of Ockham, John Stuart Mill, and George Berkeley, but there is an important issue for the Nominalist to sort out. It is often referred to as the indispensability argument, popularized by Hilary Putnam and Willard Quine.[39] In a nutshell, the argument points out that mathematics is amazingly applicable to the physical world. One might even say that it is indispensable for science. That being the case, there is good reason to believe in the existence of mathematical entities. It is hard to imagine that something nonexistent in reality can nevertheless apply so well to the physical world.

The Nominalist Hartry Field took this point seriously. His response to the indispensability argument is the work *Science without Numbers*. In it he attempts to show that, so far as their applications go, mathematical theories need not refer to objectively real objects. Instead, the theories merely need to be "conservative" in the sense that they must be consistent and satisfy a few other minimal conditions.[40] Field then develops "nominalistic axioms" that he claims are sufficient for doing science. Many mathematicians, when looking at these axioms, are unconvinced by the argument. To them, the theory that Field built up looks like another form of mathematics, and a very abstract form at that.[41]

Ontology and the Continuum Hypothesis

The continuum hypothesis is due to the work of Georg Cantor (1845–1919), who was the first mathematician to formalize the concept of infinity. Acting out of obedience to carry out his understanding of God's will, Cantor developed a theory of transfinite numbers. It was vigorously opposed by well-known mathematicians such as Leopold Kronecker, who, like Brouwer, was an Intuitionist (see Section 1.1, Logic). According to Joseph Dauben,

> Cantor believed that God endowed the transfinite numbers with a reality making them very special. Despite all the opposition and misgivings of mathematicians in Germany and elsewhere, he would never be persuaded that his results could be imperfect. This belief in the absolute and necessary truth of his theory was doubtless an asset, but it also constituted for Cantor an imperative of sorts. He could not allow the likes of Kronecker to beat him down, to quiet him forever. He felt a duty to keep on, in the face of all adversity, to bring the insights he had been given as God's messenger to mathematicians everywhere.[42]

Cantor showed that infinite sets can be of different sizes. Two infinite sets are the same size (technically, cardinality) if there is a one-to-one correspondence between their elements. Thus, the set of natural numbers ($N = \{1, 2, 3, \ldots\}$) has the same size as the set of even natural numbers ($2N = \{2, 4, 6, \ldots\}$) because there is a one-to-one correspondence between the two sets: $n \leftrightarrow 2n$.

From that standpoint, it seems at face value that all infinite sets would be of the same size, but Cantor showed otherwise. Remarkably, the set A of all real numbers between zero and one cannot be put into a one-to-one correspondence with N. Mathematicians use the symbol "aleph-null" ($\aleph_0$) to designate the cardinality of N, and c (for "continuum") to designate the cardinality of A.

The *continuum hypothesis* (CH) is the assertion that there is no set whose cardinality is between $\aleph_0$ and c. Cantor spent a great deal of effort trying to show that CH is true. At one point, he thought that he had a proof, but he found an error in it. At another point, he thought he had a proof that the hypothesis was false, but again he found an error. He died without knowing the answer.

In 1940 Kurt Gödel took a big step in proving the CH. He showed that, if ZFC is consistent (ZFC is the axiom set discussed in Section 1.1, Logic), then so is the axiom set ZFC + CH. In 1963 the Stanford logician Paul Cohen (1934–2007) finally put the issue to rest, at least in the context of ZFC. Using a technique known as forcing, he showed that, if ZFC is consistent, then so is ZFC + ¬CH (i.e., ZFC + the negation of CH).[43] Collectively, the results of Gödel and Cohen demonstrate that, if ZFC is consistent, then CH can be neither proved nor disproved within that system.

Thus, the question "Is the continuum hypothesis true or false?" actually has four possible answers depending on one's philosophical outlook: (1) Yes, mathematical objects are objectively real entities, so CH must be either true or false, and I think it is true; (2) Yes, CH is either true or false, and I think it is false; (3) Yes, CH is either true or false, but I have no inkling as to what the true situation is; and (4) No, mathematical objects are not objectively real entities, so there is no universal truth of the matter. Gödel and Cohen collectively have shown that, at least under ZFC, CH is neither true nor false.

The outlook people have on the above question is a good indicator of their ontological viewpoint. In some concluding remarks about CH, textbook author Steven Lay writes,

> Thus the continuum hypothesis is undecidable on the basis of the currently accepted axioms for set theory … It remains to be seen whether new axioms will be found that will enable future mathematicians finally to settle the issue.[44]

The thought that the issue can be "settled" probably reveals the author's realist view of mathematical objects.

2. Worldview Issues

Holmes lists four characteristics that comprise a Christian worldview: (1) holistic and integrational (looking at the "big picture"); (2) exploratory (an endless undertaking because a Christian worldview entails that human finiteness is unlikely to exhaust any subject); (3) pluralistic (because Christians, knowing their fallibility, should welcome a variety of perspectives); and (4) confessional or perspectival (a Christian worldview starts with an admixture of beliefs, attitudes, and values).[45]

Some of the topics discussed in the previous section could well qualify as being worldview issues. In what follows we highlight a sampling of additional aspects of mathematics that relate to a Christian worldview.

Unreasonable Effectiveness?

In 1960 the physicist (and eventual Nobel Laureate) Eugene Wigner published an article that has exerted a considerable amount of influence, especially in the past several years.[46] He saw no satisfactory explanation for the phenomenal success that mathematics seemed to enjoy in the quantum world. Matrix procedures that had been successful with the hydrogen atom were abstracted and applied to the helium atom. Wigner states that there was no warrant for this move because the calculation rules were meaningless in this new context. Yet, the application turned out to be miraculous:

> The miracle occurred … [when] the calculation of the lowest energy level of helium … [agreed] with the experimental data within the accuracy of the observations, which is one part in ten million … Surely, in this case we "got something out" of the equations that we did not put in.[47]

Wigner cites other examples and finally concludes by saying,

> The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning.[48]

Wigner finally received a response from the mathematical community in 1980. The computer scientist Richard Hamming published an article in *The American Mathematical Monthly* in which he gave four "partial explanations" that could account for the success of mathematics: (1) mathematicians craft postulates that conform to things they already have observed, so the implications of those postulates would naturally bear success; (2) mathematicians deliberately select the kind of mathematics that, ahead of time, seems appropriate for a given situation, so the success of mathematics is really no surprise; (3) science (and by implication mathematics) answers comparatively few problems, so there is no big success story here; and (4) evolutionary accounts can explain why human reasoning power is successful.[49]

Hamming concludes by saying that his analysis might account for some of the success of mathematics, but does not fully explain it. Given Wigner's experience with the hydrogen-helium story, he would probably take issue with Hamming's second point in any case.

In 2008 the logician and mathematical historian Ivor Grattan-Guinness gave a more thorough response to Wigner. He pays careful attention to how different philosophical schools might view the status of theories: as mere devices for calculation, for example, some forms of positivism; or as explanatory agents, for example, some forms of Platonism. He then argues that, for the most part, mathematical theories develop in a cultural context, are influenced by other theories already in place, and arise in conjunction with "worldly demands." Referencing Karl Popper, he indicates that there may be an element in science that is guesswork. Sometimes one "hits the bullseye," and that just might have been Wigner's situation in the early stages of quantum theory development.[50] This approach is not necessarily at odds with that of Thomas Kuhn, the proponent of paradigm shifts in science.[51] While Grattan-Guinness is not especially sympathetic with Kuhn's explanation of the structure of scientific revolutions, he does "… accept his advocacy of the Gestalt nature of the change."[52]

Grattan-Guinness may have overstated his case somewhat. For example, no physical phenomena guided the formation of complex analysis—a key tool for Wigner. Nevertheless, his case is a powerful one, and it reinforces the danger of the "you can't explain this" attitude that sometimes accompanies the Wigner discussion. It is somewhat reminiscent of "God of the Gaps" theories. A problem with them for Christian apologetics is that, potentially, the gaps that seem to exist with current theories may someday be closed up.

Other attempts to answer Wigner's question from a Christian or theistic perspective are more in line with cosmological "fine-tuning" arguments, some kind of gap/fine-tuning hybrid approach, or an "inference to the best explanation" argument. Mark Steiner agrees with Grattan-Guinness in that he criticizes Wigner for ignoring the failures in science, but nevertheless sees the success of mathematics in science as an argument against naturalism. If guesswork is involved in science, it is interesting that, as a grand strategy, the bullseye so often is hit when the method employed rests on mathematical theories that invariably grew out of human aesthetic criteria. As Brian Green observes, "Physicists … tend to elevate symmetry principles to a place of prominence by putting them squarely on the pedestal of explanation."[53] Steiner sees this outcome as evidence of some sort of privilege that befalls the human species. It makes the universe appear to be "user friendly" and thus of an anthropocentric character. And any form of naturalism, for Steiner, is *ipso facto* nonanthropocentric.[54] The author of this article has produced a fuller elaboration of these aesthetic considerations in the edited volume *C. S. Lewis as Philosopher: Truth, Goodness, and Beauty*.[55]

Aesthetics

What aesthetic principles apply in mathematical theory formation? G. H. Hardy developed several ideas in his book *A Mathematician's Apology*. He states that criteria governing "good" mathematics include economy of expression, depth, unexpectedness, inevitability, and seriousness—qualities that also seem to form standards for good poetry.[56] Two of these standards—inevitability and unexpectedness—seem in conflict: how can something inevitable also be unexpected? In a beautiful mathematical proof, however, there is almost always a clever idea that takes the reader by surprise. The idea often reveals a new insight in a similar way that a brilliant move might reveal an opponent's weakness in a chess match. Then, often with other clever ideas, the proof proceeds to a conclusion that in retrospect is inevitable. A similar line of reasoning might apply to the reading of a beautiful poem. It will contain many phrases or nuances that are delightfully new or unexpected. Yet, at the end—paradoxically—there is a feeling that the prose had to be stated the way it was.

What are some Christian perspectives on mathematical aesthetics? Matt Delong and Kristen Schemmerhorn have produced a short piece,[57] and more work in this area would be welcome.

Chance

In 1998 William Dembski published *The Design Inference*, which is a revision of his PhD dissertation in philosophy for the University of Illinois at Chicago. In it he maps out a mathematical theory for detecting design, and thus can legitimately be considered as a founder of the "Intelligent Design" movement. Essentially, the theory makes use of a "design filter," which operates by asking two questions about phenomena that evidently have no natural law explanations: whether they are statistically very unlikely, and whether they contain independently detectable patterns. If the answer to both questions is yes, then design may be reasonably inferred. Dembski tackles problems, such as determining how unlikely something must be to pass the filter's test, and indicates that the general thrust of his approach conforms with what people do all the time in attributing design to things they encounter.[58]

Dembski's work has generated a considerable amount of controversy—not so much relating to his filter per se, but in his applications of it. An opponent of standard evolutionary explanations for the emergence of life, he is a leading proponent for allowing the teaching of intelligent design as part of the science curriculum in public schools. Along with others, he cites numerous examples of biological systems that purportedly exhibit design as determined by the filter.[59]

Dealing with randomness is awkward for those who view God as sovereign, and also for those who see the universe as a closed, deterministic system. Recently, however, Christian thinkers such as Keith Ward[60] and David Bartholomew[61] have explored the possibility that God may use chance or randomness in fulfilling his purposes for creation. Bartholomew contrasts his thinking with Dembski in the following way:

> The main thesis of the Intelligent Design movement runs counter to the central argument of this book. Here I am arguing that chance in the world should be seen as within the providence of God. That is, chance is a necessary and desirable aspect of natural and social processes which greatly enriches the potentialities of the creation. Many, however, including Sproul, Overman and Dembski, see things in exactly the opposite way. To them, belief in the sovereignty of God requires that God be in total control of every detail and that the presence of chance rules out any possibility of design or of a Designer.[62]

It is not clear that Bartholomew is correct in his description of Dembski's apparent opposition to chance; the main point here, however, is to illustrate two very different approaches to a philosophy of chance that Christian thinkers might take.

The topic of chance has become so important that the Templeton Foundation recently made funding available to help facilitate scholars in their thinking about the issue.[63] James Bradley, the project director for this grant, has listed some interesting examples of randomness that may hint at divine providence.[64] Here are two: (1) The process of diffusion, which involves random molecular motion, delivers nutrients to the approximately ten trillion cells in the human body. Thus, randomness serves a purpose in this instance. (2) Some dynamical systems, for example, Julia sets, produce stable outcomes from random inputs, and other such examples can be found in genetic algorithms and quantum randomness. Thus, order and randomness in these instances are not mutually exclusive.

Bradley has also written about chance for this journal,[65] and for more general readers.[66] Dillard Faries has also published on the topic in this journal.[67] Any additional output that Christian mathematicians might produce in this area will be a welcome contribution to worldview issues.

Culture

Mathematics has had a profound influence on human culture. For example, it can be argued that a significant amount of modern philosophy has been driven by ontological and other problems raised by the practice of mathematics. A portion of a work edited by Howell and Bradley traces this influence from a Christian perspective.[68] Vladimir Tasić has produced a 157-page volume focusing on a single issue: how mathematics has influenced postmodern thought.[69]

Both accounts paint with broad strokes, but the grounds are fertile for Christians expounding on more-targeted influences of mathematics, influences of which the general public might not be aware. Recent articulations of significant issues are not hard to find. To illustrate, Carlos Bovel has argued that a clause in the Westminster Confession can be traced to the geometrical methodological approach in philosophy launched by René Descartes.[70] On a more popular level, a sophisticated mathematical algorithm is the basis for the search engine used by Google. Articles on this topic have appeared in mathematical journals, of course,[71] but accessible books are now on the market, at least for readers with some degree of mathematical sophistication.[72] Information (though outdated) on the process that Google employs is even available for public consumption on Wikipedia.[73]

3. Ethical Issues

The practices by internet companies such as Google have rightly undergone ethical scrutiny by the public. According to Holmes, the values that Christians have will show up—consciously or unconsciously—in their work. In the ethical sphere, an important component for integrating a discipline with the Christian faith involves what ethicists term "middle-level" concepts, which are the mediators between the "facts" uncovered by a discipline and the biblical values of justice and love. This section explores some possibilities for ethical integration in mathematics.

Disciplinary Worth

Christian educators do not all share the same degree of freedom in the profession of their disciplines. The latitude endorsed by their guilds in determining appropriate choice of topics and assigned readings varies considerably. In mathematics, the curricular expectations at the undergraduate level are fairly narrowly focused. Nevertheless, all disciplines share a common concern: whether the discipline itself is worth pursuing.

Two of the standard responses for the worth of mathematics are the *aesthetic argument* (mathematical theories, like great art, have worth simply because of their beauty), and the *future-value argument* (even if a current mathematical theory has no apparent use, theories of mathematics have—historically—eventually resulted in important practical applications). The increasing specialization of mathematics, however, makes these arguments more difficult to sustain. Often, for some highly technical mathematical results, only a dozen or so people fully understand them. If that is the case, the aesthetic and future-value arguments are at least threatened: the value of beautiful things that can be appreciated by only a handful of people can be questioned, and mathematical results must have a certain amount of dissemination if they are to have a reasonable chance of one day finding an application. Michael Veatch has written on this conundrum,[74] and further work from a Christian perspective would be welcome.

Disciplinary Apology

Related to the question of disciplinary worth is the need for Christians to develop an apology for the study of mathematics. The section on aesthetics mentioned an apology by G. H. Hardy. It contains many valuable insights, but was written from a secular perspective and published prior to World War II. Many changes have occurred since then that would no doubt have influenced Hardy's analysis.[75] This author has produced a short apology from a Christian perspective,[76] but a more substantial contribution would render a valuable service to the Christian community.

Disciplinary Pedagogy

The past several years have seen an explosion in pedagogical ideas. In part, it has been driven by the technological revolution. One hears of discussions about MOOCs (massive online open courses), flipped classrooms, IBL (inquiry based learning) practices, and the like. David Klanderman, who specializes in mathematics education, has written on the influence of constructivism in public education,[77] but additional Christian perspectives are needed in evaluating the ever-increasing approaches to education. What, for example, should a Christian response be to pressing factual observations such as the so-called achievement gap in mathematics between various ethnic and social groups? What middle-level concepts can promulgate the biblical values of justice and love in helping overcome the "stereotype threat" that many identifiable groups experience in the mathematical arena?[78]

Should Ethics Influence Mathematics?

Some may claim that ethical considerations should have no bearing on the practice of mathematics. Vern Poythress argues that such a judgment is self-refuting.[79] To see why, label that statement as "C : ethical considerations should have no bearing on the practice of mathematics." Following that as an axiom if you will, it follows that mathematical practice ought not to be influenced by the ethical claim C , which is a self-refuting statement.

4. Attitudinal Issues

Christian mathematicians (indeed, all Christian thinkers) should exhibit practices and affections that grow out of Christian values. According to Holmes,

> If I were teaching symbolic logic, which is as close as a philosopher comes to mathematics, my Christianity would come through with my attitude and integrity far more than in the actual content of the course. A positive, inquiring attitude and a persistent discipline of time and ability express the value I find in learning because of my theology and my Christian commitment.[80]

Holmes goes on to say that these attitudes should affect more than how Christians pursue truth. Their reverence and love for God should also motivate them toward justice (giving all people what they are due, including God), and a desire to act out in practical ways their conviction that every area in the liberal arts—including mathematics—has to do with God.

David Smith gives some nice illustrations of how such attitudes can be played out in teaching the grammar of a foreign language, a subject that is on a similar plane of abstraction as mathematics. He shows how Christian perspectives can be brought to bear in the choice of assigned writing exercises and dialogues used for classroom practice.[81]

Christian mathematics educators can profitably follow Smith's model. Standard exercises in differential equations, for example, can easily be morphed to model phenomena that relate to issues such as ecology or carbon dating that are ripe for Christian involvement. Certain topics by themselves can also serve as springboards for discussion. For example, Wayne Iba has used his training in artificial intelligence to study the proper way in which software programs should render service.[82] What other creative options are possible for Christian mathematicians?

5. Pranalogical Issues

In addition to the four approaches that Holmes delineates, two gospel narratives collectively suggest a fifth category for integrating faith and learning. They share a common feature in that the principles involved are commended by Jesus for their faith.

Pranalogy Defined

The first one, found in Matthew 15:21–28, is the story of the Syrophoenician woman. Her daughter is demon possessed. She begs Jesus for help. In an unusual response, Jesus says, "It is not good to take the children's bread and throw it to the dogs." The woman replies, "Yes, Lord; but even the dogs feed on the crumbs that fall from their masters' table." Jesus then says, "O woman, your faith is great; it shall be done for you as you wish."

The second instance (and the inference that can be drawn from it—see the following paragraph) was highlighted in a chapel address given by Robert Brabenec, in which he referred to an account recorded in Luke 7:1–10.[83] It is the story of a Roman soldier whose servant is desperately ill. In the parallel account given in Matthew 8:5–13, the soldier comes to Jesus and says,

> "Lord, I am not worthy for You to come under my roof, but just say the word, and my servant will be healed. For I also am a man under authority, with soldiers under me; and I say to this one, 'Go!' and he goes, and to another, 'Come!' and he comes, and to my slave, 'Do this!' and he does it." Jesus then says to those around him, "Truly I say to you, I have not found such great faith with anyone in Israel."

Then he heals the servant.

In addition to the praise given by Jesus in these accounts, there is something else that they have in common. The faith of both petitioners came, in part, from their ability to glean a practical spiritual truth by drawing an analogy from what they had learned by experience. The woman did so from behavior she observed among dogs. The soldier likewise understood the implications of having authority by virtue of his occupation, and he applied that knowledge to a trust in the authority that Jesus would have to heal.

This analysis gives rise to an additional category for integrating faith and learning. For lack of a better word, it should probably be called the *pranalogical* because it involves a practical application of an analogy gleaned from one's discipline or life experience. Such an application is the proposed definition of *pranalogy*, a word obtained by combining *pr*actical and *analogy*.

There are several potential pranalogical applications of mathematics that can relate to and even enhance one's Christian faith. Following are some suggestions.

Pranalogical Examples

First, as indicated in Section 1.2, Ontology, Cantor showed that there are actually different sizes of infinity. If the teacher of this theory draws the proper connections, it seems inevitable that, once students see and understand the proof of this result, their notion of God as being infinitely wise, infinitely powerful, or infinitely good, takes on a new and richer meaning, a meaning that would not be possible without seeing that proof.

Of course, other applications involving the infinite are possible. The work of Benoit Mandelbrot and others in developing fractal geometry has led to bizarre sets exhibiting self-similarity and infinite detail.[84] Orbits of points whose starting locations are arbitrarily close together are nevertheless radically different. What pranalogies might Christians meaningfully draw from these ideas?

The second application was brought to light long ago by Bishop George Berkeley. In 1734 he composed an essay entitled "The Analyst; or a Discourse Addressed to an Infidel Mathematician."[85] It is at once a critique of the foundations of calculus and a rebuke of those scientists who deride people of faith for believing in "mysteries," such as the Trinity, that just do not seem to add up. His work closes with a series of 67 pithy rhetorical queries, one of which is "Whether such Mathematicians as cry out against Mysteries, have ever examined their own Principles?"

In other words, Berkeley asserts that, even in mathematics, there are paradoxes. The foundations of calculus have been shored up since Berkeley's time, but paradoxes nevertheless remain. For example, using the axiom of choice, Banach and Tarski were able to show that it is possible to decompose a sphere into only five sections. Then they can be reassembled—without distorting any of the sections in any way—into two completely contiguous spheres of identical size to the first.[86] Surely that is both a mystery and a paradox.[87]

Returning briefly to Cantor's work, the following facts, when put together, are also paradoxical: (1) between any two rational numbers there is an irrational number; (2) between any two irrational numbers there is an irrational number; and (3) these two sets of numbers have no one-to-one correspondence. Thus, there are infinitely more irrational numbers than rational numbers, though infinitely many of both. If pressed to explain this issue, a mathematician might say something like, "Well, that's just how things work when dealing with mysterious concepts like infinity."

Indeed, and if things can get so convoluted in a logically precise, carefully defined system such as mathematics, it should be no surprise when paradoxical ideas arise in the Christian faith. The study of mathematics can thus help cope with these faith paradoxes.

A Pranalogical Caveat

Developing useful pranalogies from one's field of study can be fruitful, but there lurks an obvious danger. In part, it is a danger that accompanies all analogies, but it is especially prominent in mathematics: it is easy to draw analogies that are careless and trite. A well-known mathematician once remarked that the sensitivity of orbits to initial starting locations that Mandelbrot discovered illustrates how God created freedom. Of course, that argument does not hold up. The resulting orbits may be sensitively dependent on their starting locations, and in principle the differences in starting locations may be beyond the capabilities of measurement per the Heisenberg uncertainty principle. Nevertheless, the orbits are still absolutely determined by their starting locations.

Thus, in developing pranalogies one must keep in mind the limits of any model, and in dealing with mysteries ultimately return to Paul's statement in 1 Corinthians 13:12: "For now we see in a glass darkly, but then face to face; now I know in part, but then I will know fully just as I also have been fully known." ◈

Notes

[1]Emil Brunner, *Revelation and Reason* (Philadelphia, PA: Westminster, 1946), 383. Similar statements can be found in other writings by Brunner. See, for example, *The Christian Doctrine of Creation and Redemption* (Cambridge, UK: Clarke, 2002).

[2]A modern axiomatic system has five components: undefined terms (the basic syntactical strings); definitions (composed of undefined terms); axioms (the unquestioned assumptions from which results will be derived); propositions or theorems (the results so obtained); and rules of reasoning (the methods by which axioms and previously proved theorems will be combined to produce new results).

[3]James Bradley, "The Big Questions," in *Mathematics through the Eyes of Faith*, ed. James Bradley and Russell Howell (San Francisco, CA: HarperOne, 2011), 1–14, begins by listing nine such questions.

[4]Constance Reid, *Hilbert* (New York: Springer-Verlag, 1996), 16–17.

[5]Arthur F. Holmes, *The Idea of A Christian College* (Grand Rapids, MI: Eerdmans, 1987). I have altered Holmes's original alphabetical listing to correspond with the order presented in this article.

[6]For Christian colleges, Holmes lists a third area: theology.

[7]Gottlob Frege, *Translations from the Philosophical Writings of Gottlob Frege*, ed. Peter Geach and Max Black (Oxford: Blackwell, 1985), 214.

[8]Technically two of these axioms are actually axiom schemata, each of which contains infinitely many instances. Going into details about this idea, however, is not necessary for the purposes of this article.

[9]The word axiom comes from the Greek *axios* (ἄξιος), meaning worthy.

[10]Note the difference between this statement and the law of bivalence, which says that, for any proposition P, either P is true or P is false.

[11]For further details, see Stewart Shapiro, "Intuitionism: Is Something Wrong with Our Logic?," in *Thinking about Mathematics: The Philosophy of Mathematics* (Oxford: Oxford University Press, 2000), 172–200.

[12]All scripture is from the New American Standard Bible.

[13]This possibility is suggested in James Bradley, Russell Howell, and Kevin Vander Meulen, "Ontology," in *Mathematics through the Eyes of Faith*, ed. Bradley and Howell, 221–44.

[14]Michael Dummett confirmed this fact to me in a private conversation over lunch and tea at Wolfson College, Oxford, in June 2007.

[15]For details, see John Byl, "A Christian View of Mathematics," in *The Divine Challenge: On Matter, Mind, Math and Meaning* (Edinburgh: Banner of Truth, 2004), 254–78.

[16]Vern S. Poythress, "A Biblical View of Mathematics," in *Foundations of Christian Scholarship: Essays in the Van Til Perspective*, ed. Gary North (Vallecito, CA: Ross House, 1976), 158–88.

[17]See Kurt Gödel, "On Formally Undecidable Propositions of *Principia Mathematica* and Related Systems," in *Kurt Gödel: Collected Works,* vol. 1, *Publications 1929–1936*, ed. Solomon Feferman et al. (Oxford: Oxford University Press, 1986). It contains Gödel's original 1931 paper in German with an English translation on the facing pages.

[18]In fact, a careful popularized account by two respected academicians was criticized by Gödel himself: that by Ernest Nagel and James Newman, *Gödel's Proof* (New York: New York University Press, 2001).

[19]The model most mathematicians had in mind at the time of Gödel was probably the axiomatic scheme contained in the (eventual) three-volume work, *Principia Mathematica*, by Alfred North Whitehead and Bertrand Russell (Cambridge: Cambridge University Press, 1963).

[20]Very loosely, a "well-formed" proposition is one created by properly obeying the syntactical rules of the axiomatic system being used.

[21]John Lucas, "Minds, Machines, and Gödel," *Philosophy* 36, no. 137 (1961): 112–27, http://users.ox.ac.uk/~jrlucas/mmg.html.

[22]See Sir Roger Penrose, *Shadows of the Mind* (Oxford: Oxford University Press, 1994).

[23]See Kurt Gödel, "Ontological Proof," in *Kurt Gödel: Collected Work*s, vol. 3: *Unpublished Essays and Letters*, ed. Solomon Feferman et al. (Oxford: Oxford University Press, 2001).

[24]See David Knight, "Computer Scientists 'Prove' God Exists," Spiegel Online, abcnews.com, October 27, 2013, http://abcnews.go.com/Technology/computer-scientists-prove-god-exists/story?id=20678984.

[25]Anselm, *Proslogium*, available in a variety of publications and online at http://www.fordham.edu/halsall/basis/anselm-proslogium.asp.

[26]See Christoph Benzmüller and Bruno Woltzenlogel Paleo, "Formalization, Mechanization and Automation of Gödel's Proof of God's Existence," September 10, 2013, http://arxiv.org/pdf/1308.4526v4.pdf.

[27]See Ludwig Wittgenstein, *Remarks on the Foundations of Mathematics* Part III, rev. ed. (Cambridge, MA: MIT, 1983). By *surveyability,* Wittgenstein means that mathematical proofs should be able to be reproduced in the same manner in which an artist can reproduce a picture.

[28]The author of this article was privileged to be at this conference; the reporting of the questions and answers that follow are from memory.

[29]See Martin Gardner, *The Night Is Large: Collected Essays, 1938–1995* (New York: St. Martin's, 1997).

[30]For further elaboration, see Glen Van Brummelen, "Mathematical Truth: A Cultural Study," in *Mathematics in a Postmodern Age: A Christian Perspective*, ed. Russell W. Howell and W. James Bradley (Grand Rapids, MI: Eerdmans, 2001), 45–64.

[31]For details, see Roger Penrose, *The Emperor's New Mind* (Oxford: Oxford University Press, 1989) and Roger Penrose, *Shadows of the Mind* (Oxford: Oxford University Press, 1994).

[32]For example, see Rick Grush and Patricia Churchland, "Gaps in Penrose's Toilings," *Journal of Consciousness Studies* 2, no. 1 (1995): 10–29; Roger Penrose and Stuart Hameroff, "What 'Gaps'?," *Journal of Consciousness Studies* 2, no. 2 (1995): 99–112.

[33]Jean-Pierre Changeux and Alain Connes, *Conversations on Mind, Matter, and Mathematics* (Princeton, NJ: Princeton University Press, 1995).

[34]For a nice elaboration of Augustine's ideas, see James Bradley, "An Augustinian Perspective on the Philosophy of Mathematics," *Journal of the ACMS* (2007), http://www.acmsonline.org/journal/2007/Bradley-Augustinian.htm.

[35]Christopher Menzel, "God and Mathematical Objects," in *Mathematics in a Postmodern Age*, ed. Howell and Bradley, 65–97.

[36]Roy A. Clouser, *The Myth of Religious Neutrality: An Essay on the Hidden Role of Religious Belief in Theories* (South Bend, IN: University of Notre Dame Press, 2005), 229.

[37]See Menzel, "God and Mathematical Objects," in *Mathematics in a Postmodern Age*, ed. Howell and Bradley, footnote 42, 93–94.

[38]Technically, a universal is something that has many instances, such as redness, whereas abstract objects, such as numbers and sets, do not exist in the physical world. This distinction will not be elaborated on any further.

[39]See, for example, Hilary Putnam, "What Is Mathematical Truth?," in *Mathematics, Matter and Method,* vol. 1, *Philosophical Papers*, 2nd ed. (Cambridge: Cambridge University Press, 1979), 60–68; or Willard van Orman Quine, "Things and Their Place in Theories," in *Theories and Things* (Cambridge, MA: Harvard University Press, 1981), 1–23.

[40]See Hartry H. Field, *Science without Numbers: A Defense of Nominalism* (Princeton, NJ: Princeton University Press, 1980).

[41]Anecdotally, when Field's book first came out, many parents of children who feared the subject of mathematics rushed to buy the book because of its title. They hoped it would serve as a "gentler, kinder" introduction to science. They were extremely disappointed, however, when they saw the dense notation!

[42]Joseph Warren Dauben, *Georg Cantor: His Mathematics and Philosophy of the Infinite* (Princeton, NJ: Princeton University Press, 1979), 291.

[43]Paul Cohen, "The Independence of the Continuum Hypothesis," *Proceedings of the National Academy of Sciences* 50 (1963): 1143–48, http://www.pnas.org/content/50/6/1143.

[44]Steven Lay, *Analysis with an Introduction to Proof,* 5th ed. (Upper Saddle River, NJ: Pearson, 2014), 91.

[45]Holmes, *The Idea of a Christian College*, 58–59.

[46]The Nobel prize was awarded three years after Wigner published his essay. According to the Royal Swedish Academy of Sciences, the basis for Wigner's selection was "his contributions to the theory of the atomic nucleus and the elementary particles, particularly through the discovery and application of fundamental symmetry principles."

[47]Eugene Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," *Communications on Pure and Applied Mathematics* 13, no. 1 (1960): 9.

[48]Ibid., 14.

[49]See Richard Wesley Hamming, "The Unreasonable Effectiveness of Mathematics," *American Mathematical Monthly* 87, no. 2 (1980): 81–90.

[50]Ivor Grattan-Guinness, "Solving Wigner's Mystery: The Reasonable (Though Perhaps Limited) Effectiveness of Mathematics in the Natural Sciences," *Springer Science+Business Media* 30, no. 3 (2008): 7–17, http://www.sfu.ca/~rpyke/cafe/reasonable.pdf.

[51]Thomas Kuhn, *The Structure of Scientific Revolutions* (Chicago, IL: The University of Chicago Press, 1996).

[52]Ivor Grattan-Guinness, "The Place of the Notion of Corroboration in Karl Popper's Philosophy of Science," in *Induction and Deduction in the Sciences* (Norwell, MA: Kluwer, 2004), 253.

[53]Brian Green, *The Elegant Universe* (New York: Norton, 1999), 374.

[54]Mark Steiner, *The Applicability of Mathematics as a Philosophical Problem* (Cambridge, MA: Harvard University Press, 1998).

[55]Russell W. Howell, "Lewis's *Miracles* and Mathematical Elegance," in *C. S. Lewis as Philosopher*, ed. Dave Bagit, Jerry Walls, and Gary Habermas (Downers Grove, IL: InterVarsity, 2008), 211–27.

[56]Godfrey Harold Hardy, *A Mathematician's Apology* (Cambridge: Cambridge University Press, 1967) .

[57]For one example, see Matt Delong and Kristen Schemmerhorn, "Beauty," in *Mathematics through the Eyes of Faith*, ed. Bradley and Howell, 143–70.

[58]See William A. Dembski, *The Design Inference: Eliminating Chance through Small Probabilities* (New York: Cambridge University Press, 1988). Dembski also addresses other important issues, such as what it means to have an independently detectable pattern.

[59]See, for instance, Michael J. Behe, *Darwin's Black Box: The Biochemical Challenge to Evolution* (New York: Free Press, 2006).

[60]Keith Ward, *God, Chance and Necessity* (Oxford: Oneworld, 1996).

[61]David J. Bartholomew, *God, Chance and Purpose: Can God Have It Both Ways?* (Cambridge: Cambridge University Press, 2008).

[62]Ibid., 99.

[63]For details, visit Randomness and Divine Providence: Request for Proposals, supported by the John Templeton Foundation, October 1, 2012, http://www.calvin.edu/mathematics/randomnessproject/rfp.html.

[64]See Randomness and Divine Providence: Examples, http://www.calvin.edu/mathematics/randomnessproject/examples.html.

[65]James Bradley, "Randomness and God's Nature," *Perspectives on Science and Christian Faith* 64, no. 2 (2012): 75–89.

[66]See James Bradley, "Chance," in *Mathematics through the Eyes of Faith*, ed. Bradley and Howell, 93–114.

[67]See Dillard W. Faries, "A Personal God, Chance, and Randomness in Quantum Physics," *Perspectives on Science and Christian Faith* 66, no. 1 (2014): 13–22.

[68]See Glen Van Brummelen, "Mathematization in the Premodern Period," in *Mathematics in a Postmodern Age*, ed. Howell and Bradley, 133–222.

[69]Vladimir Tasić, *Mathematics and the Roots of Postmodern Thought* (New York: Oxford University Press, 2001).

[70]Carlos R. Bovell, *By Good and Necessary Consequence: A Preliminary Genealogy of Biblicist Foundationalism* (Eugene, OR: Wipf & Stock, 2009). For an evaluation of this book, see Russell W. Howell, "Inerrancy: A Cartesian *Faux Pas*?," *Books and Culture* (November/December 2014): 8–10.

[71]For example, see Michael Rempe, "Google and the Mathematics of Web Search," in *Eighteenth Biennial Conference Proceedings of the ACMS*, ed. Russell W. Howell (2011): 131–37, http://www.acmsonline.org/conferences/2011/proceedings-2011.pdf.

[72]For example, see Amy N. Langville and Carl D. Meyer, *Google's PageRank and Beyond: The Science of Search Engine Rankings* (Princeton, NJ: Princeton University Press, 2012).

[73]See "PageRank," in *Wikipedia: The Free Encyclopedia*, http://en.wikipedia.org/wiki/PageRank.

[74]See Michael Veatch, "Mathematics and Values," in *Mathematics in a Postmodern Age*, ed. Howell and Bradley, 223–49.

[75]Hardy takes great pride, for example, that no applications will ever be found in his area of research. Ironically, his number-theoretic results are now very useful in modern-day encryption systems.

[76]See Russell Howell, "An Apology," in *Mathematics through the Eyes of Faith*, ed. Bradley and Howell, 245–58.

[77]See David Klanderman, "Teaching and Learning Mathematics: The Influence of Constructivism," in *Mathematics in a Postmodern Age: A Christian Perspective*, ed. Howell and Bradley, 338–59.

[78]For a detailed account of "stereotype threat," see Claude M. Steele, *Whistling Vivaldi: And Other Clues to How Stereotypes Affect Us* (New York: W. W. Norton, 2010).

[79]See Vern S. Poythress, "A Biblical View of Mathematics," in *Foundations of Christian Scholarship: Essays in the Van Til Perspective*, ed. Gary North (Vallecito, CA: Ross House Books, 1976), 158–88. Poythress actually argues for the incoherence of the claim that mathematics should not be influenced by religious belief. I have only slightly adapted his argument for ethics.

[80]Holmes, *The Idea of a Christian College*, 47.

[81]David Smith, "Does God Dwell in the Detail? How Faith Affects (Language) Teaching Processes," in *Engaging the Culture: Christians at Work in Education*, ed. Richard Edlin, Jill Ireland, and Geoff Beech (Blacktown, AU: National Institute for Christian Education, 2014), 131–52.

[82]See Wayne Iba, "Before We Get There, Where Are We Going?," in *Your Virtual Butler: The Making-of (Lecture Notes in Computer Science/Lecture Notes in Artificial Intelligence)*, ed. Robert Trappl (New York: Springer, 2013), 54–69.

[83]The address was given at Wheaton College on March 25, 2009.

[84]For details, see Benoit B. Mandelbrot, *The Fractal Geometry of Nature* (New York: W. H. Freeman, 2002).

[85]George Berkeley, "The Analyst; Or, A Discourse Addressed to an Infidel Mathematician. Wherein it is examined whether the Object, Principles, and Inferences of the modern Analysis are more distinctly conceived, or more evidently deduced, than Religious Mysteries and Points of Faith" (1734), ed. David R. Wilkins, available from Kessinger Publishing's Rare Reprints collection and at http://www.maths.tcd.ie/pub/HistMath/People/Berkeley/Analyst/Analyst.html.

[86]For readers of French, see Stefan Banach and Alfred Tarski, "Sur la décomposition des ensembles de points en parties respectivement congruentes," *Fundamenta Mathematicae* 6 (1924): 244–77.

[87]One may object to the claim of paradox on the grounds that the two spheres are each nonmeasurable sets with respect to Lebesgue measure. But that just pushes the conundrum back one step to the question of how there could be such bizarre things as nonmeasurable sets in the first place.

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Article

Douglas C. Phillippy

A Pranalogical Approach to Faith-Integration with Students

Douglas C. Phillippy

This article is written in response to Russell Howell's "The Matter of Mathematics," an essay intended to describe some of the latest challenges for scholars investigating the relationship between mathematics and the Christian faith.[1] *In his essay, Howell asks, "Does faith matter in mathematics?" His answer is "yes" (at least at the metalevel), and he uses the four categories of faith-integration suggested by Arthur Holmes in his book,* The Idea of a Christian College, *as the framework for his thoughts.*[2] *Howell supplements these four categories (foundational, worldview, ethical, and attitudinal) with a fifth, the pranalogical, a term which he defines. In my response, I suggest a strategy for involving undergraduate students in the conversation about faith and mathematics. After highlighting some of the pitfalls of trying to achieve this goal within the four categories of faith-integration suggested by Holmes, I will argue that the fifth category, the pranalogical, has potential to draw students into the conversation.*

My first experience in Christian higher education followed twenty-five years of secular education. After twenty-plus years of training followed by several years of teaching at secular institutions, I was confronted with a concept that was entirely new to me, and the confrontation could not have taken place at a less opportune time. I was being interviewed by a former dean of the college where I am currently employed, and he asked me a question that caught me completely off guard. His question: "What connections do you see between your faith and mathematics?" Today, I do not remember how I answered that question, but I do remember the anxiety I felt as a fumbled my way through an answer. Why was I anxious? Although I had been raised in a Christian family, and had made a personal commitment to Jesus Christ as a young boy, and even though I had spent eleven-plus years being trained as a mathematician and had already taught for two years at two different institutions, I had not put a lot of thought into the relationship between my faith and mathematics.

In regard to my discipline, I thought, as Harry Blamires defined it, "secularly." He said, "To think secularly is to think within a frame of reference bounded by the limits of our life on earth; it is to keep one's calculations rooted in this-worldly criteria."[3] This is not to say that when I was confronted with ideas that directly opposed my Christian upbringing that I simply abandoned my biblical convictions and accepted what passed as scientific theory, or even fact, in the secular community. It is to say, however, that when it came to mathematics, recognizing God as the all-knowing, omnipotent Creator of all that exists, and other ideas essential to the Christian—eternity, heaven and hell, sin and forgiveness, and the fallen state of humanity and its need for a savior—did not enter into my thought process. In short, my faith did

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not matter when it came to my understanding of mathematics.

My inability to articulate a mature answer to the dean's question was a direct result of my education. Unfortunately, this applies to many, if not all, Christians who have obtained an education from secular institutions. In contrast to my training, the college at which I currently teach is a Christian college that makes the claim "Christ is preeminent." The mission statement of this college includes the following proclamation:

> Our mission is to educate men and women toward maturity of intellect, character and Christian faith in preparation for lives of service, leadership and reconciliation in church and society.[4]

In addition, the department in which I teach, the Information and Mathematical Sciences Department, has its own mission statement which includes the following objective: "to challenge students to live out their faith in their vocation as they become servant-leaders in society, church, and the world."[5] These statements suggest that both the college and the department for which I teach take seriously the importance of pursuing a career in light of the Christian faith and its teachings. A natural question to ask then concerns how the goals and objectives that are alluded to in these mission statements make their way into the classroom. That is, how are the students in my classroom challenged to "live out their faith in their vocation," or to "become servant-leaders in the world" by the instruction I give them?

It should be noted that we cannot assume that just because the setting is a Christian college with fine-sounding mission statements that this type of learning is actually taking place. In particular, we cannot assume that this type of instruction naturally takes place just because there is a Christian professor at the front of the classroom. In fact, I would suggest that for a professor like me, who has had no formal instruction in this type of thought, the task of challenging students to think about their education and future career in light of the Christian faith is not an easy thing to do. The discipline of mathematics makes this especially hard, because as Howell suggests, if one plays the game of mathematics, one agrees to play by its rules, resulting in a practice that is "world-viewishly" neutral.[6] The reality is that a teacher who has studied the discipline of mathematics from a secular perspective for many years is not likely to have thought much about what it means to teach mathematics from a Christian perspective.

In his book, *Faith and Learning on the Edge,* David Claerbaut recounts his experience at a Christian college, an undergraduate institution that claimed to teach its courses from a "Christian perspective." According to Claerbaut, "apparently, that teaching occurred in classes I cut or slept through, because I recall scarcely a single class devoted entirely to providing an overtly Christian perspective from which to view the material studied."[7] Instead of professors who taught from a Christian perspective, he encountered "rebellious, agnostic students—many of whom had been forced by their parents to attend a Christian college—boldly proclaiming their unbelieving views in dormitory bull sessions."[8] Claerbaut suggests that his education left him unprepared to answer some of the questions that were raised by these agnostic students. In the end, he says his college education left him "intellectually unarmed, devoid of any ammunition" to confront the examples of unbelief that he encountered even on his Christian college campus.[9]

Before my department had developed a mission statement, the only place in its curriculum that formally attempted to address the idea of faith and learning as it relates to pursuing a career in mathematics was the capstone course for our majors. This meant that our mathematics majors had to wait until their last semester of college before they were required to deal with these issues and the questions they might raise. This is not to say that there were no other opportunities to address faith-related issues, but such issues as discussed in the classroom were usually devotional in nature, and rather intermittently dispersed throughout the curriculum. In many ways then, mathematics majors at my college had a similar experience (at least in terms of their major courses) to the experience Claerbaut had at his college.

If a Christian college does not prepare its students to confront unbelief and also to recognize erroneous beliefs within the academic disciplines, then what is the advantage of a Christian education? Can a Christian college or university expect its graduates to challenge secular thought that contradicts a Christian worldview if it fails to include faith-related topics in its curriculum? The answers to these questions seem obvious to me and motivate self-examination. How do the stated faith-related goals and objectives implied by the mission statements of my college, school, and department make their way into my classroom? Does the instruction that I offer my students arm them with ammunition not only to confront examples of unbelief that they may encounter, but also to prompt them to ask and seek answers to questions regarding the discipline of mathematics as it relates to their Christian faith?

At this point, I would like to begin to argue why I think that the pranalogical category introduced by Howell is a welcome addition to the categories suggested by Arthur Holmes. In particular, I want to suggest that this category allows mentors to develop a contextual framework that is appropriate to drawing undergraduate students into the conversation regarding faith and mathematics. To do so, I will use the language of *faith-integration* that Howell also uses in his essay. After defining what I mean by faith-integration, I hope to describe an appropriate strategy for faith-integration within the discipline of mathematics and then argue why I think that the pranalogical category is better suited to undergraduate participation than the other categories mentioned by Holmes (and Howell). I will conclude this article with a brief discussion of some of my own work in this area.

William Hasker describes faith-learning integration as "a scholarly project whose goal is to ascertain and to develop integral relationships which exist between the Christian faith and human knowledge, particularly as expressed in the various academic disciplines."[10] In general, when I use the language of faith-integration, I mean any attempt by both educators and students alike to relate the academic disciplines (not just an individual's major or specialty) to a biblical worldview. This attempt could be as simple as a devotional that uses a concept or fact within a discipline to illustrate a spiritual truth, or it could be much more complex with the very nature of the discipline itself depending on the faith assumptions that are either held or not held. For the purposes of this article, I am interested in making connections that are deeper than just devotional in nature. However, I need to express a word of caution here. Since the focus of this article is on undergraduate participation in faith-integration, a greater emphasis must be placed on the process of faith-integration rather than any final product that may result. I agree with Claerbaut who says that initial attempts at faith-integration need not be particularly good.[11] This approach is valid because, when attempts at faith-integration are made in the company of fellow scholars, not only will there be the opportunity for constructive criticism, but these very attempts may also stimulate further attempts which are actually better than the original.

A Strategy for Faith-Integration within the Discipline of Mathematics

I grew up learning about God from my parents, pastors, and Sunday school teachers. Among other things, I was taught that God is beyond anything I could imagine. I learned that he is eternal, existing outside of time. I learned that he is omniscient, knowing not only the number of hairs on my head, but also the number of hairs on every head of every human being that ever lived. I also learned that he is omnipresent, present wherever two or three are gathered together in his name. In short, I learned about the infinite nature of God, a concept that is difficult for my finite mind to grasp. Moreover, as I learned about these attributes of God, I was challenged with concepts such as the triune nature of God and paradoxes of the faith such as the "first will be last and the last will be first."[12] All of these ideas are foundational to my faith, and yet because they are rooted in the infinite nature of God, they are difficult for me to understand.

The concept of infinity is also foundational to my study of mathematics and its inclusion in my studies has proven to have its own difficulties. It was not until I studied calculus as a high school senior that I really began to deal with the concept of infinity in a mathematics classroom. Prior to that, infinity was just an idea, but in my calculus class, I was actually expected to use that idea in my calculations. Limits brought me infinitely close to a point without ever actually getting me there. My study of infinite series taught me how to add up an infinite number of terms, most of the time not finding a sum but only knowing whether the sum was in fact finite. Moreover, as I studied calculus, I learned that certain mathematical properties that I thought to be universally true, such as the commutative property of addition, did not necessarily hold true in the realm of the infinite.[13] All of these ideas were hard to grasp as a high school senior, and even today, after twenty-plus years of teaching the subject, I am still mystified by some of the outcomes that are a result of using the infinite in my calculations.

Because both mathematics and theology seek to describe the infinite, one might ask if there is any relationship between the insights gained from these two different perspectives in the search for truth. Unfortunately, in my case, this question never entered my mind. For me, theology and mathematics were disjoint. I learned about the infinite God in church and through reading my Bible, while I learned about the mathematical concept of infinity in my calculus classes. In my mind, these two manifestations of the infinite were unrelated. I had what Richard Bube would call a "compartmentalized" view of these disciplines.[14] This view holds that mathematics and theology tell us "different kinds of things about different things."[15] One who holds this view believes that mathematics and theology deal with two totally unrelated aspects of reality and therefore have no common ground. I believe that, practically speaking, most Christian mathematics majors enter college with this compartmentalized view of their faith and the discipline they intend to pursue.

What is the actual relationship between these two representations of the infinite? More importantly, if a compartmentalized view of faith and mathematics produces a limited understanding of truth, how does one move away from it toward a view that more accurately reflects reality? In an attempt to answer these questions, I will use some of the language that is found in the literature to describe the relationship that exists between faith and various disciplines. Richard Bube mentions seven patterns for relating science to the Christian faith,[16] one of which is the aforementioned "compartmentalized." Of the seven patterns that he mentions, none seems to fit mathematics (and in particular our discussion of the infinite) perfectly. However, certain aspects of the "complementary" and "new synthesis" patterns seem to form a basis for a strategy of integration that is appropriate for our current discussion of the infinite.

The complementary pattern suggests that mathematics and theology *can* tell us "different kinds of things about the same things."[17] That is, both mathematics and theology can provide valid insights into the nature of the infinite, but they do so from different perspectives and therefore tell us different things. Similarly, the "new synthesis" pattern suggests that mathematics and theology *should* tell us the "same kind of things about the same things," but the present status of both disciplines makes this impossible. Both of these strategies are flawed when it comes to relating mathematics and faith. The problem with the complementary view is that it stresses the differences in knowledge obtained from the two contributing perspectives. Although mathematics and theology may tell "*different* kinds of things about the same things," I believe that it is also possible that they tell "the *same* kind of things about the same things." This is more in line with the "new synthesis" pattern.[18] Unfortunately, this pattern holds that the current states of theology and mathematics do not allow for integration to take place and therefore calls for a radical transformation of theology, mathematics, or both. I do not believe that the current states of mathematics and theology disallow integration, and therefore I reject the need for radical transformation.

Instead, I believe that when it comes to mathematics and faith, secular thinking has contributed to the tendency to compartmentalize knowledge. Therefore, it is the Christian scholar's task in integration to "decompartmentalize" this knowledge and to link it in some integral way. Attempts at connecting the mathematical and theological concepts of infinity should thus not require major reconstructions of either of these ideas, but rather should focus on how one of these concepts can shed light on the other. Such a strategy is the compatibilist strategy suggested by Ronald Nelson in *The Reality of Christian Learning.* This approach assumes that the integrity of both the faith and the discipline are intact, and that the scholar's task is to show how the shared assumptions and concerns of the discipline and faith can be profitably linked.[19]

In regard to my discussion of the infinite at the beginning of this section, there is no reason to believe that an infinite God and the idea of a mathematical infinity are in conflict. The compatibilist strategy recognizes this as fact and seeks to link the two in some way. Howell's essay clearly takes a compatibilist approach to faith-integration within the discipline of mathematics. He hints at this when he suggests that he seeks to analyze mathematics at the metalevel. In his essay, he notes that the axiomatic paradigm that defines mathematical practice has been in place for several centuries.[20] The purpose of his article is not to question this paradigm, but as is stated by Howell in his introduction to Foundational Issues, to delineate "a sampling of perspectives that lead to important interactions with the Christian faith."[21] In the next section, I will consider some of these perspectives in light of the initiative to include undergraduate students in the conversation.

The Difficult Task of Integrating Faith and Mathematics

I believe that the task of integrating faith and a discipline should be a two-way process. That is, I believe that my faith should affect the way I approach my discipline, and the study of my discipline should enhance my understanding of truth and therefore benefit my understanding of faith. In his discussion of faith-integration, Hasker refers *both* to the insights of a Christian worldview that are relevant to the discipline, and to the contributions of the discipline to the Christian view of reality.[22] Likewise, in describing integration, Holmes states,

> Integration is concerned ... with the positive contributions of human learning to an understanding of the faith and to the development of the Christian worldview, and with the positive contribution of the Christian faith to all the arts and sciences.[23]

It is clear from this statement that Holmes recognizes that faith-integration allows for contributions both from learning to faith and from faith to learning, making it a two-way process.

Nevertheless, much of the literature seems to emphasize faith's impact on learning. For example, after stating that "learning has contributed from all fields to the church's understanding and propagation of its faith," Holmes adds that the Christian college must recognize that "faith affects learning far more deeply than learning affects faith."[24] In making this statement, Holmes makes a distinction between the two directions of integration. He identifies one direction of integration as being "deeper" than the other.

Of Hasker's four major dimensions of integration within the theoretical disciplines, only one, the worldview contribution, clearly emphasizes a discipline's contribution to the Christian view of reality. Of this view Hasker says, "[the] worldview contribution is the one which has been least emphasized in the literature … so it may be worthwhile saying a few things in defense of its inclusion."[25] In making this statement, Hasker recognizes that academia has had little to say about the contributions a theoretical discipline makes to the Christian view of reality. Howell also seems to imply this in his essay suggesting that when analyzing mathematics at the metalevel, "faith perspectives will surely influence the conclusions one comes to on important questions about mathematics."[26]

If integration is restricted to the influences of faith on learning, the mathematician loses a dimension of integration that is full of many rich possibilities. This is unfortunate because before any restrictions are made, the integration process is already not easy or natural for the mathematician. In speaking of disciplines within higher education which superficially seem to have no integral relationship with Christianity, Holmes includes mathematics.[27] Of the three approaches to integration mentioned by Gene Chase (applicational, incarnational, and philosophical), he states that with respect to mathematics, two "seem inadequate" and one seems "difficult."[28]

The mathematician who restricts faith-integration to a scholarly project that examines faith's impact on his discipline is, in reality, asking if there is a *Christian mathematics*, that is, a type of mathematics that is different from the rest of mathematics because of the influence of Christianity. Many mathematicians, even Christian mathematicians, would argue that the answer to this question is no. Hasker notes that

> the mathematician can deny, with some plausibility, that his Christian faith makes or ought to make a substantive difference to the way he conducts the study of his field: there is no "Christian Mathematics"; the problems and methodologies of mathematics are the same for the believer and the nonbeliever.[29]

Howell agrees with this conclusion in his essay, but suggests that not all is lost; that one can still participate in faith-integration at the metalevel, where analysis and criticism of the discipline can take place.[30] He then goes on to propose several faith-related questions in each of the four categories suggested by Holmes, as well as in his own "pranalogical" category.

Howell's discussion of the relationship that exists between mathematics and faith in these five areas is excellent. However, in the following paragraphs, I would like to make a distinction between the first four categories defined by Holmes (foundational, worldview, ethical, and attitudinal)[31] and the fifth category added by Howell, the pranalogical. The problem that I see with faith-integration in Holmes's four areas is that it tends to rely on faith's impact on the discipline and therefore gravitates toward a discussion that either is not directly dependent on the discipline of mathematics, or relies heavily on philosophical and historical arguments. The pranalogical, on the other hand, seems to consider the impact that mathematics can have on faith and as such provides a different perspective from which to do faith-integration. Howell himself seems to use a different language when he talks about the pranalogical. In his essay he states, "pranalogical applications of mathematics can relate to and even enhance one's Christian faith."[32] Note the direction of impact, namely, mathematics on faith.

What does the attitudinal approach to integration entail? Speaking of this view of integration, Chase says that there is a strong version of this view which claims that "there is a Christian mathematics only insofar as there are Christians who are mathematicians."[33] Holmes describes this approach by saying that "the attitude of the teacher or student is the initial and perhaps most salient point of contact with the Christian faith."[34] Holmes then implies that the attitudinal approach would be extremely significant if he were to teach a mathematics course. He states,

> … my Christianity would come through in my attitude and my intellectual integrity more than in the actual content of the course. A positive, inquiring attitude and a persistent discipline of time and availability express the value I find in learning because of my theology and my Christian commitment.[35]

This is an example of faith-integration that seems "inadequate" to Chase.[36] Hasker goes further and says "cultivation of personal living on the part of the faculty member" is not faith-learning integration.[37]

I agree to some extent with Chase and Hasker that a deeper type of faith-integration exists, one that is more directly connected to the discipline of interest. Attitudinal issues exist across disciplines and are not unique to the study of mathematics. Still, even if one takes this further and suggests, as Howell does, that attitude should influence the types of assignments that mathematics instructors make, one could argue that this is more of a faith-integration exercise for the instructor than the student. Nevertheless, I would argue that the attitudinal approach is a necessary component of faith-learning integration; from a practical viewpoint, it is probably the most important approach to faith-learning integration an individual can take. In fact, I believe that unless an individual takes this approach to faith-integration, all other attempts at doing it will be merely academic. For this reason, all mathematicians should seek to work at faith-integration at the attitudinal level.

With regard to ethics, Howell lists three possibilities for integration in mathematics: disciplinary worth, apology, and pedagogy.[38] These are topics that undergraduate students certainly can write about. In fact, I have my first-year students write an apology of their own after attending my first-year seminar for mathematics majors. Their assignment is to write a letter to a friend who is considering a major in mathematics expressing why a Christian should indeed pursue a career in mathematics. My only problem with this as an example of faith-integration within the discipline of mathematics is that the resulting discussion is not unique to the discipline of mathematics. Howell's own apology which appears in *Mathematics through the Eyes of Faith* ends with these words:

> Thus, whether you choose to use your gift in mathematics—*or any field* (emphasis mine)—as a vehicle for your Christian vocation depends on several factors. Do you like it? Are you good at it? Does the world need it? Do others encourage you in it?[39]

As such, the integral relationship that is being developed here is more between vocation and Christianity and not so much between mathematics and Christianity. With the exception of the question, "Does the world need it?" the answer is not so much dependent on the discipline, but more on the individual who is asking the questions. This is not to say that this is an inappropriate exercise; I believe that it is an appropriate exercise, and I believe that it is faith-integration. However, I do not think that it is the best example of faith-integration that emphasizes mathematics.

Faith can also influence mathematics at the foundational level. What does this type of integration require? Interpreting the use of the words "Christian mathematics" by the Dutch philosopher Herman Dooyeweerd, Holmes says "yet he is thinking not of proofs and procedures but rather of the foundations of mathematics and the fact that God and the law-governed nature of his creation make mathematics possible at all."[40] That is, there is a Christian mathematics when one recognizes that the foundations of the subject are dependent upon the structure that God built into the universe. Thus, this type of integration is typically done from a philosophical point of view and requires an examination of the assumptions that underlie the discipline in view of an individual's faith. Much of what is done in terms of scholarly integration projects within the field of mathematics is done at this level. Howell's essay certainly validates this claim. He begins his discussion with Foundational Issues, noting that "mathematics has a particularly rich tradition" regarding "the historical and philosophical components that have shaped its practices, procedures, and paradigms," and almost half of his essay is devoted to these issues.[41]

Integration at this level can pose problems though. Hasker notes that the "foundation of mathematics is a primary concern for only a rather small percentage of mathematicians and for virtually no undergraduate students."[42] Because of this, he suggests that it would seem to have, at best, limited relevance, a statement with which I agree. To compound the issue, many mathematicians do not have a very strong knowledge base in philosophy or theology. Holmes notes that a scientist can come out of the best graduate school with little more than an eighth-grade knowledge of theology, and perhaps less of philosophy.[43]

Howell's discussion of worldview issues is also, at least to some degree, related to philosophy. He begins by noting that some of the topics discussed in the foundational issues category could just as well qualify as worldview issues.[44] In each of the topics that he introduces in this category, with the exception of aesthetics, there is some connection to philosophy or philosophical argument. Regarding unreasonable effectiveness, a topic that, in my opinion, is by nature very philosophical, he notes that attention was paid to how "different philosophical schools might view the status of theories."[45] In his discussion of chance, he refers to "two very different approaches to a philosophy of chance that Christian thinkers might take." Regarding culture, he refers to several works that describe how mathematics has shaped modern philosophy and thought.[46] Certainly, discussion of the topics that Howell presents in the worldview section is not limited to the philosophical arena, but much of the discussion initiated by Howell seems to have a philosophical taste to it.

How can Christians participate in the faith-integration process with integrity if they are forced to go outside of their own interests and knowledge? One approach would have students strengthen their philosophical and theological understanding. The very nature of scholarly work suggests that this should be the case, but for an undergraduate student or even an established applied mathematician who is more interested in procedures and methodologies than the assumptions that underlie them, it would seem that the integration process would be better suited at the procedural level than at the foundational level. Connections involving faith more naturally occur in an area of interest to an individual. This is true for the teacher of mathematics as well. Teachers typically interact with students whose primary interest in mathematics is not at the foundational level. If teachers are to model the integration process to their students, it would best occur at the level where the teaching occurs. For both the teacher and the working mathematician, integration at the practitioner's level of mathematics does not pose the problems of interest and knowledge that occur at the foundational level.

A Pranalogical Approach to Integration

Integration can and should be done at the functional level of mathematics, that is, where it is practiced, taught, and learned by most individuals. This type of integration depends on the functionality of mathematics and therefore usually considers the discipline's impact on faith. The dimension of "worldview contribution" suggested by Hasker seems to fit well here.[47] This facet of faith-integration seeks to identify how the study of mathematics contributes to an understanding of the world God has created. In particular, it asks how the Christian who has been trained in mathematics views reality differently than the Christian who has had no mathematical training. Mathematicians who answer this question will have a better understanding of their discipline's relevance to their faith, and teachers of mathematics who answer this question will have a tool to motivate their students in their study of mathematics.

How does the study of mathematics contribute to an understanding of the world God has created? I would submit that one of the primary means through which contributions are made is the modeling process. Mathematical models attempt to describe reality abstractly. For this reason, the modeling process seems to fit the worldview dimension of integration mentioned by Hasker well. Giordano and Weir define a mathematical model as a "mathematical construct designed to study a particular real-world system or phenomenon."[48] This definition implies that the goal of mathematical modeling is to study and gain insight into some aspect of reality. By modeling some real-world system, I can gain insight into how that system actually works, and thus have a better understanding of the world God has created.

Caution needs to be used here. Just because a model contributes to a better understanding of God's creation does not make it an example of faith-integration. This understanding can be sought after for a variety of purposes and ultimately used in a variety of ways. It can be used for destructive purposes, or it can be used to improve the quality of life on this earth. Even in the case in which the quality of life is improved, if no attempt is made to relate the model to a biblical worldview, it is not an example of faith-integration. For example, manufacturers of a wide variety of commodities, from shoes to airplanes, use mathematical models to improve existing products and develop new ones.[49] In doing so, these manufacturers improve the safety and performance of their products. However, if such improvements are motivated only by profit or other self-serving outcomes, and there is no discernible connection to a biblical worldview, these models are not examples of faith-integration. So once again this type of faith-integration is closely tied to the motives and attitudes of the model maker. For that reason, this kind of faith-integration also faces the criticism that it is not uniquely related to the discipline of mathematics.

If, however, one seeks to use a mathematical model to better understand some theological concept such as the consistency of God or his infinite nature, the relationship between mathematics and faith is much deeper than the attitude-dependent relationship described above. Here the relationship does not focus so much on the attitude of the modeler (that is not to say that attitude is irrelevant), but on the mathematics and its relationship to faith. This type of integration is what Howell refers to as the "pranalogical," that is, "the practical application of an analogy gleaned from one's discipline or life experience."[50] Consider our earlier discussion of the infinite. Regarding Georg Cantor's discussion on different sizes of infinity, Howell says,

> If the teacher of this theory draws the proper connections it seems inevitable that, once students see and understand the proof of this result, their notion of God being infinitely wise, infinitely powerful, or infinitely good, takes on a new and richer meaning, a meaning that would not be possible without seeing that proof.[51]

In other words, students who have studied mathematical infinities will have a better understanding of the infinite nature of God than if they had not.

The pranalogical approach to integrating faith and a discipline is not without its own potential problems. Too often attempts at this type of integration are only devotional or illustrative in nature. In describing pseudo-integration, David Wolfe cites an example from an article that was written to illustrate the difference between teaching in public day schools and Christian day schools: "Two and two is always four ... and God is always the same; you can depend on him."[52] Both Hasker and Wolfe argue that this is not faith-learning integration.

Although my definition of faith-integration would allow for such an example, it is not the type of faith-integration that is the subject of this article. The above example uses a mathematical "fact" to illustrate a spiritual truth. It considers two unrelated concepts—addition and the immutability of God—and leaves them as separate. Nothing is done to bring the two concepts together. One concept simply illustrates the other. While addition and the immutability of God may not be internally shared by both mathematics and the Christian faith, the concept of consistency that is the main point in the above illustration is internal to both. The worldview dimension of integration asks how the study of mathematics contributes to the Christian's understanding of consistency. It is only when this question has been asked, and the relationship between faith and discipline in the context of consistency has been considered, that genuine integration has taken place.

In summary, modeling gives the mathematician several avenues from which to practice faith-integration. Guided by Christian principles, the mathematician can construct models of reality with the hope of better understanding God's creation in order to improve the quality of life here on this earth. When this happens, mathematics becomes a tool through which mathematicians can love their neighbors as themselves, yet another example of faith influencing the practice of mathematics. But modeling can also be used to gain insight into things that are more directly related to the Christian faith, such as the infinite nature of God. When this happens, mathematics serves as a pranalogical tool that can actually help to shape a proper biblical worldview. Moreover, the insight gained into the biblical worldview is at least directly related if not unique to the study of mathematics.

A Faith-Integration Project for Students

I conclude this article by describing my attempt at including undergraduate students in the faith-integration conversation. I am currently writing a text, now in its third draft, that includes a collection of what I have called "faith-integration projects." These projects provide opportunities for the reader to practice faith-integration by encouraging dialogue. To accomplish this, I begin the conversation with some of my own thoughts on a particular topic. These thoughts are intended only to initiate the dialogue, not to provide the reader with an expert's final analysis of the topic. In particular, each project consists of a short essay that is an attempt on my part to relate faith and mathematics in some way. These essays discuss a variety of mathematical topics appropriate for undergraduate students; many of them are pranalogical in nature.

Because the essays are designed to promote discussion, my hope is that they will provide a basis for further work in the area of faith-integration. In other words, the essay is only part of the project. Each project has the potential for reader participation. Each project begins with a question and includes some of my thoughts as to how that question might be answered. As such, my discussion provides an opinion and not "the answer" to the question. The key to these projects really is the reader's response. My role is only to begin the conversation. The reader's response may be a critique of my essay, or it may be the reader's own answer to the question posed by the project, or it may be both. It may even be the reader's initial thoughts to some other question that the essay prompted her to consider. In any case, the goal of each essay is to engage the reader in connecting faith and mathematics.

While the primary goal of the essay portion of each project is to begin a conversation with the reader regarding faith and mathematics, my writing serves an additional purpose. In particular, my essay serves as a pattern of the type of work that is expected to enter into the dialogue. At a minimum, the dialogue should be a response to some of my comments. At a more serious level, the dialogue might be original work, not a follow-up to discussion in the essay. Ultimately, the purpose of these projects is to help the reader think deeply about mathematics and faith, whether by responding to the author's thoughts or by producing original work. In either case, the discussion should include appropriate worked-out mathematical examples as well as an overview of the topic being considered, including pertinent definitions and theorems. Discussion should include references to scripture and appropriate faith-related definitions. It also might include what others have written and said about the topic. A student project need not include all of the above elements, but it should contain some of them.

One such project in my text is entitled "The Infinite and Intuition." It investigates the following question, "Can the study of the infinite in mathematics help a Christian develop intuition with regard to understanding God and eternity?" To help answer this question, students are first asked to consider their intuition with regard to the infinite in mathematics. They are asked to guess at the percentage of whole numbers that have at least one "3" in their decimal representation; they are then guided through the calculations that show that this percentage approaches

100% as the number of digits in the whole number approaches infinity. Most students are surprised by this result. Students also encounter an infinite set of blocks that, when stacked one upon the other, have an infinite height, and yet can fit in a 2 inch by 2 inch corner of a desk drawer. After making these observations, I state,

> The exercises in this project were offered to illustrate two principles regarding human intuition as it relates to the infinite. First, because human intuition is grounded in an experience in a finite world, and because that experience is often in the context of quantities that are relatively small, human intuition with respect to the infinite is unlikely to be something that has had opportunity to develop. Second, when it comes to the infinite, some outcomes do not seem to make sense, much less be intuitive.

Students are then asked to respond to my essay with their own essay; they are asked several questions to prompt their thoughts.

1. How does the author answer the question, "Can the study of mathematics help a Christian develop intuition with regard to understanding God and eternity?" Do you agree or disagree with his thoughts?
2. Identify one belief that you hold about God which you do not fully understand. In what ways is this belief related to God's infinite nature? Has the discussion in this chapter given you any insight regarding this belief?
3. Read 1 Corinthians 2. Analyze the claims that the author makes in this chapter in light of what this passage says about understanding things related to God.
4. Identify one surprising mathematical result that you have encountered which is based in the infinite (not mentioned in this project). Does this result give you any insight into spiritual things?
5. Has your intuition ever failed you when it comes to thinking about God? In what ways is God's infinite nature related to this failure?

Involving students in the conversation about faith and mathematics sharpens that conversation and increases their understanding of truth. By making use of the pranalogical in projects like the one described above, students can be drawn into the conversation. They enter the conversation not as individuals forced to consider philosophical arguments that are of no interest to them, or perhaps even beyond their understanding, but as a part of a community of scholars in the context of the mathematics that they are currently studying. More importantly, not only is this conversation relevant to and attainable by undergraduate students, but it also may actually strengthen the faith of all who are involved in the conversation. ◈

Notes

[1]Howell defines the pranalogical as the practical application of an analogy gleaned from one's discipline or life experience. Russell Howell, "The Matter of Mathematics," *Perspectives on Science and Christian Faith* 67, no. 2 (2015): 74–88.

[2]Arthur F. Holmes, *The Idea of a Christian College* (Grand Rapids, MI: Wm. B. Eerdmans, 1975).

[3]Harry Blamires, *The Christian Mind* (Ann Arbor, MI: Servant,1997), 3.

[4]The author of this article teaches at Messiah College. The quote is a part of the college's mission statement.

[5]A part of the mission statement of Messiah College's Information and Mathematical Sciences Department.

[6]Howell, "The Matter of Mathematics," 74.

[7]Claerbaut, *Faith and Learning on the Edge*, 14.

[8]Ibid., 14.

[9]Ibid.

[10]William Hasker, "Faith-Learning Integration, An Overview," *Christian Scholar's Review* 21, no. 3 (1992): 234.

[11]Claerbaut, *Faith and Learning on the Edge*, 139.

[12]Matthew 20:26.

[13]For example, see Ron Larson and Bruce Edwards, *Calculus* (Belmont, CA: Brooks/Cole, 2009), 637–38.

[14]Richard H. Bube, "Pattern 3: Science and Christian Theology are Unrelated," chap. 6 in *Putting It All Together: Seven Patterns for Relating Science and the Christian Faith* (New York: University Press of America, 1995).

[15]Ibid., 95.

[16]Ibid., "Pattern 1: Science Has Destroyed Christian Theology"; "Pattern 2: Christian Theology in Spite of Science"; "Pattern 3: Science and Christian Theology are Unrelated"; "Pattern 4: Science Demands Christian Theology"; "Pattern 5: Science Redefines Christian Theology"; "Pattern 6: A New Synthesis of Science and Christian Theology"; and "Pattern 7: Christian Theology and Science: Complementary Insights," chapters 4–10.

[17]Ibid., 167.

[18]Ibid., "Pattern 6: A New Synthesis of Science and Christian Theology," chap. 9.

[19]Ronald R. Nelson, "Faith-Discipline Integration: Compatibilist, Reconstructionalist, and Transformationalist Strategies," in *The Reality of Christian Learning: Strategies for Faith-Discipline Integration,* ed. Harold Heie and David L. Wolfe (Grand Rapids, MI: Wm. B. Eerdmans, 1987), 320.

[20]Howell, "The Matter of Mathematics," 75.

[21]Ibid.

[22]Hasker, "Faith-Learning Integration, An Overview," 236.

[23]Arthur F. Holmes, *The Idea of a Christian College* (Grand Rapids, MI: Wm. B. Eerdmans, 1975), 46.

[24]Ibid.

[25]Hasker, "Faith-Learning Integration, An Overview," 245.
[26]Howell, "The Matter of Mathematics," 74.
[27]Holmes, *The Idea of a Christian College*, 47.
[28]Gene Chase, "Complementarity as a Christian Philosophy of Mathematics," in *The Reality of Christian Learning*, ed. Heie and Wolfe, 231.
[29]Hasker, "Faith-Learning Integration, An Overview," 246.
[30]Howell, "The Matter of Mathematics," 75.
[31]Holmes, "Integrating Faith and Learning," in *The Idea of a Christian College*, 45–60.
[32]Howell, "The Matter of Mathematics," 85.
[33]Chase, "Complementarity as a Christian Philosophy of Mathematics," 231.
[34]Holmes, *The Idea of a Christian College*, 47.
[35]Ibid.
[36]One could argue that this approach fits both the applicational and incarnational approaches described by Chase.
[37]Hasker, "Faith-Learning Integration, An Overview," 235–36.
[38]Howell, "The Matter of Mathematics," 84.
[39]Russell Howell, "An Apology," in *Mathematics through the Eyes of Faith*, ed. James Bradley and Russell Howell (New York: HarperCollins, 2011), 245–54.
[40]Cited in Holmes, *The Idea of a Christian College*, 47.
[41]Howell, "The Matter of Mathematics," 75–81.
[42]Hasker, "Faith-Learning Integration, An Overview," 246.
[43]Holmes, *The Idea of a Christian*, 56.
[44]Howell, "The Matter of Mathematics," 81.
[45]Ibid.
[46]Russell W. Howell and W. James Bradley, eds., *Mathematics in a Postmodern Age: A Christian Perspective* (Grand Rapids, MI: Wm. B. Eerdmans, 2001); and Vladimir Tasić, *Mathematics and the Roots of Postmodern Thought* (New York: Oxford University Press, 2001).
[47]Hasker, "Faith-Learning Integration, An Overview," 245, 246.
[48]Frank R. Giordano, William P. Fox, and Steven B. Horton, *A First Course in Mathematical Modeling* (Pacific Grove, CA: Brooks/Cole, 2003), 54.
[49]The author of this article has developed mathematical models of the foot and shoe while working for a major shoe company.
[50]Howell, "The Matter of Mathematics," 85.
[51]Ibid.
[52]Cited in David L. Wolfe, "The Line of Demarcation between Integration and Pseudointegration," in *The Reality of Christian Learning*, ed. Heie and Wolfe, 4.

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Jason Wilson

Article

Integration of Faith and Mathematics from the Perspectives of Truth, Beauty, and Goodness

Jason Wilson

How should the enterprise of mathematics-faith integration be classified? In his essay, "The Matter of Mathematics," Russell Howell groups contemporary mathematics-faith integration into five categories: foundational, worldview, ethical, attitudinal, and pranalogical. In this article, an alternative approach is proposed using Alister McGrath's scheme of truth, beauty, and goodness. While Howell's categories are somewhat mutually exclusive, truth, beauty, and goodness are viewed as different perspectives of the same mathematical phenomena. In addition, throughout this article, the faith-learning integration scheme of John Coe is applied to the subject matter. Coe asserts that there are conceptual, methodological, and teleological dimensions to all faith-learning integration. The complementary approaches are intended to enrich the project of mathematics-faith integration, and help apply it not only to the head but also to the heart. The perspectives and dimensions described may be viewed as providing mathematics educators with ways to go beyond the usual secularized mathematical content and connect it with the Creator and the students' relationship with him.

> *The fear of the LORD is the beginning of knowledge;*
> *Fools despise wisdom and instruction.* (Proverbs 1:7[1])

> *Now God gave Solomon wisdom and very great discernment and breadth of mind, like the sand that is on the seashore. Solomon's wisdom surpassed the wisdom of all the sons of the east and all the wisdom of Egypt. For he was wiser than all men, than Ethan the Ezrahite, Heman, Calcol and Darda, the sons of Mahol; and his fame was known in all the surrounding nations. He also spoke 3,000 proverbs, and his songs were 1,005. He spoke of trees, from the cedar that is in Lebanon even to the hyssop that grows on the wall; he spoke also of animals and birds and creeping things and fish. Men came from all peoples to hear the wisdom of Solomon, from all the kings of the earth who had heard of his wisdom.* (1 Kings 4:29–34)

The attainment of wisdom attributed to Solomon occurred under the Old Covenant. How much more then should we be able to grasp the wisdom of God, which is Christ himself (1 Corinthians 1:24, 30), since we have the "mind of Christ" in the New Covenant (1 Corinthians 2:16)? I believe that the kind and manner of insight divinely given to Solomon is available to us today in Christ, and that it is not limited to the ethics, hymn-making, and biology of 1 Kings 4:29–34. Rather, in this article, let us consider the possibility that it is available for multi-faceted discernment in all knowledge, including the teaching and research of mathematics and the sciences.

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The occasion of this article is a response to the broad and thought-provoking lead essay, "The Matter of Mathematics" by Russell Howell.[2] To structure his essay, Howell employed Arthur Holmes's four categories of faith-learning integration: foundational, worldview, ethical, and attitudinal. To these he added a new category: pranalogy (= practical analogy). Within each of these five categories, Howell discussed many of the major contemporary areas of mathematics-faith integration in an attempt to provide a foundation for advancing the scholarly Christian thought in this area. This article seeks to make three contributions to this advance: (1) develop an alternative, but complementary, categorization of the entire mathematics-faith integration enterprise, (2) develop and illustrate three different dimensions of viewing the enterprise, and (3) offer a novel subcategory within Howell's pranalogy.

In a bold appeal to the theological community, theologian Alistair McGrath brilliantly calls the church to recast our natural theology.[3] He argues that the classical view of natural theology was almost exclusively focused on the cognitive-rational-ontological part of life, to the exclusion of the affective (emotions) and enactive (practical outworking). As such, he proposes an intentional re-envisioning of natural theology around the three classical themes of the *Platonic triad*: truth, beauty, and goodness, which correspond to the cognitive, affective, and enactive aspects of life, respectively. After reflecting on McGrath's work, this author has been challenged to see that his conception of mathematics-faith integration has largely been subject to the same narrow focus on the cognitive that McGrath warns against. How fitting is mathematician Howell's timely essay that guides mathematics-faith integration forward in this direction.[4]

In an independent line of inquiry, a different way of viewing faith-learning integration is provided by John Coe, Director of the Institute for Spiritual Formation at Biola University. Coe describes three dimensions of faith-learning integration in education: conceptual, methodological, and teleological.[5] The *conceptual* dimension is the harmonization of the subject matter content with the Christian worldview. In the *methodological* dimension, students bring their disciplines before the Lord in prayer and ask him to teach them in it, using such questions as "Lord, what does this truth prompt in my heart?" and "Is my attitude about this area right before You?" The *teleological* dimension asks the Lord, "How does this apply to my life?" and "What should I do as a result of this teaching?"

The conceptual dimension is the primary kind of mathematics-faith integration that has been done by Christian mathematicians. In fact, the conceptual dimension has been so strongly emphasized that Howell provocatively opens his essay with Emil Brunner's statement that "it is meaningless to speak of a *Christian Mathematics*." The quote implies not only that are there no methodological and teleological dimensions to mathematics-faith integration, but also that the conceptual dimension of mathematics is so untainted by sin that there is no distinction between what would otherwise be a secular vs. a Christian mathematics. Similar to McGrath's enlarging the faith-learning enterprise by considering the additional perspectives of beauty and goodness, Coe enlarges the faith-learning enterprise by considering the additional dimensions of methodological and teleological.

The primary thrust of this article lies in expanding the discussion of the categories for approaching the mathematics-faith integration enterprise. In addition to advancing scholarship, the expanded categories can be useful for teaching. The first contribution intended with this article is to provide an alternative way to classify mathematics-faith integration by using McGrath's categories of truth, beauty, and goodness. While not stated as such, Howell's categories appear to be intended as a somewhat mutually exclusive classification. By contrast, McGrath's categories comprise three different perspectives on the one reality of mathematics. Howell's five categories are still considered useful, and the alternative approach explored in this article should be viewed as complementary. The three perspectives form the titles of the three main sections of this article. By viewing mathematical phenomena from different perspectives, students are able to obtain a more well-rounded view of mathematics-faith integration. In particular, the beauty and goodness perspectives legitimize inquiry in fresh directions as well as providing connections with other disciplines.

The second contribution intended with this article is to provide three different dimensions of integration by applying Coe's dimensions. The rationale behind the two less-discussed dimensions, methodological and teleological, is similar to that behind Howell's invention of his fifth category of "pranalogy," a "practical application of an analogy gleaned from one's discipline or life experience."[5] That is, there is much truth in mathematics, but what ought one do with it, spiritually speaking? Exploring answers to this question has proven to be a fruitful source of motivation in the author's classroom. Methodological and teleological integration will be modeled in each of the three main sections through scripture quotations, discussion of quotations from student papers who practiced it,[6] and occasional reflection prompts. The first prompt offers the following suggestions both for personal use with this article and for future use with students: (1) pause to reflect on the section, waiting on the Lord; (2) consider the student's response in the quotation; and (3) ask The Teacher if he has anything for you at that point (1 John 2:27).

The third contribution intended with this article is the proposal of a novel biblical type of a mathematical phenomenon, which may be classified as a Howellian pranalogy. It is given in Section 2.2 Images of Divine Things. The other examples of mathematics-faith integration throughout this article are less detailed. They are crafted primarily to illustrate the mode of approaching the entire enterprise of mathematics-faith integration from the three perspectives of truth, beauty, and goodness, and the three dimensions of conceptual, methodological, and teleological.

1. Truth

> *Buy truth, and do not sell it,*
> *Get wisdom and instruction and understanding.*
> (Proverbs 23:23)

This verse highlights the well-worn path of those who think about mathematics-faith integration today. Section 1 Foundations and section 2 Worldview of Howell's article intersected this area, comprising about one-half of his material, on the topics of logic, ontology, and chance. Howell's book, *Mathematics through the Eyes of Faith,*[7] co-edited with James Bradley, provides accessible quality coverage of additional mathematics-faith integration questions on truth in chapters entitled "Infinity," "Dimension," "Chance," "Proof and Truth," and "Ontology." Since truth is the most widely covered perspective of mathematics-faith integration, this section is limited to one remark on one truth topic from Howell's article. It is included as a full section in order to provide an illustration of truth as a perspective in relationship to the beauty and goodness perspectives later.

Howell succinctly summarized Gödel's mathematical incompleteness theorems, which state that no consistent axiomatic system can demonstrate its own consistency.[8] Call this *mathematical incompleteness*. In other words, mathematical incompleteness finds consistent axiomatic systems that require information from the outside to determine whether they are true.

Consider another form of incompleteness:

> Christian theology provides an ontological foundation which confirms and consolidates otherwise fleeting, fragmentary glimpses of a greater reality, gained from the exploration of nature without an attending theoretical framework. A traditional natural theology can be thought of as drawing aside a veil briefly, partially, and tantalizingly, eliciting an awareness of potential insight, and creating a longing to be able to grasp and possess whatever is being intimated.[9]

Call this *natural theology incompleteness*. In other words, natural theology incompleteness finds internally consistent systems of natural theology that require outside information to determine whether they are true.

Could an analogy be made from mathematical incompleteness to natural theology incompleteness? It could be along these lines: *As formal mathematical systems require outside information to determine whether they are true, so differing natural theologies require outside information/revelation to determine if they are true*.

Gödel made the mathematical argument rigorous. Could theologians utilize an analogy of this sort to gain further insight into the general vs. special revelation issue by leveraging the mathematical insights?

It is generally held that many mathematical axiomatic systems are true, for example, the Zermelo-Fraenkel axioms of choice, Euclidian geometry (local scale), and the Kolmogorov's axioms of probability. However, Gödel demonstrated that they cannot be *proved* true within the system itself. The manner of escaping the mathematical incompleteness trap to arrive at the truthfulness of mathematics was discussed in Howell's section 1.2 Ontology and section 2 Worldview Issues.[10] The former was in a discussion of the competing philosophies of mathematics. The latter was in the subsection Unreasonable Effectiveness? in which the remarkable fit between the abstract world of mathematics and the real world is discussed. This provides our first contrast between Howell's five categories and McGrath's three perspectives. Here two different categories were referenced in response to one question. By contrast, the question about these concepts arises from the perspective of truth (Is a particular axiomatic system true? Does a particular mathematical concept "fit" the real world?). The concepts are further elucidated from the perspective of beauty (To what degree are the properties of competing axiomatic systems beautiful and what is their meaning? What are the implications of the unreasonable effectiveness of mathematics?). The concepts are yet further elaborated from the perspective of goodness (What is the axiomatic system good for? How can the mathematical concept be used to help humankind?).

Let us now shift to Coe's categories. Up to this point, the dimension of this section has been conceptual. The following two quotations are from student papers: the first response depicts the methodological dimension; the second, the teleological dimension.

> I have always been taught that physics, not mathematics, is the natural law that defines what we observe in nature. Though mathematics is a crucial element of physics, it was interesting to consider the field as distinct from the laws of physics … Here the author establishes a solid argument for the link between divine nature and created order. Our God has made a covenant, a binding contract, with the nature that he himself created. In so doing, God reveals his glory to us and receives the praise for the intricate work of his hands. Mathematical equations that have been developed by humankind reveal the divine nature of God to humans in natural law, thus proving that God has intricately designed them.

Methodological integration is seen in the student's gaining a vision of the "link between divine nature and created order" and seeing God's glory.

> This section is reminiscent of the *Centuries* by Thomas Traherne in describing the gift and worship that is called upon by the glories of the cosmos. A particularly relevant aspect of this participation in the plan of God is the explicitly glorious nature of "nature" itself, not for itself, but in its expression. With the informed position that nature may teach of God and that it is made by his wisdom, participation in the divine nature changes the very way we engage with and perceive life as well as encouraging us to call upon the divine, as the cosmos itself is an orchestration of God's purpose.[11]

Teleological integration is seen in that the student is prompted to make connections with readings in other courses, and then pray ("call upon the divine").

2. Beauty

> *[Wisdom] will place on your head a garland of grace;*
> *She will present you with a crown of beauty.*
> (Proverbs 4:9)

Mathematics contains numerous beautiful phenomena. This has been known by mathematicians for thousands of years, but to this day it is still largely unknown by the public at large. As history has progressed, the power of mathematics has become more widely known, and math occupies an authoritative place in curricula from kindergarten through college. Nevertheless, the power and authority of mathematics are often viewed as lifeless, being felt by people more as a bully than as a ballet dancer. The author has been embroiled in conversations similar to the following countless times:

> "What do you do?"
> "I teach mathematics."
> "Oh. [Memory of pain appears on face]
> The farthest I ever got was …"

It is culturally acceptable to put mathematics in a separate box from the rest of learning and be bad at it, or not like it. This attitude ought not to be! Would a wider public awareness of the beauty perspective of mathematics help?

The beauty of mathematics, and of scientific theories that are expressed in the language of mathematics, is well known throughout the mathematical community, as Howell describes in the subsection Aesthetics.[12] For many, it is even a guiding principle: when confronted with two possible choices, whether results, expressions, proofs, and so forth, people will invariably choose the more beautiful, if possible. Only when the more beautiful option is definitively shown to be incorrect or otherwise inferior will they move to the less beautiful. But what is beauty in mathematics? It is elegance, awe-strikingness, symmetry, power, simplicity, generality, complexity, profundity. Beauty is a nonessential characteristic of mathematics that so regularly characterizes it. But why? What is it doing there?

In 2004, James Bradley founding editor of the *Journal of Christians in the Mathematical Sciences (JACMS),* wrote in his inaugural letter fourteen questions the community needed to address. Question 10 asks,

> Mathematicians frequently state that one of their principal motivations for their work is that they find mathematics of great beauty. What is the concept of aesthetics being used here? How does it compare and contrast with aesthetic concepts in the visual arts and other fields? Christian thinkers have often emphasized the beauty of God. Is there a relationship between these concepts of beauty? If so, what is it?[13]

Trolling through *JACMS* archives reveals references to the relationship between the beauty of mathematics and God, such as mathematical beauty inspiring worship of God, but they do not provide detailed elaboration. The chapter "Beauty" in *Mathematics through the Eyes of Faith*, edited by Bradley and Howell, has perhaps the most extensive Christian discussion of the beautiful mathematical content, including quotes on the relationship by Nicholas Wolterstorff, C. S. Lewis, and Abraham Kuyper. Nevertheless, the actual relationship between mathematical beauty and God is not elaborated beyond the following most explicit quote, "… beauty derives from the beauty of God and that our sense of beauty may derive from our being made in the image of God."[14] As such, the beauty within mathematics is a reflection of the nature of God and, as such, can and should be viewed as a window through which to give the awe/worship to its proper source, which is God.[15] Howell calls for more work in this area.[16] The first subsection below offers an approach to explaining what the beauty means. The second subsection is a lengthy exposition on a theological approach to aesthetics, or interpreting the beauty of God, which may be construed as a Howellian pranalogy.

2.1 Beautiful Mathematics

It is the glory of God to conceal a matter;
But the glory of kings is to search out a matter.
(Proverbs 25:2)

Given any [Euclidian] triangle ABC, is it not amazing that the median of each side intersects at a single point called the centroid? And the perpendicular bisectors of each side intersect at a single point called the circumcenter? And the altitudes of each side intersect at a single point called the orthocenter? And these three centers lie on a single line called the Euler line? And on the Euler line, the distance from the orthocenter to the centroid is always twice the distance from the centroid to the circumcenter? This is stunning because one could conceive of a triangle whose medians (or perpendicular bisectors or altitudes) did not connect at a single point. And even if the three centers were all points, it is surprising that these points would always have such a simple and elegant relationship.[17] The successive combination of so many phenomena, each amazing on its own, presents to the soul a profound sense of awe not unlike the scene of an exquisite waterfall on a magnificent mountainside amidst a gorgeous forest. Any one of these beautiful scenes would amaze, but a superlative effect emerges when they combine. Their united exponential beauty is further enhanced by the absence of the contrary, for example, if the forest were brown, or if the altitudes failed to converge at a point.

Consider another illustration. How is it that π is not merely the ratio of the circumference of a circle to its diameter, but also is the sum of the innocuous looking Leibniz infinite series

$$\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots\right)$$

and the area under this curve

$$\pi = \int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}\,dx$$

and Vieta's irrational product

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdot \cdots$$

and part of the exact scaling constant needed for the famous bell-shaped curve density function

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

and part of Euler's fundamental equation

$$e^{i\pi} + 1 = 0$$

and many, many more diverse phenomena?[18] The extent to which π penetrates mathematics and the sciences is mind-blowing and widely known, yet without a good explanation. What does it mean? Surely it reveals an underlying connectedness and order within the universe, which begs the question of its origin.[19] From a Christian worldview, the origin is aptly understood to be God.

Or why does the Fibonacci sequence describe not only certain population growths (honeybees), but also plant taxonomy (phyllotaxis), music (number of rhythms with one- and two-beat notes), the golden ratio, the golden spiral (galactic spirals), and so on? This is not a random, but a parsimonious, multifaceted, ubiquitous pattern. It is parsimonious because such classes of phenomena are easily envisioned with different, even random, sequences. It is multifaceted and ubiquitous because the complete pattern occurs throughout diverse abstract and real-world realms. Such phenomena are very pleasant, and even fun, to discover and to behold. They again point to a profound order.

Going deeper than triangles and constants and sequences, there is another profound beauty. Why do those phenomena in the world match abstract equations? Why does mathematics "fit" the world so well? See Howell's discussion, particularly the interaction between Wigner and Hamming.[20] While Hamming's naturalistic approach has some explanatory power, it falls short and is devoid of life. The "fit" is beautiful because it resonates with the soul upon viewing it the right way. To those without understanding, it is a mystery that invites them on a quest.[21] To the Christian, it is a corollary of the doctrine of the *Imago Dei*, humankind created in the image of God.

Again, the "fit" is beautiful because it yields an ennobling power: enabling humankind not only to meet their need, but also to serve, explore, and expand through such means as science, engineering, and technology. Without the sublime correspondence between the abstract and concrete realms—if the mathematics "did not work"—none of these outcomes would be possible. Again, this belief finds theological support within the Christian worldview in the creation mandate of Genesis 1:26. It became one of the fundamental assumptions that led to the scientific revolution of the seventeenth century.[22]

The abstract realm is beautiful and mysterious. It has an allure that draws the mathematician in, spurring him or her to make even more discoveries. What can be done to make such beautiful phenomena more visible and appreciated by nonmathematicians? And again, why are these connections there? What do they mean? Many Christians would agree with general propositions such as "God put them there" and "The beauty and order reflect God's nature as in Romans 1:20."[23] These answer, why? but not, what does it mean? Theologically, an answer was discussed: this means that the world is profoundly ordered, that there is a God, that humankind is created to perceive mathematical beauty, and that the world was intentionally created with the abstract-concrete "fit" to benefit humankind.

In the preceding, we have attempted to elaborate on some of the ways in which mathematics is beautiful, and used the following words or phrases: amazing, stunning, surprising, simple, elegant, profound, sense of awe, superlative, exponential, infinite, mind-blowing, parsimonious, multifaceted, ubiquitous, pleasant, fun, ordered, resonating, inviting, mysterious, ennobling, and sublime. While they properly refer to mathematics, each may also be applied, in some sense, to God. Such beautiful phenomena, and the questions they elicit, are not only a treasure, but also a treasure map leading to the Ultimate Treasure.[24]

2.2 Images of Divine Things

> *One thing I have asked from the Lord, that I shall seek:*
> *That I may dwell in the house of the Lord all the days of my life,*
> *To behold the beauty of the Lord*
> *And to meditate in His temple*. (Psalm 27:4)

In section 5 Pranalogical Issues, Howell introduces his fifth category of pranalogy (=practical analogy). He cites the different infinities and mathematical paradoxes as fabulous examples of mathematical phenomena which are known to be true, and by analogy make theological phenomena more understandable or believable.[25] From the perspectives of truth, beauty, and goodness, pranalogies might be perceived in each one.

But what if, instead of our using the intellect to draw parallels between known earthly things to unknown spiritual things, we go in the reverse? That is, we "see" that God has placed in the world signs (types) which were intended to reveal divine things (antitypes). This is called typology. The study of the typology in the Bible is biblical typology. An example would be marriage. According to Paul, marriage was created by God to teach humankind about the mystery of Christ and the church (Ephesians 5:32). This is to be contrasted with the analogy approach the author had always held, namely, that Paul cleverly seized upon this deep and multidimensional part of the world to teach about Christ and the church. William Wainwright gives a particularly lucid discussion of this issue.[26] Of course, there is a fundamental question for analogies: at what point do they break down (because they depend upon human creativity)? For a type, however, there is a related fundamental question: is it real (because it depends on divine creation)?

Jonathan Edwards, the great Puritan preacher of the First Great Awakening, a founding father of Evangelicalism, and called the "most brilliant of all American theologians," discusses this issue.[27] Augustine believed that "God has left traces of the divine identity, character, and nature in the created order, in addition to the explicit, ostensive acts of revelation, culminating in Jesus Christ" and that "these *signa naturalia* are clearly distinct from the *signa data* of divine revelation."[28] Augustine also believed that "God has provided us with a richly textured and signed world which we may enjoy, while at the same time allowing it to denote and signify its original creator and its ultimate goal."[29] In addition to Augustine, an entire Christian tradition viewed the world typologically:

> The Syriac tradition regarded the typology found in Scripture as a particular manifestation of the nature of things. Types, symbols, and mysteries are at the core of Creation itself. The Syriac world view affirms that the world was created by the Word of God and thus is revelatory by nature. It further claims that the Incarnation is the summit of Creation, and was prepared for throughout history. Therefore, the typology found in nature and in Scripture is not just an interpretive tool, but is of the very essence of things.[30]

What follows is part of an answer to the aesthetics question, "What does [this particular beautiful mathematical phenomenon] mean?" It is the suggestion that there could be typological significance in mathematical phenomena. This may be construed as a subcategory of Howellian pranalogy from the perspective of beauty. It is from the standpoint of a Christian who holds that the Bible is the written word of God, profitable for our instruction today; this was also the position of Edwards when he wrote in defense of extrabiblical types. Consider the following two fascinating quotations from Edwards that articulate the position:

> Types are a certain sort of language, as it were, in which God is wont to speak to us. And there is, as it were a certain idiom in that language which is to be learnt the same that the idiom of any language is … Great care should be used, and we should endeavor to be well and thoroughly acquainted, or we shall never understand [or] have a right notion of the idiom of the language. If we go to interpret divine types without this, we shall be just like one that pretends to speak any language that han't thoroughly learnt it … God han't expressly explained all the types of Scriptures, but has done so much as is sufficient to teach us the language.[31]

> I expect by very ridicule and contempt to be called a man of a very fruitful brain and copious fancy, but they are welcome to it. I am not ashamed to own that I believe the whole universe, heaven and earth, air and seas, and the divine constitution and history of the holy Scriptures, be full of images of divine things, as full as a language is of words; and that the multitude of those things that I have mentioned are but a very small part of what is really intended to be signified and typified by these things: but that there is room for persons to be learning more and more of this language and seeing more of that which is declared in it to the end of the world without discovering all.[32]

Thus Edwards describes his belief that the Bible does not exhaust all true types, but that the whole world signifies or typifies divine things. The case is sketched in his notebook.[33] An example of a clear spiritual type never made explicit in the Bible is that the lampstand in the Tabernacle represents the Holy Spirit.[34] An example of a plausible nature type not explicit in the Bible is that the sun is an image of Christ.[35]

To propose a hermeneutic (set of interpretive rules) for identifying an extrabiblical type, consider again Edwards. He wrote two entire volumes analyzing the biblical data on the subject,[36] in addition to his notebook containing his abstracted thoughts.[37] The vast majority of Edwards's types were taken directly from the Bible, although he did provide a theological and philosophical basis for expanding the set of types from explicitly biblical types to include extrabiblical types, particularly from nature and history, of other spiritual realities.[38]

> First, to lay down that persons ought to be exceeding careful in interpreting of types, that they don't give way to a wild fancy; not to fix an interpretation unless warranted by some hint in the New Testament of its being the true interpretation, or a lively figure and representation contained or warranted by an analogy to other types that we interpret on sure grounds.[39]

This gives rise to

Rule #1: *Extrabiblical types are permitted if there is warrant by the New Testament or an analogy can be made to a sure biblical type.*

The following additional rule is proposed:

Rule #2: *The role of extrabiblical types should be limited to enhancing Christian experience, such as inspiring awe or worship, or explaining theology, but not to developing new theology or new biblical interpretation.*

This limits extrabiblical types to the beauty perspective, or aesthetics. To tie in the methodological and teleological dimension, it is recommended that candidate extrabiblical types be sought, studied, or pondered in an atmosphere like that practiced in the tabernacle of David, with its 24/7 praise and worship established by God, from which many of the Psalms came.[40]

In closing this section, if it is true that phenomena in nature—and history—are images or shadows of divine things, then perhaps mathematical phenomena would point not just to the orderliness and beauty of God's nature, but to something more. Howell's pranalogies of infinity and paradoxes might be construed this way. Could it be that Gödel's incompleteness theorems go beyond mere analogy and are a sign of the limitations of even our theological (conceptual) beliefs about God? And what about the geometric theorems, and π, and the Fibonacci phenomena? Could they be images/shadows of divine things as well?

Having focused on Coe's conceptual dimension of integration in beauty, as in truth, consider the following quotes to illustrate methodological integration.

> My mind was able to wrap around the idea of His beauty and glory in other forms of life. For example, I was able to envision the trees reaching towards God, the flowers blooming towards the heavens, and the fiber of our brains which allow us to embrace the reality and the existence of God. Through the prayer, God answered by showing me the impossible that was also made possible, and gave me visions of his beauty and glory all around the earth.
>
> Something that really touched me and kept me thinking was that natural law is a reflection of God's nature. The order that is in the world and is seen by scientists, and described by mathematical equations is not only because of the design of the Creator, but it is most importantly a revelation of the divine nature of the Creator. With this, God showed me that he is Almighty God, who created me and has my life in his hands. This made me think of Psalm 121 … This passage touched my heart in a way that I cannot describe.[41]

This is methodological integration: responding to the truths of God. It is fitting to end this subsection with a final quote from Edwards.

> The enjoyment of [God] … is the only happiness with which our souls can be satisfied … Fathers and mothers, husbands, wives, or children, or the company of any, or all earthly friends … are but shadows; but the enjoyment of God is the substance. These are but scattered beams, but God is the fountain. These are but drops. But God is the ocean.[42]

Having walked the paths of truth and beauty, we round the corner to traverse the third and final perspective of McGrath's platonic triad.

3. Goodness

> *He who gets wisdom loves his own soul;*
> *He who keeps understanding will find good.*
> (Proverbs 19:8)

At the turn of the twentieth century, mathematics was a unified discipline. Then, the famous G.H. Hardy encapsulated the tragic split between pure vs. applied mathematics with his aphorism, "Nothing I have ever done is of the slightest practical use."

Pure mathematicians tend to focus on the abstract and are at home discovering truth. Applied mathematicians, however, want their mathematics to solve problems in the real world—using it for good.

In the 1920s, modern statistics emerged as a separate discipline from mathematics, although at most universities today statistics is taught by mathematics departments and is widely viewed as a branch of mathematics. Statistics presents another case of using mathematics to effect good in the world. Taking a broader view, then, mathematics, particularly through its applied and statistical forms, can be an incredible force for good in the world. The application of mathematics and statistics for "the good" is part of the fulfillment of the creation mandate of Genesis 1:28. This is seen through engineers who design things for people, actuaries who create financial models to keep people insured, statisticians who analyze data to improve processes, and so on. Such good is well known to much of the Christian church.

The teleological dimension arises here. For the Christian mathematician, all work, whether applied or not, should be for the glory of God. The Christian who has this belief should experience enhanced motivation beyond his or her non-Christian counterpart (Colossians 3:17). This comports with Howell's remarks on both attitudinal and ethical issues.[43]

Another "good" would be a mathematics education that brings students into all three dimensions of integration. With this outcome, it is seen that the third and final perspective on integration is well known (engineering, education, etc.) and widely discussed even in non-Christian circles.

Having focused on Coe's conceptual dimension of integration in goodness, as done previously in truth and beauty, consider the following student quotes to illustrate teleological integration.

> God showed me that I need to trust him in the little things, and that nothing is too small that it escapes his attention. If he is truly sovereign over even the most miniscule molecular forces, how much more is he sovereign over my life! This gives me great peace knowing that whatever happens in this life, I still have the promise of living with my Creator forever. And my eternal life doesn't start when I die, but it started on the day that I surrendered my life to him. I am so thankful for what God has shown me through this paper. Before, I wasn't aware that learning about God's sovereignty over natural law could have these implications for my life.

> Honestly, after reading this paper, I feel a prompting to improve the quality of my time in praise toward God. I do already praise him, but after reading this I was reminded of how insignificant my praise really is. I see his invisible qualities all around me every day, even when I am not looking at anything; the laws of gravity are holding me down, as an echo of his steadfast and steady love. It is a love that never lets up, or wanes in its intensity. As a part of the praise I feel prompted to begin, I also feel prompted to be more aware of the world around me. The Lord has his invisible attributes in everything and I should want to be constantly seeking these out.[44]

Teleological integration is the application of biblical truth. Here, students were challenged to trust God more and improve their praise quality.

Conclusion

> *The fear of the Lord is the beginning of wisdom;*
> *And the knowledge of the Holy One is understanding.*
> (Proverbs 9:10)

All things in mathematics may be seen to find their end in Christ, as has been implied on this walk through the perspectives of truth, beauty, and goodness in mathematics. Howell framed mathematics-faith integration in terms of five different categories. To illustrate, he described ontology as a foundational issue, while chance was described as a worldview issue. Using the three perspectives, a different approach emerged. Both ontology and chance may be viewed from the perspective of truth. If so, the exposition of ontology would remain the same, while chance might shift to more of the technical details. Going further, ontology and chance could be viewed from the perspective of beauty. Are the different proposed mathematical ontologies beautiful? What properties of beauty do they possess? What do these elicit in the viewer? Lastly, ontology and chance may be viewed from the perspective of goodness. What good can be done with the different mathematical ontologies? For chance and goodness, the innumerable applications of probability and statistics have been harnessed in the service of the Lord and humankind. Using the three perspectives of the

Platonic triad to broaden mathematics-faith integration as a complementary alternative to Howell's five categories was the first contribution intended in this article.

The second contribution was to provide a paradigm for how Christian mathematicians can obtain a deeper spiritual engagement with the subject. This was conducted with Coe's three dimensions of faith-learning integration applied to mathematics. While most of the focus remained on the conceptual, scripture quotations, excerpts from student work of methodological and teleological integration, and occasional prayer remarks were provided to model these dimensions of mathematics-faith integration.

The third contribution was to suggest that some mathematical phenomena may be discovered to signify divine things as Edwardsian types. A hermeneutic for developing such types was provided, which included limiting such Edwardsian mathematics-types to the beauty perspective. All three contributions can be useful for teaching because they provide ways to go beyond the usual secularized mathematical content and connect it with the Creator and the students' relationship with him.

The introduction opened with Proverbs 1:7 and this conclusion closes with Proverbs 9:10. Both verses begin with, "*The fear of the* L*ORD* *is the beginning of* …," but 1:7 says "knowledge," while 9:10 says "wisdom." In mathematics, we need both. In the truth, beauty, and goodness sections, we quoted a Proverb connecting wisdom with each perspective. Thus, it is only through the fear of the Lord that we can obtain true knowledge and wisdom, from which truth, beauty, and goodness are only fully comprehended by the mind of Christ (1 Corinthians 2:16). To achieve this is a spiritual attainment, not of our own strength (1 Corinthians 2:6–3:1). Mathematical truth itself reflects the ordered nature of God (Romans 1:20). Goodness is an attribute of God and, as such, all good ultimately has its origin in him. Beauty is another attribute of God so that, similar to goodness, all beauty has its origin in him. Therefore, when we are enabled to see truths of mathematics such as the beauty of π embodied in the Creator's world and used for the good of humankind through the bell curve, let us increasingly endeavor to do it in the fear of the Lord. Is it not God's will that we see *through* the truth, beauty, and goodness of the mathematical phenomena to see him?[45] Let us then seek him above all and pray that his Wisdom, which is Christ (Proverbs 8:22–35; John 1:1–4) would be manifested through mathematics and our teaching as it was through ethics, hymn-making, and biology in Solomon's day (1 Kings 4:29–34). ◊

Notes

[1]All scripture are from the New American Standard Bible.
[2]Russell Howell, "The Matter of Mathematics," *Perspectives on Science and Christian Faith* 67, no. 2 (2015): 74–88.
[3]McGrath is well-poised for such a call as the Andreas Idreos Professor of Science and Religion at Oxford. Alister E. McGrath, *The Open Secret: A New Vision for Natural Theology* (Oxford: Blackwell, 2008). McGrath's book is cited approvingly in the introductory chapter of W. J. Bradley and R. W. Howell, eds., *Mathematics through the Eyes of Faith* (San Francisco, CA: HarperOne, 2011), 11.
[4]John Coe, "Intentional Spiritual Formation in the Classroom: Making Space for the Spirit in the University," *Christian Education Journal* 4NS, no. 2 (2000): 96–97.
[5]Howell, "The Matter of Mathematics," 87–88.
[6]Quotes are from my Biostatistics class, Spring 2014, responding to integration prompts after reading Jason Wilson, "The Laws of Nature in the Natural Versus Spiritual Mind," *Journal of the Association of Christians in the Mathematical Sciences* (2012–2013), http://www.acmsonline.org/journal/2012_2013/Wilson2012.htm.
[7]Bradley and Howell, eds., *Mathematics through the Eyes of Faith*.
[8]Howell, "The Matter of Mathematics," 76–78.
[9]McGrath, *Open Secret,* 248.
[10]Howell, "The Matter of Mathematics," 78–83.
[11]From students' papers in Biostatistics, Spring 2014.
[12]Howell, "The Matter of Mathematics," 82–83.
[13]James Bradley, "Letter from the Founding Editor," *Journal of the Association of Christians in the Mathematical Sciences (ACMS)* (2004), http://www.acmsonline.org/journal/letterfromeditor.htm.
[14]Matt Delong and Kristen Schemmerhorn, "Beauty," in *Mathematics through the Eyes of Faith*, Bradley and Howell, eds., 165–67.
[15]See Romans 1:20 and Wilson, "The Laws of Nature in the Natural Versus Spiritual Mind."
[16]Howell, "The Matter of Mathematics," 84.
[17]See Peter Woo's beautiful applet depicting this relation along with many others, http://woobiola.net/math/organs.htm. See also the proofs, http://woobiola.net/math/gbook/ch01d.htm and http://woobiola.net/math/gbook/ch03a.htm.
[18]See the collection at http://en.wikipedia.org/wiki/List_of_formulae_involving_%CF%80.
[19]Wilson, *Natural Law,* 1–3.
[20]Howell, "The Matter of Mathematics," 81–82.
[21]This is part of Proverbs 25:2, cited at the beginning of this section. See also the quotes in Wilson, *Natural Law,* 1–3; Howell, "The Matter of Mathematics," 81–82.
[22]Nancy R. Pearcey and Charles B. Thaxton, *The Soul of Science: Christian Faith and Natural Philosophy* (Wheaton, IL: Crossway Books, 1994).
[23]Wilson, *Natural Law,* 4–7.

[24]Proverbs 2:4–5.

[25]For a rich, spiritual look at the use of analogy to penetrate the divine, see Thomas Dubay, *The Evidential Power of Beauty* (San Francisco, CA: Ignatius Press, 1999), 294–96.

[26]William J. Wainwright, "Jonathan Edwards and the Language of God," *Journal of the American Academy of Religion* 48, no. 4 (1980): 524.

[27]George Marsden, *Jonathan Edwards: A Life* (New Haven, CT: Yale University Press, 2004), 1. Unfortunately, Edwards's ideas on the subject were not picked up by his successors because they were not published after his untimely passing in 1758. It would not be until 1948 that the first of his notebooks on typology was published in Perry Miller, *Images or Shadows of Divine Things* (New Haven, CT: Yale University Press, 1948). It would not be until 1993 that the notebook which succinctly articulated his views was published under the title *Types* in Jonathan Edwards, *Typological Writings*, vol. 11 of *The Works of Jonathan Edwards* (New Haven, CT: Yale University Press, 1993), see remark on p. 145.

[28]McGrath, *The Open Secret*, 258.

[29]Ibid.

[30]Joseph Beggiani Seely, "The Typological Approach of Syriac Sacramental Theology," *Theological Studies* 64 (2003): 543–44.

[31]Edwards, *Typological Writings*, 151.

[32]Ibid., 152.

[33]Ibid., 146–53.

[34]See Exodus 25:31–35; Hebrews 8:1–5; cf. Revelation 1:4, 4:5; Zechariah 4:2–6.

[35]Edwards, *Typological Writings*, no. 4, 52.

[36]"Images or Shadows of Divine Things" and "Types of the Messiah," in Edwards, *Typological Writings*.

[37]Ibid., 146–53.

[38]Ibid. Throughout the work, Edwards discusses Psalms 78:2; 125:1–2; John 9:7; 6:31–32; Romans 5:14; Galatians 4:21–23; 1 Corinthians 9:9–10; 10:1–4, 6, 11; 13:2; Hebrews 4:3; 5:6, 11; 7; 8:2, 4–5; 9:1–4, 5, 8–11, 22–24; 10:1; 11:19; 13:11–13; 2 Corinthians 3:13–14; and Colossians 2:16–17. This list includes all Bible references in *Types*, except those in his argument from the permutation of names, p. 150. Nevertheless, Edwards's catalog was primarily biblical types (on his view), though he did include some extra-biblical, including the numbers 76, 95, 142, 146, 156, 196, and 147.

[39]Edwards, *Typological Writings*, 148.

[40]See 1 Chronicles 16:37–42; 23:5; 25:7; 28:13, 19, 21.

[41]Biostatisics students, Spring 2014.

[42]Jonathan Edwards, *Sermons and Discourses, 1730–1733*, vol. 17 of *The Works of Jonathan Edwards* (New Haven, CT: Yale University Press, 1993), 394.

[43]Howell, "The Matter of Mathematics," 83–85.

[44]Biostatistics students, Spring 2014.

[45]Wilson, *Natural Law*, 7ff.

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Joshua B. Wilkerson

Cultivating Mathematical Affections: The Influence of Christian Faith on Mathematics Pedagogy

Joshua B. Wilkerson

The goal of this article is to make the case that Christian faith has an opportunity to impact the discussion on best practices in mathematics, not primarily through the cognitive discussion on objectives and standards, but through the affective discussion on the formation of values, the cultivation of mathematical affections—not merely knowing, but also loving, and practicing the truth, beauty, and goodness inherent in mathematics. First, I will outline the work being done on affect in mathematics education, examining what values are actually endorsed by the community of mathematics educators. After summarizing this work on affect, it will be clear that, even in the words of leading researchers, the field is lacking any cohesive, formal approach to analyzing and assessing the affective domain of learning. Secondly, I will argue the thesis that Christian faith offers solutions to the frustrations and shortcomings admitted by researchers on affect in mathematics education. Christian faith offers insight into how mathematical affections might actually be shaped. Here I will draw heavily on the work of philosopher James K.A. Smith and make explicit connection between his work and the mathematics classroom. Finally, I will conclude with a call to action discussing how we as Christian educators might begin to have fruitful contributions to and dialogue with the current research being done in mathematics education.

"When am I ever going to use this?" is a statement that is often on the ears of every mathematics teacher. Please notice that I referred to this as a statement and not as a question. It has been my experience as an educator (and validated through many conversations with fellow colleagues in the profession) that the true nature of "When am I ever going to use this?" is typically not a legitimate inquiry as to the appropriate timeframe in which the student will eventually apply the material at hand in a "real-life" scenario. Rather, the phrase more often arises as a statement. It is a statement of frustration. It is the culmination of confusion and stress, and it usually serves as an exclamation of their withdrawal from the mental activity at hand. In other words, the answer to the question, "When am I ever going to use this?" has already formed in the student's mind as "I will never use this, so learning it is a waste of time."

The real issue being raised by students is not one of application, but rather one of *values*. I have found that the best response to such a statement/question is to translate it into what I believe the student truly meant to express: "Why should I *value* this?" I believe that this is the question of ultimate concern in the mathematics classroom, and this is the question upon which the Christian faith exerts the greatest influence on the pedagogy of mathematics.

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In his introductory essay, Russell Howell notes the recent explosion of pedagogical practices in mathematics being driven by the technological revolution. Howell calls for Christian perspectives in evaluating these current trends in education. This article is meant to answer that call by suggesting that a Christian perspective can steer the analysis of pedagogical practices from a technological/application lens to a lens focused on the values inherent in mathematics education.

Let us begin by considering why students might phrase their value judgment in terms of the practicality of mathematics? Utility is the language in which the students' culture—including their teachers—has conditioned them to speak. Now, to be sure, application is certainly important to consider in the teaching of mathematics as a powerful pedagogical tool. Application should not be ignored. The goal of this article is simply to call our attention to the deeper issue: our students' desperate desire to find something of value in this world and specifically in the mathematics classroom.

As educators, we play a central role in the formation of students' value systems. As Christian educators, the framework of inculcating values in students and the pedagogical steps we take to achieve this goal are motivated and guided by the transforming grace of the gospel and the historical tenets of the Christian faith. I would argue that as Christian mathematics educators we are afforded a unique venue to act missionaly in contributing answers to a very real need in mathematics education research and practice. I will argue that the question most in need of addressing in the mathematics classroom today is not on the level of cognition—it is not a question of what information (be it in the form of national standards or daily class learning objectives) needs to be passed on to our students. Rather, the question most in need of addressing in the mathematics classroom today it is on the level of the affections—it is a question of formation, of what type of people we desire our students to be, of how we answer, "Why should I *value* this?"

From a Christian perspective, learning has little meaning unless it produces a sustained and substantial influence not only on the way people think, but also on how they act, feel, and ultimately worship. There is ample opportunity now, perhaps more so than ever, for Christian mathematics educators to influence the development of what I will term mathematical affections: not merely knowing, but also loving, and practicing the truth, beauty, and goodness inherent in mathematics.

Values in Mathematics Education: Neglecting Mathematical Affections

Education is inherently value laden.[2] There might be some educators who feel that discussion of values and virtues has no place in an academic setting, especially a public/secular one. The mathematics classroom even more so has a tendency to be seen as values neutral. If we as Christian educators are going to be in a dialogue with secular mathematics educators in any meaningful way, it is important to first make clear that education, and specifically mathematics education, is inherently value laden. It is not a question of "*Are* you teaching values?" but, rather, "*Which* values are you teaching?" Even the statement "We should not be focusing on values in the classroom" is itself a value-based statement.[3] The good news is that the door is open, so to speak, for this values-in-mathematics conversation to begin in a substantive manner.

Noted philosopher of mathematics education Paul Ernest dedicates an entire chapter of his book *The Philosophy of Mathematics Education* to demonstrating the value-laden nature of mathematics, noting that "within mathematics there are implicit values."[3] Now, *where* exactly those values derive from may be up for debate, but that is beyond the scope of this article.[4] For our purposes, the simple recognition that values exist in mathematics (and by extension in the mathematics classroom) is a foundational starting point.

Beyond Ernest, value language is scattered throughout national policy documents on the teaching of mathematics.[5] We see this language in national standards such as the NCTM (National Council of Teachers of Mathematics) *Professional Standards for Teaching Mathematics* (1991): "Being mathematically literate includes having an appreciation of the *value* and beauty of mathematics as well as being able and inclined to appraise and use quantitative information" (emphasis added).[6] Mathematical literacy, according to the NCTM, involves not merely *using* quantitative information, but also giving the discipline of mathematics its proper *value*. Another national policy document, *Adding It Up: Helping Children Learn Mathematics*, a report published by the National Research Council, argues that mathematical proficiency has five strands, one of which is termed "productive disposition." Productive disposition is defined as "the habitual inclination to see mathematics as sensible, useful, and worthwhile."[7] The current Common Core State Standards Initiative grounds its standards for mathematical practice in part upon the same five proficiency strands proposed by the National Research Council.[8] To be mathematically *proficient* (not just literate), the valuation of mathematics must lead to a habit of seeing mathematics as worthwhile—that is, valuable to justify time or effort spent. Mathematics education is inherently value laden.

So the conversation now moves from addressing the existence of values to the questions, *which* values? *where* do they come from? and *how* do educators instill them into students? It is this last question, how to instill values into students (or, in other words, how to cultivate mathematical affections), which this article focuses on.

In examining the current perspectives on affect in mathematics education, I will construct my argument as follows: (1) research on affect in mathematics education tends to misrepresent what affect actually is; (2) this misrepresentation leads to a body of research that largely attempts to address affect in terms of cognition; and (3) the confusion that exists in 1 and 2 results in a shaky foundation (if any at all) for building a discussion as to how to go about cultivating mathematical affections in students. This will set the stage for discussing the impact of Christian faith upon this issue later in this article.

As a first step, consider a foundational document for composing the learning objectives and outcomes of an academic course: Bloom's Taxonomy (figure 1).[9]

	Cognitive Domain (Mental Activity)	Affective Domain (Character and Conscience)	Psychomotor Domain (Physical Activity)
Behaviors from simple to complex	Creating (compose, originate, design, invent)	Characterizing (revise, require, rate, avoid, resist, manage, resolve)	Originating (arrange, build, construct, initiate)
	Evaluating (judge, criticize, evaluate, appraise, recommend)	Organizing (discuss, theorize, formulate, balance, prioritize)	Adapting (alter, rearrange, vary, revise)
	Analyzing (compare, classify, rank, infer, extrapolate)	Valuing (measure proficiency, subsidize, support, debate)	Mechanizing (assemble, calibrate, fasten, measure, mend)
	Applying (organize, solve, generalize, produce)	Responding (comply, follow, commend, volunteer, acclaim, engage in)	Guided Responding (copy, trace, reproduce, react)
	Understanding (explain, infer, interpret, summarize, paraphrase)	Receiving (differentiate, accept, listen for, respond to)	Setting (begin, move, show, state)
	Remembering (recite, quote, list, define)		Perceiving (choose, identify, relate, select)

Figure 1. Bloom's Taxonomy.

The title "Bloom's Taxonomy" is typically used only in reference to the cognitive (mental/knowledge) domain of learning,[10] while the affective (heart/feeling) domain of learning is more specifically referred to as "Krathwohl's Taxonomy," due to the work of David Krathwohl.[11] The affective domain is not simply based on subjective emotions (though emotion may play a small part in affective learning); rather, it is about demonstrated behavior, attitude, and characteristics of the learner[12]—all of which are deeply rooted to success in the mathematics classroom, and all of which are largely misunderstood in mathematics education research.

A quick glance at this chart will reveal that "application" falls under the cognitive domain of learning while "valuing" falls under the affective domain of learning. So when a student asks, "When am I ever going to use this?" (but really means, "Why should I value this?") and a teacher responds to the surface level application question without digging any deeper, the student receives a cognitive response to an affective question. Such a reply also implicitly reinforces in the student's mind that value stems from utility. No wonder students are confused as to why they should value mathematics: their teachers, by and large, are confused as well. Why? Because, even though affective language permeates national published standards on the teaching of mathematics as an ideal that we should strive to inculcate into students, there is little discussion on *how* to go about accomplishing this task.

Affective learning tends to be seen as subjective and emotional; therefore it does not fit well with the objective mindset we have about mathematics teaching and learning. In a special issue of *Educational Studies in Mathematics* devoted entirely to affect in mathematics education, Rosetta Zan states:

> Affect has been a focus of increasing interest in mathematics education research. However, affect has generally been seen as "other" than mathematical thinking, as just not part of it. Indeed, throughout modern history, reasoning has normally seemed to require the suppression, or the control of, emotion.[13]

This quote reveals the tendency in mathematics education to see affect as equivalent with emotions. If affect is indeed synonymous with emotions (or at least viewed that way by the teacher), then it is a very subjective domain and much trickier to navigate than the (at least seemingly) objective cognitive domain. Application of mathematical concepts is much more objective, and something educators are much more familiar with, in the context of mathematics teaching as compared with values. So why do students not, by and large, value mathematics for its own sake, for the beauty, truth, and goodness it reveals? Why do students not look beyond utility to find value? Because their teachers, following the lead of their own teacher preparatory programs and mathematics education research, have taught them otherwise.

The misconception of what affect actually is, and has been historically defined as, has led to a body of research that approaches affect primarily through the lens of cognition—an area that can be analyzed and assessed much more tangibly and objectively. I have organized my summary of this research to follow the levels of Krathwohl's affective domain of learning as illustrated in figure 1: receiving, responding, valuing, organizing, and characterizing. As Christian educators, I believe that it may be more appropriate to view Krathwohl's levels as being grouped into two strands: instilling values and practicing virtues.

In a foundational article on affective learning in mathematics in the *Handbook of Research on Mathematics Teaching and Learning*, Douglas McLeod states:

> Affective issues play a central role in mathematics learning and instruction. When teachers talk about their mathematics classes, they seem just as likely to mention their students' enthusiasm or hostility toward mathematics as to report their cognitive achievements. Similarly, inquiries of students are just as likely to produce affective as cognitive responses, comments about liking (or hating) mathematics are as common as reports of instructional activities. These informal observations support the view that affect plays a significant role in mathematics learning and instruction. Although affect is a central concern of students and teachers, research on affect in mathematics education continues to reside on the periphery of the field … All research in mathematics education can be strengthened if researchers will integrate affective issues into studies of cognition and instruction.[14]

This 1992 article is still applicable today. McLeod goes on to cite efforts to reform mathematics curriculum and those reform efforts' emphasis on the role of affect. The specific documents he cites are the

NCTM *Professional Standards for School Mathematics* (1989) and the National Research Council's report on mathematics education titled *Everybody Counts* (1989). A shift forward in time to statements made in the NCTM's *Professional Standards for Teaching Mathematics* (1991 and 2000) and the National Research Council's report *Adding It Up: Helping Children Learn Mathematics* (2001), reveals that a strikingly similar argument to McLeod's can be made today, with noticeably unchanging language of national published standards, and the similar situations of finding research on affect "on the periphery." It can be argued that McLeod's work has yielded few results and is in need of an adjustment. You will also notice the concluding remark on integrating the study of affect into "studies of cognition." As we will see below, this is the dominant approach taken by researchers in the field and the primary reason that McLeod's work has yielded little by way of results.

The strand of "values" that I propose for organizing our thoughts on affect covers Krathwohl's categories of receiving (the student's willingness to attend to particular phenomena of stimuli), responding (active participation on the part of the student), and valuing (the worth or value a student attaches to a particular object, phenomenon, or behavior). The term Values is essentially referring to developing an attitude toward a particular subject (in this case mathematics). Support for offering this classification of values stems from the NCTM *Professional Standards for Teaching Mathematics* (1991) quoted above. To see how much of the work being done under the strand of instilling values is motivated primarily by cognitive issues, we can turn to another quote from McLeod:

> The emphasis on affective issues (in the U.S. reform movement in mathematics education) is related to the importance that the reform movement attaches to higher-order thinking. If students are going to be active learners of mathematics who willingly attack non-routine problems, their affective responses to mathematics are going to be much more intense than if they are merely expected to achieve satisfactory levels of performance in low-level computations' skills.[15]

This quote as well as numerous examples from research being done on affect[16] seem to indicate a trend that much of the research on developing values[17] in the mathematics classroom is largely driven by increased attention to higher-order cognitive thinking and its impact on the affections of students, rather than vice versa. This ordering of the cognitive as primary and the affective as subservient to the cognitive tends to lead to discrepancies in actually defining what we are talking about (namely, "beliefs" language is classified under affective research, though in actuality it can be argued that beliefs are much more cognitive in nature).[18] In light of this body of research, Anna Sfard writes:

> Finally, the self-sustained "essences" implied in reifying terms such as knowledge, beliefs, and attitudes constitute rather shaky ground for either empirical research or pedagogical practices—a factor of which neither research nor teachers seem fully aware.[19]

It is difficult to develop a robust body of research on affect when it is unclear what exactly affect is and what terminology should be used.

The next strand of affective learning that I proposed, "virtues," is sadly not on any stronger footing in current research than that of "values." The proposed strand of "virtues" covers Krathwohl's categories of organization and characterization. "Virtues" simply refers to allowing values to inform practices—to form habits based on proper values. We can find this language present in "the habitual inclination to see mathematics as … worthwhile" from *Adding It Up: Helping Children Learn Mathematics*.[20] In discussing practicing virtues in the mathematics classroom, I am most interested in exploring research that takes seriously the last two stages of Krathwohl's taxonomy of the affective domain of learning: organizing (bringing together different values, resolving conflicts between them, and beginning the building of an internally consistent value system), and characterizing by value or value set (individual has a value system that has controlled his or her behavior for a sufficiently long time for him or her to develop a characteristic "life style"—thus the behavior is pervasive, consistent, and predictable).

There seems to be very little, if any, research in mathematics education that is focused on the practicing of virtues (the actual demonstration of values through actions).There are several reasons for the dearth of material in this area; I would like to mention two of them. First, as the quote offered by Zan mentioned above indicates, there has been a separation of affect from mathematics research. Second, due to the variability in the research on instilling values, very little groundwork has been laid to take the research a step further: to analyze the practice of those values on a consistent basis. One fruitful point of research on addressing these two issues is offered by Marja-Liisa Malmivuori,[21] who builds off the work of McLeod. While there may be some underlying issues in McLeod's approach as discussed above, we have here an example of continuing work on the level of organization and characterization of student affections in mathematics classes. Malmivuori presents affect as an essential aspect of a student's self-reflection and self-regulation (which fits well with Krathwohl's "organization" category in the affective domain[22]). Students are viewed as agents who constantly interpret and evaluate their experiences and regulate their behavior, in interaction with their mathematics learning environment. Not only are students organizing a value system in mathematics, but they are also evaluating it and allowing it to inform their behavior and habits. However, this research is still being driven largely through the lens of cognition—in this case, focusing on affect in the context of mathematical problem solving.

In summary, there is very little research available with regard to developing the organization and characterization levels of the affective domain in mathematics apart from viewing affect as secondary to cognition. By seeing cognition as the primary goal of the mathematics classroom, there is confusion in defining what exactly we mean when we speak of affect: are we discussing beliefs, or emotions, or attitudes, or values? If space allowed for further study, we would find that work on affect in mathematics can largely be classified as trying to reconcile these various models for understanding what affections actually are, and attempting to explain the complex interaction between affect (whether that is termed as attitudes, or beliefs, or values, or something else) and cognition. Without a solid base of understanding affections, little has been done to analyze at a practical level *how* we as educators might go about cultivating mathematical affections. Removing cognition as the primary lens through which affect is analyzed in mathematics education is no easy task. As Gerald Goldin notes:

> Mathematics educators who set out to modify existing, strongly held belief structures of their students are not likely to be successful addressing only the content of their students' beliefs … it will be important to provide experiences that are sufficiently rich, varied, and powerful in their emotional content to foster students' construction of new meta-affect. This is a difficult challenge indeed.[23]

By "meta-affect" Goldin is referencing affect about affect—or, in other words, how one feels about feeling. For instance, one might experience the feeling of fear when attending a horror movie, but find it enjoyable to do so. This meta-affect level at which students determine what emotions, attitudes, and beliefs are preferable to others is akin to our discussion of value formation, and hence the aptness of Goldin's quote. Values are not going to be modified simply by focusing on content and cognition. The experience of the student needs to change in order to see growth in this area. As we will now see in the next section, a Christian perspective on the teaching of mathematics is up to Goldin's challenge.

Values in Christian Faith: Cultivating Mathematical Affections

What we are really talking about when discussing the affective domain of learning are the habits of our students, how they are instilled, how they are encouraged (or discouraged), and how they are evaluated. For believers, Christian faith will have an obvious impact on this discussion; however, the purpose of this section is to examine how Christians might influence the work being done on affect in mathematics education in a way that would be accepted by all practicing researchers, regardless of their faith commitments. I will begin by briefly summarizing some of the key work that has focused on a Christian approach to mathematics pedagogy, and clarify how what is being proposed here differs from the work that has already been done and how it contributes to this much-needed body of research and resources. Then I will make use of James K. A. Smith's work in *Desiring the Kingdom* to demonstrate how the specific frustrations of researchers in the field of affect in mathematics education can be addressed from a Christian perspective, by ultimately viewing human beings as primarily affective (and secondarily cognitive) creatures. Finally, I will conclude with some practical suggestions for cultivating mathematical affections in the classroom and offer a call to action for developing more resources along these lines.

Let me take a moment to define more clearly what I mean by mathematical affections. The title of this article is in homage to Jonathan Edwards's *Treatise Concerning Religious Affections*.[24] Edwards's goal was to discern the true nature of religion, and in so doing, dissuade his congregation from merely participating in a Christian culture (a mimicked outward expression) and motivate them to long for true Christian conversion (an inward reality of authentic Christian character). The purpose of this article is to engage us as educators in discerning the true nature of mathematical pedagogy and in determining how we as Christian educators can approach the teaching and learning of mathematics: does it simply mimic the modern culture of utility by requiring outward demonstrations of knowledge retention and application, or does it aim deeper at analyzing true inward character formation?

For Edwards, affections were not synonymous with emotions as they tend to be in today's culture (or in today's mathematics education research as noted by Zan). Edwards understood affections as aesthetics—a way of orienting your life via a mechanism that determines what is beautiful and worthwhile. Affections are character producing and habit forming. It is Edwards's definition of affections (orientation of life, determining worth) that actually appears in policy documents that we have cited.

Consider once more that being mathematically literate involves having an appreciation of the value and beauty of mathematics, and being mathematically proficient involves a habitual inclination to see mathematics as worthwhile. Foundational documents in the area of mathematics education plainly portray mathematics as beautiful, of value, and affecting the habits of the learner to see mathematics as worthwhile. However, as we have seen, none of these documents develops *how* we as teachers are to go about accomplishing this task. It is almost as if these phrases are included in these documents as a courtesy—as a way of saying, "This is how we teachers feel about mathematics, and it would be nice for our students to feel this way too. But feeling is subjective, so there is no real way for us to instruct objectively, or to assess students in this regard." This is a point of connection that we as Christian educators can make with the educational system as a whole—we can answer the questions of *how*. We have much to contribute here, and we do not have to be overtly religious in the presentation.

Now let us return to our initial question, "Why should I value this?" and consider how we might respond from a Christian perspective. Michael Veatch notes,

> There is a prevalent attitude that one learns what is good mathematics by seeing and doing it, not by discussing values. The knowledge needed by the person entering the field will rub off on her. The classroom clearly reflects this attitude.[25]

As it stands, our current methods of teaching mathematics are producing untold numbers of students who see mathematics as more a function of natural ability rather than effort, who are willing to accept poor performance in mathematics, who often openly proclaim their ignorance of mathematics without embarrassment, and who treat their lack of accomplishment in mathematics as a permanent state over which they have little control.[26] The reason for this is that we have given values (affections) a backseat in the mathematics classroom.

In *The Abolition of Man*, C.S. Lewis writes, "Education without values, as useful as it is, tends to make man a more clever devil."[27] This is a fairly accurate statement of the modern-day system of mathematics education. If we do not focus on values, if we do not focus on the affective learning of our students, then their education will still be useful in the sense that they will increase in cognitive ability and learn to apply their thinking. But is that outcome really valuable in and of itself? Without a proper sense of values to guide their application, are we not really just making students "more clever devils"?

As we have already noted, education is inherently value laden, so values cannot actually be removed from education. Lewis's point is that the value we instill in education should be affective—loving learning for its own sake and valuing wisdom. If you do not focus on affections, then you still have usefulness, but is that really beneficial? In the words of the Bishop in Victor Hugo's *Les Misérables*: "The beautiful is as useful as the useful … Perhaps more so."[28] Aesthetics can be more useful than utility. I have defined mathematical affections not simply as knowing, but also as loving, and as practicing the truth, beauty, and goodness inherent in mathematics. A Christian perspective on the pedagogy of mathematics has much to offer in this regard.

While there are many resources that examine a Christian perspective of mathematics pedagogy (that is, the teaching of mathematics from a Christian perspective, not just an understanding of mathematics from a Christian perspective), there are three that I would like to briefly mention.[29] David Klanderman addresses a Christian response to the constructivism espoused by Ernest above. The goal is to analyze constructivism as a philosophy of mathematics and offer it as an example of how Christians might form their own thinking and offer their own justifications for teaching decisions within the mathematics classroom. Klanderman focuses on the formation of a broader philosophical base from which to approach the teaching and learning of mathematics, rather than addressing specific pedagogical practices and their outcomes, though he does address many of the national policy documents and published standards. He concludes,

> In the areas of teaching and learning of mathematics (Christian) perspectives may result in policies that are similar to those espoused by people with differing views, but for very different reasons.[30]

Although Christians have no right to expect explicitly Christian standards to be proposed by a publicly funded and supported organization such as NCTM, we nonetheless need to have these conversations in the context of Christian community. Where this article differs from Klanderman is that I believe, if argued appropriately, new standards on affect in mathematics that are rooted in an explicitly Christian worldview could indeed be drafted by organizations such as NCTM and implemented across a variety of classrooms, not only Christian ones.

Harold Heie describes the Christian motivation behind the pedagogical strategy of posing integrative questions. By integrative question, he means a question that cannot be addressed without formulating coherent relationships between academic disciplinary knowledge and biblical/theological knowledge.[31] While certainly a valuable tool, and a highly recommended teaching strategy, integrative questions still only target cognition in students. Heie argues for a Christian pedagogy based on shaping beliefs and worldview.[32]

James Nickel notes the need to move beyond "thought" in developing objectives for a biblical Christian mathematics curriculum, noting that mathematical thought, from a Christian perspective, is meant to further God's purposes of redemption and dominion, and thus move us to action.[33] While Nickel does encourage moving beyond thought (or cognition) in determining our teaching practices, his focus tends to be more along the lines of the utility discussed in the introduction, motivated simply from a Christian worldview—or cognitive perspective. One could argue that there is still an underlying assumption that affections are formed primarily through a cognitive understanding of the Christian faith. If this is the fullest approach we take to teaching mathematics from a Christian perspective, we as Christian educators will face the same dilemmas encountered by secular researchers in attempting to examine how to cultivate mathematical affections in students.

The preceding works by Klanderman, Heie, and Nickel contribute greatly to a Christian understanding of what it means to teach mathematics well. However, as beneficial as those resources are for those teaching in explicitly Christian contexts, they lose their value in secular contexts that are extremely unlikely to adopt their underlying faith commitments.[34] It is my contention that integrating the work of James K. A. Smith into mathematics education has the potential to produce research on affect in mathematics that can be accepted broadly by all mathematics educators. Smith urges Christian educators to move beyond worldview and belief language, as such language tends to result in pedagogies that still operate on the level of disseminating information.[35] While space may not allow for a complete analysis of Smith's work,[36] I want to highlight some of the main themes. Then I believe it will be apparent how his distinctly Christian perspective to what human beings are and how they learn, provides some answers that researchers on affect in mathematics education are searching for.

"Behind every pedagogy is a philosophical anthropology."[37] Before you can teach a human being you must first have a notion of what a human being is. Smith notes that a pedagogy that focuses on cognition, that sees education as primarily disseminating information, tends to assume human beings are primarily "thinking things" and cognitive machines.[38]

Smith's thesis is that human beings are primarily affective beings before they are cognitive beings, and this anthropology bears itself out in our current educational system regardless of whether we recognize it. As Smith describes education:

> Education is not primarily a heady project concerned with providing information; rather, education is most fundamentally a matter of formation, a task of shaping and creating a certain kind of people. What makes them a distinctive kind of people is what they love or desire—what they envision as "the good life" or the ideal picture of human flourishing. An education, then, is a constellation of practices, rituals, and routines that inculcates a particular vision of the good life by inscribing or infusing that vision into the heart (the gut) by means of material, embodied practices. And this will be true even of the most instrumentalist, pragmatic programs of education (such as those that now tend to dominate public schools and universities bent on churning out "skilled workers") that see their task primarily as providing information, because behind this is a vision of the good life that understands human flourishing primarily in terms of production and consumption. Behind the veneer of a "value-free" education concerned with providing skills, knowledge, and information is an educational vision that remains formative. There is no neutral, nonformative education; in short, there is no such thing as a "secular" education.[39]

For Smith we are first and foremost creatures of desire before we are creatures of thought or even creatures of belief. Our affections pull us through life toward our vision of "the good life" rather than our cognitions rationally pacing out our steps. We are creatures of love, and love requires practice.[40] In other words, our affections are shaped by the practices/habits/rituals that we are immersed in. Smith refers to these as liturgies—rituals of ultimate concern: rituals that are formative for identity, that inculcate particular visions of the good life, and do so in a way that means to trump other ritual formations.[41] While Smith offers much to unpack for Christian educators, for our purposes of examining affect in mathematics education, the following points are significant to note: (1) the argument that human beings are primarily affective rather than cognitive beings, and (2) the argument that our affections are shaped by practices (liturgies).

What if human beings are primarily affective learners and only secondarily cognitive learners? All of the research cited above treats the affective domain of learning as needing to be interconnected with the cognitive domain (a position which Smith would agree with), but none of the research (with the possible exception of Goldin's work—though this needs to be explore in greater depth) argues for the primacy of the affective domain. Smith would argue that, as Christian educators, we should advance this point further in the research of our respective academic fields. What is refreshing is that Smith notes how this ancient Christian understanding of human beings as creatures of love is finding support in contemporary philosophy and psychology. Therefore there is a base from which to further research on affect in mathematics education (and really in all education) that does not require explicit Christian faith commitments in order to be accepted.

Smith notes that much work has been done in the last century to suggest shifting the center of gravity of the human person from the cognitive to the noncognitive—from the cerebral head to the affective region of the body.[42] The reference "affective region of the body" is a significant one. Often the affective dimension of the human person is associated with the heart and emotion (as we saw in our analysis on affect above). However, Smith's work seems to support the notion that it is the actions/habits of the body that work to form and portray our affections.

This philosophical notion seems to be confirmed by contemporary work in cognitive science as well. It is bodily practices that train the body (including the brain) to develop habits or dispositions to respond automatically in certain situations and environments. Claims regarding material, bodily formation of our noncognitive dispositions are as old as Aristotle, but now they receive support and evidence from contemporary neuroscience and cognitive science.[43] Christian Smith, in his methodological manifesto for the social sciences, noted that the dominant paradigms of social sciences reflect human beings as rational machines, and he calls for a more holistic understanding of humans as believing (affective) or what he terms "narratological" animals, that is, creatures driven by story at an affective level rather than by logic and rationality at a cognitive level.[44]

Charles Taylor notes that what we as humans think about is just the tip of the iceberg and cognition cannot fully or adequately account for how or why we make our way in the world. For Taylor, there is something beneath the cognitive, what he terms "the imaginary"—defined as the way ordinary people imagine their social surroundings that is not expressed in theoretical terms but is carried in images, stories, and legends.[45] Here Taylor uses "imaginary," not in the romantic sense of invention, but, rather, in reference to a precognitive framework or lens through which we view and interact with the world. All of this research is summarized here to note the potential for Christian mathematics educators to build an argument for the primacy of affect in education from a foundation that does not necessarily attach itself to Christian faith commitments and thereby does not lack transference into secular research.

While much of the above work in philosophy and cognitive science needs to be developed in more explicit detail as it pertains to mathematics education, it nonetheless establishes the groundwork that such academic work on the primacy of affections is out there and is, in fact, growing. The key question then seems to be, "What if human beings are primarily affective learners and only secondarily cognitive learners?" If this work is indeed true, and it changes the way we see human beings, then it necessarily must change the way we teach human beings. The majority of research on affect proceeds with an (often unstated) assumption that we are primarily cognitive beings, and the results of that research bear this point out as we have seen—framing arguments that focus on cognition, confusing terminology and learning objectives, and so forth. As Christian mathematics educators, we have the opportunity to contribute the following analysis to work on affect: if human beings are primarily affective learners, *how* then do we develop the affections? As we have seen, James K. A. Smith argues that this occurs through the liturgies of the classroom. Before moving to this last point to discuss some possible ways in which we might cultivate mathematical affections in students, allow me to make note of several other studies on affect in mathematics education in light of the preceding discussion on philosophy and psychology.

Some work being done in the research of mathematics education takes these ideas into account. Such work aims to produce a new unit of analysis for the study of mathematical activity, integrating affectivity and cognition.[46] While this is certainly a step in the right direction, integrating the affective and cognitive, it does not go the extra step to suggest the primacy of the affective.

A stronger statement with regard to the primacy of affective learning is made by Markku Hannula.[47] In examining motivation in the mathematics classroom, Hannula notes that, in order to understand student behavior in classrooms, we need to increase our understanding of what motivation is and how it is regulated. The first relevant issue that he discusses is the importance of the unconscious (or preconscious) in motivation. He also goes on to note that, as a potential, motivation cannot be directly observed, but rather it is only observable as it manifests itself in affect and cognition (for example as beliefs, values, and emotional reactions). Goldin discusses a research-based theoretical framework based on affect as an internal representational system.[48] Key ideas include the concepts of meta-affect and affective structures, and the constructs of mathematical intimacy and mathematical integrity. Goldin understands these as fundamental to powerful mathematical problem solving, and deserving of closer attention by educators. We see in Hannula a recognition of the pre-conscious (and hence precognitive) place of motivation that then influences students' affective actions. In Goldin we find an approach that sees affect as an internalized organization structure which is necessary for students to succeed in the cognitive task of mathematical problem solving.[49]

Finally, let us consider *how* one goes about cultivating mathematical affections. I will offer a few ideas, focused from Smith's notion of liturgies, and drawn specifically from the mathematics classroom. However, this is the area in which we as Christian mathematics educators need to do more work. This article is meant to serve largely as a call to action—a realization of the opportunity we have before us to contribute to a much-needed body of research on affect. There are three brief examples I wish to discuss in light of everything that has been discussed thus far.

1. More consideration needs to be given to assessment. The NCTM *Assessment Standards for School Mathematics* (1995) states, "It is through assess-

ment that we communicate to students what mathematics are valued." If our goal is to cultivate mathematical affections (values) in students, assessment is the primary means by which we do so. We need to consider what liturgies of assessment we participate in at both the formative and summative levels. For instance, is the emphasis on correctness of a student response? Perhaps a teacher poses a question to the class and a student answers incorrectly. The teacher responds with a simple "no" and moves on to call upon another student who they know will provide the right answer and move the lesson along. If we fall into this pattern (liturgy) of formative assessment, we are instilling into students the notion that mathematics is only about getting to a correct answer, and we are ignoring the productive struggle that it takes to get there. At a summative level, as long as high-stakes standardized exams exist in which the main goal is to achieve a certain percentage of correct responses, we will always be fighting an uphill battle in getting students to value mathematics for its creative processes.

2. More consideration needs to be given to technology. We need to be careful not to implement the newest technological accessories in our classroom just because students are used to having technology in their lives outside of school. If we are trying to offer up mathematics as being the technologically savvy discipline and, therefore, worth the interest of students, I would argue that we are largely going to lose that battle. We are offering mathematics as a competing interest against the newest apps, games, and electronic devices that students are inundated with on a daily basis. As much as I love mathematics, I know that this is a competition it will not win. What if instead we focused on technological liturgies in the classroom that utilized mathematics as a way of examining and critiquing technological advancements rather than simply using those advancements to try to make mathematics more fun? What if these liturgies could instill in students a sense of mathematics (and education as a whole) as something other than just a competing product for their attention and, rather, a foundation for their life that informs the product choices and decisions they make? What if we stopped feeding the culture of immediacy that technology has engrained in us and purposefully use the classroom as a time to step back and reflect? Perhaps then students would not automatically jump to the calculator when faced with a difficult problem and proceed to give up if the answer is not achieved in under a minute.

3. More consideration needs to be given to service. There is much that can be contributed to service-learning in mathematics. Personally, after I began implementing service-learning projects in all of my classes, I was amazed at the impact it had on students on both a cognitive and affective level. Service to the community turns the focus away from individualistic goals of education (such as what grade the student receives) to the more altruistic aims of education. In their reflection from a recent project, one of my students wrote "The service-based aspect of the project made it more engaging because we met new people and we had the mindset that we could actually help someone by completing this project." By comparison, Matthew 20:26–28 states, "Whoever wishes to become great among you shall be your servant, and whoever wishes to be first among you shall be your slave; just as the Son of Man did not come to be served, but to serve, and to give his life a ransom for many." If the goal of education is the formation of a certain type of person, then the more that we can get students to express sentiments rooted in scripture as the result of their experience in the math classroom, then the more likely it is that we are heading in the right direction. More resources need to be produced in this regard.[50]

In summary, I believe that there is a need for more work to be done on developing values in students apart from a primarily cognitive approach, and I am convinced that Christian faith has much to offer in this regard. Though cognition and affection are certainly interrelated, more research needs to be done on the assumption of the affections as primary to the students' learning process. There is ample opportunity now, perhaps more so than ever, for Christian mathematics educators to have a major influence on the cultivation of mathematical affections: not merely knowing, but also loving, and practicing the truth, beauty, and goodness inherent in mathematics. ◊

Notes

[1]For an excellent discussion of this point, see C. S. Lewis, *The Abolition of Man* (New York: HarperOne, 2009).

[2]As noted by Russell Howell, "The Matter of Mathematics," *Perspectives on Science and Christian Faith* 67, no. 2

(2015): 84, citing Vern S. Poythress, "A Biblical View of Mathematics," in *Foundations of Christian Scholarship: Essays in the Van Til Perspective*, ed. Gary North (Vallecito, CA: Ross House Books, 1976), 158–88.

[3]P. Ernest, *The Philosophy of Mathematics Education* (New York: Falmer Press, 1991), 259.

[4]Ernest, in presenting his philosophy of mathematics education known as social constructivism, roots these values in the cultural contexts of the human creators of mathematics (ibid., 261). For a thorough discussion of a distinctly Christian perspective on values in mathematics, I highly recommend M. Veatch, "Mathematics and Values," in *Mathematics in a Postmodern Age: A Christian Perspective,* ed. R. Howell and J. Bradley (Grand Rapids, MI: Eerdmans, 2001), 223–49. For a distinctly Christian perspective on Ernest's social constructivism and its impact on the math classroom, I recommend D. Klanderman, "Teaching and Learning Mathematics: The Influence of Constructivism," in *Mathematics in a Postmodern Age*, ed. Howell and Bradley, 338–59. Both Veatch's and Klanderman's work will be referenced throughout this article.

[5]National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics* (Reston, VA: NCTM, 1989); National Council of Teachers of Mathematics, *Professional Standards for Teaching Mathematics* (Reston, VA: NCTM, 1991); National Council of Teachers of Mathematics, *Assessment Standards for School Mathematics* (Reston, VA: NCTM, 1995); National Research Council, *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (Washington, DC: National Academies Press, 1989); and Mathematics Learning Study Committee, *Adding It Up: Helping Children Learn Mathematics* (Washington, DC: National Academies Press, 2001).

[6]National Council of Teachers of Mathematics, *Professional Standards for Teaching Mathematics* (1991), http://www.fayar.net/east/teacher.web/math/standards/previous/ProfStds/TeachMath.htm.

[7]Mathematics Learning Study Committee, *Adding It Up*, 116.

[8]Common Core State Standards Initiative, *Standards for Mathematical Practice*, http://www.corestandards.org/Math/Practice/.

[9]Composed based on B. S. Bloom, M. D. Englehart, E. J. Furst, W. H. Hill, and D. R. Krathwohl, *Taxonomy of Educational Objectives: Handbook I. Cognitive Domain* (New York: McKay, 1956); and D. R. Krathwohl, B. S. Bloom, and B. B. Masia, *Taxonomy of Educational Objectives: Handbook II. Affective Domain* (New York: Longman, 1964). Underlying graphic credit goes to C. L. Jorgensen, http://cljorgensen.com/tag/blooms-taxonomy/.

[10]Bloom et al., *Taxonomy of Educational Objectives: Handbook I. Cognitive Domain;* and L. W. Anderson and D. R. Krathwohl, *A Taxonomy for Learning, Teaching, and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives* (New York: Pearson, 2000).

[11]Krathwohl et al., *Taxonomy of Educational Objectives: Handbook II. Affective Domain*.

[12]Ibid.

[13]R. Zan, L. Brown, J. Evans, and M. S. Hannula, "Affect in Mathematics Education: An Introduction," *Educational Studies in Mathematics* 63, no. 2 (2006): 113.

[14]D. B. McLeod, "Research on Affect in Mathematics Education: A Reconceptualization," in *Handbook of Research on Mathematics Teaching and Learning*, ed. D. A. Grouws (New York: Macmillan, 1992), 575.

[15]Ibid.

[16]See, for example, T. Haladyna, J. Shaughnessy, and J. M. Shaughnessy, "A Causal Analysis of Attitude toward Mathematics," *Journal for Research in Mathematics Education* 14 (1983): 19–29; D. B. McLeod, "Affective Issues in Mathematical Problem Solving: Some Theoretical Considerations," *Journal for Research in Mathematics Education* 19 (1988): 134–41; D. B. McLeod, "The Role of Affect in Mathematical Problem Solving," in *Affect and Mathematical Problem Solving: A New Perspective*, ed. McLeod and Adams, 20–36; D. B. McLeod, "Beliefs, Attitudes, and Emotions: New Views of Affect in Mathematics Education" in Ibid., 245–58; D. B. McLeod, "Research on Affect and Mathematics Learning," *Journal for Research in Mathematics Education* 25 (1994): 637–47; R. S. Prawat and A. L. H. Anderson, "The Affective Experiences of Children during Mathematics," *Journal of Mathematical Behavior* 13, no. 2 (1994): 201–21; I. M. Gomez-Chacon, "Affective Influences in the Knowledge of Mathematics," *Educational Studies in Mathematics* 43 (2000): 149–68; M. L. Malmivuori, "The Dynamics of Affect, Cognition, and Social Environment in the Regulation of Personal Learning Processes: The Case of Mathematics" (PhD diss., University of Helsinki, 2001); G. C. Leder, E. Pehkonen, and G. Törner, eds., *Beliefs: A Hidden Variable in Mathematics Education?* (The Netherlands: Kluwer Academic Publishers, 2002); X. Ma, "Cognitive and Affective Changes as Determinants for Taking Advanced Mathematics Courses in High School," *American Journal of Education* 113 (2006): 123–49; M. L. Malmivuori, "Affect and Self-regulation," *Educational Studies in Mathematics* 63 (2006): 149–64; J. Maaß and W. Schölglmann, *Beliefs and Attitudes in Mathematics Education: New Research Results* (Rotterdam: Sense Publishers, 2009); M. Cotic and M. V. Zuljan, "Problem-Based Instruction in Mathematics and Its Impact on the Cognitive Results of the Students and on Affective-Motivational Aspects," *Educational Studies* 35, no. 3 (2009): 297–310; M. Lebens, M. Graff, and P. Mayer, "The Affective Dimensions of Mathematical Difficulties in Schoolchildren," *Education Research International* (2011): 1–13; and J. Molera-Botella, "Is There a Relationship in Primary Education between Affective Factors in Mathematics and Academic Performance?," *Estudios sobre educación* 23 (2012): 141–55.

[17]To be clear, the term "values" is rarely used explicitly in research on affect. D. B. McLeod in "Beliefs, Attitudes, and Emotions: New Views of Affect in Mathematics Education," in *Affect and Mathematical Problem Solving: A New Perspective*, ed. McLeod and Adams, 245–58; "Research on Affect in Mathematics Education: A Reconceptualization," in *Handbook of Research on Mathematics Teaching and Learning*, ed. Grouws; and "Research on Affect and Mathematics Learning," *Journal for Research in Mathematics Education* 25 (1994): 637–47, divides the affective domain into emotions, attitudes, and beliefs—listed in increasing order with a connection to cognition. V. A. DeBellis and G. A. Goldin in "Affect and Meta-affect in Mathematical Problem Solving: A Representational Perspective," *Educational Studies in Mathematics* 63, no. 2 (2006): 131–47, add a fourth domain: "values, ethics, and morals." It is my position that the "value" language from policy standards has all of these domains in mind, and so I have placed research that addresses any one of these domains under the larger umbrella of "values."

[18]J. K. A. Smith, *Desiring the Kingdom: Worship, Worldview, and Cultural Formation* (Grand Rapids, MI: Baker Academic, 2009).

[19]A. Sfard, *Thinking as Communicating: Human Development, the Growth of Discourses, and Mathematizing* (Cambridge: Cambridge University Press, 2008), 156.

[20]Mathematics Learning Study Committee, *Adding It Up*, 116.

[21]M. Malmivuori, "The Dynamics of Affect, Cognition, and Social Environment in the Regulation of Personal Learning Processes: The Case of Mathematics" (PhD diss., University of Helsinki, 2001); and M. Malmivuori, "Affect and Self-regulation," *Educational Studies in Mathematics* 63 (2006): 149–64.

[22]Krathwohl et al., *Taxonomy of Educational Objectives: Handbook II. Affective Domain*.

[23]G. A. Goldin, "Affect, Meta-affect, and Mathematical Belief Structures," in *Beliefs: A Hidden Variable in Mathematics Education?*, ed. Leder, Pehkonen, and Törner, 71. Emphasis added.

[24]Jonathan Edwards, *A Treatise concerning Religious Affections* (New York: Cosimo Classics, 2007).

[25]Veatch, "Mathematics and Values," in *Mathematics in a Postmodern Age: A Christian Perspective*, ed. Howell and Bradley, 243.

[26]McLeod, "Research on Affect in Mathematics Education: A Reconceptualization"—And still just as true today.

[27]Quote attributed to C. S. Lewis though there is no direct evidence of him having written it. The quote does aptly summarize a major point of Lewis's treatise on education, *The Abolition of Man*, http://lanternhollowpress.com/2012/07/15/meditations-with-c-s-lewis-a-more-clever-devil/.

[28]Victor Hugo, *Les Misérables* (New York: Penguin Group, 1987), 23.

[29]My focus will be on the pedagogy of mathematics and not the content of specific mathematics courses. There are numerous resources that address how Christianity impacts our understanding of mathematics as a discipline and how we might modify the content of courses to underscore Christian faith commitments, many of which have appeared in this journal. Recommended in this regard are Howell and Bradley, eds., *Mathematics in a Postmodern Age*; J. Bradley and R. Howell, eds., *Mathematics through the Eyes of Faith* (New York: HarperOne, 2011); J. Mays, "Why Math Works: Answering Wigner et al.," *Association of Classical and Christian Schools* 19, no. 4 (2012): 14–16; D. F. M. Strauss, "A Historical Analysis of the Role of Beliefs in the Three Foundational Crises in Mathematics," *Facets of Faith and Science*, vol. 2, ed. J. M. van der Meer (New York: University Press of America, 1996); "Mathematics: A Christian Perspective," a curriculum developed by the Kuyers Institute at Calvin College.

[30]Klanderman, "Teaching and Learning Mathematics: The Influence of Constructivism," 359.

[31]H. Heie, "Developing a Christian Perspective on the Nature of Mathematics," in *Teaching as an Act of Faith*, ed. A. C. Migliazzo (Bronx, NY: Fordham University Press, 2002), 101–102.

[32]Ibid., 99.

[33]J. Nickel, *Mathematics: Is God Silent?* (Vallecito, CA: Ross House Books, 2001), 235.

[34]See also J. Barrett and D. Klanderman, "A Christian Constructivist? The Impact of Worldview on Learning Theories and the Mathematics Education Research Community," *Journal of the Association of Christians in the Mathematical Sciences* (2006), http://www.acmsonline.org/journal/2006/KlandermanBarrett.pdf, for a discussion of attempts to reconcile competing theories of learning mathematics within a Christian context falling on the deaf ears of the secular research community.

[35]Smith, *Desiring the Kingdom*, 18.

[36]Smith himself notes that in analyzing how our affections are shaped by something like teaching, the form of the teaching is inextricably linked to the message behind the teaching. So for me to summarize his work is doing it a great injustice, and I therefore strongly encourage a full reading of *Desiring the Kingdom*. My apologies to Dr. Smith.

[37]Smith, *Desiring the Kingdom*, 27.

[38]Ibid.

[39]Ibid., 26–27.

[40]Ibid., 76ff.

[41]Ibid., 86.

[42]See M. Heidegger, *Being and Time*, trans. John Macquarrie and Edward Robinson (New York: Harper and Row, 1966); and E. Brann, "Are Humans Ultimately Affective?," *Expositions: Interdisciplinary Studies in the Humanities* 1 (2007): 53–70.

[43]See J. A. Bargh and T. L. Chartrand, "The Unbearable Automaticity of Being," *American Psychologist* 54 (1999): 462–79; and T. D. Wilson, *Strangers to Ourselves: Discovering the Adaptive Unconscious* (Boston, MA: Harvard University Press, 2004).

[44]C. Smith, *Moral, Believing Animals* (Oxford: Oxford University Press, 2003).

[45]C. Taylor, *Modern Social Imaginaries* (Durham, NC: Duke University Press, 2004).

[46]See C. R. Arujo, F. Andrade, I. Hazin, J. T. R. Falcao, J. C. do Nascimento, and M. M. L. Lessa, "Affective Aspects on Mathematics Conceptualization: From Dichotomies to an Integrated Approach" (paper presented at the 27th International Group for the Psychology of Mathematics Education Conference held jointly with the 25th PME-NA Conference, Honolulu, HI, July 13–18, 2003), v2, 269–76.

[47]M. S. Hannula, "Motivation in Mathematics: Goals Reflected in Emotions," *Educational Studies in Mathematics* 63 (2006), 165–78.

[48]DeBellis and Goldin, "Affect and Meta-affect in Mathematical Problem Solving."

[49]Another area of emerging research that I believe is fruitful in regard to cultivating affections and instilling values that is not discussed at great length here is the area of aesthetics. See N. Sinclair, *Mathematics and Beauty: Aesthetic Approaches to Teaching Children* (New York: Teachers College Press, 2006).

[50]Some resources that currently exist are C. R. Hadlock, ed., *Mathematics in Service to the Community: Concepts and Models for Service-Learning in the Mathematical Sciences*, no. 66 (Washington, DC: Mathematical Association of America, 2005); K. Black, K. Crisman, and D. Jardine, "Introductory Editorial: Special Issue on Service-Learning," *PRIMUS* 23, no. 6 (2013): 497–99; and Service-Learning and Mathematics webpage, http://www.math-cs.gordon.edu/~kcrisman/.

Valorie Zonnefeld

Article

Practical Applications of an Integrally Christian Approach to Teaching Mathematics

Valorie Zonnefeld

Descriptions of various frameworks and approaches to integrating Christian faith in the mathematics classroom are explored, as well as examples and techniques. In particular, a subject-centered approach is advocated in contrast to the traditional teacher-centered approach or, more recently, the student-centered approach.

Teaching Christianly has been a passion of mine since I first felt the call to teach. Unfortunately, connections to the spiritual realm are less overt in mathematics than in other disciplines. Throughout my teaching at the middle school, high school, and now collegiate level, I have wrestled with finding a distinctively Christian approach to teaching mathematics. When I first began teaching, I knew in the back of my mind that what I taught was no different from that at secular institutions. Math concepts do not change from school to school. The fact that a triangle has 180 degrees is the same in the secular and the religious school. I comforted myself that the math was the same, but the atmosphere that I created made my classroom distinctive. I became increasingly uncomfortable with this response, with nagging thoughts that there must be more to a Christian approach to teaching mathematics than this.

I have been given opportunities to work with both pre-service and practicing teachers to explore their thoughts regarding the integration of faith and mathematics. In both settings I have asked, "What does Christian mathematics teaching look like?" Pre-service and practicing teachers alike readily offered their insights into the topic. Responses have included patience, creating a community of learning, caring for students, acknowledging each student's individuality, kindness, and honesty. I immediately followed this question by asking which of the responses represented distinctively Christian teaching and which represented qualities of any good teacher. It soon became apparent that many of the qualities that were valued as *distinctively Christian* also described good teaching in general.

Christian educators do not hold a monopoly on good teaching, as many unbelieving teachers also display strong teaching qualities through common grace. While it is true that a Christ-like attitude and the fruit of the spirit (love, joy, peace, patience, kindness, goodness, faithfulness, gentleness, and self-control) are character traits that Christian educators should display, I have come to believe that this view limits the possibilities for integration. Math is not neutral. As a Christian educator, I have realized that there are more opportunities to integrate faith in mathematics than I once believed. Harold Heie, retired senior fellow at the Center for Christian Studies at Gordon College, has aptly stated that if God is the Creator of all that is true, there ought to be connections between our faith and mathematics.[1]

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Distinctively Christian mathematics teaching goes beyond the teacher's treatment of the students and the classroom environment. In this response, I will outline the journey that I have taken regarding my approach to having faith integral to mathematics teaching: the purpose of teaching mathematics from a Christian perspective, frameworks that have been used to describe approaches to faith integration, and effective teaching techniques that I have found for integrating faith and mathematics.

When I started teaching classes at a Christian college, I was forced to reexamine my belief that my classroom environment and treatment of students fulfilled my obligation to teach from a Christian perspective. I thoroughly enjoyed teaching the classes, but one disappointment was the students' responses to the last question of the course evaluation: "*How has your faith or biblical perspective been shaped or deepened by this course or your instructor?*" Answers included blanks, "N/A," "It's a math course," "Not really, but it's math, so that's fine," and "Not really, it's just math." I was heartbroken that few students acknowledged any deepened understanding or even that I had made an effort in my teaching to acknowledge the Lordship of Christ in mathematics. My love for the Lord made no apparent impact on my class or students. I was "deepening the world's hunger rather than helping to alleviate it" in my teaching of mathematics.[2] This experience pushed me to search for a more faithful way to teach mathematics from a Christian perspective, and it led me to David Smith.

The Purpose of Teaching Math

Like Russell Howell,[3] Smith has also been a resource for me in considering the integration of faith and mathematics. I had the opportunity to hear Smith speak twice in 2008; he played an important part in deepening my understanding, inspiring me to view my curriculum planning and teaching in a new light.[4] The question that he repeatedly asked was, "What would spiritual development look like if it showed up in your salad?" He pushed me to examine what spiritual development would look like in my classroom. This question forced me to reevaluate the goals for my classroom. Smith prompted me to dream about my ideal Christian mathematics classroom: a classroom community of learners striving to learn more about the mysteries, beauty, and usefulness that God has interwoven in the spatial and physical dimensions of reality, an environment which prompts students to ask, "Lord, what would you have me do for you with this knowledge?"

The purpose of learning mathematics plays an important role in creating a distinctively Christian approach to teaching mathematics. David Huizenga writes, "The purposes of mathematics can clearly distinguish the Christian school classroom from its secular counterpart," in that secular academics can hold knowledge as a tool to manipulate and control for individual gain.[5] The implicit goal of mathematics in this environment is all too often to "get ahead" and "make lots of money." This misguided purpose for mathematics lies in opposition to the goal of Christian education and many of the mission statements of Christian institutions.

Christian educators must renounce this abuse of mathematics and boldly reclaim mathematics education for Christ. Abraham Kuyper stated, "There is not a square inch of creation of which Christ does not say 'It is mine!'"[6] This includes the square inch represented by mathematics education. Richard Russell uses the understanding of the sovereignty of our Lord over creation to describe the responsibility of Christian educators: "Our task as ambassadors of the Kingdom in the field of education is to reclaim every area of educational thought, learning and practice" for Christ.[7] This reclamation process causes educators to examine all aspects of educating, from assessment to discourse and curriculum. Given the enormity of this task, how does a teacher begin this reclamation process?

A solid understanding of the *purpose* of teaching mathematics is an important foundation for Christian teachers. Parker Palmer describes education as guiding students "on an inner journey toward more truthful ways of seeing and being in the world."[8] When the above is applied to mathematics, students will see the purpose of mathematics not as an avenue for personal gain, but as a tool to carry out their God-given calling.[9] Mathematics is "a tool for redemption" and directs students "toward the Creator rather than toward the created."[10] This distinction between the Creator and created has been a helpful tool for me in identifying educational purposes that have gone wayward.

The "aha" moments when the class is amazed at the mathematical beauty that God has built into the placement of leaves on a tree or the use of hexagons in honeycombs are wonderful, but practically speaking, not every lesson inspires students to a greater appreciation for God. In fact, I have often experienced the opposite, as my students have expressed disgust for algebra or integrals. This is the time when it is vitally important that teachers understand the purpose of mathematics and the importance of an understanding of the numerical and spatial aspects of creation as we exercise our dominion over creation. This also makes it imperative that Christian teachers be able to answer the "When will we ever have to use this" question. At times, the answer to this question may be that we can learn more about creation and our Creator, but students also need to see the practical applications of the mathematics that they are learning. Some students do not readily see the beauty in mathematics, but they may be drawn to its incredible utility.

The purpose that a teacher holds for mathematics may also be communicated implicitly through the topics selected to illustrate mathematical concepts. Are the problems all about maximizing profit and minimizing expense in a materialistic sense, or do they examine problems from contexts such as decreasing pollution, stewardship of resources, or understanding the spread of a deadly virus? A steady flow of problems solely focused on personal gain sends an unspoken message to students that mathematics is not a tool for redemption, but for personal advancement.

As has been argued, the purpose that a teacher holds for teaching mathematics is revealed through subtle differences that provide overtones throughout a class. Next, we examine various frameworks for education that Christian educators have used to integrate faith and mathematics.

Frameworks

Multiple approaches to addressing faith and mathematics are used by Christian educators. This section will start with an examination of the term "integration," followed by James Nickel's three approaches to integration.[11] The section closes with Smith's spectra of integration.[12]

Integration or Integral?

I have used the term *integration* of faith and mathematics knowing that it may lead to a misunderstanding. To integrate implies connecting two things that are separate parts like combining peanut butter and chocolate for a recipe.[13] Mathematics and faith are intimately connected and need not be integrated. More appropriately stated, faith is integral to mathematics. To follow the food analogy, mathematics without faith is the equivalent of skim milk: the faith (or fat) has been removed. As Howell has so aptly argued in his essay, faith and mathematics are intimately interwoven.[14] Unfortunately, educators have sought to teach mathematical concepts in isolation, losing their connections to reality and, consequently, to faith issues. Despite my misgivings with the word integration, it is the most commonly used word for Christian educators. For these reasons, I will continue to use it with the caveat that I see integration as rejoining things that were originally joined and meant to be seen as unified aspects of reality.

Approaches to Integration

In *Mathematics: Is God Silent?,* Nickel describes three approaches that Christian educators have used to integrate their faith and mathematics.[15] The first is mathematics as usual. A dualism is present in this approach in which the Bible is sacred, but mathematics is secular. An educator who uses this approach would expect no difference between the mathematics classrooms of a believer or an unbeliever, since mathematics is secular. A Christian school that adopts this philosophy hangs its faith integration on activities such as chapel, morning devotions, and Bible class while leaving the subjects themselves untouched. Howell makes a beautiful argument against this separation in both the lead article in this issue and the book *Mathematics through the Eyes of Faith* that he coedited with James Bradley.[16] Mathematics is not secular, but clearly displays the beauty and structure of our Creator.

The second approach that Nickel describes is "baptizing mathematics." In this approach, spirituality is sprinkled on mathematics without really affecting the subject or the class. Examples of baptizing mathematics include tacking a scripture onto a lesson or offering a prayer before class with little connection to the subject or activities. One of the Christian mathematics curricula currently available looks no different than traditional curricula, with the exception of the Bible verse on the top of each worksheet—a verse largely unconnected to the topic of the lesson. While baptizing is an easy approach to implement, it also displays dualism since the actual mathematics and the spiritual act are disconnected.

This second approach has also aptly been described as the *frosting approach* with mathematics providing the cake and faith, the frosting over it. Similar to a cake and frosting, the cake inside (mathematics) remains unaffected by the frosting (spirituality). Huizenga states that faith integration must go "beyond a devotional or an opening prayer, [to] search for and unveil Christ in every concept, every formula, every proof, [and] every operation."[17]

The final approach outlined by Nickel uses the all-encompassing integration described by Huizenga, recognizing God as the foundation of all knowledge. In this approach, everything visible and invisible reflects God. In studying mathematics, we learn more about the nature of our God. In this third approach,

> teachers of mathematics … bring to the attention of their students the power and beauty of mathematics. [Letting] the students not only know what math can do, but [also letting] them admire it for its elegance and order, and [giving] glory to God for what he has revealed to man through it.[18]

What a beautiful picture of an approach to mathematics that is integrally spiritual. It is this final comprehensive approach that I desire for my classroom, but find challenging to accomplish. If faith is integral to mathematics, it moves beyond a Bible verse at the start of a lesson to affect not only the purpose for learning math and the types of problems chosen, but also the classroom dynamic. In the next section, we will explore comprehensive integration that is coherent, grounded, and authentic.

Integration Spectra

Smith has offered three helpful spectra to consider when examining curricula that integrate faith and learning.[19] Each spectrum offers a continuum of one descriptor versus the second with a goal of reaching the second descriptor. These spectra have helped me reflect on my own classes' faith integration. The first spectrum is *fragmented* versus *coherent*. In a fragmented curriculum, the scripture does not change the heart. Including a spiritual reference or verse allows the teacher to check off faith integration and move along with mathematics class as usual. This is in contrast to the integral use of biblical concepts to enlighten the learning. An example of coherent faith integration is examining the ratio of doctors to people in different areas of the United States and the world. Issues of justice and caring for downcast members of society are powerfully demonstrated while still learning valuable knowledge about ratios.

The second spectrum is *spiritualized* versus *grounded*. In the spiritualized approach, faith issues are introduced, but quickly drift away from mathematics with no real connection. A spiritualized approach finds a weak connection between mathematics and faith, and shifts from learning about math to a spiritual discussion. An example of a spiritualized approach might occur when teaching the quadratic formula. A teacher introducing the discriminant would follow with a sermonette on how Christians, too, should be discriminating. The connection between the quadratic equation and wise choices is tenuous at best. Issues including justice, stewardship, the spread of diseases, and human behaviors offer depth and vital connections between faith and mathematics that are both spiritual, yet grounded in mathematics and students' daily lives. Hilgeman states, "Integration must always be meaningful, or students will develop a lack of respect for God's truth."[20] Students need to see practical, grounded applications of faith in mathematics.

The third spectrum is *decorative* versus *authentic*. In a decorative approach, the Bible is stripped of its authority as it is brought in, but never really used. An example of the decorative approach is exploring applications of geometry using instructions for building the temple in 1 Kings. Similar to Nickel's description of baptism, the Bible is used, but spiritual matters do not really change anything.[21] Authentic integration of faith and learning may still use 1 Kings in a geometry lesson, but would not stop short of authentic integration. Closing questions could bring the integration from decorative to authentic; for example, what is God communicating through these passages? how does God view worship? and does this change how you view your church building? Authentic integration affects the heart as students and teacher alike are stirred by the power of the scripture.

At this point, you may be thinking that designing a classroom and curriculum where faith is integral to mathematics is difficult. While it is difficult, everybody can take small steps to more faithfully unfold mathematics in their classrooms. The next section will describe examples of practical techniques to integrate faith and mathematics.

Integration Techniques

Teacher-Centered

A false dichotomy has been built in recent decades pitting teacher-centered approaches against student-centered approaches. Teacher-centered approaches have traditionally been the norm in mathematics classrooms. This approach is characterized as *teaching by telling*. The educator disseminates knowledge of procedures while the students absorb it.

> It assumes that the teacher has all the knowledge and the students have little or none, that the teacher must give and the students must take, that the teacher sets all the standards and the students must measure up.[22]

This approach has received increased criticism from educators and educational researchers who believe that students learn mathematics by doing math. Thus, the only person learning in a teacher-centered classroom is the teacher.[23] As a result, a growing number of mathematics educators have pushed for student-centered approaches.

Student-Centered

Student-centered approaches move away from the *sage on the stage* model in teacher-centered approaches to place the teacher as more of a *guide on the side*. I spent the better part of a decade moving my classroom from a teacher-centered to a student-centered approach, believing that it was a better method of teaching students. I worked to incorporate pedagogies that allowed students to scaffold their learning by constructing knowledge and assimilating it to prior knowledge. I assimilated many constructivist pedagogies including incorporating jigsaw techniques, fostering student discourse, and crafting guiding questions. All of these techniques allowed students to be more deeply involved in their learning. I believed that compared to the traditional, teacher-centered classroom, a student-centered classroom was a better and more respectful approach to working with students as image bearers of God. Students are not minds to be filled, but unique persons who learn in multiple ways. I focused on meeting students' needs emotionally, physically, and developmentally so they could be actively involved and engaged in their learning.

While I have not abandoned involving students in their learning, I became increasingly uncomfortable with the philosophical underpinnings of student-centered approaches. My fear was, and remains, that educational theorists including John Dewey, Maria Montessori, and Ernst von Glasersfeld go too far with constructivism and student-centered learning by allowing students to construct their own knowledge. Dewey sees students not as constructing ideas from their environment, but as "observers, participants, and agents who actively generate and transform the patterns through which they construct the realities that fit them."[24] Taken to an extreme, student-centered approaches allow a student to decide that 2 + 2 = 5. This is a dangerous step toward social constructivism in which the bedrock beliefs of Christianity become irrelevant as students construct their own realities. This is inconsistent with Christian beliefs of absolute truth.

Another difficulty that I faced with a student-centered approach was how it fed the individualism present in our society. Christians value individuals as each is created in God's image. Unfortunately, Western culture has distorted and elevated the value of the individual resulting in students, and eventually adults, who are self-serving and self-promoting. Ideas of community and working for the benefit of all, take a back seat when individuals believe that they are number one.

Subject-Centered

It is against these misgivings that I read Palmer and later Maryellen Weimer who promote a fresh approach to pedagogy.[25] Weimer criticized the false dichotomy that juxtaposes teacher-centered and student-centered approaches as pitting teaching versus learning; she states, "The best teaching is not one or the other, but a combination of both."[26] Palmer concurs with this, suggesting a subject-centered classroom.[27] This was a breath of fresh air to me, since I was not comfortable with either the student- or teacher-centered approaches.

Since that time, I have worked to adjust the focus of my classroom from students to the subject, seeking to lead my students to uncover the truth God has placed in mathematics. God's truth takes center stage. Similar to the student-centered approach, I remain the "guide on the side" and still plan learning experiences that encourage my students to be actively involved in their learning. One of the advantages of guiding students is that when students discover a concept on their own, they internalize it and learn it at a deeper level with greater retention.

In a subject-centered classroom, both the students and the teacher are actively involved, but it is the subject that takes center stage. Curiosity along with cognitive dissonance are harnessed to draw students in to learn more about topics in mathematics. For example, a lesson on odd and even numbers may be motivated by the question, "can you think of any four odd numbers that add up to 19?"[28] Students will explore possible combinations of numbers and make guesses until they realize that pairs of odd numbers always have even sums. Similarly, pairs of even numbers also have even sums. In this example, it is curiosity about mathematics that propels the subject to the center of learning, giving students "direct access to the energy of learning and of life."[29] I believe that a subject-centered approach is a more faithful way of unfolding the beauty and mystery that God has created in mathematics with students. Through a focus on the created (mathematics), students learn more about the Creator (God). A subject-centered approach avoids the overemphasis on either teachers or students and focuses on the truths of the concepts that often challenge students and teachers to a deeper understanding.

Teacher and Student Roles

The roles of both teachers and students are important in a subject-centered approach and take time to establish. Teachers are responsible for orchestrating opportunities for students to immerse themselves in the subject and for guiding students as they wrestle for greater understanding. A weakness of some student-centered approaches is that they can emphasize students so much that they reduce the authority and knowledge of the educator in the room. The teacher holds a unique role as an expert who can point students in the right direction and guide them to resources and materials to further their learning.

The "lawful regularity of creation" is particularly pronounced in mathematics.[30] This makes mathematics especially suitable for a subject-centered approach as the subject itself, through its regularity, guides students unlike other subjects in which conclusions may be more ambiguous. The regularity of mathematical rules and conclusions, along with the importance of students internalizing mathematics, is why I believe that teaching mathematics is unique: helping students less, often results in more learning. If teachers say too much, they diminish the learning opportunity and decrease the cognitive demand. It is through cognitive dissonance that students seek to organize their learning and pursue answers to their questions. This is the reason why an important aspect of high quality mathematics teaching is diagnosing students' level of understanding and guiding them to the point where they can make connections to the learning at hand.

In a sense, diagnosing and guiding is similar to playing the game Catch Phrase® in which the clue giver (teacher) guides their team (the class) to say the secret word without actually saying the word themselves. As the team guesses, the clue giver continues to improve the clues given in response, pointing them closer to the secret word and guiding them away from distractors. On some level, mathematics educators play this game on a daily basis in a subject-centered approach, guiding students toward understanding without saying too much and diminishing learning opportunities. By setting the subject at the center, the teacher's job is to connect the student to opportunities and resources for learning about the subject, that is, learning about an aspect of the creation.

The imagery of a team playing a game and working together for a common outcome is an apt description of my ideal classroom. I want the students in my classroom to collaborate with both the teacher and other classmates as they work together to enhance each member's learning. This collaboration is a reflection of what God desires for his body as individuals work together to learn more about the intricacies he has woven throughout mathematics.

A subject-centered classroom is more comfortable for both teachers and students. In a teacher-centered classroom, the teacher is a performer. This increases the expectation of a flawless performance.

For teachers in this setting, "getting caught in a contradiction feels like a failure."[31] In a subject-centered classroom, students understand that the teacher is an expert, but is also still learning alongside the students about the vast intricacies that God has concealed throughout mathematics for humans to uncover. In this setting, a mistake does not signify a failure, but rather an opportunity for learning. In classes where I have most successfully designed a supportive, subject-centered atmosphere, students take my missteps as opportunities that challenge them not only to learn more about mathematics, but also to help push forward the learning for the community. Palmer describes it well:

> In a subject-centered classroom, gathered around a great thing, getting caught in contradiction can signify success: now I know that the great thing has such a vivid presence among us that any student who pays attention to it can check and correct me … students have direct, unmediated access to the subject, and they can use their knowledge to challenge my claims.[32]

It is likely in a subject-centered approach that unexpected turns will more frequently reveal areas that are unknown to the educator, including mistakes. In a collaborative classroom community, teacher mistakes no longer represent weaknesses, but an opportunity for teachers to model not only the Christian virtue of humility, but the fact that they, too, are life-long learners.

The traditional, teacher-centered approach, in which educators present already-worked, error-free material, leaves many students with the incorrect notion that those who understand mathematics never make mistakes. Unfortunately, this facade of perfection causes many students to believe that they are not part of the mathematics community because they frequently make mistakes as they master new concepts. Mistakes are a natural part of mathematics, and educators need to model that they, too, make mistakes and do not have answers to every question. This humility and honesty that can be so lacking in mathematics classrooms is strikingly similar to the humility and honesty necessary as we progress in our own faith journeys.

An additional advantage of a subject-centered approach is that it does not force students to enter the teacher's domain in a teacher-centered approach or similarly force teachers to enter the students' territory in a student-centered approach. Both students and teachers maintain their identity and unique roles, as they gather around the subject as learners.

A subject-centered classroom is also an easier setting in which to practice the Christian virtue of hospitality. In a subject-centered classroom, the community of learners can be more comfortable for students since the instructor is no longer seen as the possessor of all knowledge and evaluator of the student. Granted, assessment will need to occur at some point, but a relationship of working together with the teacher as guide to uncover mathematical knowledge is more inviting to students who hold anxiety toward the subject. The teacher no longer grants access to mathematics since mathematics is the center of all work. Recent technological advancements support a subject-centered approach as students now have more methods to access mathematics than was traditionally available with only the teacher and textbook. This environment, in which all are seeking to deepen their knowledge of mathematics and in which competition is not emphasized, is a more hospitable environment for students to learn about math and its Creator.

Conceptual Teaching

Closely connected to a subject-centered approach is the importance of conceptual teaching: teaching in which students learn not only *how* a concept works, but also *why*. The saying "an ounce of understanding is worth a ton of memorization" supports this.[33] A conceptual approach is in conflict with the current push for high-stakes testing which pressures educators to cover every area of their field, often at the expense of a deeper, more conceptual understanding. The result is shallow knowledge of many topics that, unfortunately, does not last. Palmer suggests that instead of telling students everything they need to know, "information they will neither retain nor know how to use," teachers need to bring students into the circle of practitioners.[34] In other words, students need to be introduced to how mathematicians think and relate in a community of truth. Palmer states that in doing this "we do not abandon the ethic that drives us to cover the field—we honor it more deeply."[35]

Huizenga agrees with this approach, stating that shallow learning gives students only the human descriptors of God's truth in mathematics.

> When we insist (by the very way that we structure lessons and assignments) that students attain and display a measure of real understanding of mathematical relationships, we bring them into contact with divine truth and beauty.[36]

This face-to-face meeting with God's divine truth in mathematics is what I desire for my mathematics classroom.

A Caveat

A subject-centered approach does not imply that lectures are eliminated and every class period will consist of circle time around a mathematical topic. Weimer emphasizes the importance of recognizing "when 'teaching by telling' effectively advances the learning agenda."[37] The difference between a teacher-centered and a subject-centered class is that a lecture is selected when it is the most effective means of learning more about the subject.

A subject-centered approach chooses from both teacher-centered and student-centered approaches, including "lectures, lab exercises, fieldwork, service learning, electronic media, and many other pedagogies, [both] traditional and experimental" to find the best pedagogy to learn more about the topic.[38]

The recent push for student-centered classrooms has given lecture a bad rap; some of this criticism is warranted given the over-dependence mathematics education has had on lecture. Yet, the baby should not be thrown out with the bath water as there is a time and a place for lecture. Though beyond the scope of this article, there are also methods that make lecturing more effective and engaging to students. What is important is that the teacher orchestrate learning experiences that most effectively allow the subject to be the center of the class, those pedagogies that most faithfully allow the truth of mathematics to be seen by students. If teaching by telling is the most effective method for that topic, then a well-designed and implemented lecture is the natural response. The key to choosing a technique is that "at the center … is a subject that continually calls [students and teachers] deeper into its secret, a subject that refuses to be reduced."[39]

Questioning

Another technique that I have found fertile for integrating faith and mathematics is the use of essential questions and significant questions.

Essential Questions

Essential questions are overarching questions that guide the course or unit. Each course that I teach includes essential questions that not only give a big picture of the objectives, but also integrate perspectival connections. Examples of course-wide, essential questions that include a spiritual connection are, where does math come from? is math created or discovered? what does God reveal to us in math? what role do we have as image bearers of God in math? how are Christians to use math? and what does God communicate through mathematics? Unit-based, essential questions are more focused, but still give a macroview of the concepts and skills; for example, how can algebra describe creational phenomena? what laws of probability has God built into creation? and how can I use statistics to honor or dishonor my Creator?

Starting with essential questions grounds the course or unit in its place in God's creation. Unfortunately, many students see mathematics as a set of hard-to-reach, abstract rules or tricks, with little meaning in their daily lives. The framework of essential questions allows me to reflect on an elegant solution or beautiful pattern as more than a coincidence; it is also an opportunity to learn about the beauty and organization that God has built into mathematics. The essential questions are also a method to remind students that when we learn about mathematics, we learn more about the Creator and his creation.

Significant Questions

A second questioning technique that is useful for integrating faith and learning is significant questions. This technique also stems from Smith's work.[40] Howell mentioned Smith's work briefly in his coverage of attitudinal issues.[41] I would like to examine Smith's emphasis on a curriculum that gives opportunities for spiritual growth in greater depth.

Smith gives an example of squirrels and trees to demonstrate a curriculum that is fertile for faith integration. Squirrels climb trees. Trees were not explicitly made for squirrels and squirrels were not explicitly made for trees. However, God made trees with rough bark, and he made squirrels with claws to climb. As a result, squirrels are constantly climbing trees. If the trees were smooth or slippery, the squirrels would not climb them. The trees allow affordances for squirrels to climb. Likewise, as mathematics educators, we can design our curricula to allow affordances for spiritual and moral growth.

As a foreign language educator, Smith worked to recontextualize his teaching so that it allowed affordances for spiritual growth. He still taught the same concepts as the textbook, but in a different context, one fertile for faith integration.

Recontextualizing mathematics is an interesting and motivating way to teach. Too often, schools present a fragmented reality. Aspects of creation are distilled in 45-minute allotments with little connection from one course to the next. Not only are courses disjointed, but also mathematics itself is often disconnected from reality. Hilgeman warns, "Students who learn principles without their application to life will never consider math important."[42] Recontextualizing mathematics offers applications of mathematics as well as possible opportunities for perspectival issues and faith integration.

Examples of Integration

The remainder of this article will give examples of significant questions and recontextualizing mathematics. A majority of my teaching experience is with secondary mathematics and entry-level undergraduate courses. While the reader may teach more-advanced classes, I believe that these examples will stimulate others to imagine applications for their specific courses.

Personal Finance

My first attempt at recontextualizing mathematics was a unit I developed on personal finance for high school students that raised issues of poverty and justice.[43] As my first attempt at significant questions, this unit took a fair amount of time to develop. After this experience, I found it more natural to introduce questions into my lesson plans, and I was surprised at how frequently significant questions naturally arose throughout classes without previous preparation.

Converting Rates

An unexpected significant question occurred as I taught my students how to convert rates. Previously, I had demonstrated several examples using the typical questions of inches per year, miles per gallon, and so forth. On this particular day, I asked the class to estimate the number of seconds per life the average student will spend in church. Students were impressed by the large number and responded with surprise about the length of time they spent in church. If I had stopped here, I would have simply baptized the concept with religious language. Their response, however, provided a perfect lead-in to questions such as the following: what if we calculated the number of seconds playing basketball or listening to music? and would an examination of your calendar make it clear what is important in your life?

The context of time allowed the students to learn not only about rates, but also about how time reflects our priorities. One class, in response to the rate calculations for time in church, recognized that worshiping God occurs in other ways and places that were not accounted for, such as personal devotions and activities done to God's glory, including activities such as planting flowers and even sitting in math class. The beauty of a significant question is that it has the potential to evoke a heart response in students. Using this approach to teaching rates took a few extra minutes; however, I found that students learned the material at a deeper level. Compared to the previous years that I had taught rate conversions, students understood rates at a more profound, conceptual level because they were engaged with a recontextualized use of mathematics that was relevant to their lives. More importantly, this new approach to teaching rates allowed affordances for spiritual growth.

History

Math history can be a useful vehicle to integrate faith and mathematics. For example, the slow development of probability theory and its roots in gambling help students understand how humans took a beautiful aspect of creation and distorted it for financial gain. Similarly, when teaching the Pythagorean theorem, students love to hear about the Pythagoreans' strange practices and their worship of numbers. It is an ideal time to share how the Pythagoreans distorted reality by worshiping the created (numbers) instead of the Creator (God). Students are shocked to hear the extreme measures the Pythagoreans took to protect their worship of numbers. It provides the teachable moment to ask if there is something that is out of balance in students' lives. Are they worshiping the created instead of the Creator? Including the history of mathematics gives a context to mathematics. It helps students understand humanity's role in uncovering the elegance and order that God has designed within creation.

Ratios and Proportions

To teach ratios, I have students measure various parts of their bodies including sections of their fingers, the height and width of their head, their wingspan, and height. Students are surprised to find that the ratios of each student's body parts are so similar. I then introduce students to phi and the golden ratio describing how humans value objects that display the golden ratio as beautiful, including the many features present in their own bodies. Students are then amazed to see the many applications of the golden ratio and the golden spiral that God has embedded throughout nature.

Action figures and Barbies® are also a great resource for teaching proportions. Students are asked to measure various body parts of their figure. These measurements are then converted using proportions and the height of the average male or female to find what the dimensions of a life-size action figure or Barbie® would be. This activity not only gives students a realistic problem to practice proportions with, but it also brings up perspectival issues. Students understand that the image of action heroes and Barbies® presents a vision of strength and beauty that is physically unattainable. This is an ideal opportunity for educators to reemphasize the importance of a positive body image and the beauty that God has given to each student.

Types of Numbers

Although a simple example, I have found teaching about domains often brings up opportunities for the definition of human life. Typical questions include the best type of number to use to describe each situation. I include an example that results in an answer of 15.7 people. When I ask, "Is 15.7 a good domain for describing people?" I leave plenty of wait time for student responses. Without fail, one student will sometimes jokingly, or seriously, ask if 15.7 is accurate to describe a group that includes a person who is missing a body part or limb. I redirect this question back to the class, and they conclude that a person missing a body part is still a person. As Christians, we believe that it is the soul that constitutes personhood. Although a small example, this domain problem reemphasizes the importance of each human being, regardless of their physical state.

Conclusion

These are just a few suggestions for faith integration. The possibilities are limited only by your imagination. As Galileo noted, "God wrote the universe in the language of mathematics."[44] From the patterns of seashells to pinecones and the ocean waves, God has covered creation with his mathematical fingerprint. The number of ways to teach lessons and demonstrate mathematics is infinite. This points to another aspect of God that we can learn from mathematics, that he is infinite as well.

It is exhilarating to show students how God has imprinted his personality, beauty, creativity, and orderliness in the area of mathematics. I long for a classroom and curriculum that acknowledges that God is sovereign over *all* creation. I want a curriculum that causes students to delight in the concepts being studied and in which students and faculty are seen as image bearers who work in concert to build a learning community in which Christ's sovereignty is acknowledged throughout. Students are asked to answer the question "How can I use mathematical knowledge to help redeem every inch of creation for the glory of God?" That is the same question that I have struggled to answer throughout this article and throughout my journey to teach more integrally. "How can I use my mathematics classroom to help redeem every inch of creation for the glory of God?" *Soli Deo Gloria!* ◊

Notes

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[3]Russell W. Howell, "The Matter of Mathematics," *Perspectives on Science and Christian Faith* 67, no. 2 (2015): 74–88.

[4]David Smith, "Fostering Moral and Spiritual Development in the Mathematics Classroom" (lecture, Kuyers Institute for Christian Teaching and Learning at Calvin College,

Grand Rapids, MI, 2008); David Smith, "The Bible in the Classroom" (lecture, Heartland Christian Educators Conference at Dordt College, Sioux Center, IA, 2008).

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[33]Huizenga, "Integrating Faith and Math," 21.

[34]Palmer, *The Courage to Teach*, 122.

[35]Ibid., 123.

[36]Huizenga, "Integrating Faith and Math," 21.

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[38]Palmer, *The Courage to Teach*, 115.

[39]Ibid., 105.

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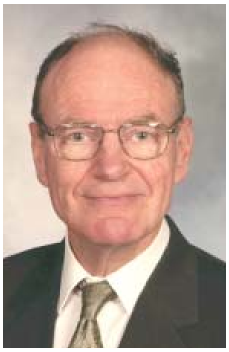

Donald C. Morton

Climate Science Continued

Donald C. Morton

The December 2014 issue of *Perspectives on Science and Christian Faith* carried two articles on current climate science. The present author challenged some of the basic assumptions and the conclusions following from them,[1] while Thomas Ackerman presented the familiar consensus position of the reports of the Intergovernmental Panel on Climate Change (IPCC).[2] Now I would like to respond to Ackerman and further emphasize why we should not depend on the predictions of climate models used in the 2013 Report, which I will refer to as IPCC2013.[3]

There is no doubt that our climate is changing, as it always has. Also, I am sure that adding CO_2 and the other minor absorbing gases (CH_4, N_2O, and CFC's) to our atmosphere increases the earth's temperature and that temperature has risen during the last 250 years. The central issue is how much of the temperature rise from 1970 to 1998 is due to natural causes. The abrupt slope changes in the global surface temperature curve in figure 1 of my previous article[4] show that these effects must be important and most of the hypotheses to explain the present plateau in the temperature attribute it to various natural phenomena absent from the models. The IPCC statements that human activity is the dominant cause of the temperature rise are based on comparing models with and without the anthropogenic gases, but now we know that the models omitted many possible natural causes. In any case, this wide range is not very useful. If the fraction is 95% anthropogenic, we have a serious problem, but if it is close to 50%, we very likely can adapt without major economic disruption.

We are told that the predictions of disastrous global warming caused by human activity are based solidly on science, so it is appropriate to review that science. Central to the scientific method is the development of a theory to explain some aspect of the natural world, and then testing it by predicting new results of experiments or observations not used in the formulation of the theory. In the case of the climate models used by the IPCC, simply reproducing past observations is not a test because these models depend on hundreds of parameters to represent phenomena too complicated to put into the computer codes. These parameters are calibrated by comparisons with past observations.

A high-priority goal of climate models is to predict how the mean global surface temperature anomaly changes with the rising concentrations of CO_2 and similar gases, but the lack of any temperature increase since 1998 continues to challenge the models. Ackerman explains the divergence by the stochastic nature of climate

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and recognizes that these processes are not occurring in the models in the same way as in the observations. In models tested with perturbations, the perturbations seem to average out after a decade or more, but it remains a hypothesis that our climate will do the same because it depends on many stochastic phenomena omitted from the models. The proposed time scale necessary to see the global climate average keeps increasing with the duration of the temperature plateau. Furthermore, the wiggles in the IPCC plots for individual models have a pattern rather different from the almost constant temperature of the twenty-first century.

Besides being stochastic, climate is expected to be chaotic in that it jumps from one approximately stable state to another, rather like the observed temperature. IPCC2013 recognizes the problem by the statement, "There are fundamental limits to just how precisely annual temperatures can be projected, because of the chaotic nature of the climate system" (FAQ 1.1, p. 140). However, there is no indication of how long the models are valid even though predictions often are shown to the year 2100.

As we continue to add CO_2 to our atmosphere, global temperatures eventually could start to rise again, or they could fall if the present weak solar activity continues. Until we understand the cause of the plateau we will not know how much of the rise is due to human activity. Whatever happens to future temperatures, there remain serious difficulties with the present climate models. The physics of climate requires a multitude of nonlinear differential equations, yet the models assume without justification that linear approximations are valid for predicting the future.

Ackerman described climate models "as straightforward applications of the laws of physics and chemistry."[5] This is true in a broad sense, but the physics quickly is overwhelmed by the adjustment (tuning) of hundreds of parameters to match a model to the real world. According to IPCC2013,

> Model tuning aims to match observed climate system behavior and so is connected to judgments as to what constitutes a skillful representation of the Earth's climate. For instance, maintaining the global-mean top-of-the-atmosphere energy balance in a simulation of pre-industrial climate is essential to prevent the climate system from drifting to an unrealistic state. The models used in this report almost universally contain adjustments to parameters in their treatment of clouds to fulfill this important constraint of the climate system. (Box 9.1, p. 749)

Clouds are a fundamental component of any climate system because they influence how much sunlight is scattered back to space, but they enter simply as parameters.

> The simulation of clouds in modern climate models involves several parameterizations that must work in unison. These include parameterization of turbulence, cumulus convection, microphysical processes, radiative transfer, and the resulting cloud amount (including the vertical overlap between different grid levels), as well as subgrid-scale transport of aerosol and chemical species. The system of parameterizations must balance simplicity, realism, computational stability and efficiency. Many cloud processes are unrealistic in current GCMs, and as such their cloud response to climate change remains uncertain. (IPCC2013, Sec. 7.2.3.1, p. 584)

IPCC2013 further elaborates the challenges of parameterization, stating,

> With very few exceptions modeling centres do not routinely describe in detail how they tune their models. Therefore the complete list of observational constraints toward which a particular model is tuned is generally not available … It has been shown for at least one model that the tuning process does not necessarily lead to a single, unique set of parameters for a given model, but that different combinations of parameters can yield equally plausible models. (Box 9.1, pp. 749–50)

Parameters are necessary in complex climate modeling, but they have the risk of producing a false model that happens to fit existing observations but incorrectly predicts future conditions. The parameters for most of the present IPCC models were largely influenced by data from 1961 to 1990 when temperatures were rising faster than the average, so it is not surprising that the response of the models to CO_2 is excessive.

The IPCC reports claim that the averages of models or ensembles of models with small variations in their parameters provide a useful guide to the uncertainty in the predictions, but the samples are not random.

Referring to these multimodel ensembles (MME), IPCC2013 states,

> the sample size of MME's is small, and is confounded because some climate models have been developed by sharing model components leading to shared biases. Thus, MME members cannot be treated as purely independent. (Sec. 9.2.2.1, p. 755)

The IPCC report continues with

> As a result, collections such as the CMIP5 MME cannot be considered a random sample of independent models. This complexity creates challenges for how best to make quantitative inferences of future climate. (Sec. 9.2.2.3, p. 755)

It is regrettable that such important details about the climate models were not included in the Summary for Policy Makers (SPM).

One might ask whether the SPM writers deliberately tried to hide the difficult details of the climate models. I expect that brevity was the primary reason, but in the same way that climatologists expect biased contributions from anyone funded by an oil company, there is always the possibility of some authors choosing words that do not displease government and IPCC sponsors already committed to mitigating anthropogenic global warming. Government and IPCC representatives were involved in the final preparation of the IPCC report.

What should we do now? In my view as Christians and as scientists, we should state the whole truth about the uncertainties in the climate models, including the fraction of warming actually due to human activity. It should not be necessary for everyone trying to evaluate the predictions to have to read a thousand of pages of IPCC reports in order to learn about the fundamental inadequacies of the models described there. Certainly we should respect God's creation and not be wasteful of all the wonderful sources of energy he has provided, but the present evidence of danger is not so compelling that we must stop flying to conferences in distant places. Certainly we should terminate bad policies such as the mandatory use of biofuels, transporting petroleum products by rail where a pipeline is possible, or destroying jungle habitat to grow palm oil.

Also we should take time to thoroughly review proposed policies, particularly questioning their impact on the poor in developed countries and on everyone in poor countries seeking a better life. For example, we ought to reject the claim that climate change is the world's most serious environmental problem, and encourage countries to give priority to reducing real pollution that is affecting people's health. If there were some reduction in the generation of CO_2, that would be a useful byproduct, but not the primary goal. Some people will remain concerned about the more pessimistic predictions, and so will prefer the precautionary principle and advocate a severe reduction in the use of fossil fuels, but they should not claim that their choice is being driven by the science.

Finally, remember that consensus on a scientific issue proves nothing. Ackerman extrapolates from consensus on thermodynamics, electromagnetic wave propagation and fluid mechanics, but each of these have earned their consensus status through more than a century of successful predictions. The climate models are not there yet. Science progresses by questioning everything, and this includes comparing theory with experiment and observation. ◊

Notes

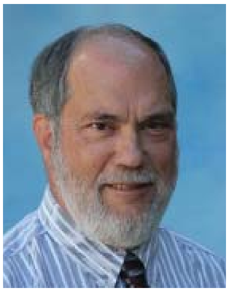
[1] D. C. Morton, "Climate Science and the Dilemma for Christians," *Perspectives on Science and Christian Faith* 66, no. 4 (2014): 236–41.

[2] T. P. Ackerman, "Christian Action in the Face of Climate Change," ibid., 242–47.

[3] IPCC, *Climate Change 2013: The Physical Science Basis.* Contribution of Working Group I to the Fifth Assessment Report (AR5) of the Intergovernmental Panel on Climate Change (New York: Cambridge University Press, 2013), http://www.ipcc.ch.

[4] Morton, "Climate Science and the Dilemma for Christians," 237.

[5] Ackerman, "Christian Action in the Face of Climate Change," 242.



Thomas P. Ackerman

Response to Donald Morton

Thomas P. Ackerman

Donald Morton wrote an article for *Perspectives on Science and Christian Faith*, to which I was asked to respond in a companion article.[1] Following the publication of these two articles, Morton responded with a shorter piece that included quite a few comments challenging the reality of human-induced climate change and the reliability of climate models.[2] As I struggled to find an appropriate response to Morton's comments, I began to feel that I had been assigned a new "labor of Hercules."

The crux of the matter is that, given a limited print space, it is far easier to raise issues and ask questions than to answer them, because adequate answers always take more words than the questions themselves. So, I find myself with a dilemma: I can write a short textbook on climate science or I can write a handful of very brief rebuttal statements. If I do the former, it will be too long to publish in this journal. If I do the latter, Morton (and perhaps other readers) will see my response as inadequate and perhaps even arrogant, because I must of necessity appeal to expert knowledge without providing detailed explanations of that knowledge. So what to do? Instead of responding to all of Morton's questions, I have tried to respond to a few of these, but focus on what I see is the core issue: Should we as Christians be actors in combating climate change or should we be passive watchers, or perhaps a spirited opposition?

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The Summary for Policy Makers (SPM) prepared by the International Panel on Climate Change (IPCC) in the most recent of their periodic reports (the Fifth Assessment Report or AR5) provides the following two conclusions.[3]

> SPM 1.2: Anthropogenic greenhouse gas emissions have increased since the pre-industrial era, driven largely by economic and population growth, and are now higher than ever. This has led to atmospheric concentrations of carbon dioxide, methane and nitrous oxide that are unprecedented in at least the last 800,000 years. Their effects, together with those of other anthropogenic drivers, have been detected throughout the climate system and are extremely likely to have been the dominant cause of the observed warming since the mid-20th century. (p. 4)

> SPM 2.2: Surface temperature is projected to rise over the 21st century under all assessed emission scenarios. It is very likely that heat waves will occur more often and last longer, and that extreme precipitation events will become more intense and frequent in many regions. The ocean will continue to warm and acidify, and global mean sea level to rise. (p. 10)

We climate scientists have tried to make these statements as clear and compelling as possible. Earth surface temperature has warmed significantly in the last one hundred years; greenhouse gas concentrations have increased during this same period to levels not seen in the last 800,000 years (the extent of our reliable ice core measurements); increasing greenhouse gas concentrations are due to human activity; and the overwhelming consensus of our science community is that these greenhouse gases are "extremely likely to have been the dominant cause of the observed warming." Furthermore, our carefully considered conclusion is that warming will continue throughout the twenty-first century leading to climatic extremes in the atmosphere and ocean. This is as close as we can get to the "dependable estimate" that Morton requests. The statements are both scientifically accurate and unambiguous.

Our primary measure of climate change is an increase in the global surface air temperature over the last century, but many other impacts of climate change are well documented. These include the loss of Arctic sea ice, an acceleration in the rate of sea level rise, melting glaciers, heat trapping in the ocean, increased thawing of permafrost, and the poleward migration of species. These effects can only become more severe over the next few decades.

One of Morton's contentions is that conclusions cannot be safely drawn from climate models because their uncertainties are too large. Morton supports this statement by extracting from a very long document (over 1,500 pages) a few sentences that discuss uncertainties in climate models. Uncertainty is an integral part of all science. The IPCC author teams are fully aware of the uncertainties in the models. Nonetheless, these author teams came to the conclusions I quoted. How do we reconcile the conclusions with the uncertainties? Is the climate science community being deliberately deceitful and trying to hide these issues? Morton suggests that the authors might be unduly influenced by government or IPCC "sponsors."[4] Hardly so. The SPM is drawn directly from the detailed report that summarizes the current state of our understanding of climate and climate change. A look through the IPCC volume on the physical science basis will convince one that there is enormous breadth in peer-reviewed climate research and that uncertainties are taken seriously in those research papers and the IPCC report. The climate science community has in fact considered the very issues that Morton raises and has concluded that, while important, they do not stand in the way of the conclusion that *human activity has changed and is changing our climate in ways that will negatively impact our future and the future of our children and grandchildren*.

What can I say in answer to Morton's specific questions about climate models? Actually, I and my colleagues can say a great deal, as evidenced by the hundreds of articles cited in the IPCC report. The climate models that we use to study current and future climate are not perfect, but they are very good, particularly when we use them to study changes in the global climate and changes in climate on large regional scales such as the United States. This applies to current climate, changes in past climate, and projections of future climate. Many of the uncertain parts of climate science are about specific types of outcomes (for example, storm frequency and intensity) and projections of changes in smaller regional climate patterns (such as monsoon rainfall).

In this context of prediction, Morton raises the very interesting subjects of the stochastic nature of the climate system and the role of chaos in climate science and projection. These issues are challenging to understand because of their complexity. But, they are receiving due attention from the climate science community and our collective understanding is included in the IPCC report. When Morton cites the IPCC report as stating, "There are fundamental limits to just how precisely annual temperatures can be projected, because of the chaotic nature of the climate system," he draws the unfounded conclusion that we have no understanding of the validity of the timescales of climate projections. No reputable climate scientist argues that climate models (or weather models) can be used to predict the global annual temperature precisely, because the randomness (stochastic nature) of internal variability in the climate system prevents us from doing so. Climate models are able to simulate temperature rise over the past century (albeit with certain caveats) but at the decadal (10-year) or longer timescale, which is what we expect. The statement quoted by Morton is not a concession by scientists of a flaw in climate models; it is an explanation of the model limits. In addition, the role of chaos in the climate system is much more nuanced than Morton suggests. While there appear to be attractors (relatively stable states) in the system, there are many of them and the transitions between states are relatively smooth.[5] The fact that we do not yet completely understand some of these complex issues does not detract from the conclusions of the IPCC report. In the opinion of the climate community, the report conclusions are not materially affected by the uncertainty remaining at this point.[6]

Morton spends considerable time criticizing the representation of clouds in climate models, particularly the use of parameterizations. Climate scientists have invested an enormous amount of time and effort on this problem in the last twenty years. Despite Morton's assertion, clouds in climate models do not "enter simply as parameters." Climate-model clouds form, produce precipitation, and decay similarly to what occurs in the real world, based on mathematical expressions for the chemistry, physics, and thermodynamics of a wet atmosphere (one containing water vapor). These complex equations contain parameters, which are variables that need to be specified because they cannot be calculated within the model, generally because of a lack of computer time. One example of a parameter might be the average size of a cloud droplet or ice particle. The values of these parameters are determined by comparison with data from large atmospheric field programs, time series data from ground observing sites, and satellite data records. Most of these data have been acquired in the last 15–20 years by a constellation of complex instruments in space and on the ground. So, we might use global satellite measurements to say that the average droplet diameter in boundary layer clouds over the ocean is 15 micrometers, which is consistent with field observations (made from aircraft and ships) as well. Because clouds are highly reflective, we might find that droplets that are slightly smaller (say 13 micrometers) give better agreement with satellite measurements of top-of-atmosphere reflected solar radiation. Since a diameter of 13 micrometers is within the uncertainty of our measurements, we prefer to use 13 instead of 15. This is the extent of the "tuning" of models and is hardly the huge problem suggested by Morton. (Cloud properties and their effect on the earth energy budget is one of my ongoing scientific interests, so I am figuratively biting my tongue at this point, trying hard not to add another few pages!)

Detailed responses to Morton's assertions about clouds and parameterizations can be found in the IPCC report, volume 1 (particularly chapter 7 on clouds and chapter 9 on the evaluation of climate models) and in the references cited there as well. Uncertainties are discussed at length. Contrary to Morton's assertions, climate models are not "linear approximations" to past data, nor are parameters set to arbitrary values. There is no evidence for his statement that "the physics quickly is overwhelmed by the adjustment (tuning) of hundreds of parameters …" In fact, as I just discussed, such tuning does not occur in the manner that he suggests and these parameters are part of the physics, not some afterthought.

Our current climate models solve a set of coupled and fully nonlinear differential equations in both atmosphere and ocean. We simulate the future by forcing these equations with projections of increasing greenhouse gas concentrations. We also simulate the effects of possible changes in solar activity, volcanic activity, and human-generated air pollution. Finally, we continue to test our models against current observations and against past data. As we learn more about how climate science works, we continually improve our models to make them the best representation of climate that we can.

Now let's move on to what I see is the issue at the core of Morton's comments. If climate change science is correct and our current actions are creating a situation that threatens human lives and ecosystems around the world, then as Christians we must respond to that situation by altering our behavior and working to alter the behavior of our broader society. If one can argue, however, that climate science is terribly uncertain, then we do not have to do anything. Suppose that climate scientists are wrong in our estimates of anticipated climate change effects over this century. How wrong do we have to be before this issue no longer demands an ethical response? If what actually happens to climate over the next fifty to one hundred years is somewhat less than climate scientists are currently predicting, would that absolve us from taking action now? If what happens is only half as bad, would that absolve us? Do we then do nothing? Apparently Morton thinks so, because he claims that if 50% of the warming to date is anthropogenic, then we have nothing to worry about. As far as I know, this statement is completely unsupported by any evidence.

Furthermore, uncertainty is a two-edged sword. What if what happens is actually worse than what we are predicting (an equally plausible outcome)? It is possible that global warming may alter the climate much more than we currently expect. How should we behave when that outcome is a risk? The best science on the problem of climate change says that we are driving our planet toward a very uncertain future and that future is most problematic for the poor, the generations yet to be born, and the ecosystems on which we depend. You can see these arguments and evidences fleshed out in the second volume of the IPCC report on *Impacts, Adaptation and Vulnerability*. The National Academy of Sciences has also produced numerous reports on climate change, the science, uncertainties and likely outcomes (for example, the multi-volume work *America's Climate Choices*).[7] Reports arriving at similar conclusions have been written by learned societies in other countries.

Morton ends his commentary with three things that we should do. The first is that "we should state the whole truth about the uncertainties in the climate models including the fraction of warming actually due to human activity." I (and my scientific colleagues) absolutely agree. That is why the IPCC report runs to 1,500 pages. Because these models are complex, stating the whole that is known about their uncertainties requires hundreds of pages. However, we can summarize what we know in shorter format. The most recent and currently best summary on climate change and its uncertainties, as well as the relative contributions of natural variability and human activity to that change, can be found in the IPCC report, volume 1, "Summary for Policy Makers" (p. 14). Morton's insistence on knowing the precise fraction due to human activity is a "red herring," distracting us from the essential point that human activity is causing, and will continue to cause, global warming, unless we reduce human emissions of greenhouse gases.

I certainly agree with Morton's point about not wasting energy. I am a strong advocate of using renewable energy resources, in part because that allows us to conserve fossil fuels for other uses than simply burning them and avoids a whole range of associated environmental problems. While I can and do practice energy conservation in my daily life, there are many actions that can be taken only at the societal level. I look forward to the Christian community taking an active lead in promoting energy conservation and the use of renewable energy in North America.

My position is that we need to take action now, because every day that we delay makes the problem worse, given the very long lifetime (hundreds of years at least) of carbon dioxide in our atmosphere. In 2013 a team of 31 international researchers who modeled a number of emissions projections concluded that, even if we could reach zero carbon emissions within fifty years (a very difficult task), it would take centuries to return ocean and surface temperatures to current conditions. This effect is called the "climate commitment." The actions we take today will play out over long periods of time.[8]

While neither Morton nor I are economists, analyses of the economic and social risks associated with climate change have been carried out. They show that the potential costs of waiting to change are high and that these costs can be reduced by acting now. Many economists disagree with the conclusions Morton raises, even in the face of scientific uncertainty. Some researchers have argued that "effective mitigation action must be started decades before the climate changes of concern are actually observed,"[9] and that "in general, uncertainty about a problem may indicate the need for more, or less, action to address it, depending on the nature of the unknowns."[10]

Certainly, there are costs to changing our reliance on fossil fuels. There are also costs to not doing so, and conversely, benefits from making such changes. The US government and many parts of the private sector recognize the costs of inaction. For example, FEMA is requiring states to have climate change action plans before they receive disaster aid in the future, because of potential costs from sea-level rise and other problems. The Department of Defense also disagrees strongly with Morton's position, in part because they see climate change as a national security issue.[11] The insurance industry has come down on the side of climate change mitigation because the costs of climate change to industries and municipalities are significant. On the other hand, the costs of making changes in order to lower greenhouse gas emissions can provide co-benefits. Cleaning up the large atmospheric brown clouds that harm human health and crop growth over much of Asia would save human lives as well as slow greenhouse gas emissions. Promoting urban vegetation would absorb greenhouse gases but would also absorb dangerous particulates and lower local temperatures, two outcomes that would significantly improve human health in cities. Actions such as lowering food waste, something the U.N. has called for, would both help provide food for growing numbers of people, and lower carbon emissions.

Joel Pett of the *Lexington Herald-Leader* drew an often-reproduced cartoon that shows people at a climate summit conference listening to a speaker providing a list of benefits obtained by moving away from reliance on carbon-based fuels. These benefits include cleaner air and water, healthier children, sustainability, renewable energy, and so forth. A grumpy individual in the audience then opines: "What if it's all a big hoax and we create a better world for nothing?" To some extent, this is my reply to Morton. Global warming is not a hoax. But, it is a major component of the multiple human impacts on the environment that are working together to damage our atmosphere, ocean, and land, as well as human health and agricultural productivity.

Creating a better world means first acknowledging that our actions are changing global climate, and then taking responsibility for those actions. It means leading our society toward a solution to the problem of climate change and toward a sustainable future. To do so, we must lower carbon emissions, and we need to start now. To be blunt, we should have started years ago. The few actions taken by the governments of the USA and Canada over the past two decades have had a largely negligible impact on CO_2 emissions and global warming. This is not because we cannot do anything, but because we will not. I agree with Morton that we need to think carefully about the costs and benefits of various actions, such as the use of biofuels to lower emissions, and that we should "terminate bad policies," but we cannot use "thinking" as an excuse to do nothing. Christians profess God's love for the world and for all God's children. We must show this love through our actions. We must be leaders, not reluctant followers, in the struggle to reduce carbon emissions and stabilize climate, not only for the sake of our generation, but for the sake of our children and our children's children. ◈

Notes

[1]D. C. Morton, "Climate Science and the Dilemma for Christians," *Perspectives on Science and Christian Faith* 66, no. 4 (2014): 236–41; and T. P. Ackerman, "Christian Action in the Face of Climate Change," ibid., 242–47.

[2]D. C. Morton, "Climate Science Continued," *Perspectives on Science and Christian Faith* 67, no. 2 (2015): 135–37.

[3]The IPCC "Summary for Policy Makers" is a condensation of each of the IPCC reports from three working groups: *I. The Physical Science Basis*; *II. Impacts, Adaptation and Vulnerability*; and *III. Mitigation of Climate Change*. These volumes, along with a "Synthesis Report" were prepared as part of the Fifth Assessment Report published during 2013–2014 (found at http://www.ipcc.ch/). These reports are the result of an exhaustive process of summarizing the existing scientific literature, reaching consensus on the conclusions of the literature, and thoroughly vetting those conclusions throughout the scientific community before publication. I strongly encourage those interested in this subject to read the "Summary for Policy Makers" (SPM). In his comments, Morton quotes from the IPCC report, *Climate Change 2013, The Physical Science Basis*, which provides the foundation for the SPM. The "Technical Summary" of this document is an excellent resource for the scientifically and technically inclined reader.

[4]This comment reflects a severe misunderstanding of the IPCC process. The IPCC was created by the United Nations Environmental Programme and the World Meteorological Organization to provide assessment reports. It does not fund the time of scientists involved in the assessments or provide research money, although it does fund travel expenses to IPCC meetings. The US government, as well as most countries, does not provide funding for IPCC assessment activities either. The result is that IPCC assessments require a substantial commitment of scientists' time with no remuneration of any kind. As a participant in WMO activities over the years, I can assure the readers that these activities have been a drain on my personal time and resources that has not been reimbursed in any way. Scientists such as I agree to do these activities because they are a public good, not because they provide personal rewards.

[5]Research published by our group at the University of Washington uses cluster analysis to show that atmospheric weather patterns can be sorted into similar states or clusters. The weather system moves smoothly and, in some cases, predictably from one clustered state to another. The number of clusters is not unique but depends on the amount of data available and the mathematical analysis being employed. See, for example, S. Evans, R. T. Marchand, and T. P. Ackerman, "Variability of the Australian Monsoon and Precipitation Trends at Darwin," *Journal of Climate* 27 (2014): 8487–8500. doi: http://dx.doi.org/10.1175/JCLI-D-13-00422.1.

[6]It is interesting to note that the word "chaos" does not even appear in volume 1 of the most recent IPCC report. There are, however, extensive discussions of natural climate variability, which is the expression of chaos in the climate system, and of climate-model ensembles (groups of identical model simulations starting with different initial conditions), which is one of the effective ways to study the effects of chaos. One might conclude from my first statement that climate scientists are ignoring chaos, but that is far from the truth. Our study of scientific chaos theory has led us to a deeper understanding of the climate system and how to simulate that behavior.

[7]National Research Council of the National Academy of Sciences, *America's Climate Choices* (Washington, DC: National Academies Press, 2011).

[8]K. Zickfeld et al., "Long-Term Climate Change Commitment and Reversibility: An EMIC Intercomparison," *Journal of Climate* 26 (2013): 5782–809.

[9]M. Webster et al., "Uncertainty Analysis of Climate Change and Policy Response," *Climatic Change* 61 (2003): 295–320.

[10]Congressional Budget Office (CBO), *The Economics of Climate Change: A Primer* (Washington, DC: Congress of the US CBO, 2003), 32.

[11]The Department of Defense (DoD) reports such as the "2014 Climate Change Adaptation Roadmap," http://www.scribd.com/doc/242845848/Read-DoD-report-2014Climate-Change-Adaptation-Roadmap; and the "FY 2012 Strategic Sustainability Performance Plan," http://www.acq.osd.mil/ie/download/green_energy/dod_sustainability/2012/DoD%20SSPP%20FY12-FINAL.PDF, provide insight into DoD thinking on climate change.

ENVIRONMENT

INTRODUCING EVANGELICAL ECOTHEOLOGY: Foundations in Scripture, Theology, History, and Praxis by Daniel L. Brunner, Jennifer L. Butler, and A. J. Swoboda. Grand Rapids, MI: Baker Academic, 2014. 262 pages. Paperback; $26.99. ISBN: 9780801049651.

Given this title, you might think that you are picking up a textbook. The title *does* accurately reflect the themes of the book, but it substantially undersells the voice(s). The content is solid, and the book serves as an introduction to the topic as well as a "shaking of hands" with a rich pool of historical and contemporary writers and thinkers. It is a treasure trove of quotes and opens up the scope of the larger discussion surrounding our theology of the created world. Coming out of the Reformed tradition and being familiar with the likes of Cal DeWitt, Fred Van Dyke, Wendell Berry, and Doris Longacre, I was pleased to be introduced to a diversity of other voices, including those from Eastern Orthodox and Pentecostal traditions.

This book feels like an invitation to conversation. The self-introduction of each of the authors and their description of their writing process leaves one with the feeling that you are listening in on a very careful, gracious, thought-provoking, and impassioned discussion. In the first section, they explain to the reader their motivations for writing the book, their hermeneutical approaches, and core biblical reasons for caring for the earth. The intentional inclusion of "Tension Points" among the authors lends depth to the book and further invites the reader to the discussion. They even go so far as to explicitly ask the reader to consider their own opinion on a variety of issues, both theological and practical. This is a great strength of the book. It is not a reference manual or textbook, though I have already used it in the classroom. The book has a bit of an episodic feel as the voices of each of the writers emerge. It feels a bit like eating a fruit salad; the flavors blend, yet heterogeneity is maintained. Rather than being disruptive, this promotes a reflective engagement with the material.

Throughout the second section of the book, "Exploring Ecotheology," historical views of Christians are presented evenly. The ambiguity and periodic ambivalence of thousands of years of Christian thought on the relationship of human beings to nature is not reconstructed to portray Christianity and the church through time as a model of ecological sensitivity and creation care. They do an excellent job of clarifying and critiquing the roots of that ambiguity and show the interweaving of threads of many contemporary Christian positions throughout our theological heritage. They make it very clear that ecotheology is not some new fad but, rather, as they quote Sallie McFague, "… nothing less than a return to our Hebrew and Christian roots" (p. 126).

The authors' commitment to a gracious critique of history is obvious; they point out that as contexts and the needs of the world change, so must the church's emphasis. The authors state, "Good theology … is always resituating itself in response to the current situation of the planet and humanity" (p. 125). The authors are convinced and convicted that the multiplicity of ecological crises is the "next great work facing both humanity and the Christian faith" (p. 16) and that we bear responsibility for where we are and where we will go in the future. They quote Wendell Berry as stating, "The culpability of Christianity in the destruction of the natural world and uselessness of Christianity in any effort to correct that destruction are now established clichés of the conservation movement" (pp. 46–47). This must change.

Throughout the book, but particularly in the second section, the authors make it clear that our view of the creation is deeply interwoven throughout our broader theological understanding. The authors touch on the interplay between our theology of nature and theological concepts such as the image of God, the transcendence and immanence of God, the humanity and divinity of Christ, the trinity, sin, soteriology, eschatology, the problem of gnosticism, pneumatology, covenant, and many other theological topics. Importantly, the various parts of this discussion closely tie our desire for orthodoxy to our love for our neighbor, our calling to stewardship, and discipleship. Throughout the book, the authors substantiate their claim that "our common call to earthkeeping is a part of our call to discipleship, and our call to discipleship is nothing more than a call to Jesus Christ" (p. 5). This is another great strength of the book. The conversational feel of the book situates all of the addressed issues within real, whole people. Even though treated separately, theology, discipleship, practice, and experience are all tied together. The term "evangelical" in the title appropriately highlights overarching themes of conversionism, activism, biblicism, and crucicentrism.

The third section of the book, "Doing Ecotheology," outlines the contours of activity situated in our love for God and all that he has made: our neighbors and our *oikos*—our home. The authors clearly articulate that orthopraxis shapes orthodoxy as much as the other way around; knowledge is insufficient. They cite research which demonstrates that there is no direct relationship between having more information and being more ecologically conscious. In parallel to the theological and philosophical connections made previously, the authors clarify the interrelationships between the *practices* of stewarding the creation, caring for our neighbor, and loving God repeatedly throughout the section.

The "Last Things" section is only last in the sequential sense; in many ways it is the most critical. While most sections of this book are clearly directed to the initiate, this section will be worth revisiting over and over again throughout a life filled with confrontations with the degradation of God's good creation. It is a calling to continually live in hope and a concise articulation of what that means.

This book is a valuable tool for Christians seeking to respond in love to a rapidly changing world and the ecological crises before us. It is unfortunate that, like many other well-written and timely books, it will likely not be read by those who could most benefit from it. The opening pages describe the impact of climate change on the well-being of a small community in the Gaza province of Mozambique. For climate change deniers, these could very well be the last pages that they read. Given the title, they may have never picked up the book in the first place. Hopefully, those who do read and embrace the message of discipleship deeply threaded throughout the text will put this book, or its message, into the hands and hearts of those who need it most.

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Ethics

TO TOUCH THE FACE OF GOD: The Sacred, the Profane, and the American Space Program, 1957–1975 by Kendrick Oliver. Baltimore, MD: The Johns Hopkins University Press, 2013. 248 pages. Hardcover; $39.95. ISBN: 9781421407883.

This recent book examines several possible connections between religious thought and the exploration of space, more specifically Christianity and the American space program through the Apollo moon landings and shortly thereafter. Having thought about this topic myself—and dismissed any such connection—I was not expecting much from this work. However, although the book shows that these connections might not be present or robust, it explores the issues with depth and insight.

In considering these connections, we can focus our thinking around a simple question: did religion motivate the space program, or provide a post hoc framework for its interpretation? The former might be seen in a general ethos on the part of key leaders or individuals in the trenches. This motivation is unlikely. Even Charlie Duke, who walked on the moon during the Apollo 16 mission and later became an active Christian evangelist, separates any religious drive from his role in the program (*Moonwalker*; Thomas Nelson, 1990). His religious conversion came later. He was driven, as were many of the astronauts and engineers in the early years of the program, by the need to push boundaries; the motivation was as simple as that. Consider also that the Soviet Union was in space first, and now China has a very active space program—neither of these are known for overt religiosity on the governmental or institutional level (although admittedly we cannot know the inner motivations of the individuals involved). The second part of the question is of more interest and potential relevance: was the exploration of space, driven by whatever motivations, later interpreted through the lens of a spiritual or even religious quest? This is a question explored throughout most of the book.

The book's introduction is an astute, literate, and readable setting of the culture of the time (the 1960s). This material is not fundamentally new, but it is presented with a different emphasis than in other works, and well done. More generally, the author is willing to look past simple answers. For example, the invocation of religious language (Kennedy asks God's blessing at the start of the Apollo program) could well reflect cultural/political views, not religious views in any real sense. The author marshals an impressive array of research exploring many related and some tangential areas, such as the rise of evangelism and a look back to a time when technology was seen as a redemptive force for humanity. The religious question is raised early on: is our quest into space performed in praise of God, or rather does it preclude the need for God since we can now reach for the heavens on our own? This is the essential duality explored here: casting off the need for God through our technological prowess, or coming closer to him through our push into the heavens.

The overall modus operandi of the book is to present an example where religion seemed closely and uniquely connected to the space program, and then show that that connection is illusory, superficial, or transient. This is demonstrated through several key themes: invocation of religious language as a motivating force for human exploration of space, use of religious imagery to interpret the experience of space exploration, the religious experiences of the astronauts themselves, and the marshaling of public support for religious expression in the space program. In each case, it is shown that these connections between space and religion are tenuous at best, and history has shown them to be temporary. This is not to denigrate this approach, for it works well in keeping the reader's attention by connecting with aspects of the program that were publicly visible and easily noted (by those who were paying attention to space through to the mid-1970s). In following these trains of apparent connection, the author brings to bear a wide range of sociological work on American religious and technical culture of the time. Despite the occasional tendency to academic jargon and convoluted sentences—par for the course in much academic writing—the book serves its purpose well, even for a lay audience (provided that the readership is truly interested in religion and/or space exploration).

The first major theme, the role of religion as a source of motivation for the exploration of space, is a legitimate one in which to couch part of the book's development. By most personal accounts those who were most heavily involved—as administrators, scientists, engineers, or astronauts—were driven by secular and practical reasons. This is something that any engineer would find to be a moot point. Faster, higher, further, the next big thing—these are the drivers for many of those with a technical or adventurous mindset. The heady enthusiasm from being a part of such an adventure is known by anyone fortunate enough to have been involved in something of this stature, and the space program by virtue of its sheer size and visibility particularly lends itself to this sense of grand adventure. But this transcendent feeling on the part of the players is also present in sports and many other activities: the search for something larger than one's self. It is afterward that it may be seen as a religious quest, if so inclined. In fact, it was those left on the ground who seemed most anxious to infuse the endeavor with meaning (spiritual or otherwise)—to little avail, in the long run.

The second theme is the invocation of religious imagery to interpret space exploration—to place it in a larger context and meaning. The impact here is less than clear. Did access to space alter old concepts of the heavens? Was this a voyage to look for God or to destroy him (Norman Mailer poses this very question in *Of a Fire on the Moon*; Signet, 1971)? The question bears on the issue of extraterrestrial beings, UFOs, and the anthropic principle. What does the incarnation mean if there is life elsewhere? What about salvation? Space exploration raised questions, but again it seems there was little lasting change in the debate or in the concepts. (Again, see Mailer.) There was an "anticipation of cosmological effects" more so than actual effects (p. 70).

Theme three is centered on the experiences of the astronauts themselves. As wayfarers in the heavens—humanity's surrogate travelers to otherworldly realms—they were expected to convey back to us the immensity of the experience. But, for the most part, the astronauts were lacking in a language to match the grandeur of the undertaking. Any tendency to connect with larger issues was inevitably limited: they were technologists, not poets. The idea throughout is that they brought back what they took with them, and although there were many occasions to express wonder and awe, these were quickly subsumed by the operational tasks at hand and tended toward the sense of Earth as a protective home rather than one of divine inspiration (with exceptions as noted below).

As support for this idea, it is interesting to observe that for the lunar landing missions of the Apollo program, it is the lunar module pilots, and not the commanders (both of whom landed on the moon in each mission), in whose lives one might see reference to any form of spiritual experience. (A point raised by Andrew Smith in *Moondust*; Harper Perennial, 2006.) For one thing, the lunar module pilots did not have the burden of command. More so perhaps, the commanders were temperamentally better suited not to have any such type of "ephemeral" experience, but rather to concentrate on the immediate needs of the mission. It is telling that of those Apollo astronauts who had the most overt spiritual or religious quests on their return to Earth, none were commanders and all appeared to have had at least some thread of a connection to religious or spiritual sensibilities before their journeys: most notable in this case was Jim Irwin who started the High Flight Foundation ministry, and Edgar Mitchell who began the Institute of Noetic Sciences. Again, in each case, they brought back what they took with them.

The final theme, public support for the space program, shows that people felt a religious component of space exploration was worth protecting, but perhaps due more to a general sense that, among other things, the battle for school prayer had been lost and that government would give in again if not put on alert. This then was not necessarily an effort to protect the religious component of space exploration per se but rather an effort to protect religion in public life.

In the end, Apollo and related efforts of the time were larger in *quantity* but not in *quality* than other events and activities. This explains why there was little-lasting effect on religious thought. It seems that there *should be* a space-religion connection but it is continually seen to be superficial or nonexistent. "The human condition has not been transcended by the passage to new worlds" (p. 134). Travel in space, in fact, resulted in a turn toward Earth, spurring the ecology movement through images such as the iconic Earthrise from Apollo 8 (a point made also by Robert Poole in *Earthrise: How Man First Saw the Earth*; Yale University Press, 2010). "It was the assumption of a cosmic destiny for mankind, not the claims of conventional faith, that now seemed most open to doubt" (p 168).

Overall, the book presents an even-handed view. The author seems to come to the conclusion reluctantly— as have I—that there simply is no fundamental connection between space exploration and religion. On a superficial level, perhaps. Books continue to come out on this topic: the technology, the politics, the people of the early space program. We continue to see something special in the Apollo program, but that could be because of the other many fascinating aspects, not solely religious or spiritual. In some sense, a program like Apollo (and its precursors) is so large and so unique that it looms in history like a spiritual quest. But in many ways the event—and the entire Space Age of the 1960s and early 1970s—was out of context. (This phrase is used to good effect in Al Worden's recent book *Falling to Earth* [Smithsonian Books, 2011], regarding the personal effort to deal with the return to mundane earthly life after a trip to the moon. The best approach is to place it in perspective as something that had no logical predecessor or successor: *sui generis*.) Fundamental and permanent cultural changes resulting from the space program have been—so far—rare, the ecology movement, as noted, being one possible exception. The fundamental point is that we take from space exploration what we bring to it, a religious connection that is fleeting at best, and exploration that has so far caused more of a turn to Earth than to God.

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iDISORDER: Understanding Our Obsession with Technology and Overcoming Its Hold on Us by Larry D. Rosen. New York: Palgrave Macmillan, 2012. 256 pages, notes, index. Paperback; $16.00. ISBN: 9781137278319.

iDisorder, by Larry D. Rosen, is a short book with an intriguing premise: the extensive use of modern technology causes many people to exhibit symptoms of classical, common, psychiatric disorders. The book systematically goes through these disorders—communication disorders, ADHD, depression, obsessive-compulsive disorder, narcissistic personality disorder, hypochondriasis, schizoaffective and schizotypal disorders, body dysmorphia, voyeurism, and addiction—and cites countless studies demonstrating how technology enhances or draws out the symptoms of the disorders. As a professor of psychology at California State University, Dominguez Hills, Rosen is well acquainted with these disorders.

Note that the author does not argue that technology causes these disorders. He only argues that technology can cause (or enhance) symptoms that match the symptoms of people diagnosed with these classical disorders. Since the Diagnostic and Statistical Manual (DSM) does not cite technology and media as contributing factors to these disorders, the author refers to them as "iDisorders."

One of the most accessible and convincing chapters is "Obsessively Checking in with Your Technology ... 24/7" in which the author describes how technology (especially the cell phone) often leads to compulsive behaviors. The author describes how people, including himself, compulsively check their cell phones for new messages, new texts, or missed calls. The chapter contains multiple anecdotes on how individuals get anxious when they travel into an area without cell phone reception. Some even refuse to travel when they know they will be "off the grid" for a time. The chapter relates results of multiple polls and surveys on technology usage during vacations, individuals' "FOMO" (fear of missing out) and "disconnectivity anxiety." The chapter then compares these symptoms to those of classically defined panic disorder and obsessive-compulsive disorder. It ends by giving useful advice on how to deal with an obsessive-compulsive iDisorder. This advice includes using settings on your devices to reduce the number of notifications the device triggers and doing a four-step process of Rethink, Reboot, Reconnect, and Revitalize (p. 58).

The other chapters follow this same pattern. The author defines the symptoms of the classical disorder, relates some anecdotes of how the symptoms are brought out by extensive use of technology (computers, cell phones, tablets, social media, etc.), cites studies and surveys implying a connection between the technology and the symptoms, and finishes with a refreshing section on how to identify, avoid, and/or treat the iDisorder in oneself or someone one knows. Some chapters also contain surveys to help readers gauge their own tendency to having an iDisorder. Each chapter is rife with citations—the endnotes of the book contain twenty-one pages of bibliographic references to journal articles, conference presentations, books, and websites.

The advice for treating an iDisorder is generally quite predictable. First, measure your dependence on the technology (how much time or money is spent using this technology each day) and determine how you feel when you do not have access to your technology. Then, avoid situations which may trigger symptoms of the iDisorder. Use technological tools (e.g., apps or plug-ins) to limit or change your use of the technology. Set aside time intentionally to meet others face-to-face, put away technology, and nurture real-world relationships. Be accountable to others as you try to change your behavior. In some cases, seek help from a therapist.

Rosen is not antitechnology and, in fact, stresses that he is a thorough and early adopter of many technologies. It is refreshing that he uses his own behaviors in some chapters as examples of symptoms of iDisorders (p. 50).

Rosen does not make any Christian or spiritual commentary on iDisorders. However, the book is relevant to Christians because it exposes and addresses the symptoms of these iDisorders, which can be exhibited by Christians and non-Christians alike. A common theme of these iDisorders is a person prioritizing relationships with technology and media over relationships with others (and for Christians this includes God). Knowledge of these iDisorders is useful for Christians to evaluate their own behavior. This knowledge may expose a Christian's dependence on technology instead of complete dependence on God. Christians might also discover that they exhibit behaviors which diminish their ability to minister to, have empathy for, and serve others in this technology-heavy world. For example, they may realize they are becoming less able to carry on long conversations with someone, they increasingly evaluate people by their looks, or they are becoming increasingly unable to meet appointments because of excessive time spent online.

The author makes the claim that the use of technology is irresistible. Thus, he never suggests that people avoid the iDisorders by simply getting rid of their cell phones, data plans, or social networking accounts. Calling technology adoption "irresistible" is controversial from a Christian perspective, because Christians are called to exercise freedom and responsibility. With God's help, a person can resist the negative impacts of technology. On the other hand, we Christians are called to engage, reform, and redeem culture, so avoiding all technology may hamper our ability to be witnesses of Christ in this world. Thus, a thorough investigation of the possible impact of technology on our thoughts and behaviors may be very useful, so that technology use does not become an idol but is instead used in service of God in our walk and work in this world.

I recommend this book. It is short and quite readable, apart from occasions when the author lapses into the use of psychology jargon that would not be understood by the average reader. The large bibliography may be a useful reference for anyone interested in exploring this area further.

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History of Science

INFINITESIMAL: How a Dangerous Mathematical Theory Shaped the Modern World by Amir Alexander. New York: Scientific American/Farrar, Straus and Giroux, 2014. 368 pages, biographical summary of key historical figures, timeline, endnotes, index. Hardcover; $27.00. ISBN: 9780374176815.

Five men in flowing black robes convened a meeting in the Collegio Romano to pronounce judgment on a dangerous idea which they feared might plunge their world into chaos. On August 10, 1632, the Jesuit fathers condemned and prohibited the dangerous and subversive doctrine of infinitesimals, the proposition that a continuous line is composed of distinct and infinitely tiny parts. Their opposition to this mathematical theory was based on the belief that the world was an orderly place, governed by a strict and unchanging set of rules. Infinitesimals threatened to undermine the authority of established religious and political order.

In *Infinitesimal*, the author weaves a historical drama, with all the intrigue of an adventure novel, set in the context of the mathematics of the infinitely small. Its key actors include many well-known philosophers, religious leaders, mathematicians, and scientists of antiquity through the Scientific Revolution, from Plato to Thomas Hobbes, Martin Luther to the Jesuits, Pythagoras to John Wallis, and Archimedes to Galileo Galilei and Isaac Newton. It is a fascinating read, connecting the dots between the religious turmoil of the Protestant Reformation, the consequent political upheavals that swept through Europe, and the birth of the modern scientific movement, including the religious ban on the heliocentric astronomy of Galileo and Nicolaus Copernicus, and leading to the development of modern calculus by Isaac Newton and Gottfried Leibniz. The debate over infinitesimals, while relatively unknown in comparison with the controversy regarding heliocentrism, occupied the same historical and intellectual space and involved many of the same religious and philosophical concerns.

The concept of infinitesimals is that, just as a cloth is composed of many layers of fine threads, an object of two-dimensional shape can be thought of as a collection of an infinite number of infinitely small but discrete lines. A solid surface can be considered an infinite number of two-dimensional planes, while a one-dimensional line can be divided into an infinite number of points. For modern scientists and mathematicians, this concept seems obvious because we have grown up with calculus involving the summations of the infinitely small. But in the sixteenth and seventeenth centuries, this concept was the subject of an intense and vigorous debate, with the outcome affecting no less than the stability of the social order and the authority of the church.

Why was this mathematical theory, which is standard curriculum today, considered so dangerous back then? Martin Luther launched the Protestant Reformation in 1517 by posting his Ninety-Five Theses on the door of Wittenberg's Castle Church and openly defended his stand against Catholic authority in 1521 at the Diet of Worms. The Protestant Revolution that followed plunged Europe into a series of religious and political conflicts that seemed to rock the very foundations of the civilized order. In order to counteract the chaos and uncertainty caused by the schisms and to restore allegiance to the authority of the church, the Papacy founded the Society of Jesus, known as the Jesuits. Among their missions was a strong emphasis on education; and thanks to the efforts of Christopher Clavius, their mathematical teachings were firmly established on the principles of Euclidian geometry.

Clavius believed that Euclidian geometry held the secret for restoring order to society and re-establishing the absolute authority of the church. He held that Euclid's theorems impose a rigorous order of logical proof that could establish the truth with undeniable certainty, proceeding from simple statements to ever-more-complex questions. This was in harmony with what the Jesuits were trying to accomplish—to impose a true, eternal, and unchallengeable order upon a seemingly chaotic reality—and Clavius believed that geometry held the key to other intractable problems in the scientific and religious debates of the time.

But infinitesimals threatened to challenge that rigid system of logical proofs. By dividing a line into an infinite series of infinitely small points, paradoxes and logical contradictions arose that defied the desired rationality and order of Euclidian geometry. If the parts are infinitely small, then the sum of their lengths should be zero. Or if they are not infinitely small, then an infinite number of them should be infinitely large rather than of a finite size. Comparing "all the lines" making up one shape with "all the lines" of another shape requires comparing infinity with infinity, which had been considered mathematically off-limits since the days of Zeno of Elea in the fifth century BCE. Consequently, in a series of judgments from 1601 to 1651, the Jesuit "Revisors General" denounced and finally banned as anathema the doctrine of infinitesimals.

Although in many ways an exact contrast to the Jesuits, Thomas Hobbes, philosopher and mathematician of the 1600s, also opposed infinitesimals for much the same reason. In Hobbes's philosophy (expressed in *Leviathan* and other works), the disorder in society needed to be brought under the control of an absolute ruler who would maintain and impose the sovereign will of society upon its dissenters. Hobbes opposed religious intrusion into matters of the state and held particular spite toward the Jesuits and the Catholic hierarchy, but agreed with them on the subject of mathematics: Euclidian geometry represented the solution of a rigid, unchanging certainty, order, and stability; but the problems with infinitesimals threatened that order and certainty, because their paradoxical results could just as easily lead one to a false result as to a true one.

On the other side of the question were leading intellectuals of the early modern age, including Galileo, whose final book in 1633 expounded the theory of "indivisibles"; Bonaventura Cavalieri, whose name and books on geometry were often cited by later mathematicians; Evangelista Torricelli, who provided a rigorous defense and series of proofs using indivisibles; and John Wallis, who sparred with Hobbes's philosophies and mathematical claims for several decades through a series of books and pamphlets.

One interesting resolution to the paradox of the infinitely small was proposed by Torricelli. Construct a rectangle ABCD with a diagonal BD. Then construct a series of horizontal and vertical lines intersecting at a point E along the length of the diagonal, forming an infinite series of smaller and smaller rectangles. The number of horizontal and vertical intersecting lines is equal to one another, yet the horizontal or vertical space occupied by the lines in each dimension is different because of the differing length of the sides of rectangle ABCD. Torricelli boldly asserted that the answer to this paradox was that the intersecting lines, although infinitely small, were thicker in one dimension than the other, in proportion to the difference in the sides of the rectangle. He went on to apply this technique by constructing lines intersecting a parabola, enabling him to calculate the slope of the tangent at every point on the infinite parabola. Rather than avoiding the paradoxes, Torricelli sought to understand their mysteries and employ them in the development of a powerful mathematical tool. A generation later, the "method of indivisibles" would be transformed into the differential and integral calculus of Leibniz and Newton, revolutionizing the mathematical foundation of the modern scientific landscape. The book concludes with the establishment of the Royal Society of London and the lengthy intellectual debate between John Wallis and Thomas Hobbes, ending with Hobbes's death in 1679. Appendices provide short biographies of the key players involved in the struggle, plus a timeline of key events.

Although the development of calculus is mentioned in the book, this reader was left hoping for another chapter or two describing in more detail how Newton and Leibniz each used infinitesimally small divisions to finally develop the formal methods of calculus. For instance, what were the differences and rationale behind their approaches? Why did Newton employ infinitesimals but shy away from their use in his formulations, whereas Leibniz made them a central component of his notation? Another concern is that the author characterizes the subject not only as an intellectual controversy, but as an anti-Catholic and perhaps antireligious screed. The reader is left with the impression that Roman Catholic Italy was plunged into intellectual stagnation by rejecting modernity through its insistence on eternal and unchanging truths, whereas England became the bastion of scientific, intellectual, and economic progress due to its openness to dissent and lack of strict religious doctrine. This caricature of post-Renaissance Italy (and by extension, religious conservatism in general) is certainly lacking in historical and philosophical nuance and may aid in perpetuating the modern "warfare model" of the science/religion dialogue.

But despite these relatively minor complaints, I would highly recommend this intriguing book to all who are interested in mathematics or the history of the modern scientific era.

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EARTH'S DEEP HISTORY: How It Was Discovered and Why It Matters by Martin J. S. Rudwick. Chicago, IL: The University of Chicago Press, 2014. 315 pages plus glossary, a section "for further reading," and bibliography. Hardcover; $30.00. ISBN: 9780226203935.

In 1972, British brachiopod paleontologist Martin Rudwick penned a judicious and revelatory volume, *The Meaning of Fossils: Episodes in the History of Palaeontology*. This book (now 2nd ed., University of Chicago Press, 1985) remains a treasure store of insight into the impact of discovery—as well as the communication of discovery—upon many individuals of talent during the sixteenth through nineteenth centuries. Many of these historical protagonists were devout Christians (for example, Conrad Gesner, John Ray). Rudwick explored their ponderings and their fraternal debates as to just what these remains meant.

More books followed; I count nine, including the volume under review. These included a volume of translation, from the French, of Georges Cuvier's work on fossils (*ossemens fossiles*)—arguably the birth of vertebrate paleontology—and also a volume (*Scenes from Deep Time*, 1992) analyzing the impact of illustrations of "former worlds" revealed by these exhumed remains, during the eighteenth and nineteenth centuries. The scope of Rudwick's coverage broadened, to include the history of fieldwork and deliberation upon the history of Earth as well as that of life. Collectively, his writings now comprise the most significant single-author corpus analyzing the history of the earth sciences. Rudwick brings his Christian faith to his scholarship.

The present volume, *Earth's Deep History*, summarizes the development of a history of Earth. It is written in an accessible style and sparkles with nearly one hundred illustrations, mostly reproductions of original illustrations or text pages from significant individuals ranging from James Ussher to contemporary astrogeologists. Along the way, the geological time-scale develops until it reaches its current scope and detail.

Rudwick painstakingly demonstrates why historical thinking is an essential component of Earth comprehension. Earth and its parts are four-dimensional objects. Rudwick cleanly narrates the step-by-step realization that Earth was an object with a long history. The explanatory power and practical utility of time in analyses were appreciated for two centuries prior to the development of radiometric dating techniques. In fact, through several incidents, Rudwick explicates how spatially—and geometrically—commonsense interpretations of the rock record demanded large volumes of time, and this in the face of opposition based on the "absence of a mechanism." An example would be the development, over the course of several decades, of what would eventually become known as "plate tectonics" prior to the acceptance of the driving mechanism, mantle convection.

The apprehension of deep time during the eighteenth and nineteenth centuries, far from presenting obstacles to faith, was regarded as an ally:

> Closely related to this sense of the providential designfulness of the natural world was a sense of wonder at the romance of vanished deep past that the geologists' research was disclosing. So, for example, Mantell—who had discovered the *Iguanodon*, the first of the fossil reptiles to be classed later as a dinosaur—exploited a profitable vein of popular science by describing the *Wonders of Geology* (1838). The sheer scale and unanticipated strangeness of the earth's long history was often treated as welcome evidence for the grandeur of God's creation. Far from geology being in intrinsic conflict with religious faith, the science was widely regarded in the early nineteenth century as its ally and supporter. (p. 163)

A thread running through *Earth's Deep History* is the participation of earnest Christians in the development of the historical Earth sciences. Contrary to the wishes of some contemporary vocal atheists as well as some equally vocal Christians, faith and science have never been at war.

> What is certainly untenable is any claim that the discovery of the Earth's deep history has in the past been retarded or obstructed by "Religion" … In the history of the discovery of the earth's own history, as in the history of many other aspects of the sciences, the idea of a perennial and intrinsic "conflict" between "Science" and "Religion"—so essential to the rhetoric of modern fundamentalists, both religious and atheistic—fails to stand up to historical scrutiny. (pp. 306–7)

At several points during *Earth's Deep History*, Rudwick takes fellow geologists, or popular science writers, to task for falling prey to the temptation to frame a historical narrative in terms of a manufactured conflict metaphor.

As a coda to this manufactured war, Rudwick provides a brief appendix on the late twentieth-century "young-Earth geology" movement. Having thoroughly documented the hard toil, physical and mental, of sincere and gifted Christians in the recovery of Earth's deep history, he is taken aback at the "startling reinvention of the idea of a 'young Earth,' which the sciences of the earth outgrew for very good reasons back in the eighteenth century" (p. 309). He concludes, "Sadly, creationists are utterly out of their depth" (p. 315; last sentence of the volume).

For its comprehensive scope, intelligibility, delightful illustrations, and at times bluntly personal approach, this volume is a treat. I highly recommend it as a solitary read or as an introduction to Martin Rudwick's other authoritative works.

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Philosophy & Theology

PROCESS AND PROVIDENCE: The Evolution Question at Princeton, 1845–1929 by Bradley J. Gundlach. Grand Rapids, MI: Eerdmans, 2013. 374 pages, including bibliography and index. Paperback; $39.00. ISBN: 9780802868985.

One does not have to be directly involved in science or religion to have been affected by the often divisive discussions surrounding the topic of creation versus evolution. It is a topic that has captivated western culture for nearly two centuries. For the most part, this debate is depicted as a battle between atheistic, rational science versus an antiquated religious folklore about the existence of a higher creative being. Having degrees in biology and geology as well as theology, I have been in the middle—often a target—of both sides of this conversation. The book reviewed herein elucidates how committed Christians have responded to this conflict from the genesis of the controversy.

One of the points of contention is the debate over evolution as a natural process versus God's directional providence. It is these two supposed antithetical ideas that Bradley J. Gundlach, Professor of History at Trinity International University, Deerfield, Illinois, draws from for the title of his book *Process and Providence*. Gundlach takes a historical look at the rising cultural interest in evolution beginning in the mid-nineteenth century. He frames his exploration in the context of the variety of responses from faculty at Princeton, both seminary and university, between 1845 and 1929. Princeton was chosen, according to the author, because "Princeton was the most important center of conservative Protestant thinking on matters of science and religion in America" (p. 6). Gundlach notes that his approach to history was less of a systematic analysis and more of a historical narrative. He introduces the cultural context in each of the decades, outlining the emerging scientific ideas in evolution and the social implications arising from natural science's philosophical conclusion that God can be rejected. As the book works through the emerging issues, Gundlach highlights key individuals at Princeton and presents, uncritically, their responses based on letters, lectures, and publications as well as Princeton's larger reactions through faculty hirings.

Through this process, Gundlach's book highlights the manner in which professors at Princeton—in the disciplines of both theology and natural science—avoided a reactionary, confrontational clash, but instead sought a collegial, critical dialogue with the direct and indirect issues arising in popular culture as a result of the proposed theory of evolution. Rather than rejecting outright these new proposals, as many Christians were doing, faculty at Princeton sought to affirm the scientific method and consider evolution, while at the same time upholding God's providence. Even by the late 1860s, in the aftermath of Darwin's *Origin*, Gundlach points out,

> Only reluctantly did the Princetonians describe the relations of science and religion in terms of conflict. After all, their whole apologetical point was that knowledge was no enemy to faith, that the two were neither hostile nor indifferent to each other, but the closest of friends. (p. 51)

Gundlach even notes that the mechanism of progression was embraced, not only for changes seen in plant and animal life but also for interpreting developments in the biblical text as well as culture as a whole.

As thinkers began to draw philosophical conclusions from evolutionary thought, Princeton's faculty sought to engage the metaphysical and epistemological implications (including the loss of teleology for creation and the rise of atheism along with the deterioration of long-standing morals and values). In an effort to encourage the church to confront the potential sociological ramifications of evolutionary theory, the military metaphor of war was used to describe this struggle. The counter-offensive to "science's" destruction of Christian foundations consisted of five strategies: watch, detect, expose, confront, and overpower. The remainder of the book explores how this tactic played itself out over the next sixty years, focusing predominantly on the roles played by Princeton's leading figures—Charles Hodge, James McCosh, and their "Bright Young Men"—as they continued to wage the war for a Christian perspective on evolution by "taking the best that science had to offer and bringing it back 'under God' at Princeton" (p. 160).

Princeton was, for the most part, successful in showing how careful thought about evolution did not betray the biblical narrative about God and God's providential role in creation. However, in the early quarter of the twentieth century, a renewed angst toward evolution arose from within the fundamentalist movement. With the death of people like B. B. Warfield and the departure of other Princeton scholars who were open to considering the positive nature of evolution, Gundlach outlines the "highly polarized situation of the 1920s uncongenial to the Old Princeton views of science and religion" (p. 273). He describes the multitude of underlying issues that pressured Princeton's faculty into taking a more conservative stand as the Scopes Monkey Trial neared. Gundlach concludes by recounting how, by 1929, the battle plan which began in 1865 was forcibly ended by the restructuring of the seminary by the Presbyterian Church in the United States of America over concerns about denomination strife due to theological error.

Process and Providence excels at elaborating the underlying issues of each time period as well as introducing the individuals who were important contributors to the discussion. These nuances help the reader understand the significance of the discussions that took place as Princeton sought to deal with evolution in a thoughtful, welcoming, but theologically critical manner. Gundlach also succeeds in allowing the historical record of the Princetonians to define and answer the question of evolution at their institution. While Gundlach abstains from offering simplistic answers or a systematized presentation of opinions from the highlighted faculty, it was obvious that despite there never being a clear consensus at Princeton on the question of evolution, the concern for all was finding a balance in the relationship between process and providence. However, even with a close reading of the text, the narrative was, at times, difficult to untangle. To clarify the intricate web of relationships, Gundlach would have done well to include a summary of this information in a series of tables.

Process and Providence is a dense read in terms of quantity of material, which could make reading it overwhelming for the historically, biologically, or theologically uninitiated. While this text would be best suited to those with a specific interest and background in one or more of those three topics as it relates to the question of evolution, it is nevertheless accessible enough to the more generalized reader who wants to explore the topic in greater detail. Furthermore, it could serve as an encouragement for those, like myself, that have found themselves in the middle of what has too often has become a one-side-or-the-other debate. Gundlach reminds us that we can stand on the shoulders of a cloud of witnesses who did not sacrifice their belief in God's providence in order to accept the possibility of natural processes.

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FROM NOTHING: A Theology of Creation by Ian A. McFarland. Louisville, KY: Westminster John Knox Press, 2014. 212 pages. Paperback; $35.00. ISBN: 9780664238193.

In a memorable episode from the hit television series *Seinfeld*, Jerry and George are presented with the daunting task of pitching their pilot for "a show about nothing" to the executives of NBC. One suspects that Ian McFarland may have had a somewhat easier time convincing the editors of Westminster John Knox Press to publish his book, because in attempting to retrieve the classic doctrine of creation *ex nihilo* (from nothing), he has actually produced a book about everything that is and the God who freely creates out of the plenitude of the life that has been eternally shared between the Father and the Son in the Holy Spirit.

From Nothing: A Theology of Creation is a work of "systematic theology" in the best sense of the term. McFarland draws upon a chorus of voices from across the Christian theological tradition (e.g., Irenaeus, Maximus the Confessor, Anselm, Thomas Aquinas, Karl Barth) to present a nuanced and compelling defense of the doctrine of creation *ex nihilo*. The symmetry and elegance of the book's organization reflect something of both the marvellous ordering of creation and the book's central material conviction that the doctrine of creation from nothing is best understood within the context of the doctrine of the Trinity. The book is divided into two parts and, fittingly, each part is divided into three chapters. The first part is given the superscription *Exitus* (outflow), as it is primarily concerned with the rootedness of creation within the life of God. The three chapters in the first part are devoted to unpacking in succession the component parts of the statement, "God creates from nothing." Part Two, *Reditus* (return), marks a "shift from creation's rootedness *in* God to the contours of its existence *under* God" (p. xiv) and includes chapters entitled "Evil," "Providence," and "Glory." The two parts are bookended by a substantial introduction and a brief conclusion; the latter is followed by a thorough bibliography and helpful scripture and subject indices.

Following an introductory chapter that outlines some of the exegetical, historical, and contemporary challenges associated with the doctrine of creation *ex nihilo*, McFarland turns in the second chapter to the question of the identity of the God who creates from nothing. McFarland's recourse to the doctrine of the Trinity at this point will seem relatively uncontroversial to those trained in theology following the great Trinitarian revival of the twentieth century. However, this identification of the Triune God as the Creator and the corresponding implications of this identification are frequently overlooked or obscured in debates surrounding creation and the relationship between faith and science as they play out at a more popular level. While the doctrine of creation from nothing affirms that God was under no compulsion to create, the affirmation that the Creator is Triune God, who is intrinsically living, productive, and present, allows one to see that there is a certain fittingness to God's creative work, which helps to counter charges of divine arbitrariness and divine determinacy.

The existence of creatures called into being from nothing by the Triune God is characterized by a contingency marked by movement and place. The radical dependence of each created being upon the Creator is the great ontological equalizer, as reflected in the refrain of John of Damascus, which recurs throughout the book: "All things are distant from God not by place, but by nature." Echoing the diversity in unity which marks the life of the Triune God, God's desire to create naturally results in a glorious diversity of created beings which, in faith, can be perceived as participating in a larger and harmonious whole. This Trinitarian construal of creation from nothing allows McFarland to acknowledge the distinctive role assigned to human beings in the divine economy in a way that does not diminish the integrity and value of the nonhuman creation. The first part of the book concludes with a chapter that stands as the outworking of the Trinitarian commitments articulated in the second chapter through the lens of Christology.

If God is in no way limited in his creative work, as the doctrine of creation from nothing affirms, how then do we account for a world, which, as scientific evidence suggests, has been characterized by suffering and death from long before the first human beings appeared on the scene? The second part of the book begins with an exploration of this question. While McFarland contends that theodicies (attempts to provide a solution to the problem of evil) are mistaken, he does find in the biblical books of Proverbs, Job, and Ecclesiastes three distinct and mutually enriching accounts of evil from within the context of the doctrine of creation.

God's resistance to evil in the present for the sake of the creatures' attainment of their proper ends has historically been treated under the doctrine of providence and is the subject of chapter six. McFarland draws upon the scholastic categories of *conservatio* (preservation), *concursus* (accompaniment), and *gubernatio* (direction) to explicate God's providential activity. McFarland's exploration of the issues raised as a result of a wholehearted commitment to *both* divine sovereignty *and* creaturely integrity may make this the most interesting chapter of the book for readers of this journal. For example, in his treatment of *concursus,* McFarland stresses that a proper understanding of the doctrine requires the recognition of the metaphysical discontinuity between God and creation. Recognition of this discontinuity allows for a noncompetitive understanding of divine and creaturely causation that allows us to speak of primary and secondary causation. This distinction can be brought to bear on Einstein's famous dictum that God does not play dice with the universe. In terms of primary causation, Einstein's assertion is obviously true, since all that exists depends upon God for its continuing existence. But from the perspective of secondary causation, God could very well play dice with the universe by bringing about created effects in the absence of any created cause, or what modern science has identified as the truly random event.

Since creation has been created for an end that lies beyond its inherent capacities, namely sharing in the life of the Triune God, McFarland includes a brief chapter devoted to the topic of glory. The glorification of creation is not merely an event that awaits us in the future. Even now, a part of the creation—heaven—is transparent to the glory of God. Eastern iconography and the Eucharist also serve as case studies for exploring a vision of glorified matter and the presence of glorified matter in the midst of the not-yet-glorified earth, respectively. As a result of this investigation, it becomes apparent that "the point of glory is not to negate the present form of creation but to perfect it" (p. 180).

At the very outset, McFarland makes clear that his intent is to provide a theological account of the doctrine of creation from nothing. As a result, he has very little interest in staking out a position within debates surrounding temporal origins. According to McFarland, the doctrine of creation *ex nihilo* is not the description of a process, but fundamentally "a proposal about the character of God's relationship to the world" (p. xiv). However, this does not mean that McFarland has no interest in the fruit of scientific exploration. At various points in both the body of the text and perhaps even more frequently in the footnotes, he is informed by and drawn into dialogue with the findings of various scientific disciplines. In fact, one of his major emphases in the book's conclusion is that a commitment to scientific investigation into the conditions of creaturely flourishing is a necessary correlate to the affirmation of creation from nothing. The reader lacking theological training may find *From Nothing* to be demanding reading, but for those who persevere, the theological payout is far from nothing.

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ACTUAL CONSCIOUSNESS by Ted Honderich. New York: Oxford University Press, 2014. 402 pages. Hardcover; $55.00. ISBN: 9780198714385.

Often known as "the hard problem," consciousness has been an issue of debate in recent years in the fields of religion and science as well as philosophy. In *Actual Consciousness*, Ted Honderich presents a summary of the major theories and discussions available, while working toward a possible solution. He sets his tone early in the book:

> The informality of style, not always serious and impersonal enough for all professional philosophers in their working hours, is partly owed to and a reminder of the fact that the inquiry must be a kind of joint and mutual enterprise … That's life, baby. (p. xv)

The book does follow this convention, and is quite conversational, from its scattered references to Tottenham Hotspur to its direct addresses to the reader.

The book is divided into eleven chapters, which build the author's argument while at the same time summarizing popular viewpoints on consciousness. The chapters read as follows:

- "Need for an Adequate Initial Clarification";
- "Five Leading Ideas about Consciousness";
- "Something's Being Actual";
- "Dualisms, Functionalisms, Consciousness-Criteria";
- "Other Consciousness Theories, Criteria Again";
- "What Is it to Be Objectively Physical?";
- "Perceptual Consciousness—What Is and What Isn't Actual";
- "Perceptual Consciousness—Being Actual Is Being Subjectively Physical";
- "Cognitive and Affective Consciousness—Theories, and What Is and What Isn't Actual";
- "Cognitive and Affective Consciousness—Being Actual Is Being Differently Subjectively Physical"; and
- "Conclusions Past and Present."

With all that said, the book covers topics you would expect in a book on consciousness, such as a discussion of dualism, functionalism, and linguistic theory. All the while Honderich is building the argument for "actual consciousness." On pages 67–68, Honderich presents a list of the characteristics of consciousness. He follows up with this description of actual consciousness:

> We need a summary description for the characteristics assembled. Ordinary consciousness taken as having these characteristics, I shall henceforth say, is actual consciousness, consciousness as something's being actual, consciousness as the actuality of something. Whatever else may be the case with conscious states and events—for example the quite different fact that we have a hold on our own conscious states and events—they have this nature. (p. 69)

The approach to the discussion and the organization is helpful in a few ways. First, the book serves as a summary of current discussion on consciousness. Second, the text has many organized lists scattered throughout its pages, such as the one mentioned above, reminding the reader of what we do know about particular topics. For example, there are checklists on pages 184–86 and 231–32 that list the characteristics of objective and subjective physical worlds, respectively (two individual lists in the first case). These sorts of lists allow the reader to review quickly what is known about a subject and to follow the overall argument of the author, which at times can be difficult in a large monograph. On pages 328–29, there is a chart with lists that bring this all together. Third, the text is a long-running dialogue, and although a difficult topic, it does bring the reader into conversation with the author, keeping things a bit more engaged than many other monographs.

Overall I believe this to be a helpful book for those interested in the issue of consciousness, both in the professional and academic realms, as the author has intended (p. xv). Consciousness is a key issue in religion and science dialogue, specifically in the Christian tradition due to the long-standing ways in which theology has been tied to dualist conceptions of the person. In moving beyond (or defending) a dualistic conception of the person, a full knowledge of the field of consciousness is essential. For theologians, the issue is so connected to the idea of soul that it becomes a foundational point for argumentation. The text weaves both science and philosophy together in a way that leaves the reader feeling that the issues have been discussed, the author has made his case, and the argument holds. With that said, not everyone will agree with the author's conclusions, but hopefully all readers are left with a better sense of what the subject of consciousness entails and what subissues are relevant to this discussion. Make no mistake, this is a philosophy book, not a religion and science text, but readers in the field of religion and science may find it useful and an excellent resource.

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Along with all their other contributions, many members of ASA and CSCA publish important works. As space permits, *PSCF* plans to list recently published books and peer-reviewed articles related to science and Christian faith that are written by our members and brought to our attention. To let us know of such works, please write to patrick.franklin@prov.ca.

Ψ PSYCHOLOGY

SEXUALITY AND SEX THERAPY: A Comprehensive Christian Appraisal by Mark A. Yarhouse and Erica S. N. Tan. Downers Grove, IL: InterVarsity Press, 2014. 356 pages, references, index. Hardcover; $35.00. ISBN: 9780830828531.

Christians often flounder when faced with challenging questions about sexuality in themselves, their families, their communities, and seek clarity and guidance from psychotherapists or pastors. In *Sexuality and Sex Therapy*, Yarhouse and Tan survey and evaluate the current state of research and clinical practice, grounded in Christian theological beliefs in the goodness of our sexuality, the distortions of sin, and the promise of redemption.

The authors are leaders in sex research, theory, clinical practice, and training within the broad evangelical Christian umbrella. They are well qualified to write this book, and for the most part write it clearly, accessibly, and with a professional, thoughtful, compassionate, holistic approach.

The book is divided into four sections. The first lays out foundational perspectives—theological, sociocultural, biological, and clinical—and the last section returns explicitly to these worldview questions. These are the most helpful parts for readers who are not practicing clinicians. The middle two sections focus on the problems clients bring to sex therapy—sexual disorders and dysfunctions—and also issues around gender and sexuality identity.

In a combination text and workbook style, within each chapter the authors provide "application boxes" that raise issues and ask key questions, helping readers to identify their own attitudes, values, and beliefs around sexuality and to consider how these affect their interaction with clients. While the chapters that address specific sexual issues are a bit repetitive for someone reading from cover-to-cover, they are structured so clinicians can extract guidelines and ideas without re-reading the entire book.

The authors are up to date in their knowledge of research on sexuality, and have also included the most recent diagnostic categories for sexual disorders from the latest edition of the Diagnostic and Statistical Manual for Mental Disorders (DSM-5). They handle this information deftly and with a thoughtful, critical eye. Their sensitivity to limits and complexity will be of help to therapists and pastors who may not be equipped to do that evaluation themselves.

For people interested in science and religion dialogue, the book provides an example of how a particular set of Christian beliefs around sexuality might be expressed in the context of sex therapy. Yarhouse and Tan are not prescriptive about specifics; rather, they challenge readers to think about their own values, beliefs, and assumptions, and to consider how these affect their approach to clients seeking help in the area of sexuality. They also model good clinical practice in that they encourage readers to focus on their *clients'* values, struggles, and needs.

The authors are attempting to do several things with this book: provide an overview of current understandings of sexuality and its disorders and dysfunctions, possible treatments, appropriate professional practice guidelines, and the implications of Christian perspectives. It is impossible to do all of these areas justice in a single volume. The result is that in places the book is frustratingly vague. Descriptions of sexual dysfunctions, their possible causes, and treatment options are not sufficiently detailed for therapists who do not already have background in these areas. These sections might actually be more helpful for people experiencing these dysfunctions themselves or for pastors who wish for a bit of an overview. More importantly for readers interested in science and religion is the fact that the foundational chapters at the beginning and the end are not deeply integrated with the specific sexual issues described in the middle. To their credit, the authors structure the book to encourage readers to engage in their own processes of integration. Nevertheless, it would be helpful for the authors to be more explicit about the ways in which they see Christian faith making a difference in sex therapy.

The chapter on "Sexual Identity Conflicts" is an interesting and distinctive take on the topic of sexual orientation. Questions of sexual identity are highly conflicted and contested, especially among Christians. Yarhouse and Tan attempt to tread lightly, thoughtfully, and compassionately while at the same time suggesting that the claims of major US mental health organizations, such as the American Psychiatric Association and the American Psychological Association, are oversimplified. Such claims include the beliefs that sexual orientation is innate and immutable, something that one discovers rather than develops, and that attempts at prevention or treatment are harmful and unethical. Consistent with Yarhouse's previous work on sexual identity, the authors distinguish among sexual attractions, sexual orientation, and sexual identity. They point out that people have choices about how they understand and respond to their sexual attractions, and when persons have sexual desires that expressed would put them in conflict with their own or their faith community's beliefs, they may need support to navigate this struggle. Yarhouse and Tan are careful to acknowledge that therapy rarely results in significant change in a person's sexual orientation or attractions. They focus instead on the importance of recognizing clients' social and cultural contexts, and helping them toward an integrated identity, one that takes into account the clients' beliefs, values, context, and desires.

While there is much to applaud in Yarhouse and Tan's deft exploration of Christian perspectives on sexuality and sex therapy, they clearly evince a view of Christian sexual ethics that is pretty standard evangelical American fare: Sexual expression is ideally exclusive to those in heterosexual marriage. For example, their discussion of sexual identity quite clearly implies that sexual attractions to any but the "other" sex are problematic to committed Christians and must be dealt with—not, to be sure, through enforced treatment, and not without deep compassion. While they encourage therapists to be open to the possibility that their clients may choose to adopt a "gay identity," the main message suggests that, for the committed Christian, there are better alternatives. Though I deeply appreciate their nuancing of the issues involved, I wish they had extended their focus slightly to include the communities within which people struggle with questions of sexuality. There is little here that acknowledges the profound anguish, heartache, and family struggles that are often associated with a Christian daughter or son identifying as gay. All the attention is on the person struggling with their sexual desires, when there may also be an important place for Christian therapists to speak to families and communities whose fears, attitudes, and beliefs often contribute significantly to their clients' pain.

This focus on the individual or couple is a general weakness of the book. While therapists' and pastors' primary concern is for the person or couple seeking their support, these clients live in a context, as Yarhouse and Tan repeatedly acknowledge. Yet nowhere in the book is there much suggestion that perhaps the context, not the client, is the problem. I was also disappointed that the foundational material in the first four and the last chapters did not make reference to some truly excellent work by scholars such as Margaret Farley or Lisa Sowle Cahill. While it is impossible to cite everyone who has weighed in on these topics, a broader range of perspectives would have deepened this material, and provided some food for thought regarding the possible limits of the "standard" evangelical view of sexuality.

Overall, this is a fairly accessible book that would be of use to Christian pastors and therapists who occasionally deal with clients struggling with sexuality. For those interested in science and Christian faith discussions and their implications for the "culture wars" around sexuality, this book is worth considering. The thought exercises should stimulate critical engagement with the issues and help readers to thoughtfully digest other excellent books on these topics.

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Religion & Science

SEX ITSELF: The Search for Male and Female in the Human Genome by Sarah S. Richardson. Chicago, IL: The University of Chicago Press, 2013. 320 pages. Hardcover; $45.00. ISBN: 9780226084688.

It seems that humans have an intrinsic compulsion to classify elements in God's creation: to express a taxonomic urge. Perhaps this urge is a result of God instructing humans to name each living creature from the beginning (Genesis 2:19–20), or perhaps it is a natural reaction to the overwhelming diversity in creation. Regardless of the origin or intent, the taxonomic urge includes classifying sex and gender. Sociologists attempt to determine the influence these two components have upon individuals in society. Psychologists attempt to assess differences in male and female brains. Biologists attempt to describe the molecular mechanisms involved in forming males and females.

Predictably, the topics of sex and gender have not escaped the church. Many of the major controversies in the Christian community circle around these topics, perhaps more now than ever before. Many denominations continue a multi-decade conversation wrestling with the implications of sex and gender for ordination and sexual orientation. Heading into these conversations, it is important to realize that even the scientific methods of defining the mechanisms of how we become male or female are blurred, perhaps more than the general population realizes. Our attempts have tended to work toward reducing complex issues into overly simplistic categorizations. In the book, *Sex Itself: The Search for Male and Female in the Human Genome*, Sarah Richardson highlights the biases and inadequacies that have influenced the formation of what it means to be male and female—even at a genetic level.

Trained as a developmental biologist, I began reading Richardson's book hoping for an in-depth examination of the genetic cases that defy current bifurcated gender categories. Instead, however, Richardson succeeds in accomplishing something different. Utilizing reviews of historical, philosophical, and gender studies, she brings to the forefront of the discussion the evidence that science is not immune to the influences of culture and society. She boldly argues that females have been portrayed as secondary to males, even in scientific attempts to elucidate the biochemical mechanisms which define the development of females from males.

She points out that the genomic approach of studying the sex chromosomes is too limited and riddled with gender politics. Such gender politics permeates the words we use to describe genetic pathways that cause differentiation of males and females. Terms such as dominant and default state have a hierarchical ring. Despite much talk about gender, "discourses around gender, discourses often framed by the expectation that the facts of biology would help to settle the matter of the hierarchy of the sexes once and for all" (p. 71), Richardson gives multiple examples showing that science, unfortunately, has had a hand in enabling negative gender stereotypes.

Richardson provides a helpful review and critique of how the approaches to assessing the nature of gender bifurcation among humans are riddled with biases. Specifically, she addresses several major areas including whether the X and Y chromosomes are appropriately named "sex chromosomes," the claim that Y chromosome is shrinking, and that from a genomic perspective, men and women are not that different—certainly not different enough to consider each sex as having their own distinct genome.

Sex Itself is a great primer to begin examining our history and current academic approaches pertaining to defining sex and gender from a genomic perspective through a historical and philosophical lens. To be aware that we explore genomics and molecular mechanisms of development with a bias is only the first step, however. By placing humans into a dichotomy that is attempting to explain a spectrum of sex or trying to undermine one end of the spectrum over the other, we do all a disservice. This book leaves us with a challenge to critique how current paradigms fall short.

Whether an individual is perceived to be male or female impacts what one experiences from a physical, reproductive, psychological, and social perspective. Our gender labels influence who we perceive ourselves to be and can influence the limits and goals we set for ourselves. Should Christians then focus our analytical abilities on the mechanisms that generate phenotypic differences in sex? Should we carefully examine whether there are distinct God-intended roles for men and women? Are we doing a disservice to ourselves and future generations by continually bifurcating ourselves into one of two categories? If these questions intrigue you, you should read *Sex Itself*.

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Technology

BIG DATA: A Revolution That Will Transform How We Live, Work, and Think by Viktor Mayer-Schönberger and Kenneth Cukier. Boston, MA: Eamon Dolan/Mariner Books, 2014. 252 pages. Paperback; $15.95. ISBN: 9780544227750.

The data trails we create do not disappear. They also do not remain dormant. Instead, they are aggregated and harvested to serve multiple purposes, many far different from the reason the data was first produced. This aggregated data can be used to predict flu outbreaks, predict who might be a potential terrorist, or locate city dwellings that have been illegally subdivided into multiple units. These applications and many more are possible due to the massive amount of data that exists. The culture and techniques that have recently appeared in this context are often called *big data*.

The book *Big Data*, written by Viktor Mayer-Schönberger (professor of Internet Governance and Regulation at Oxford University) and Kenneth Cukier (data editor of *The Economist*), is intended as an introduction for a general audience. It includes many interesting examples of how big data techniques are being used.

Rather than provide a precise definition of big data, the authors work from a more general statement:

> Big data refers to things one can do at a large scale that cannot be done at a smaller one, to extract new insights or create new forms of value, in ways that change markets, organizations, the relationship between citizens and governments, and more. (p. 6)

They assert that big data is making fundamental changes in how we operate. Three changes are emphasized.

First, in the past, data was difficult or expensive to collect. Consequently, we used samples and sophisticated statistical analysis to reach meaningful conclusions. With big data, we are able to use data sets that approach comprehensive collections. For example, in 2004, Walmart used data-mining techniques to examine their old sales receipts for interesting correlations. They discovered that sales of strawberry pop-tarts increased seven-fold shortly before a hurricane. This discovery was possible because they looked for correlations in massive amounts of data with no preconception of what they were seeking.

Second, in the past, data needed to be collected carefully in order to minimize bias and increase the accuracy of the predictions. Big data can tolerate imprecise data and also data that are stored in different formats or using different units. The errors tend to neutralize each other in the large mass of data that is processed. For example, in 2008 MIT economists Alberto Cavallo and Roberto Rigobon used web-crawling software to gather half a million US product prices each day. Comparing prices for common items is not easy since different web pages may describe the products using different words or phrases. Nevertheless, they used this mass of data to detect a deflationary trend in prices right after Lehman Brothers filed for bankruptcy in September 2008. The more traditionally derived CPI data was not able to detect this significant event until the November 2008 numbers were available.

Third, perhaps the most profound change is a diminishment in the search for causation. Instead, the big data culture seeks correlations. Sometimes this is sufficient; in other cases, causation may be explored once an important correlation is found. The authors state, "Knowing why may be pleasant, but it's unimportant for stimulating sales" (p. 52).

The book develops these ideas and also explores their consequences. The authors consider some potential societal risks and offer proposals to prevent or minimize the negative consequences. Although the book is not primarily focused on ethical issues, the authors do take a strong stand on the potential for using big data to predict the behavior of individuals. They are quite uncomfortable with using big data correlations for making a preemptive arrest of a particular person based solely on a high predicted probability that a crime will be committed. After noting that such a prediction can never be disproved (since the arrest occurs before any actual crime), they state:

> Perhaps with such a system society would be safer or more efficient, but an essential part of what makes us human—our ability to choose the actions we take and be held accountable for them—would be destroyed. Big data would have become a tool to collectivize human choice and abandon free will in our society. (p. 162)

This strong assertion about the value of human free will is not grounded in any religious or ethical presuppositions or arguments; it is just assumed to be a universal value.

The authors state that "a single version of the truth" is no longer a useful goal. This assertion is made in the context of being able to query a data collection multiple times and get a consistent result, so we should not assume that they would make a similar claim about more profound kinds of truth. Nevertheless, in this context they state, "We are beginning to realize not only that it may be impossible for a single version of the truth to exist, but also that its pursuit is a distraction" (p. 44). I suspect that many readers may temporarily forget the context and interpret this as a general assertion. That would be unfortunate since the biblical record is quite clear that truth matters. Jesus claimed to be the truth (John 14:6). In 1 Corinthians 15:12–19, Paul makes a strong case that the validity of our beliefs matters. He would not affirm the radical postmodern sentiment, "if it makes you feel good, it can be a truth for you."

There is passing mention of a few other topics that might be of interest to readers who are interested in the interplay of Christian faith and the big data culture. These include the nature (or existence) of causality, whether data-driven decisions may maximize profits but suppress creativity and artistic/human merit, resulting in a culture of mediocrity and a shift in our worldview. The worldview shift is to see information as primary: "With the help of big data, we will no longer regard our world as a string of happenings that we explain as natural or social phenomena, but as a universe comprised essentially of information" (p. 96). Readers who want an in-depth examination of this topic should read *The Information: A History, A Theory, A Flood* by James Gleick.

The assertions about big data in this book highlight the notion that technology is not neutral. How we collect data, how we analyze it, and what we do with the results are all shaped by our worldview. But the culture of big data will also modify worldviews and reshape society. For instance, collections of data may become one of the most valuable resources a company or institution owns. In some cases, it may be the *most* valuable asset. If their warning against preemptive arrests is not heeded, big data may also reshape our understanding of legal culpability.

This book is a quick, nontechnical, but useful introduction to the culture of big data. For those wishing to investigate more thoroughly, there is an index and extensive endnotes and a detailed bibliography. However, you will need to provide your own religious and ethical framework from which to consider the impact of big data.

Reviewed by Eric Gossett, Department of Mathematics and Computer Science, Bethel University, St. Paul, MN 55112.

Letters

If Adam Did Not Exist, Who Else Did Not?

"Adam never existed" is the bold statement made by Denis Lamoureux in his article, "Beyond Original Sin: Is a Theological Paradigm Shift Inevitable" (*PSCF* 67, no. 1 [2015]: 35–49, 40). With Adam and Eve relegated to mythology, where does one place the people listed in the genealogies of Genesis in chapters 4–6? How far down the list of names must one go after Adam and Eve to encounter the first historical person? For example, is it Abraham? Or is he also part of ancient history? How about Enoch, mentioned once in Genesis 4 and twice in the New Testament (Hebrews 11 and in Jude)? Noah and the flood are referred to in the New Testament by our Lord, and again with all other Old Testament heroes of faith listed in Hebrews 11. Are these real people or so-so stories? What criteria do we use to make that distinction?

This is not a rhetorical question. For me, it is the logical follow-on to the claims that Adam and Eve never existed. Once you argue yourself out of Adam (an Adam who did exist), what chapter in Genesis starts to become historical? For example, C. S. Lewis considered the first eleven chapters of Genesis as myth.

In my opinion, creationists ignore legitimate scientific explanations and try to force-fit them into Genesis 1 and 2. On the other hand, evolutionary creationists consider accounts recorded in Genesis 1 and 2 as ancient stories and try to re-interpret them in the light of the "proven facts" of Darwinian evolution.

Ultimately, we should show deference to our brothers and sisters in Christ, and humbly admit that we will never have the full picture of creation, this side of eternity.

Ken Touryan
Fellow of the American Scientific Affiliation

Response to Touryan

I am grateful to Ken Touryan for his letter because he raises some significant issues. I believe that real history in the Bible begins roughly around Genesis 12 with Abraham. Like many other evangelical theologians, I view Genesis 1–11 as a unique type of literature (literary genre) that is distinct from the rest of the Bible. So from my perspective, was Abraham a real person? Yes. Was there a King David in the tenth century BC? Yes. Were the Jews deported to Babylon in the sixth century BC? Yes. Was there really a man named Jesus in the first century AD? Yes. Are the gospels eyewitness accounts of actual historical events, including the Lord's teaching and miracles, and especially his physical resurrection from the dead? *Absolutely yes!* Even though I do not believe that Adam was historical, I thoroughly believe in the historicity of Jesus and the biblical testimonies of his life. See 1 John 1:1–3; 2 Peter 1:16–18; Luke 1:1–4; and Acts 1:1–19. Also see Richard Bauckham, *Jesus and the Eyewitnesses* (2006).

Now an important clarification and correction needs to be made regarding Touryan's comment that "evolutionary creationists consider accounts recorded in Genesis 1 and 2 as ancient stories and try to re-interpret them in the light of the 'proven facts' of Darwinian evolution."

This is an absolutely false assertion. I have *never* interpreted scripture in the light of evolution. I interpret scripture in the light of scripture and ancient Near Eastern literature. As my article shows, the *de novo* creation of humans is an ancient conceptualization that is no different than the *de novo* origin of the firmament, the heavenly sea, and the sun, moon, and stars placed in the firmament. I reject scientific concordism for biblical reasons, *not* because of evolution. In fact, my PhD in evangelical theology came before my PhD in evolutionary biology. I rejected the historicity of Genesis 1–11 and concordist interpretations of these chapters in seminary when I was still a thoroughly committed anti-evolutionist.

It does concern me that an ASA Fellow uses scare quotes in the phrase "the 'proven facts' of Darwinian evolution." First, evolution is a fact. For those of us who have actually studied evolutionary biology to the PhD level, there is no debate because the evidence for evolution is *overwhelming*. In fact, a 2009 Pew study reveals that 97% of scientists accept evolution. Second, those of us who have actually published on evolutionary topics in refereed scientific journals rarely qualify evolution as "Darwinian." Does Touryan as an aeronautical engineer refer to gravity as Newtonian?

Finally, and most disturbing to me, is Touryan's final sentence in his letter: "Ultimately, we should show deference to our brothers and sisters in Christ, and humbly admit that we will never have the full picture of creation, this side of eternity."

Earlier Touryan accuses me of making a "bold statement" with regard to my denying the historicity of Adam. But I believe I offered a reasonable argument in my article—the Bible has an ancient understanding of the origin of the heavens and earth; it stands to reason that this is also the case with the origin of living organisms, including humans. And ancient Near Eastern creation accounts confirm my contention.

In contrast, Touryan's final sentence is merely a "bold" proclamation with no academic substantiation whatsoever. It is this type of anti-intellectualism that plagues evangelical Christianity, and it has been a stumbling block to many of our young people who have lost their faith once they see the evidence for evolution in university.

The name of our organization has the word “scientific” in it. I believe that members of the American Scientific Affiliation should show “deference to our brothers and sisters in Christ” who have actually studied evolutionary biology. And for those ASA members who have never held a fossil in their hand, or worked at an outcrop, or published a refereed paper on evolution, I believe they should “humbly admit” that they are not competent to comment on the scientific theory of evolution in public.

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Historical Adam?

As one who has labored in the tar pits of the Bible, science, and history dispute for thirty years and counting, I was pleased to see yet another adventurer in the debate. In his abstract of “Genetics, the Nephilim, and the Historicity of Adam” (*PSCF* 67, no. 1 [2015]: 24–34), Davidson uses “first” three times, such as “first human pair,” as if that designation is a necessary component to a historical Adam and Eve. Here are pertinent questions: Is biblical history also human history? If not, is it at least compatible?

In the interest of shedding historical light on the issue, an exegetical mistake with major consequences befell the early church. When Paul set out on missionary trips, he would visit synagogues seeking out Jews who would listen to the good news that the Messiah had come. Largely he was rejected. Although the emperor of Rome was proclaimed to be a god and Greeks had many gods, Jews knew only one God. A human god was blasphemous to the Jews, yet Paul found a few Jews who would listen and took his message to heart.

Not committed to a one-God concept, Greeks and Romans proved more receptive, and they became an integral part of early congregations. Followers of “The Way,” as the early church was called, consisted of Romans, Greeks, and converted Jews who would pray, take communion, and read the scriptures aloud at Sunday meetings. Although a letter or two may have been in their possession, the Greek Septuagint version of the Old Testament was an object of weekly reading, and the first book, Genesis, would be a likely starting place. Listening to the stories of Adam, Cain and Abel, and Noah read aloud, Gentiles in the group of believers would have had no reason to think Jewish history wasn’t their own history too. Thus the mistake was born that persists to this day. Jewish history was perceived as human history.

In 1611 when the King James translators produced an English version of the Bible, they labored under the same mindset as early believers. They thought that the entire human race derived from Adam and Eve, that the flood was worldwide with only a family of eight surviving, and that all humans gathered at Babel and scattered in small groups, speaking foreign tongues. This total misunderstanding skewed the translation and virtually canonized the tradition that had arisen 1,600 years earlier and has survived to this day among many conservative Christians. A liberal response has been to assign Genesis to a “genre” bereft of historical accuracy. Thus the conundrum: “Is Genesis 2–11 true human history, bogus human history replete with theological lessons, or legitimate Semitic history with theological content implicit therein?” Sufficient evidence gleaned from thirty years of digging leads me to conclude that Genesis was written by Semites, for Semites, and about Semites. Gentiles may peruse Semitic history in Genesis and are free to wonder why our own ancestors did not leave us a historical record of our own.

As to the biblical text, recent translations have modernized English equivalents of Hebrew words to some extent, but because of insufficient knowledge of the history of the ancient Near East and its relevance, tradition marches on undeterred by an abundance of contrary evidence. Only within the last two hundred years has the scholarly world been in possession of some of the history of the ancient Near East inscribed on cuneiform tablets in Akkadian and Sumerian languages. This newfound evidence could revolutionize how we understand Genesis.

In his article, Davidson waded into a 2,000-year-old quagmire that has engulfed many gallant exegetes and expositors with a model similar in many respects to Denis Alexander’s “*Homo divinus*” model. Both models fail to address adequately a “blinding glimpse of the obvious” that struck me in 1986, when my article, suggesting Adam was “injected” into a populated world, was published in the *Washington Post*. Clearly, Adam belongs to the Neolithic Period (that is, mention of tents, livestock, musical instruments, and implements of bronze and iron in Gen. 4:20–22), thus appearing no earlier than 10,000 years ago. *Homo sapiens*, however, has a 200,000-year history. Any conceivable “first man” in biological terms, even if one could be found, cannot possibly be our man, Adam.

Dick Fischer, MDiv
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Response to Dick Fischer

My understanding of Fischer's position is that he believes that the biblical, archaeological, and scientific evidence leads to an obvious conclusion that Adam was a Neolithic human, living among a large and widely dispersed population of other humans, whom God selected as the progenitor of the Jews (or, more broadly, of Semitic people). The early church was mistaken in their belief that the Genesis account was not just a description of the origin of the Jews, but of all humanity. He does not argue for a liberal interpretation of the early chapters of Genesis that assigns it to the realm of myth, but does believe it to be grossly misinterpreted as describing the history of all humans. His letter leaves the question of the origin of the soul and how Adam's sin relates to non-Jews unaddressed.

The model proposed in "Genetics, the Nephilim, and the Historicity of Adam" is not dependent on a particular date in the past, so the only substantive difference with Fisher's understanding is whether Adam should be thought of as the father of only Semites, or an earlier father of all humanity. I will briefly compare the strengths of the two positions from genetic, archaeological, and biblical perspectives.

1. Genetics: The proposed model and Fischer's are equally plausible. Both have Adam and Eve existing among other hominids/humans with interbreeding among their offspring. A relatively recent common ancestor of Semitic peoples and an older common ancestor of all humans are supported by population genetics.

2. Archaeology: The descriptions in the first eleven chapters of Genesis do indeed fit within the period of recorded history from the Ancient Near East, as Fischer argues. However, the unique manner in which the human experience is recorded in these first chapters has led some to refer to it as proto-history, wherein theologians differ on whether the geography and industry represent the period in which the events occurred, or the period in which the history was written. In the latter case, modernized language may be used to represent events from a more distant past. The common-ancestor-of-all model does not require this to be the case, but does allow for the possibility.

3. Bible: On this point, I will argue that the common-ancestor-of-all model requires less biblical massaging. The verses of greatest theological concern are found in Romans 5 (which Fischer did not mention), where Paul makes a bold claim that sin and death came to all men through one man, Adam. If Paul were addressing the church in Jerusalem, one might reasonably argue he was referring only to Jewish history, but he was writing to the church in Rome, populated principally by Gentile believers. If early Gentile believers mistakenly interpreted Adam to be their own forefather, as Fischer says, the source of the error must be pinned on Paul. Indeed, some theologians, such as Denis Lamoureux, assert that Paul was mistaken in his own view of Adam. The only alternative is a theological construct that allows a host of pre-existing and co-existing humans to have lived and died without the imputation of sin until the arrival of an isolated proto-Semite (or tribe, if one wishes to invoke the notion of federal headship).

Finally, there is an interesting dichotomy between the two argued views. (It is not a defense of either position, but merely a note of interest.) For the common-ancestor-of-all model, I argue for a less word-literal understanding of Genesis 1–11, and a more word-literal understanding of Romans 5. Fischer does the opposite, arguing for a more word-literal interpretation of Genesis, and, at least by implication, a less word-literal understanding of Romans.

Gregg Davidson
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