

7.91 Problems

for Section 7.1,

7-1: Convert 68°F to $^{\circ}\text{C}$ and $^{\circ}\text{K}$, -12°C to $^{\circ}\text{F}$ and $^{\circ}\text{K}$, and 377°K to $^{\circ}\text{C}$ and $^{\circ}\text{F}$.

7-2: Thermal Expansion

Almost all solids expand when their temperature increases: $\Delta L = \alpha L_0 \Delta T$, where ΔL is the amount of *thermal expansion*, α is the *coefficient of linear expansion* for a solid, L_i is initial length, and ΔT is the T change. If it helps to solve a problem, you can replace ΔL and ΔT with $L_f - L_o$ and $T_f - T_i$.

a) If a piece of steel ($\alpha = 12 \times 10^{-6} \text{ C}^{-1}$) is shaped into a cylinder (which has, at 0°C , diameter = 10.0 cm and length = 5.000 m) that is heated to 100°C , what is its final length, and change in length?

b) If this cylinder is placed between two concrete walls that are exactly 5.000 m apart at 0°C , what force would be exerted against the walls at 100°C ? What really happens? { Steel's elastic modulus is $2000 \times 10^8 \text{ N/m}^2$, and the ultimate compressional strengths of steel & concrete are $5 \times 10^8 \text{ N/m}^2$ and $2 \times 10^7 \text{ N/m}^2$, respectively. }

c) If a tape measure that is correct at 15°C is used to measure 1 m of cloth in a room at 40°C , is the buyer being cheated?

d) Liquids also expand as temperature increases. To get maximum gasoline for your money, should you buy it in the early morning or late afternoon? { Liquids usually have more thermal expansion than solids. }

for Section 7.2,

7-3: What are the molecular masses of H_2 , O_2 , N_2 , NH_3 , CO_2 ? { The atomic masses of H, C, N and O are 1, 12, 14 and 16, respectively. }

What is the mass of one O_2 molecule?

7-4: In a 1 liter cube (10 cm on each side) of solid H_2O (ice: 18.0 g/mole , $.917 \text{ g/cm}^3$), what is the mass, number of molecules, and number of atoms. What fraction of the mass is contributed by hydrogen?

What is the volume of a billion (10^9) atoms of gold? { Gold is 197.0 g/mole , 19.3 g/cm^3 . }

for Section 7.3,

7-5: A 15.0 liter tank contains 2.00 moles of iron and 5.00 moles of N_2 gas at 100°C . What pressure is exerted against the walls?

7-6: Air is a mixture (of mostly N_2 & O_2) with an average molecular mass of 28.8 g/mole . Under normal conditions, what is the total mass of air in a 6 m x 4 m x 3 m room, at 20°C ?

What is air's density at 20°C ? at 0°C ?

7-7: **a)** An expandable container has an initial volume of 5 m^3 at STP. If pressure and temperature change to 2.0 atm and 70°C , what is the final volume?

b) When a fixed-volume container changes from 170°K to 273°C , the pressure triples. What can you conclude? What are some possible reasons for this?

7-8: A scuba diver takes a deep breath from his tank at a depth of 10.3 m where the pressure is 2 atm, then holds his breath while returning to the surface. What happens to the diver?

7-9: If the volume of a gas-filled cylinder doubles because its height doubles, explain what happens (and why) to the gas pressure on its O -ends and on its side-walls. { Hint: Consider what happens to a molecule that travels \leftrightarrow between the sides, or \updownarrow between the bottom & top. }

When the volume of a gas-filled sphere doubles, what happens to the frequency of gas collisions against each cm^2 of the wall? { Hint: What happens to the sphere's diameter? What happens to its surface area? How do these changes affect the gas pressure ($= P/A$) against the sides? }

7-10: A .2 m^3 tank contains 1.5 moles of H_2 , 14 grams of N_2 , and 12.04×10^{23} molecules of CO_2 . If the total pressure is 1.6 atm, what is the pressure of each gas?

If a chemical reaction totally consumes the H_2 , what is the pressure of each gas?

7-11 Optional (use this if you are studying "vapor pressure, saturation, humidity,..."):

A 4.0 m^3 tank contains neon (an "inert" gas) and water vapor that is in equilibrium with a tiny bit of liquid water at the bottom of the tank. The initial pressures of neon and water are 30 torr and 20 torr, respectively.

If the tank volume is reduced to 2.0 m^3 (some water remains at the bottom of the tank, and temperature is the same as it was initially) what is the total pressure?

If *relative humidity* in the 2.0 m^3 tank is 80%, what is the final total pressure?

for Section 7.4,

7-12: Gases are "ideal" (with negligible molecular volume and forces) at relatively low pressure and high temperature.

The normal boiling points of oxygen (O_2) and water (H_2O) are -218°C and $+10^{\circ}\text{C}$. Predict whether each gas acts "ideally":

O_2 gas at 110°C and 1 atm,

H_2O gas at 110°C and 1 atm,

H_2O gas at 110°C and .001 atm,

H_2O gas at 300°C and 1 atm.

7-13: What is the root-mean-square speed of H_2O gas molecules at 100°C ?

7-14: A tank contains .8 mole of H_2 and .8 mole of O_2 . Which type of molecule is (on the average) faster, and by what factor?

As discussed in Section 7.3, the H_2 and O_2 produce equal pressures, even though the mass of O_2 is 16 times larger. Show why pressure is independent of mass, using the fact that pressure depends on two "partially conflicting factors": the rate of collisions and momentum change per collision.

7-15: If H_2 and O_2 are in separate tanks, do the H_2 's have a larger average speed?

In a tank that contains H_2 and O_2 , does every H_2 molecule move faster than every O_2 molecule?

If a certain H_2 is 100 m/s (much slower than average) at one instant, will it also be slower-than-average 1 second later?

7-16: In "root-mean-square averaging", the operations are done in reverse order: 1) square each number, 2) find the mean [average], 3) take the square root.

Find the "regular" and "rms" averages for 5, 7 and 9. Will the rms-average always be larger?

7-17: Use ratio logic to compare the speeds of O_2 molecules at $300^\circ K$ and $1200^\circ K$.

At $300^\circ C$, it takes 30 seconds for 5% of the molecules in a tank to escape through a tiny hole in the wall. If the tank's temperature is changed until the molecules are moving 50% faster (what is the new T ?), how long will it take for 5% of the molecules to escape?

If temperature and average molecular kinetic energy increase by a factor of 1.034, by what factor will the rate of the molecules' chemical reactions be changed?

for Section 7.5,

Free Information, to use in Section 7.5's problems.
Specific heats: ice ($2.10 J/gC^\circ$), water ($4.186 J/gC^\circ$), steam ($2.01 J/gC^\circ$), lead ($.13 J/gC^\circ$), steel ($.45 J/gC^\circ$).
{ Problem-solving strategies are identical whether you use SI or (as above) non-SI units. Just be consistent. }

Phase changes: for ice \rightarrow water ($334 J/g$), for water \rightarrow steam ($2255 J/g$). ==lead-nec?

7-18: How much heat must be removed from 20 g of ice at $115^\circ C$, to convert it into ice at $-10^\circ C$.

7-19: Find the specific heat of a metal, if a 50 g chunk at $90^\circ C$ raises the temperature of 20 g of water from $17^\circ C$ to $43^\circ C$.

7-20: What is the result (steam, water, ice, or a mixture of two of these) if 20 g of steam at $140^\circ C$ is mixed with
a) 50 g of ice at $0^\circ C$? **b)** 100 g of ice at $0^\circ C$? **c)** 200 g of ice at $0^\circ C$?

Optional: calculate numerical results.

7-21: A 500 W heater is immersed in $120 cm^3$ of $20^\circ C$ water in a cup whose heat capacity is $20 J/C^\circ$ [notice the

units]. After 1/4 minute, what is the water temperature? When does the water reach $90^\circ C$?

7-22: If 70% of the heat content in a 400 Calorie piece of cake is converted into useful energy, how long can it run a 50 watt motor?

for Section 7.6,

7-23: The bottom of a steel cylinder (7.0 cm diameter, 20 cm height) is in boiling water, and the top is slightly indented so it can hold a 55 g ice cube. After the ice cube begins to melt, how long will it take before all of it melts? { To melt ice requires $334 J/g$, and the thermal conductivity (K) of steel is $40 J/s m C^\circ$. }

7-24: You have a cup of $100^\circ C$ coffee and a small amount of $5^\circ C$ water in a refrigerator. To make the coffee as cool as possible at the end of 5 minutes, should you put the cold water into the coffee immediately or at the end of the 5 minutes?

To conserve energy when you heat a pot of water from $20^\circ C$ to $100^\circ C$, should you set an "immersion heater" at 250 W or 1000 W?

7-25: A primitive cabin has an outdoor toilet. The temperature is $5^\circ C$. Would you rather sit on a seat of wood ($K = .012 J/s m C^\circ$) or copper ($K = 380 J/s m C^\circ$)?

7-26: One end of a 16 cm long steel cylinder is at $10^\circ C$ and the other end is at $70^\circ C$. What is the temperature at 0 cm, 4 cm, 8 cm, 12 cm and 16 cm from the cold end? { Hint: This is a simple problem. Don't make it complicated. }

For the cylinder(s) shown below, what is the temperature at the • spots? { Hint: At "steady state", what can you say about the heat flow through the wide and narrow parts? Can you turn this flow-rate statement into an equation? }

7-27: Hot and Cold Animals

Imagine that "box-animals" (shown below) produce metabolic heat at a rate proportional to their mass, and lose body heat at a rate proportional to their surface area. If both animals have the same density and generate the same heat-per-kg, which one is hotter?

If they have the same body temperature, which one must produce more heat-per-kg?

Would your answers change for "ball-animals" with bodies shaped like spheres instead of cubes?

7-28: An unclothed person with $1.7 m^2$ of surface area has a skin temperature of $34^\circ C$ and emissivity = .7. In a $10^\circ C$ room, what is the person's net heat loss?

for Section 7.7,

7-29: If 40% of the energy escapes into the air, what is the final temperature of a $20^\circ C$, 500 m/s, 50 g lead bullet after it is stopped by a wall? Will any of the bullet melt? { Lead has heat capacity = $.13 J/gC^\circ$, melting point $327^\circ C$. }

7-30: How high can a 500 kg elevator be lifted at constant speed by using 10 seconds of 500 W electrical energy, burning 2 grams of gasoline (its heat-of-combustion is 4800 J/g), converting 7×10^{-14} gram of mass into 6300 J, condensing 4.0 g of steam (for water→steam, 2255 J/g is required), allowing a spring (with $k = 5000$ N/m) to recoil from a compression of 1.5 m, and gathering 25% of the solar energy (approximately 1000 W/m²) that falls on 2 m² of area during 10 s, if the elevator's motor converts 70% of this energy into useful work.

7-31: When .010 gallon of gasoline burns (heat of combustion = 1.23×10^8 J/gallon), a 500 kg elevator can be lifted 250 m at constant speed if the motor is 100% efficient.

If the motor converts 70% of the gasoline's energy into "mgh", how high can .010 gallon lift the elevator? To lift the elevator 250 m, how much gas will this 70%-motor use?

7-32: A 10 kg block slides down a 30°, 8.0 m long ramp. Discuss energy conservation for the block at the bottom of the ramp (where its speed is 7.0 m/s) and after it has come to rest on a horizontal surface past the ramp.

7-33: If heat flows into an object, does its temperature always increase? If not, give counter-examples.

7-34: a) If a system absorbs 700 J of heat and has 500 J of work done on it, what is its change of internal energy?

b) A system's internal energy increases by 1200 J while the surroundings gives up 700 J of heat. How much work did the system do?

for Section 7.8,

7-35: Step 1: In a closed/insulated system, liquid water spontaneously turns into solid water (ice) at -10°C . Does the system's entropy increase or decrease?

Step 2: If heat flow returns the system to -10°C , has the system's entropy increased (from before Step 1 to after Step 2)?

Step 2: If heat flow returns the system to -10°C , is the overall ΔS (from before Step 1 to after Step 2) positive or negative?

7-36: Can you think of a way to make a "purple" solution separate into red and blue layers? {Hint: use your imagination. }

7-37: Optional: Do this if your class (in either physics or chemistry) uses $\Delta G = \Delta H - T \Delta S$ or if you're curious about "why things happen".

Which of the two factors, stronger bonds (ΔH) and more entropy ($-T \Delta S$), favors the " $\text{H}_2\text{O}(\text{l}) \leftrightarrow \text{H}_2\text{O}(\text{g})$ " reaction in the \rightarrow direction? Which factor favors the reverse reaction, in the \leftarrow direction? Which factor is more important at high T? at relatively low T?

7-38: What is the efficiency of an engine that has 20% due to friction/... if it absorbs heat at 600°C and releases

heat at 400°K ?

What are the requirements for a heat engine that is 100% efficient? Is it possible to build such an engine?

7-39: If we ignore losses due to friction/..., how much energy is required to move 3340 J of heat (enough to freeze 10 g of water to ice) from a freezer at -5°C to the room at 20°C ? {Hint: Use the same principles as for a heat engine.}

If the motor has 30% waste due to friction, how much energy is required?==

7-40: ==[is 7-39 the last problem?]

7.92 Solutions

7-1: $68^\circ\text{F} = 20^\circ\text{C} = 293^\circ\text{K}$,
 $-12^\circ\text{C} = -4^\circ\text{F} = 261^\circ\text{K}$, $377^\circ\text{K} = 104^\circ\text{C} = 219^\circ\text{F}$.

Why? 68°F is 36°F (= $68-32$) above freezing, which is $36(5/9) = 20$ of the larger $^\circ\text{C}$ above freezing at 0°C . Then add 273 to get 293°K .

-20°C is 20°C below freezing, or $20(9/5) = 36$ of the smaller $^\circ\text{F}$ below freezing at 32°F , $32-36 = -4^\circ\text{F}$.

$377^\circ\text{K} \rightarrow (-273) \rightarrow 104^\circ\text{C}$, 4° above boiling water, or $4(9/5) = 7$ of the smaller $^\circ\text{F}$ above 212°F . {Or use "T-above-freezing logic" to get the same answer. }

7-2: a) $\Delta L = \alpha L_i \Delta T = (12 \times 10^{-6})(5.00)(100-0) = .006$ m. $L_f = L_i + \Delta L = 5.000 + .006 = 5.006$ m.

b) If the walls didn't break, compressional force between each cylinder-end and the wall (these are a "third-law force pair") would be $F = A Y \Delta L / L_o = (\pi .05^2)(2000 \times 10^8)(.006/5.000) = 1.9 \times 10^6$ N!

At 100°C , the pressure would be $P = F/A = 1.9 \times 10^6 / (\pi .05^2) = 2.4 \times 10^8$ N/m². But the wall breaks when thermal expansion makes the pressure exceed the concrete's strength of 2×10^7 N/m².

Bonus: you can substitute $F/A = 2 \times 10^7$ N/m² into $F/A = Y(\Delta L/L_o)$, solve for $\Delta L/L_o = .0001$, substitute $\Delta L/L_o$ into $\Delta L/L_o = \alpha \Delta T$, then solve for $\Delta T = 8.3^\circ$, and $T_f = T_i + \Delta T = 0^\circ\text{C} + 8.3^\circ\text{C} = 8.3^\circ\text{C}$.

c) The distance between the tape's 1 m marks is larger at 40°C , so the buyer actually gets a little (very little!) more cloth than 1 m. {Thermal expansion of cloth is negligible because cloth has "air spaces". }

d) If expansion is larger for gasoline (liquid) than for the pump (solid) that measures its volume, you get more mass-and-molecules per gallon in the morning when the gasoline temperature is lower.

7-3: H_2 : $1+1=2$ g/mole, O_2 : $16+16=32$ g/mole,
 N_2 : $14+14=28$ g/mole, NH_3 : $14+1+1+1=17$ g/mole,
 CO_2 : $12+16+16=44$ g/mole.

1 mole of O₂ molecules has a mass of 32 g, so one O₂ has $m = .032 \text{ kg} / (6.022 \times 10^{23}) = 5.3 \times 10^{-26} \text{ kg}$.

7-4: 1 liter is $(10 \text{ cm})^3 = 10^3 \text{ cm}^3$. Beginning with this "known amount", use conversion factors to find $(1000 \text{ cm}^3)(.917 \text{ g/cm}^3) = 917 \text{ g}$, which is .917 kg, $(917 \text{ g})(1 \text{ mole}/18.0 \text{ g}) = 50.9 \text{ moles of H}_2\text{O}$.

The number of H₂O molecules can be stated as either "50.9 moles" or (multiplying by 6.022×10^{23} things/mole) " 307×10^{23} molecules". Because there are 3 atoms (H,H,O) in each H₂O molecule, the number of atoms is "152.7 moles" or " 9.20×10^{25} ".

Every mole of H₂O contains 2 moles of H (this is 2 g of H) and 1 mole of O (16 g of O). The fraction of H is $(\text{g of H})/(\text{g total}) = (2 \text{ g})/(18 \text{ g}) = 1/9$. H is 2/3 of the atoms but only 1/9 of the mass, because H-atoms have less mass than O-atoms.

$$10^9 \text{ gold-atoms} \frac{1 \text{ mole}}{6.022 \times 10^{23} \text{ atoms}} \frac{197 \text{ g}}{1 \text{ mole}} \frac{1 \text{ cm}^3}{19.3 \text{ g}} = 1.70 \times 10^{-14} \text{ cm}^3. \text{ This is a very small volume!}$$

7-5: In $PV=nRT$, n is the moles of gas (not solid or liquid) so $n_{\text{gas}} = 5$. If you use SI units, convert liters to m^3 . For either set of units, convert °C to °K. $.015 \text{ m}^3$ to m. or lit-atm units (as on the right):

In SI units,	In lit-atm units,
$P(.015) = (5)(8.314)(373)$	$P(15) = (5)(.0821)(373)$
$P = 1.03 \times 10^6 \text{ Pa}$	$P = 10.2 \text{ atm}$

These P's are the same, because $101300 \text{ Pa} = 1 \text{ atm}$.

7-6: $V_{\text{room}} = (6\text{m})(4\text{m})(3\text{m}) = 72 \text{ m}^3 = 72 \times 10^3 \text{ l}$. Under normal conditions, $P \approx 101300 \text{ Pa} \approx 1 \text{ atm}$.

In SI units, $(101300)(72) = n(8.314)(293)$.
In lit-atm units, $(1)(72000) = n(.0821)(293)$

Solving either equation gives $n = 2990$ moles, so $m = (2990 \text{ moles})(.0288 \text{ g/mole}) = 86.1 \text{ kg}$.

At 20° C, $\rho_{\text{air}} = (86.1 \text{ kg})/(72 \text{ m}^3) = 1.20 \text{ kg/m}^3$.

There are two ways to find the density at 0° C. You can repeat the $PV=nRT$ calculation above, but use $T = 273^\circ \text{C}$. Or use ratio logic; if P and V are constant, n increases as T decreases, so at 0° C the air density is $1.20(293/273) = 1.29 \text{ kg/m}^3$.

7-7: In **a**, it is reasonable to assume that n remains constant (the problem can't be solved without doing so), and STP means 0° C and 1 atm. In **b**, tripled pressure can be expressed as " P_1 becomes $3P_1$ ".

For Part a,	For Part b,
$\frac{P_2 V_2}{P_1 V_1} = \frac{n_2 R T_2}{n_1 R T_1}$	$\frac{P_2 V_2}{P_1 V_1} = \frac{n_2 R T_2}{n_1 R T_1}$
$\frac{(2 \text{ atm}) V_2}{(1 \text{ atm})(5 \text{ m}^3)} = \frac{n_1 343}{n_1 273}$	$\frac{3P_1 V_1}{P_1 V_1} = \frac{n_2 546}{n_1 170}$
$V_2 = 3.1 \text{ m}^3$	$.93 = n_2/n_1$

For ratio logic, SI and non-SI units can be mixed, but only if the units are consistent for each variable. For example, it is all right if (as in "a") both of the P's are non-SI and both V's are SI, but you couldn't mix P-units by having P_1 in atm and P_2 in Pa.

For "b", n_2 is less than n_1 . Why have the number of molecules decreased? With the information given, it is impossible to know for sure; maybe some gas leaked out, or a chemical reaction occurred, or ...

7-8: While the diver surfaces, P is cut in half but n and T are constant*, so the volume of air inside his lungs tries to double. This does much damage to his lungs, and may kill him. The proper technique is to exhale the air as he rises to the surface.

* n is constant because the diver holds his breath, and T is approximately constant (it may even increase a little, adding to the diver's V-increase difficulties).

At a depth where $P = 2 \text{ atm}$, a given volume (lungs filled with air by a deep breath) contains twice as many molecules as a deep breath at the surface.==cut?

7-9: If the bottom-to-top distance doubles it takes twice as long for a molecule to travel from the bottom to top. Because of this, collisions with the bottom and top occur half as often, so P is cut in half.

The side-to-side distance stays the same, but the side-collisions are spread over an area that is twice as large, so P (which is F/A) is cut in half.

If a sphere's $V (= 4\pi r^3/3)$ doubles, its r increases by a factor of $1/\sqrt[3]{2} = .794$, so molecules hit the wall less frequently (and P decreases) by a factor of .794.

The sphere's area $(= 4\pi r^2)$ increases by a factor of $1/.794^2$ and this causes $P (= F/A)$ to decrease by a factor of $.794^2$. The combined effect of these factors is (frequency-factor)(area-factor) $= (.794)(.794^2) = .500$. When V doubles, gas pressure is cut in half.

7-10: There are 1.5 moles of H₂, $(14 \text{ g})(1 \text{ mole}/28 \text{ g}) = .5 \text{ mole of N}_2$, and $(12.02 \times 10^{23})/(6.02 \times 10^{23}) = 2.0 \text{ moles of CO}_2$. Because all three gases share the same container, they have the same V and the same T , so (as in Problem 7-A, Part 3) P -fraction = n -fraction. For H₂, n -fraction = (moles of H₂)/(total gas moles) = $(1.5)/(1.5 + .5 + 2.5) = .375$. Similarly, the mole-fractions of N₂ and CO₂ are .125 and .500. The gas pressures are $.375(3.2 \text{ atm}) = 1.2 \text{ atm}$, $.125(3.2 \text{ atm}) = .4 \text{ atm}$, and $.500(3.2 \text{ atm}) = 1.6 \text{ atm}$, respectively.

Or you can substitute into a "ratio logic" equation:

For H ₂ ,	For N ₂ ,	For CO ₂ ,
$\frac{P_{\text{H}_2}}{P_{\text{tot}}} = \frac{n_{\text{H}_2}}{n_{\text{tot}}}$	$\frac{P_{\text{N}_2}}{P_{\text{tot}}} = \frac{n_{\text{N}_2}}{n_{\text{tot}}}$	$\frac{P_{\text{CO}_2}}{P_{\text{tot}}} = \frac{n_{\text{CO}_2}}{n_{\text{tot}}}$
$\frac{P_{\text{H}_2}}{3.2} = \frac{1.5}{4.0}$	$\frac{P_{\text{N}_2}}{3.2} = \frac{.5}{4.0}$	$\frac{P_{\text{CO}_2}}{3.2} = \frac{2.0}{4.0}$
$P_{\text{H}_2} = 1.2 \text{ atm}$	$P_{\text{N}_2} = .4 \text{ atm}$	$P_{\text{CO}_2} = 1.6 \text{ atm}$

7-11: Ratio logic: P_{neon} doubles when V is cut in half, so final $P_{\text{neon}} = 60$ torr. There is water at the bottom of the tank, and T is constant, so water vapor stays at its equilibrium pressure of 20 torr. The final P_{total} is $P_{\text{neon}} + P_{\text{water}} = 60 \text{ torr} + 20 \text{ torr} = 80 \text{ torr}$.

If relative humidity is 80%, $P_{\text{H}_2\text{O}} = .80(20 \text{ torr}) = 16 \text{ torr}$, and $P_{\text{total}} = 60 \text{ torr} + 16 \text{ torr} = 76 \text{ torr}$.

7-12: At 110°C , O_2 is well above its boiling point and is almost-ideal, but H_2O has significant non-ideal behavior because it is so close to its 100°C b.p. where intermolecular attractions were strong enough to make it liquify. But at low pressure (.001 atm) or higher temperature (300°C), H_2O gas is fairly "ideal".

7-13: Solve $\frac{1}{2}mv^2 = \frac{3}{2}kT$ or $\frac{1}{2}mv^2 = \frac{3}{2}kT$ for v .

$$\frac{1}{2} \frac{.0180 \text{ kg}}{6.022 \times 10^{23}} v_{\text{rms}}^2 = \frac{3}{2} \frac{8.314}{6.022 \times 10^{23}} \quad (373)$$

$$\frac{1}{2} (.0180 \text{ kg}) v_{\text{rms}}^2 = \frac{3}{2} (8.314) \quad (373)$$

Both equations give the same v_{rms} of 719 m/s.

7-14: Ratio logic: $\frac{1}{2}mv^2 = \frac{3}{2}RT$, H_2 and O_2 have the same T (because they're in the same tank), so v is proportional to $\sqrt{1/m}$. Because O_2 is larger by a factor of 16, it is slower by a factor of $\sqrt{1/16} = 1/4$, and it hits the wall $1/4$ as often as H_2 molecules.

Now let's look at the "momentum per collision". O_2 's average mv is 4 times larger, because its m is 16 times larger while its v is smaller by a factor of $1/4$: $mv\text{-factor} = (m\text{-factor})(v\text{-factor}) = (16)(1/4) = 4$.

Pressure depends on two factors (rate of collisions, and change-of-momentum per collision) that "partially conflict". Both factors \uparrow as $v \uparrow$. But when $m \uparrow$, one factor (speed & collision rate) \downarrow while the other factor (Δ -of- mv) \uparrow . These factors cancel each other: for O_2 (compared with H_2), $P\text{-factor} = (\text{collision-rate factor})(\Delta\text{momentum factor}) = (x 1/4)(x 4) = 1$. O_2 hits the wall $1/4$ as often, but each collision produces 4 times as much momentum change, so equal amounts of O_2 and H_2 produce equal pressures. P does not depend on m . This is why $PV=nRT$ contains n (the number of molecules) but not m (the mass per molecule).

7-15: If H_2 and O_2 have the same temperature, H_2 will be faster. But if O_2 's tank-temperature is enough higher (for example, 401°K versus 100°K) the O_2 molecules can have a faster average speed.

H_2 and O_2 share the same tank so they have the same T , and H_2 is (on the average) faster. But some "slower-than-average" H_2 molecules are slower than some "faster-than-average" O_2 molecules.

Molecules collide with each other so frequently that after a fraction of a second the molecular speeds are "randomized", and a previously slow molecule could be

slow, fast, or anything in-between.

7-16: regular average [mean] = $(5+7+9)/3 = 7.0$.

$$\text{rms average} = \sqrt{(5^2 + 7^2 + 9^2)/3} = 7.19.$$

For rms-averaging, notice that the arithmetic order is smr (the reverse of rms): square ($5^2, 7^2, 9^2$), mean (add the squares and divide by the 3 because there are 3 numbers: 5,7,9), root (take the $\sqrt{\quad}$ of the mean).

Unless every number is the same (like 7,7,7 whose regular and rms averages are both 7) the rms-average is always larger. In our simple example, the large number (9) is "weighted" more heavily than the small number (5) because 9 is counted 9 times (in 9^2) while 5 is only counted 5 times (in 5^2).

For a Maxwell-Boltzmann distribution of molecular speeds, the *rms speed* [a weighted average] is larger than the *most common speed* (which is approximately equal to the *mean speed*) by a factor of 1.22. This is discussed briefly in Solution 7-B.

7-17: $v^2 \propto T$, so $v \propto \sqrt{T}$. If T increases by a factor of 4 (from 300°K to 1200°K), v increases by a factor of $\sqrt{4} = 2$. An average 1200°K molecule is moving twice as fast, not 4 times as fast.

If molecules move 1.50 times faster, they hit the wall (and the hole-in-the-wall that lets them escape) 1.50 times more often, so their escape time is smaller. The new time is $(30 \text{ s})(1/1.50) = 20 \text{ s}$.

The rate of chemical reactions \uparrow as T (and KE and v) \uparrow , but there is no "direct proportionality". If KE \uparrow by a factor of 1.034 (like from 290°K to 300°K) we might expect reaction rate to \uparrow by a factor of 1.034 (or maybe $\sqrt{1.034} = 1.017$), but for a typical reaction the rate doubles! Why is the change so dramatic? Because most chemical reactions only occur when molecules with extremely high speeds collide. A T -increase that changes v_{rms} by only 3.4% can double the number of high-speed molecules (at the high-speed end of the Maxwell-Boltzmann curve) that have enough energy to cause a chemical reaction.

7-18: $(20 \text{ g})(2.01 \text{ J/gC})(115-100) + (20 \text{ g})(2255 \text{ J/g}) + (20 \text{ g})(4.19 \text{ J/gC})(100-0) + (20 \text{ g})(334 \text{ J/g}) + (20 \text{ g})(2.10 \text{ J/gC})(0-[-10]) = (603 + 45100 + 8380 + 6680 + 420) = 61183 \text{ Joules}$. {ignoring significant figures}

7-19: $(50)(x \text{ J/gC})(90-43) = (20 \text{ g})(4.186)(43-17)$, because HEAT LOST = HEAT GAINED. .93 J/gC°

7-20: For each diagram, $(20)(2.01)(140-100) + (20)(2255) = 46700 \text{ J}$ for "140° steam \rightarrow 100° liquid".

For **a**, $(50)(334) = 16700 \text{ J}$ for "0° ice \rightarrow 0° liquid". For **b** and **c**, this heat is $(100)(334) = 33400 \text{ J}$, and $(200)(334) = 66800 \text{ J}$.

In each picture, determine the "dominant factor" at this stage: **a** ($16700 < 46700$), **b** ($33400 < 46700$), and **c** ($66800 > 46700$). In **a** and **b**, 100° steam is dominant, so you find out how much energy it takes to raise the

0° liquid to 100° liquid: $(50)(4.19)(100-0) = 20950$ J, $(100)(4.19)(100-0) = 41900$ J. In **a**, the steam can give up enough heat to do this (46700 is larger than $16700+20950$), but in **b** the steam cannot do it (46700 is smaller than $33400+41900$). In **c**, ice is dominant: 100° liquid can be cooled to 0°, because 66800 is larger than $46700 + (20)(4.19)(100-0)$.

Final results are marked with •'s:

- a) 100° water + 100° steam,
- b) water (between 0° and 100°), and
- c) 0° ice + 0° water.

Optional: Here are equations, solutions, "checks".

- a) $16700 + 20950 = 1600 + X(2255)$, then solve for $X = 16.0$ g of steam condenses (this is < 20 g, OK!).
- b) $33400 + 50(4.19)(T-0) = 46700 + 20(4.19)(100-T)$ and $T = 73.9^\circ\text{C}$ (this is between 0° and 100°, OK!).
- c) $Y(334) + 46700 + 8380$, so $Y = 165$ g of ice that melt to 0° water (this is $<$ the available 200 g, OK!)

7-21: $(120\text{ cm}^3)(1.00\text{ g/cm}^3) = 120$ g, and with 20 J/C° there is no need to multiply by the number of grams. As in Problem 7-D, cause \Rightarrow effect:

$$\begin{aligned} \text{heat supplied} &= \text{"action" produced by this heat} \\ (500\text{ J/s})(15\text{ s}) &= [(120\text{ g})(1\text{ J/gC}^\circ) + (20\text{ J/C}^\circ)] \Delta T \\ 53.6^\circ\text{C} &= \Delta T \end{aligned}$$

$$T_f = T_i + \Delta T = 20^\circ\text{C} + 53.6\text{ C}^\circ = 73.6^\circ\text{C}$$

To find the time needed to reach 90°C you can use ratio logic (the required ΔT is 70°C, which requires (15 s)(70°/53.6°) = 20 s, or solve this equation:

$$500 t = [(120)(1) + 20] [90 - 20]$$

$$\begin{aligned} \mathbf{7-22:} \quad \text{heat produced} &= \text{heat used} \\ .70(400\text{ Cal})(1000\text{ cal/Cal})(4.186\text{ J/cal}) &= (50\text{ J/s})(t\text{ s}) \\ 23400\text{ s} &= t \\ 6.5\text{ hours} &= t \end{aligned}$$

7-23: The cylinder's bottom (in boiling water) is at 100°C, and its top (with ice starting to melt) is at 0°C. Be sure everything is in SI units, substitute & solve:

$$\begin{aligned} \frac{Q}{\Delta t} &= K A \frac{T_{\text{hot}} - T_{\text{cold}}}{L} \\ \frac{(334\text{ J/g})(55\text{ g})}{\Delta t} &= (40)(\pi .035^2) \frac{100^\circ - 0^\circ}{.20} \\ 240\text{ seconds} &= \Delta t \end{aligned}$$

7-24: The cold water has the same "cooling effect" whether it is added early or late. For efficient cooling by conduction/convection/radiation, ΔT between the coffee and room should be large during the 5 minute period. Add cold water at the end of 5 minutes.

The heat needed to "do the job" is the same whether heating is fast or slow. When the water is medium-hot (between 20°C & 100°C) it is losing heat to the environment. Minimize this loss by minimizing the heating time; use the 1000 J/s heater.

7-25: Both seats have the same ΔT : the surface of each seat is near body temperature ($\approx 34^\circ\text{C}$) while faraway parts of the seat are close to 5°C. Copper has a much higher K so it draws heat from the 34° surface (and from your body) toward the 5° surface at a faster rate, and it is much less comfortable.

7-26: Heat flow is constant through each part of the cylinder, and temperature changes smoothly from one end to the other. 1/4 of the 60° T-change occurs in each 1/4 of the length: 10°, 25°, 40°, 55°, 70°.

At steady state, heat flow ($Q/\Delta t$) is the same in the fat and skinny halves of the asymmetric cylinder.

This fact can be used to write an equation:

$$\begin{aligned} (Q/\Delta t)_{\text{left half}} &= (Q/\Delta t)_{\text{right half}} \\ K(\pi .6^2) \frac{T_{\text{center}} - 10^\circ}{1.10} &= K(\pi .3^2) \frac{70 - T_{\text{center}}}{1.10} \\ .36 T_{\text{center}} - 3.6 &= 6.3 - .09 T_{\text{center}} \\ T_{\text{center}} &= 22^\circ \end{aligned}$$

The ΔT from left-to-center is $22^\circ - 10^\circ = 12^\circ$, while ΔT from center-to-right is $70^\circ - 22^\circ = 48^\circ$. Because the cylinder's right side has 1/4 of the area, it must have 4 times the ΔT , to get the same heat-flow rate.

T at the •'s are 0°, 11°, 22°, 46° and 70°.

7-27: The large box-animal has 27 times as much volume ($3 \times 3 \times 3$ versus $1 \times 1 \times 1$). If densities and heat-per-kg are equal, it also has 27 times as much mass and 27 times as much metabolic energy. But it only has 9 times as much surface area (3×3 vs. 1×1); this \Rightarrow [do more?] so it can't get eliminate its heat as effectively, and to reach *thermal equilibrium* (a *steady state* with "heat lost = heat produced") it must have a higher temperature than the small box-animal.

For both animals to have the same temperature, the small b-a must produce relatively more heat-per-kg. {Or it must decrease its rate of heat loss with insulation (fur, feathers, blubber,...) or by lowering the temperature of its outer surface, or... }

For ball-animals, V and A are still proportional to R^3 and R^2 , respectively, so we can still use the same logic that is described above.

7-28: The net rate of heat loss ($Q_{\text{lost}} - Q_{\text{gained}}$) is

$$\begin{aligned} \frac{Q_{\text{net}}}{\Delta t} &= \frac{Q_{\text{emitted}}}{\Delta t} - \frac{Q_{\text{absorbed}}}{\Delta t} \\ &= +e\sigma A(T_{\text{person}})^4 - e\sigma A(T_{\text{room}})^4 \\ &= +e\sigma A[(307)^4 - (283)^4] \\ &= +(.7)(5.67 \times 10^{-8})(1.7)[(307)^4 - (283)^4] \\ &= 170 \text{ J/s} \end{aligned}$$

$$\frac{170 \text{ J}}{\text{s}} \frac{3600 \text{ s}}{1 \text{ hour}} \frac{1 \text{ cal}}{4.186 \text{ J}} \frac{1 \text{ Cal}}{1000 \text{ cal}} \approx 140 \frac{\text{Cal}}{\text{hr}}$$

This is a large expenditure of metabolic energy!
 ==does the body cool its surface to decrease this?
 ==cut conversion to Cal/hour? ask in P-statement?

7-29: If 40% of the energy escapes, 60% of the bullet's initial forward-motion KE stays in the bullet, converted into molecular-motion KE that makes the bullet's T-increase. The equation setup is analogous to Section 7.5's "heat supplied = action produced",

$$\begin{aligned} 60\% \text{ of } KE_i &= \text{heat that causes } T \uparrow \text{ of bullet} \\ .60[\frac{1}{2}(.050)[500]^2] &= (50 \text{ g})(.13 \text{ J/gC}^\circ)(T_f - 20^\circ) \\ 3750 \text{ Joules} &= 6.5 T_f - 130 \\ 597^\circ \text{ C} &= T_f \end{aligned}$$

This T_f is above lead's melting point of 327°C , so some lead will melt. By using Section 7.5 tools* and the fact that lead's heat of fusion (for solid→liquid) is 25 J/g, you can calculate that 1.4 g of lead will melt.

*Or you can use this intuitive logic: the "extra" heat (after the lead has reached 327°C) is $[597-327][.13] = 35 \text{ J}$, which melts some lead: $[35 \text{ J}]/[25 \text{ J/g}] = 1.4 \text{ g}$.

7-30: Including all forms, the total energy input is: $(500 \text{ J/s})(10 \text{ s}) = 5000 \text{ J}$, $(2 \text{ g})(4800 \text{ J/g}) = 9600 \text{ J}$, 6300 J , $(4 \text{ g})(2255 \text{ J/g}) = 9000 \text{ J}$, $\frac{1}{2}(5000)(1.5^2) = 5600 \text{ J}$, and $.25(1000 \text{ J/s}^2)(2 \text{ m}^2)(10 \text{ s}) = 5000 \text{ J}$.

If 70% of this energy is converted into increased elevator-mgh, $.70(5000+9600+6300+9000+5600+5000) = (500)(9.8)\Delta h$, and $\Delta h = 5.8 \text{ meters}$.

7-31: If 100% of the energy → an mgh of 250 m, 70% of this energy will → $.70(250 \text{ m}) = 175 \text{ m}$.

If .010 gallon is needed for 250 m with 100% efficiency, more gasoline is needed if the engine is 70% efficient. You must divide [not multiply] by .70: $(.010 \text{ gallon})/.70 = .014 \text{ gallon}$ is needed.

7-32: The ramp height is $(8.0 \text{ m})(\sin 30^\circ) = 4.0 \text{ m}$. At the bottom, $\Delta PE = m g \Delta h = (10)(9.8)(-4.0) = -392 \text{ J}$. Some of this PE [$\frac{1}{2} m v^2 = \frac{1}{2}(10)(7^2) = 245 \text{ J}$] has become forward-motion KE, and the other 147 J has been converted (by friction and air resistance) into molecular-motion KE (thermal energy, "heat",...) that increases the temperature of the block, ramp & air.

After the block comes to rest, all 392 J of ΔPE has been converted into thermal energy, which increases the T of the ramp, horizontal surface, block & air.

7-33: No. Section 7.5 shows that sometimes heat causes a phase change (solid→liquid, or liquid→gas) instead of a T-increase.

Section 7.7's " $\Delta U = Q - W$ " describes how Q (heat) can produce ΔU (changes in "internal energy", like ΔT , phase change,...) or W (work, like volume increase, $\Delta KE_{\text{forward-motion}}$, electrical work,...).

7-34: If you use $\Delta U = Q + W$, skip to the discussion at the end of this problem.

a) Given: $Q = +700 \text{ J}$, $W = -500 \text{ J}$ [W is - because work is done on the system, not by it], so we can find $\Delta U = Q - W = (+700 \text{ J}) - (-500 \text{ J}) = +1200 \text{ J}$.

b) Given: $\Delta U = +1200 \text{ J}$, $Q = +700 \text{ J}$ [Q is + because the system gains heat that the surroundings gives up]. $W = Q - \Delta U = (+700 \text{ J}) - (+1200 \text{ J}) = -500 \text{ J}$. Q is -, showing that the system doesn't "do work", it "has work done on it".

Did you notice that Parts a & b are two descriptions of the same system-changes?

If you use $\Delta U = Q + W$, the \pm sign of W is opposite what is given above, Q and ΔU are the same, and the answers are **a)** $\Delta U = (+700 \text{ J}) + (+500 \text{ J}) = +1200 \text{ J}$, **b)** $W = \Delta U - Q = (+1200 \text{ J}) - (+700 \text{ J}) = +500 \text{ J}$, so 500 J of work is done on (not by) the system.

7-35: The key words are "closed/insulated" [so the system is like a miniature universe] and "spontaneous" [this means that S_{universe} increases] Since our closed system is like a small universe, its entropy increases.

As in Problem 7-E, this chemical reaction causes two changes: liquid→solid [which causes S_{system} to \uparrow] and heat released by the reaction makes T_{system} \uparrow [which causes S_{system} to \uparrow]. S_{system} \uparrow because the S-increase caused by the T change is larger than the S-decrease caused by the phase change.

But if the system returns to -10°C the S-decreasing change [liquid→solid] is not overcome by ΔS due to a T-change, and S_{system} decreases. ΔS is negative.

7-36: Here is one answer. Use water-soluble blue dye and oil-soluble red dye. Shake it in a bottle, and the layers are temporarily mixed to form purple. The layers then separate (spontaneously, due to chemical interactions) into red and blue layers.

7-37: The \rightarrow direction is favored by $-\Delta S$ [the "more entropy" factor]* while \leftarrow is favored by $+\Delta H$ [the "stronger bonds" factor]. At high T, $-\Delta S$ gets lots of "votes" so it is dominant and H_2O is a gas. But at medium-low T (between the melting point and boiling point of H_2O) $-\Delta S$ has fewer votes, ΔH dominates, and H_2O is a liquid.

* If we ignore the "strong bonds" factor, it is extremely improbable for H_2O molecules to be in a small volume (liquid state) when they could be freely roaming around in a much larger volume (gas state)

7-38: The engine's overall efficiency is:

$$.80 \frac{873 - 400}{873} = .80 (.54) = .43 \Rightarrow 43\%$$

A 100% efficient engine would have to eliminate all friction/... and release heat at the *absolute zero* of $0^\circ K$. According to present scientific theory, each of these goals is impossible, and so is 100% efficiency.

7-39: == nec? how to solve?

given: Q_{cold} (not Q_{hot}), T_{hot} and T_{cold}
find: W

$$\frac{W}{Q_{hot}} = \frac{T_{hot} - T_{cold}}{T_{hot}}$$

$$\frac{W}{Q_{hot}} = \frac{Q_{hot} - Q_{cold}}{Q_{hot}} = \frac{T_{hot} - T_{cold}}{T_{hot}}$$

7-40: ==is this problem necessary?