4.91 Problems

for Section 4.1,

4-1: A 14000 N car drives up a hill (10° above horizontal) at 72 km/hour. What is its kinetic energy?

4-2: After the 2000 N block in Problem 3-C reaches 2 m/s, it is pulled → an extra 12 m with the same force of 90 N directed at 20° above horizontal. What is its final speed?

4-3: For each of the five 1-second intervals, decide whether work is +, – or zero:

4-4: A 40 gram bullet traveling 300 m/s → penetrates 8 cm into a block of wood. What average force is exerted against the bullet?

What is the speed of a 40 g, 300 m/s bullet after it passes through a 5 cm block of this same type of wood?

4-5: When you exert a constant force of 400 N on a 50 kg box while pushing it 5.0 m up a 20° ramp, its speed increases from 2.0 m/s to 3.8 m/s. What is the total work, and the work done by you, gravity, friction, and normal force? What is the friction force & coefficient of friction?

4-6: An 18.0 m train moves → at 20 m/s. A woman pushes a 14 kg box with a constant force of 28 N → along half the length of the train (9.0 m) during a 3.0 s interval, and box speed [with respect to the moving train] changes from zero to 6 m/s.

Show that $W=\Delta KE$ is true for observers standing on the train and on the ground.

4-7: When the elevator in Section 3.3's Problem 3-A (m_{total} = 570 kg, v_i = 2.5 m/s ↑, T_{cable} = 6730 N ↑) has moved 5.0 m upward, what is its velocity?

4-8: How much work do you do on a 20 kg box if you a) carry it 5.0 m horizontally at constant speed? b) carry it horizonally while accelerating it from rest to 3.0 m/s? c) lift it .60 m vertically at constant speed? What total work is done on the box during this vertical movement?

4-9: A 1200 kg piledriver is used to drive a stake into the ground. If the piledriver falls from a height of 5.0 m, and the resisting force of the ground is 4×10^6 N for this stake, how far is the stake driven into the ground by each drop of the piledriver?

4-10: How much force and work is needed to push a 50 kg block at constant speed up each of these frictionless ramps?

4-11: For a block sliding down a 20° plane, show that $W_{gravity}$ is the same whether it is calculated as $\mathbf{F_{parallel}\ d}$ or $\mathbf{F_{total}\ d\ cos\text{ø}}$.

4-12: Can Bike A stop before it goes over the cliff into the alligator pit? Can B stop?

for Section 4.3,

4-13: What is the speed of each object (starting from rest) when it reaches •?

4-14: The track below is frictionless except from C to D, which has μ_k = .40. At what locations is the speed of a 2.0 kg block equal to zero, if it begins 1) at A? 2) at B?

4-15: If a 500 kg roller coaster is moving 20.0 m/s at C, what is its speed at A, B, D? Assume a frictionless track.

4-16: A box traveling 5.4 m/s at the bottom of a 40° ramp moves 1.7 m up the ramp. What is the speed of the box when it returns to the bottom of the ramp? { Hint: Use energy accountability and "symmetry". }

What is μ_k between the box and ramp?

4-17: A motorcycle stuntman zooms down a ramp, his 200 kg cycle gaining 38 kJ from the engine and losing 4 kJ to friction. If his horizontal velocity at the peak "•" is 25 m/s, how far is he above the edge of the ramp?

4-18: Just before block hits ground, what is its speed?

4-19: Four 3.4 kg bricks, 8.0 cm thick, lie on a table. How much work is done stacking them into a 4-high pile?

How much work is needed to stack 40 of these bricks vertically? { Hint: discover an easy shortcut by studying the 4-brick problem. }

A 60 cm long chain is stretched out on a horizontal frictionless table, with 20% of its length over the edge. The chain is released and it begins to pick up speed and slither off the table. What is its speed when the last link of the chain leaves the table? { Hint: What is the "average h-change" for the 20% and 80% parts of the chain, and the resulting ΔPE's? }

4-20: **optional (to use if you study springs)**
basic

4-21: optional (for springs)
drop
drop with bounce
drop/friction bounce

4-22: optional (for springs)
1986 problem

4-23:

4-24:

for Section 4.#,

4-25: If a 1200 kg car has an 80 hp engine working at full power, how long will it take the car to accelerate from rest to 13.4 m/s?
from 13.4 m/s to 26.7 m/s (which is 60 miles/hour)?
What is the car's acceleration when its speed is 6.7 m/s, and 20.1 m/s?

Why is there a longer Δt for 13.4-to-26.8 than for 0-to-13.4?

4-26: A 57 kg woman runs up stairs, gaining 5.0 m in height during 3.5 seconds. At what rate is she doing work? { Give your answer in Watts, and in hp. }

4-27: How long does it take for a 3.0 hp motor to lift a 350 kg piano 14.0 m, to a fourth floor window?

4-28: If Problem 4-D's loaded elevator (990 kg) is equipped with a "counter-weight" of 800 kg as shown below, how much work does the motor do in lifting the elevator 20 m? If this is done in 5.78 s (so v = 3.46 m/s), what power is the motor using? If the elevator is lowered at a constant 3.46 m/s, how much power is used?
If all 45 hp is used, what is this elevator's top speed, in m/s and miles/hour?

4-29: If a car uses 12 hp to cruise at a steady 60 km/hr, what is the total force (internal, tire friction, air resistance, etc) impeding the car?
For a 65 kg skydiver with a constant "terminal velocity" of 125 miles/hour (or 55.9 m/s), at what rate is energy being dissipated by air resistance? How much energy is dissipated in 5 seconds of falling, and what happens to it?

4-30: How much work is needed to push a 1400 kg box up a frictionless 10° plane at 17 m/s for 5.0 s?

If the friction/... against a 1400 kg car at 17 m/s is 550 N, how much power is needed when this car climbs a 10° hill at 17 m/s?
Using this same power, how steep a hill could this car climb at 34 m/s?

4-31: The force required to tow a boat at constant speed is approximately proportional to the speed. If it takes 8 hp to tow a certain boat at 2.0 miles/hour, how much power is required to tow it at 6.0 miles/hour?
{ Hint: you don't have to change v to SI units. }

4-32: A car coasts down a 5.0° hill at 81 km/hr. How much power does it need to climb this hill at 22.5 m/s?

4-33: **optional** (if you use calculus)
If the position of a 5.0 kg object is given by
$x = 2t^3 - 5t^2 + 8t$ [with x and t in m and m/s],
where is the object and what is its KE when t = 2 s?
What force and power act on it at 2 s?
How much work is done on it from 0 s to 2 s?

for Section 4.6,

4-31: This graph shows the force on a 5 kg object.
If v = +3.0 m/s at x = 2.0 m, what is v when x = 6.0 m?

4-32: Use this F-x graph, which shows that F_{spring} increases as x (the distance away from x_e) increases, to derive the formula that $\Delta PE_{spring} = \frac{1}{2}\, k x_f^2 - \frac{1}{2}\, k x_i^2$.

for Section 4.7,

4-33: The velocity of 5.0 kg object changes from +3.4 m/s to –7.2 m/s. How much work was done on the object?

4-31: Using full power, a 2.0 x 10^6 W locomotive accelerates a train from 5 m/s to 25 m/s in 4.0 minutes. Ignoring friction, find **a)** mass of train-plus-locomotive,
b) a formula that shows how v depends on t,
c) velocity and acceleration halfway through the interval.
d) Is acceleration constant between i & f?

4-32:

for Section 4.8,

4-33: A force of 300 N in the → direction acts for 15 ms on a 20 gram object initially at rest. What is the impulse, and what is the object's velocity and momentum?

4-31: A 145 gram baseball is thrown ← at 43 m/s. The hitter's bat exerts an average force of 12500 N and hits the ball back toward the pitcher at 43 m/s →. What is the ball's change of speed and change of velocity?
How long was the bat in contact with the ball?

4-32: **internal & external forces**
How can a car's mv be changed by its own "internal" engine or brakes?
When a block drops downward its speed increases. Does this violate the "law of momentum conservation"?
Will a boxer tire more when he misses his opponent or hits him?

You stand on top of a box and hit its side repeatedly with a sledge hammer. Can you make the box move if there is no friction? lots of friction? an in-between amount?

4-32: A 50 kg bag of sand drops 1.5 m onto a 200 kg cart initially moving at +10 m/s → on horizontal rails. After the bag comes to rest on top of the cart, what is the final velocity of cart-and-bag?

Is vertical momentum conserved? Why?

4-33: You stand on the ground and see a 100 kg train with a 70 kg cannon (and 30 kg cannonball) move → at 4 m/s on frictionless rails. What is final velocity of the train and the ball, if the cannon shoots the ball 28 m/s [with respect to the train] in these directions:
a) forward →, **b)** backward ←, **c)** sideways ↑ [as seen from ground] and **d)** sideways ↑ [as seen from train]?

4-31: **Why isn't KE also conserved?**

A bullet (40 g, v_i = 300 m/s →) travels 8 cm into a wood block (4 kg, v_i = 0). If we assume that the bullet/block force is constant during the collision [which ends when bullet & block are moving → at the same speed], **a)** during the collision, what is the Δx of the bullet? of the block? **b)** What average force acts on the bullet? on the block? **c)** How much work is done on the bullet? on the block? For each object, does W=ΔKE?

{ Hint: Draw a picture of the collision's initial and final points, use FΔt=Δp, tvvax (twice), FΔt=Δp (twice), and FΔx=ΔKE (twice). }

4-32: What is the minimum & maximum KE-retention that can occur during a totally inelastic collision? { Think of an example at each extreme. } For each example, what is the "inelastic KE-retention range".

4-33: A gun attached to a wall fires eight 40 g bullets per second, with v = 300 m/s →. What average forces do the gun & wall feel?

If the bullets are "stopped" by a wall, what average force do they exert? If they "bounce" away from the wall elastically, what force do they exert?

When these bullets hit the wall, what is the meaning of "average force"? Make a F-versus-t graph to show the real situation and the "average force" analogy.

4-31: **three types of rockets**

If the gun in Problem 4-## (m_i = 5.00 kg) is free to move on frictionless rails, what is its velocity after 3 seconds of firing? { Hint: first assume the gun's mass is constant and calculate v_f. Then calculate the Δv caused by the first bullet and by the last bullet, and estimate the actual v_f. }

What is the speed of a 5.0 kg gun that fires one .96 kg bullet at 300 m/s?

If a 5 kg rocket continuously ejects 400 g of fuel-air mixture per second at 300 m/s, what is its velocity after 3 seconds?

Will your answer change if the rocket is in outer space where there is no "air"? Can you jump higher when your feet exert force against sand, or against concrete? Why?

If a "water hose rocket" ejects water at 4 kg/s and 30 m/s, what is its speed after 3 s? ==need m of hose!

4-32: == f nec for water hose??

for Section 4.9,

4-34: A 1000 kg car moving north at 15 m/s hits a 4000 kg truck moving west at 20 m/s. If they become meshed together, what is their velocity immediately after the collision?

4-35: A 120 kg sled moves across ice (that is assumed frictionless) at 5.0 m/s →. When it fires a bullet at 212 m/s ↑ [as seen from the ground] the sled's velocity changes by 10°. What is the bullet's mass?

4-36: What is v_f for this totally inelastic collision on a frictionless horizontal surface?

4-37: After an elastic collision between equal-mass objects, one of which is initially at rest, the objects always move off at right angles to one another. When a billiard ball moving → at 4.40 m/s collides elastically with an identical stationary ball, it moves away at 2.20 m/s. What angle does it make with its original line-of-motion?
{ Hints: The collision is elastic, sin(90° − θ) = cosθ,
and ==[nec?] sinθ /cosθ = tanθ. }

for Section 4.10,

4-38: For these elastic collisions, predict the final direction of each ball:

4-39: Two 5 kg blocks collide head-on: v_1 = 4 m/s →, v_2 = 5 m/s ←. Find their final v's.
{ Hint: This is easy. Use the last sentence of 4.10. }

4-40: Study the equations and decide your algebra strategy [order of solving equations] if you are "given" all of the variables except **a)** v_{1f} & v_{2f}, **b)** v_{2i} & v_{1f}, **c)** v_{1f} & m_1, **d)** v_{1i} & v_{2i}, **e)** m_1 & m_2, **f)** v_{1i} & m_1.

{ If you want, use numbers from Problem 4-I to see what the equation looks like after substitution, and to do the algebra. }

for Section 4.11,

4-41: Find the c-of-m for this system:

4-42: If you are at rest in the middle of a lake on totally frictionless ice, is there any way you can get to the shore?

4-43: If the 150 kg iceboat in Problem 4-J is initially moving → at +4 m/s, and the 50 kg woman walks 10 m ← in 5.0 s, what displacement do you (standing on the shore) see for the boat and woman?

4-44: A 50 kg mother (at x = +7.0 m) and her 20 kg daughter (at x = +4.0 m) decide to exchange places on a stationary 90 kg iceboat. After this move, where are the mother and daughter?

4-45: Analyze Problem 4-## as if you are "riding along with the system's center of mass". How will you answer the question: What final velocity will a person on the ground observe for the train and the ball?

4-46: When t = 5 s, a 20 kg object is at x = +16 m, moving → at +8 m/s on a frictionless surface. At this instant a firecracker inside the object explodes, splitting the object into two parts. One part moves ← at 4 m/s. What is the velocity of the other part?

Is kinetic energy conserved?

When t = 9s, where is the center of mass?

4-47: A 5 kg block accelerates uniformly from 6 m/s to 14 m/s in 4 s. What is its acceleration?
What force acts on it?

The kinetic energy of an object increases from 90 J to 490 J while being pushed 40 m. What force acts on it?

How long does it take for the velocity of a 5 kg block, pushed with 10 N force, to increase from 6 m/s to 14 m/s?

FAST FOOD: a 5 kg melon falls from a 3 m building. What is its maximum speed?

4-48: The top block starts from rest, and the ramp is frictionless. After the totally inelastic collision, how high does the block rise? { First make an educated guess, then find the answer by calculation. }

4-49: A driver travels 240 km at a speed of 60 km/hr, and continues for another 240 miles at 40 km/hr. What is her average speed for the 480 km trip?

4-50: If falling from height "h" causes a maximum velocity (just before impact) of "v", what v_{max} is caused by a fall from a height of 2h? To get a v_{max} of 2v, how far must an object fall?

Using FΔt=Δmv ratio logic, what can you say about the relative travel times for falls from heights of h and 4h?

Two 1000 kg cars are pushed by a force of 2000 N for 100 m. One car starts from rest, the other has v_i = 25 m/s. Compare their increases in KE, in v^2, in mv, and v. What can you say about the Δt for each car's 100 m interval?
If each car is pushed for an equal time, which car has a larger KE increase?

4-51: Two blocks have masses of m & 3m. If both blocks have the same momentum, which has a larger KE? If they have the same KE, which has larger momentum?

4-52: A 50 g bullet is fired from a rifle into a 3.0 kg block. The bullet-and-block slide 67 cm on a horizontal surface (with μ_k = .40) before coming to rest. What is the bullet's pre-impact speed?

4-53: When a 20 kg block is moving → at +8 m/s, a firecracker explodes, producing two pieces (5 kg and 15 kg) that move in opposite ←→ directions, and supplying 480 J of extra kinetic energy. What is the final velocity of each piece, if the large piece ends up moving faster?

4-54: optional (requires use of ΔPE_{spring})

A 5 kg block moving 20 m/s → collides with a 3840 N/m spring attached to a 15 kg block (n) that is moving 4 m/s →. What is the spring's maximum compression?

4-55: Some problems in earlier chapters can be solved using FΔx=ΔKE or FΔt=Δp.

==maybe put this in Section 4.12, as general statement, refer to 3-B and 3-C (check for others in 2/2.91 & 3/3.91?

Solve Problems 3-B (second paragraph's questions) and 3-C, using strategies from Chapter 4.

4.92 Solutions

4-1: m = w/g = (14000 N)/(9.8 m/s^2) = 1430 kg. Convert v to SI units: (72 x 10^3 m)/(3600 s) = 20 m/s. KE ≡ $\frac{1}{2}$ mv^2 = $\frac{1}{2}$(1430 kg)(20 m/s)^2 = 286000 J.

{10° is a "decoy"; the v in $\frac{1}{2}$ mv^2 represents speed (magnitude), not velocity (magnitude-and-direction).}

4-2: m = 2000/9.8 = 204 kg. ø = 25° because there is a 25° angle between **F** and **d**. Use W⇒ΔKE:

$$F\,d\cos\o = \tfrac{1}{2}\,m\,v_f^2 - \tfrac{1}{2}\,m\,v_f^2$$
$$(90)(12)(\cos 25°) = \tfrac{1}{2}(204)\,v_f^2 - \tfrac{1}{2}(204)(2)^2$$
$$408 + 979 = 102\,v_f^2$$
$$3.7\text{ m/s} = v_f$$

4-3: For the five intervals: W is + (because v and KE increase), −(v & KE both ↓), 0 (v is constant), −(v ↓), and + (KE ↑ when speed ↑ from 0 to 2 m/s, even though number-line velocity ↓ from 0 to –2 m/s). As emphasized in 4-2, the v in $\frac{1}{2}$ mv^2 represents speed.

4-4:

$$
\begin{aligned}
W &= \Delta KE \\
F\ d &= \tfrac{1}{2}\ m\,v_f^2 \;-\; \tfrac{1}{2}\ m\,v_f^2 \\
F\,(.08) &= \tfrac{1}{2}(.040)(0) \;-\; \tfrac{1}{2}(.040)(300)^2 \\
F &= -22500\ \text{N}
\end{aligned}
$$

The bullet feels F in the ← direction; this is what slows it down to a stop. The third-law partner of this force pushes the block 22500 N in the → direction.

$$
\begin{aligned}
F\quad d &= \tfrac{1}{2}\ m\,v_f^2 \;-\; \tfrac{1}{2}\ m\,v_f^2 \\
(-22500)(.08) &= \tfrac{1}{2}(.040)\ v_f^2 \;-\; \tfrac{1}{2}(.040)(300)^2 \\
184\ \text{m/s} &= v_f
\end{aligned}
$$

If 8 cm of wood-force stops the bullet, 5 cm of this force takes away 5/8 of its KE, reducing it to 3/8 (less than half) of its original value. Do you see why v is still larger than half its original 300 m/s value?

4-5: d = 5.0 m for each of the forces.

W_{you} = (400 N)(5.0 m)(cos 0°) = +2000 J,
W_{grav} = $F_{parallel}$ d = (–168 N)(5.0 m) = –840 J,
W_{normal} = (460 N)(5.0 m)(cos 90°) = 0, and
W_{total} = ΔKE = $\tfrac{1}{2}(50)(3.8)^2 - \tfrac{1}{2}(50)(2.0)^2$ = +260 J.

Total non-friction work = +2000 –840 = +1160 J. To get a W_{total} of +260 J, $W_{friction}$ must be –900 J.
W = F d cosø is –900 = f_k (5.0)(cos 180°) = μ_k N(5.0)(–1) = μ_k (mg cos20°)(–5) = μ_k (460)(–5). We can solve for "friction force = f_k = 180 Newtons" and "coefficient of friction = μ_k = .39".

4-6: For a train-observer: W = Fd = (28)(9) = 252 J, and ΔKE = $\tfrac{1}{2}(14)(6)^2 - 0$ = 252 J. W = ΔKE.

During the 3 s, an observer on the ground sees the train move (3 s)(20 m/s) = 60 m →. The box moves an extra 9 m →. W = Fd = (28)(60 + 9) = 1932 J.

The observed box speed changes from 20 m/s (the same as the train) to 26 m/s, so ΔKE = $\tfrac{1}{2}(14)(26)^2 - \tfrac{1}{2}(14)(20)^2$ = 1932 J. Again, we find that the results "support" the validity of W=ΔKE.

Each observer measures a different value of W (and ΔKE), but they agree that the relationship "W=ΔKE" is true. This is what we should expect. In fact, one of the two foundations for Einstein's *theory of relativity* is that physical laws (like W=ΔKE) are the same for observers in all non-accelerated reference frames.

4-7: F is in direction of v, so W is +. W = F d = (6726 N)(5.0 m) = 33650 J. This makes KE ↑ from $\tfrac{1}{2}(570)(2.5)^2$ [= 1780 J] to 1780+33650=35430 J, so $\tfrac{1}{2}(570)\ v_f^2$ = 35430 can be solved for v_f = 11.1 m/s.

4-8: **a)** Your vertical "box-supporting force" is ⊥ to v-and-d, and ΔKE = 0, so W_{you} = 0. **b)** The box gains $\tfrac{1}{2}(20)(3)^2$ = 90 J of KE because you do 90 J of work by pushing it in the → direction; the W done by your ↑ force is still 0. **c)** If the box is unaccelerated, the F_{total} acting on it is zero because the box's ↓ mg of (20)(9.8) is balanced by your ↑ force of 196 N. W_{you} = (+196 N)(+.6 m) = +118 Nm = +118 J. The box has ΔKE = 0, so W_{total} = 0; your W of +118 J [W is + because you push in the same direction as v] is canceled by $W_{gravity}$ = –118 J [which is
– because the $F_{gravity}$ direction is opposite v-and-d].

4-9: This is similar to Problem 4-#. During each drop, $W_{gravity}$ = +(1200 x 9.8)(5.0) = 58800 J. This KE is used to do work on the stake, $(4 \times 10^6)\Delta y$ = 58800 J; each drop drives it downward a distance of Δy = .015 m = 1.5 cm.

4-10: The shallow ramp requires F = mg sin 11.5° = 97.7 N, and W = (97.7 N)(15 m) = 1470 J.
The steep ramp requires F = mg sin 36.9° = 294 N, and W = (294 N)(5 m) = 1470 J.
Both ramps produce a height increase of 3.0 m, (15.0 m)(sin 11.5°) or (5.0 m)(sin 36.9°), so the same W_{you} is needed to overcome $W_{gravity}$ in each case. The shallow ramp lets you use less force, but it makes you push the box a longer distance. This is similar to the "pulley gear" in Problem 3-#, where *leverage* lets you gain something (decreased force) by giving up something (increased distance).

Bonus: If there is friction (same μ_k for each ramp), will pushing the box up each ramp still require the same work? {Answer is after Solution 4-##.}

4-11:

Notice that ø is 70° (the angle between the vectors of force and diplacement), not 20° (this is "θ", the ramp's elevation-angle).

$W_{gravity}$ can also be calculated by multiplying the force by the "component of d in the direction of F". W = F $d_{parallel}$ = (mg)(d sin 20°) = .34 mgd. {$F_{gravity}$ is ↓, so $d_{parallel}$ is movement in the ◊ direction.}

{4-10}: No. The 11.5° ramp has W = mg(3) + (μ_k mg cos 11.5)(15 m) = mg(3) + 14.7 μ_k mg, but a 11.5° ramp has W = mg(3) + (μ_k mg cos 36.9)(5 m) = mg(3) + 4.0 μ_k mg.

It takes less force (and strength) to push a box up the shallow 11.5° ramp. But more $W_{friction}$ must be overcome, so a larger amount of work is required.

4-12: If A brakes perfectly (up to the $\mu_s N$ limit but not over it) his work is $f_s\,d = \mu_s N d = (.50)(9.8\,m)d$. To decrease KE from $\frac{1}{2}\,m(24)^2$ to 0 requires 288m of $W_{friction}$, so $288m = (.50)(9.8m)d$, and $d = 58.8$ m.

For B, $\frac{1}{2}\,m(12)^2 = (.50)(9.8m)d$, and $d = 14.7$ m.

A cannot stop (59 m is longer than the 40 m that is available), but B can (because 15 m < 20 m).

KE $\propto$ v² and **W $\propto$ d**, so **v² $\propto$ d**. If v doubles, d increases by a factor of 4 (not 2). This is one of the many reasons why driving at high speed is dangerous!

4-13: Each object falls through a Δh of 3.0 m. {For the ball-on-a-string, $\Delta h = 5.2 - (5.2\cos 65°)$.} The TWE is:

$$KE_i + PE_i = KE_f + PE_f$$
$$\tfrac{1}{2}\,m(0)^2 + mg(+3) = \tfrac{1}{2}\,m v_f^2 + mg(0)$$
$$7.7\text{ m/s} = v_f$$

A useful way to think about this situation is *energy accountability:* some PE (= mgh) is "converted" into KE ($= \frac{1}{2}\,m v^2$). ==nec?This intuitive problem-solving strategy is discussed more fully in Problems 4-##.

Notice that m cancels; v_f is independent of mass.

4-14: From A to C, the block gains a KE of mgh = (2)(9.8)(4) = 78.4 J. From C to D, it loses a KE of $W_{friction} = f_k\,d = \mu_k N\,d = \mu_k\,mg\,d = (.4)(2)(9.8)(8) = 62.7$ J, so its remaining KE is $78.4 - 62.7 = 15.7$ J, which will take the block to a height (at point E) of $h = (mgh)/mg = 15.7/(2)(9.8) = .80$ m.

On the return trip through the D-to-C region, W_{fr} eliminates the remaining 15.7 J of KE in a distance of $d = W_{fr}/\mu_k\,mg = 15.7/(.4)(2)(9.8) = 2.0$ m.

{Notice the energy conversions: mgh→KE from A to C, KE→mgh from D to E, mgh→KE from E to D, and KE→W_{fr} from D to the final resting place.}

If the block begins at B, mgh→KE only gives it a KE of (2)(9.8)(2) = 39.2 J, so it can't make it through the C-to-D region. We can write this A-to-x_f TWE,

$$KE_i + PE_i = KE_f + PE_f + W_{fr}$$
$$0 + mg(2) = 0 + mg(0) + .4\,mg\,d$$

and solve for d = 5.0 meters past C.

4-15: By playing with the heights, you'll discover that A is 20 m above C, B is 10 m below C, and D is 15 m below C. At C, the coaster's KE is $\frac{1}{2}(500)(20)^2 = 100$ kJ. At A it is lower by $mg\Delta h = (500)(9.8)(20) = 98$ kJ, $\frac{1}{2}(500)\,v^2 = 2$ kJ = 2000 J, and $v_A = 2.8$ m/s.

At B and D, KE is larger by (500)(9.8)(10) and (500)(9.8)(15), respectively. As above, we can solve $\frac{1}{2}(500)\,v^2 = 149000$ and $\frac{1}{2}(500)\,v^2 = 173500$ for $v_B = 24.4$ m/s, and $v_D = 26.3$ m/s. Do these speeds seem reasonable, compared with the 20 m/s at C?

4-16: The block's upward movement increases its $PE_{gravity}$ by $mg\Delta h = m(9.8)(1.7\sin 40°) = 10.71m$. This is less than its initial KE [$= \frac{1}{2}\,m(5.4)^2 = 14.58m$] by 3.87m, because $W_{friction} = 3.87m$. {All energies are in Joules.} Symmetry: the block loses another 3.87m due to $W_{friction}$ on its way down the ramp, so its final KE is 14.58m – 3.87m –3.87m = 6.84m, and we can solve $\frac{1}{2}\,m v_f^2 = 6.84m$ for 3.7 m/s.

Or you can set up and solve these TWE's for the bottom-to-top and top-to-bottom trips:

$\frac{1}{2}\,m(5.4)^2 + mg(0) = \frac{1}{2}\,m(0)^2 + mg(1.7\sin 40°) + W_{fr}$,

$\frac{1}{2}\,m(0)^2 + mg(1.7\sin 40°) = \frac{1}{2}\,m v_f^2 + mg(0) + W_{fr}$. Solve the first equation for W_{fr}, substitute it into the second equation and solve for v_f.

To find μ_K: $W_{friction} = \mu_k\,mg\cos\theta\,d$, $3.87m = \mu_k\,m(9.8)\cos 40°\,(1.7)$, solve for $\mu_k = .30$.

4-17: The TWE is: $\frac{1}{2}(200)(10)^2 + 200(9.8)(14) + (38 \times 10^3) = \frac{1}{2}(200)(25)^2 + (200)(9.8)h + (4 \times 10^3)$. This can be solved for h = 4.6 m above the ground, which is 2.6 m above the edge of the ramp.

4-18: The blocks' initial KE is $\frac{1}{2}(4)(2)^2 + \frac{1}{2}(5)(2)^2 = 18.0$ J. The 5 kg block will fall 1.6 m, thus adding (5)(9.8)(1.6) = 78.4 J of KE to the system. The 4 kg block rises 1.6 m, decreasing KE by (4)(9.8)(1.6) = 62.7 J. The system's final KE is +18.0 +78.4 –62.7 = 33.7 J. This equals $\frac{1}{2}(4+5)\,v_f^2$, and $v_f = 2.7$ m/s.

Another method is to write a TWE. If "x" is h_i for the 4 kg block, its h_f is "x + 1.6". The 5 kg block has $h_i = +1.6$ m, and $h_f = 0$. Notice that, as emphasized in Section 4.3, <u>↑ must be defined as the + direction for the mgh of both blocks</u>. This is different than the definitions for F=ma "matched motion", where ↑ is + for the 4-block and ↓ is + for the 5-block.

4-19: Initially, the average height of each brick is .04 m. This picture shows that the 4 bricks have Δh's of 0, .08 m, .16 m and .24 m, respectively. The total ΔPE is 3.4g(0)+3.4g(+.08)+3.4g(+.16)+3.4g(+.24) = 3.4g(.48) = 16.0 J. This is the "stacking work".

Here is a shortcut. As shown below, the average h_i of the 4 bricks is .04 m, and their average h_f is .16 m. We can treat the bricks as a single 13.6 kg object with $\Delta PE = mg\Delta h = 13.6g(.16 - .04) = 16.0$ J.

It is easy to find ΔPE (and thus W) for the 40-brick pile, using this shortcut. The average h_i is still .04 m. The average h_f is $\frac{1}{2}$(40 bricks)(.08 m/brick) = 1.6 m. $W_{you} = \Delta PE = (40 \times 3.4)(9.8)(1.6 - .04) = 2080$ J.

For the 20%, $h_{average}$ drops from –.06 m to –.54 m, and $\Delta PE = (.20m)(9.8)([-.54] - [-.06]) = -.94m$.

For the 80%, $h_{average}$ changes from 0 to –.24 m, and $\Delta PE = (.80m)(9.8)([-.24] - [0]) = -1.88m$.

For the entire chain, the ΔPE decrease of 2.82m is "converted" into KE: $\frac{1}{2}\,m v_f^2 = 2.82m$, $v_f = 2.4$ m/s.

4-20:

4-21:

4-22:

4-23:

4-24:

4-25:

$$P \quad \Delta t = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2$$
$$(80 \times 746)\Delta t = \tfrac{1}{2}(1200)(13.4)^2 - \tfrac{1}{2} m(0)^2$$
$$\Delta t = 1.8 \text{ seconds}$$

For 13.4-to-26.8, $\Delta KE = (431000 \text{ J}) - (108000 \text{ J}) = 323000$, $\Delta t = W/P = \Delta KE/P = 323000/(80 \times 746) = 5.4$ s. This is 3 times longer than the 0-to-13.4 time.

There are two good ways to explain the longer Δt.

When v = 6.7 m/s, $F = P/v = (80 \times 746)/6.7 = 8907$ N, and $a = F/m = 8907/1200 = 7.4 \text{ m/s}^2$. If the tires could provide enough friction, and acceleration is constant over the whole time interval (is it?) we would estimate a 0-to-13.4 Δt of $\Delta t = \Delta v/a = (13.4-0)/7.4 = 1.8$ seconds, just what we calculated above. But if v = 20.1 m/s, $a = (P/v)/m = [(80 \times 746)/20.1]/1200 = 2.47 \text{ m/s}^2$. If we assume constant a, $\Delta t = \Delta v/a = (26.8 - 13.4)/2.47 = 5.4$ s. One explanation: use ratio logic on P=Fv, F=ma, $\Delta v = a\Delta t$: if P is constant and v ↑, F ↓ ; this causes an ↑ of a, and ↓ of Δt.

Because $KE \propto v^2$ and 26.8 is twice 13.4, the 0-to-26.8 ΔKE (and W) is 4 times the 0-to-13.4 ΔKE, and the 13.4-to-26.8 ΔKE is 3 times the 0-to-13.4 ΔKE, so [if P is constant] 3 times as much Δt is needed.

== compare with real --- from Motor Trend, or change my numbers to match theirs

4-26: If we ignore horizontal movement and only consider work for ↑ movement,
$P = W/\Delta t = mg\Delta h/\Delta t = (57)(9.8)(5.0)/3.5 = 800$ W, or $(800 \text{ W})(1 \text{ hp}/746 \text{ W}) = 1.1$ hp.

4-27:

$$P \quad \Delta t = m \quad g \quad \Delta h$$
$$(3 \times 746)\ \Delta t = (350)\,(9.8)\,(14)$$
$$\Delta t = 21.5 \text{ seconds}$$

4-28: Work needed to lift the elevator is $mg\Delta h = 990(9.8)(20) = 194000$ J. The falling counter-weight provides 800(9.8)(20) = 157000 J, so the motor only has to do 37000 J of work. {This is the same as lifting an "effective weight" of (990 kg – 800 kg) a height of 20 m.

$P = W/\Delta t = 37000 \text{ J} / 5.78 \text{ s} = 6400 \text{ J/s} = 8.6$ hp. This is much less than the 45 hp needed in Problem 4-D without the counter-weight. {This is like lifting a 190 kg mass at constant speed (so T = mg = 1860 N) of 3.46 m/s: P = Fv = (1860)(3.46) = 6400 W.}

To lower the elevator at constant speed also requires T = 1860 N & P = 6400 N. Same F & same v, so same P.

P = Fv, so $v = P/F = (45 \times 746)/(190 \times 9.8) = 18.0$ m/s. Using the ".447 trick" from Problem 1-==, (18.0 m/s) / .447 = 40.3 miles/hour.

4-29: The car's engine is producing a "forward" → force of $F = P/v = (12 \times 746)/(60 \times 10^3/3600) = 537$ N. The car is not accelerating, so friction/... must be canceling this force by producing an equal force of 537 N in the ← direction.

The skydiver is not accelerating, so $F_{gravity}$ [= mg = 65(9.8) = 637 N ↓] must be canceled by an air resistance force of 637 N ↑, which dissipates heat at a rate of P = Fv = 637(55.9) = 35600 W = 47.7 hp.

In 5 s, $W = P\Delta t = (35600 \text{ J/s})(5 \text{ s}) = 178000$ J. As discussed in Section 7.7, $W_{gravity}$ [$mg\Delta h$] is being converted into *molecular motion KE (thermal energy)* that raises the temperature of skydiver-and-air, instead of into the *forward motion KE* of the skydiver.

4-30: In 5 s, the car moves (17 m/s)(5 s) = 85 m up the plane, $\Delta h = (85 \text{ m})\sin 10° = 14.8$ m, and $W = \Delta KE + \Delta PE = 0 + 1400(9.8)(14.8) = 203000$ J.

P is needed to overcome friction/... and climb the hill. Climbing-P is calculated using the same logic as above: $P_{climbing} = mg\,\Delta h/\Delta t = mg(v\Delta t \sin\theta)/\Delta t = mgv\sin\theta$ {by playing with geometry as in Problem 4-##, you'll see that this is just $Fv\cos\varnothing$}, and $P_{total} = P_{friction} + P_{climbing} = 550(17) + 1400(17)(9.8)(\sin 10°) = (9350 \text{ W}) + (40500 \text{ W}) = 49850 \text{ J/s} = 67$ hp.

If we assume that $F_{friction/air}$ stays the same when speed doubles [do you think this is reasonable?]*, $49850 = 550(34) + 1400(34)(9.8)\sin\theta$, and $\theta = 3.8°$. As you expect, the car goes slower up a steep hill.

* probably not; $F_{friction/air}$ is higher at 34 m/s, so the actual θ is less than 3.8°.

4-31: When v triples, F also triples (if towing-F is proportional to v), and P (= Fv) increases by a factor of (x3)(x3) = 9, so you'll need 9(8 hp) = 72 hp.

4-32: If speed is constant, $F_{friction/...}$ [which points uphill to slow the car's speed] must be just enough to cancel $F_{gravity}$ (= mg sinθ) that points down the hill.

When the car climbs it must overcome the forces of gravity and friction, which both point down the hill. 81 km/hr = 22.5 m/s, so we'll assume that $F_{friction}$ still equals $mg\sin\theta$, and $P_{total} = P_{friction} + P_{climbing} = (mg\sin\theta)v + (mg\sin\theta)v = 46100 \text{ W} = 62$ hp.

4-33: When t = 2, $x = 2(2)^3 - 5(2)^2 + 8(2) = +12$ m.

$\mathbf{v \equiv dx/dt} = 6t^2 - 10t + 8 = 6(2)^2 - 10(2) + 8 = +12$ m/s, and $\mathbf{KE = \tfrac{1}{2} mv^2} = \tfrac{1}{2}(5)(12)^2 = 360$ J.

$\mathbf{a \equiv dv/dt} = 12t - 10 = 12(2) - 10 = +14$ m/s per s, and $\mathbf{F = ma} = (5)(+14) = +70$ N.

$\mathbf{P = Fv} = (+70)(+12) = 840$ Watts.

We can find W by integrating either "dW = P dt" or "dW = F dx".

W = ∫ dW = ∫ P dt = ∫ Fv dt = ∫ ma v dt = $\int m(12t - 10)(6t^2 - 10t + 8)\,dt = \int [m(72t^3 - 180t^2 + 196t - 80]\,dt = m[18t^4 - 60t^3 + 98t^2 - 80t]$. Evaluating this from $t_i = 0$ to $t_f = 2$ gives $5[18(2)^4 - 60(2)^3 + 98(2)^2 - 80(2)] - 5[18(0)^4 - 60(0)^3 + 98(0)^2 - 80(0)] = (+200) - (0) = +200$ J.

ΔW = ∫ F dx = ∫ ma dx = $\int [m(12t - 10)]\,dx$. There is a "mismatch" [the ∫ has t and dx] so we solve "$dx/dt = 6t^2 - 10t + 8$" for "$dx = (6t^2 - 10t + 8)dt$ and substitute for dx to get $\int m(12t - 10)](6t^2 - 10t + 8)\,dt$. This is the same ∫ as above; it can also be "evaluated" as $m[\frac{1}{2}(6t^2 - 10t + 8)^2]$, which {from t_i to t_f} gives us $5(\frac{1}{2})[6(2)^2 - 10(2) + 8]^2 - 5(\frac{1}{2})[6(0)^2 - 10(0) + 8]^2 = (+360) - (+160) = +200$ J, the same W as above.

4-31: The i-to-f W from $x_i = 2$ to $x_f = 6$ is the F-x area from 2 to 6:
$(20\text{ N})(1\text{ m}) + \frac{1}{2}(20\text{ N})(1\text{ m}) + 0 + (-20\text{ N})(1\text{ m}) = +10$ J.
Substitute into W = ΔKE:
$10 = \frac{1}{2}(5\text{ kg})v_f^2 - \frac{1}{2}(5\text{ kg})(3\text{ m/s})^2$, and $v_f = 3.6$ m/s.

4-32: W = the -shaped area between x_i to x_f
= [0-to-f area,] – [0-to-x area,]
= $\frac{1}{2}(x_f - 0)(k x_f) - \frac{1}{2}(x_i - 0)(k x_i)$
= $\frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2$

4-33: There isn't enough information to solve "W = F Δx", but W is also equal to ΔKE: $W = KE_f - KE_i = \frac{1}{2}(5.0)(-7.2)^2 - \frac{1}{2}(5.0)(+3.4)^2 = +101$ J.

4-31: **a)** P is constant, so $P\,\Delta t = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$: $(2 \times 10^6)(240) = \frac{1}{2} m(25)^2 - \frac{1}{2} m(5)^2$, $m = 1.6 \times 10^6$ kg.

b) Substitute $m = 1.6 \times 10^6$ kg, $P = 2.0 \times 10^6$ W, and $v_i = 5$ m/s into $P\Delta t = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$, then solve for $v_f = \sqrt{25 + 2.5t}$.

c) When t = 2 min = 240 s, $v = \sqrt{25 + 2.5(120)} = 18.0$ m/s. Notice that v changes by 13 m/s (from 5 to 18) during the first 2 minutes, and by only 7 m/s (from 18 to 25) during the final 2 minutes.

$a = F/m = (P/v)/m = [(2.0 \times 10^6)/18.0]/(1.6 \times 10^6) = .069$ m/s per s.

d) No. P (=Fv) is constant; but as v ↑, F-and-a ↓.

4-32:

4-33: Impulse ≡ $F_{ext}\,\Delta t = (300\text{ N})(15 \times 10^{-3}\text{ s}) = 4.5$ Ns = 4.5 kg m/s in the → direction.

This impulse makes the object's momentum change from 0 to its final value of 4.5 kg m/s →.

Momentum ≡ mv, so $v_f = (mv)_f/m = (4.5\text{ kg m/s})/(.020\text{ kg}) = 225$ m/s.

This information can be summarized in an equation:

impulse	=	**final momentum**	–	initial momentum
$F_{ext}\ \Delta t$	=	$m_f\ \ v_f$	–	$m_i\ \ v_i$
(300 N)(.015 s)	=	(.020 kg)(225 m/s)	–	(.020 kg)(0)
4.5 kg m/s	=	4.5 kg m/s	–	0

4-31: The ball's initial & final speed is 43 m/s; it doesn't change. But velocity changes from –43 m/s (←) to +43 m/s (→), a change of Δv = +86 m/s. It is easy to see this change on a number line:

–43 ----------(Δv = +86)--------> +43

If the ball is our system, the bat's force is external,

$F_{ext}\ \Delta t = (mv)_f - (mv)_i$
$(12500)\ \Delta t = (.145)(+43) - (.145)(-43)$
$\Delta t = .0010$ s

4-32: The engine (or brakes) can cause Δv because **friction** between the car-tires and road is $F_{external}$.

If we choose the block as a system, its downward mv increases because "mg" is supplied by an external agent: the earth. If we define block-and-earth as our system, mv is conserved because the block's ↓ mv is canceled by the earth's ↑ mv (third law: block is pulled downward, and earth is pulled upward).

If the boxer misses, he must produce the force that stops his own arm, thus tiring himself more than if his arm is stopped by an F_{ext} (the opponent's body).

With medium friction, you can swing the hammer back slowly (so static friction holds the box in place) and forward quickly (so it hits the box hard enough to "break loose" from friction); friction [whether static or kinetic] is F_{ext}, so... If there is no friction the box moves equal amounts, in opposite directions, during the backswing and forward swing. But if there is too much friction, you cannot overcome it during the box-hitting phase of the hammering cycle.

4-32: If bag/cart is a system, the forces (◊ N and ↔ friction) between bag & cart are internal, $F_{ext} = 0$, and for motion in the horizontal direction:

$(mv)_i = (mv)_f$
$50(0) + 200(+10) = (50 + 200)\,v_f$
$+8\text{ m/s} = v_f$

Just before the bag hits the cart, the bag (and thus the system) has ↓ momentum. After collision, the system's ◊ momentum is zero.

Why? During the collision, ◊ N-force between cart and bag increases; this is F_{int} so it doesn't change the system's mv. The ↑ N-force exerted on the cart by the rails also increases (it is temporarily larger than the system's "mg"); this is F_{ext} that makes the system's ↓ momentum decrease to zero.

4-33: We'll only consider mv in the ↔ direction of the rails. The entire system has v_i = +4 m/s. If we define the train's v_f as "v", the cannonball's v_f is **a)** +v+28 [it moves → 28 m/s faster than the train], **b)** +v–28 [do you see why?], **c)** zero [you see it move ↑ but not ↔], **d)** +v [if a train rider sees the box move ↑, you see it go → at the same speed as the train]. The cannon-shot is F_{int}, since train/cannon and cannonball are both part of the system; F_{ext} = 0, so $(mv)_i = (mv)_f$. $(mv)_i$ = (100+70+30)(+4) = +800.

The equations-and-solutions are:

a) +800 = 170 v + 30(v+28), v = –.2 m/s.
{ $v_{train} \equiv$ v = –.2, v_{ball} = v+25 = +24.8 }

b) +800 = 170 v + 30(v–28), v = +8.2 m/s.
{ v_{train} = +8.2, v_{ball} = v – 28 = –19.8 }

c) +800 = 170 v + 30(0), v_f = +4.7 m/s.
{ v_{ball} = 0 }

d) +800 = 170 v + 30(v), v_f = +4.0 m/s.
{ ball moves ⁄: 28 m/s ↑ and 4 m/s → }

4-31: If we define bullet-and-block as a system, F_{ext} = 0, .040(+300) + 4(0) = (.040+4) v_f, and v_f = +2.97 m/s. Draw pictures of the i & f situations,

then make tvvax tables for the bullet and block,

Δt	=	T	Δt	=	T
v_i	=	+300	v_i	=	0
v_f	=	+2.97	v_f	=	+2.97
a	=		a	=	
Δx	=	.08 + x	Δx	=	x

choose equations that use this information, and solve:

.08 + x = ½(300+2.97) T x = ½(0+2.97) T
B) substitute this x, A) x = 1.485 T
C) 5.33 x 10^{-4} s = T D) substitute this T,
E) x = 7.92 x 10^{-4} m

Write "$F_{ext}\,\Delta t = mv_f - mv_i$" for systems of bullet-only: F(.000533) = .04(+2.97) – .04(+300), block-only: F(.000533) = 4 (+2.97) – 4 (0), and solve: $F_{on\ bullet}$ = –22290 N, $F_{on\ block}$ = +22290 N. As expected (because they are a third-law pair), these forces are equal & opposite.

We now write FΔx=ΔKE equations (for the bullet, and then for the block) to see if they are consistent with the information we calculated above:

(–22290)(+.08079) = ½(.040)(2.97) ^{2} – ½(.04)(300) ^{2}, so W (= –1801 J) and ΔKE (= –1800 J) are equal.

(+22290)(+.00079) = ½(4)(2.97) ^{2} – ½(4)(0) ^{2}, and we find that W (= +17.65 J) equals ΔKE (= +17.64 J).

Both objects feel the 22290 N equal-and-opposite force for the same Δt, so their "combined momentum" is conserved. But the bullet travels 102 times farther during the collision (.08079 m versus .00079 m), so 99% of the system-KE is lost: 1800 J → 18 J. <u>Equal Δt does not necessarily mean equal Δx</u>, so momentum can be conserved even if kinetic energy isn't.

4-32: For a totally inelastic collision, KE-retention can vary from 0 to almost-100%. Here are examples: (9 kg block, 8 m/s →) and (12 kg block, 6 m/s ←) retains 0% of its initial KE, while (9 kg, 8.1 m/s →) and (12 kg, 8.0 m/s →) becomes (21 kg, 8.04286 m/s →) with 99.996% retention of KE.

For the first example, any collision with retention of KE between 0 and 99.9% is inelastic. For the second example an inelastic collision has KE retention between 99.996% and 99.999%.

4-33: Let's think about what happens during a 3 s interval. If "all-of-the-bullets" is defined as a system, F(3 s) = (8 bullets/s)(3 s)(.040 kg/bullet)(300 m/s) – 0, and F = 96 N. A 96 N → force exerted on the bullets produces their Δmv; its "third law partner" of 96 N ← is exerted against the gun-and-wall.

{Notice that "3 s" cancels from the equation above. In this situation you can either choose a specific time (like 3 s) or a generalized time (like "t").}

If the bullets stop, Δv = (0) – (+300) = –300 m/s. But if they bounce, Δv = (–300) – (+300) = –600 m/s. You can solve the FΔt=Δp equation for each situation, to get F_{stop} = 96 N → against the wall (and 96 N ← against the bullets), and F_{bounce} = 192 N →.

These graphs show the real situation (large F for a short Δt, 8 times per second) and the constant-$F_{average}$ simplification. Both graphs have the same "F Δt area" and cause the same Δp. {But would you rather be hit by the eight bullets, or have them pressed against you with a force of 96 N (22 pounds) for 1 second?}

4-31: The gun has m_i = 5.00 kg. After 24 bullets have fired, with total mass = .96 kg, m_f = 4.04 kg. The gun's "average mass" is ½(5.00+4.04) = 4.52 kg.

If gun-and-bullets is ≡'ed as a system, F_{ext} = 0, (5.0)(0) = (4.52) v_f + (.96)(+300), and v_f = –64 m/s.

m_f = If gun-and-bullets is a system [?]

4-32: == water hose, if done

4-34: Since the question asks "immediately after" we'll assume a Δt so small that we can ignore the external force of friction. For the car/truck system, mv is conserved in the x & y directions:

x: $(1000)(0) + (4000)(-20) = (5000)v_f$, $(v_f)_x = -16$,
y: $(1000)(+15) + (4000)(0) = (5000)v_f$, $(v_f)_y = +3$.

v-magnitude is $v = \sqrt{(-16)^2 + (+3)^2} = 16.3$ m/s.
v-direction is , with $\theta = \tan^{-1}(3/16) = 10.6°$.

4-35: If bullet/block is chosen as system, $F_{ext} = 0$. The ↑ bullet has $p_x = 0$, so the sled has $(p_f)_x = (p_i)_x = (120)(+5.0) = +600$ kg m/s. Draw a p-triangle for the sled (final p_x, p_y, p_{total}) as shown below, and solve for $p_y = -600 \tan 10°$. $(p_f)_y = (p_i)_y = 0$, so the bullet must have $p_y = +600 \tan 10°$ and **m = .50 kg**.

4-36: x & y momentum are both conserved,
x: $5(+5) + 2(0) + 3(-3 \sin 40°) = 11\, v_x$, $v_x = +1.75$,
y: $5(0) + 2(-2) + 3(+3 \cos 40°) = 11\, v_y$, $v_y = +.26$.

v-magnitude is $v = \sqrt{1.75^2 + .26^2} = 1.77$ m/s.
v-direction is ⁄, with $\theta = \tan^{-1}(.26/1.75) = 10.6°$.

4-37: elastic collision ⇒ $\frac{1}{2} m(4.40)^2 = \frac{1}{2} m(2.20)^2 + \frac{1}{2} m v^2$, so $v = \sqrt{4.40^2 - 2.20^2} = 3.81$ m/s.

The angle between the balls' velocities is 90°, so

Because $p_{iy} = 0$ and $F_{ext} = 0$, $p_{fy} = 0$:

$$2.20 \sin\theta = 3.81 \sin(90° - \theta)$$
$$2.20 \sin\theta = 3.81 \cos\theta$$
$$\sin\theta/\cos\theta\ (= \tan\theta) = 3.81/2.20$$
$$\theta = \tan^{-1}(1.732) = 60.0°$$

A check: is x-momentum conserved?
$$m(4.40) + m(0) = m(2.20 \cos 60°) + m(3.81 \cos 30°)$$

The problem is ambiguous about which ball moves at 2.20 m/s, because two different collision angles give two different 60°/30° results. With the "more head-on" of these two collisions, which ball moves away at 60°? { Answer is after Solution 4-##. }

4-38: Using 4.10's intuitive principles: After the first collision both blocks move →, but after collision #2 the lightweight 3-block reverses its direction (to ←) and the 5-block moves →.

If you want, you can find the v's for #1 (+.5 m/s, +2.5 m/s) and #2 (–.5 m/s, +1.5 m/s). Notice that each collision has an incoming-speed of 2 m/s, and a separation-speed of 2 m/s.

{4-39} If the mover-ball barely grazes the sitter, the mover's v-direction and v-magnitude don't change very much. As a collision becomes closer to head-on, the mover's v-change becomes larger. The mover's direction change is 60° for the more head-on of the two collisions that produce a 30°/60° split. {For a head-on collision, the mover's final v is zero! }

4-39: The balls just exchange v's: #1 goes ← at 5 m/s, and #2 goes → at 4 m/s.

4-40: **a)** The left-side equation (#1) and right-side equation (#2) each have 1 unknown; just solve them. **b)** #2 has 1 unknown; solve it and substitute into #1 [which has 2 unknowns at the start]. **c)** Use same strategy as in b; take advantage of the fact that "$m_1 + m_2$" is on the bottom of both fractions, so they have a common denominator. **d, e & f)** #1 and #2 each contain two unknowns. Solve d & f using "leapfrog substitution" [f requires solving a quadratic equation]. In e, solving either equation gives a ratio; for example, Problem 4-I gives "$2m_1 = m_2$" because any masses in a 1:2 ratio [like 3 kg and 6 kg] will give the same collision-results as the actual 5 kg and 10 kg.

4-41: The ellipse's c-of-m is at (+8, –2.5), even though there is no mass at this location.

$$x_{cm} = \frac{2(-4) + 5(-3.5) + 4(+8)}{2 + 5 + 4} = +.59 \text{ m}$$

$$y_{cm} = \frac{2(+2) + 5(-3) + 4(-2.5)}{2 + 5 + 4} = -1.91 \text{ m}$$

4-42: Without friction, you cannot walk. But if you throw something (a coat, shoe,...) in the direction away from shore, you'll move the opposite direction and will (if friction and air resistance don't slow you down) coast all the way to the shore.

4-43: For the reasons discussed in 4-J, $F_{ext} = 0$ and v_{cm} of the system (boat+woman) is constant at +4.0 m/s. In 5 s, the c-of-m moves (4.0 m/s)(5 s) = 20 m →. As in 4-J, the woman's walk makes the boat move 1.0 m →: $\Delta x_{boat} = +20 + 1 = +21$ m →. The woman's Δx is $+20 + 1 - 10 = +11$ m →.

4-44: As in Problem 4-J, $F_{ext} = 0$, v_{cm} is constant at 0, and $(x_{cm})_i = (x_{cm})_f$. The mother is heavier, so when she moves ← the boat moves → a distance "e". Draw pictures showing the i & f positions of mother, daughter and boat, then substitute into $(x_{cm})_i = (x_{cm})_f$:

$$\frac{20(4)+90(b)+50(7)}{20 + 90 + 50} = \frac{50(4+e)+90(b+e)+20(7+e)}{50 + 90 + 20}$$

Solving gives e = .56. The mother's x_f is 4+e = 4.56 m, and daughter is at 7+e = 7.56 m.

4-45: Initially you see a ground-person move ← at –4 m/s, and the train/cannon/ball at rest. When the ball goes →, the train "recoils" backward ←. {After the cannonshot, the c-of-m is between the train and ball; it does not coincide with any real object, but you can use your imagination to make the idea of riding on the c-of-m "real in your mind".} The relative velocity of train & ball is 28 m/s; if we call the train's speed "v" so its velocity is "–v", the ball moves at "–v+28". $F_{ext} = 0$, so momentum is conserved, and
$(100+70+30)(0) = (100+70)(-v) + (30)(+28-v)$.

v_{cm} isn't changed by the F_{int} shot, so you still see a ground-person move at –4 m/s, and you know the gp sees you move at +4 m/s. Solving the equation gives "v = 4.2"; the train moves at –4.2 m/s with respect to you, so it moves at +4.0–4.2 = –.2 m/s w.r.t. the ground-person. You see $v_{ball} = -v+28 = -(4.2)+28$, so the g-p sees +4–4.2+28 = 27.8 m/s. These are, of course, the same answers we found in 4-##.

Using similar logic, we can make equations for b-d:

b) $0 = 170(+v) + 30(+v-28)$, $v = +4.2$

c) $0 = 170v + 30(-4)$, $v = +.7$

d) $0 = 170v + 30(v)$, $v = 0$

These are the same equations as in 4-##, except that mv_i is now 0 (not +800) and in c we have –4 instead of the "0" seen by gp. And after adding 4.0, we get the same answers for v_{train} as in 4-##.

4-46: The explosion is F_{int}, so $F_{ext} = 0$ and mv is conserved: $20(+8) = 5(-4) + [20-5]v$, $v = +12$ m/s.

$KE_i = \frac{1}{2}(20)\,8^2 = 640$ J. $KE_f = \frac{1}{2}(5)\,4^2 + \frac{1}{2}(15)\,12^2 = 1120$ J. KE is not conserved; the explosion increases the system's KE by 480 J.

$F_{ext} = 0$, so v_{cm} is constant at +8 m/s. $\Delta t = 9s - 5s$. $x_f = x_i + \Delta x = (16\text{ m}) + (+8\text{ m/s})(4\text{ s}) = +48$ m.

4-47: F is not given: use tvvax ($v_f - v_i = a\,t$) to solve for $a = 2$ m/s^2. Then solve F=ma for F = 10 N, or "$F\Delta t = mv_f - mv_i$" for F = 10 N.

KE & Δx are given, you are asked to find F. Solve $F\Delta x = \Delta KE$ for F = 10 N.

"How long" asks for Δt, and F is given. Solve $F\Delta t = \Delta p$ for Δt = 4 s. Or solve F=ma for a, then substitute a into tvvax ($v_f - v_i = a\,t$) and solve for t.

The melon changes height. Aha! W ⇒ ΔKE, and $5(-9.8)(-3) = \frac{1}{2}(5)\,v_f^2 - 0$, and $v_f = 7.7$ m/s.
Or use tvvax: $v_f^2 - 0^2 = 2(-9.8)(-3)$, $v_f = 7.7$ m/s.

4-48: **1) mgh → Δ(½ mv²):** Just before collision the top block has $v = \sqrt{2gh} = \sqrt{2\,g\,4}$.
2) conservation of p: If system is both-blocks, $F_{ext} = 0$, and after the collision $v = \frac{1}{2}\sqrt{\frac{1}{2}2\,g\,4}$.

3) Δ(½ mv²) → mgh: $\frac{1}{2}(2m)\,[\frac{1}{2}\frac{1}{2}\,2\,g\,4) = (2m)gh$, so $h = \frac{1}{2}(\frac{1}{4})(8) = 1$. The final h is only 1 m, not (as you may have guessed) 2 m. Why? When v is cut to 1/2 by the collision, v^2 is cut to 1/4, and the ability to cause Δh is cut to 1/4 (of the original 4 m). Or you can think "the inelastic collision reduces the PE+KE mechanical energy, so we shouldn't expect the blocks to reach a 2 m height".

4-49: Hints: The answer is not "50 miles/hour". Use the definition of $v_{average}$. {Answer given later.}

4-50: A 2h fall (which doubles mgΔh) produces twice as much ΔKE (and Δv^2) but if the object starts from rest its v is only multiplied by $\sqrt{2} = 1.414$.

To reach 2v (with 4 times as much KE) the FΔx must increase by a factor of 4, from h to 4h.

When the distance the mg-force is applied increases from h to 4h, Δv doubles: from 0-to-v to 0-to-2v. If F & m are constant, $(mv)_f - (mv)_i = m(v_f - v_i) = m\Delta v$, $F\Delta \mathbf{t} = m\Delta \mathbf{v}$, and Δt is proportional to Δv. Because a 4h fall causes twice as much Δv, it must occur for a Δt that is twice as long.

Both cars have the same FΔx, which produces the same ΔKE and the same Δv^2. But the fast car has a shorter Δt, so it has smaller Δ(mv) and Δv.

Solving $F\Delta x = \Delta(\frac{1}{2} mv^2)$ and $F\Delta t = \Delta(mv)$ shows the ratio relationships between Δx-and-v^2 & Δt-and-v:

for the slow car	for the fast car
$\Delta x = 100$	$\Delta x = 100$
$\Delta v^2 = 400$	$\Delta v^2 = 400$
$v_i^2 = 0$, $v_f^2 = 400$	$v_i^2 = 625$, $v_f^2 = 1025$
$v_i = 0$, $v_f = 20$	$v_i = 25$, $v_f = 32.02$
$\Delta v = 20.0$	$\Delta v = 7.0$
$\Delta t = 10.0$	$\Delta t = 3.5$

If F and m are constant,
Δv^2 is proportional to Δx,
Δv is proportional to Δt.

{4-49} The first 240 miles (at 60 mi/hr) requires 4 hours, the next 240 miles (40 mi/hr) takes 6 hours. $v_{avg} = \Delta x / \Delta t = 480\text{ mi} / (4\text{ hrs} + 6\text{ hrs}) = 48$ mi/hr.

If the 60 & 40 speeds were for EQUAL TIMES, average speed would be 50 miles/hour. But 60 & 40 were traveled for EQUAL DISTANCES, so...

4-51: To have equal momentum, the smaller block must have 3 times as much v, 9 times as much v^2, and (because it has 1/3 as much mass) 3 times more KE. **KE-factor** = (m-factor)(v^2-factor) = (1/3)(9) = 3.

To have equal KE, the smaller block must have 3 times more v^2, and $\sqrt{3}$ times more v. But this isn't enough to overcome its smaller mass, so it has less p. **p-factor** = (m-factor)(v-factor) = (1/3)(1.73) = .577.

4-52: Use the same strategy as in Problem 4-K. Choose three special points: before impact, v-match, v becomes 0. Solve $F\Delta t = \Delta p$ for the 1-to-2 interval: $v_2 = .0164\ v_1$. Write $W = \Delta KE$ for the 2-to-3 interval,

$$[-\mu_k N] \quad d \quad = \tfrac{1}{2}\, m\, v_3{}^2 \; - \; \tfrac{1}{2}\, m \quad v_2{}^2$$
$$-(.4)\, m\, (9.8)\, (.67) \quad = \tfrac{1}{2}\, m\, (0)^2 \; - \; \tfrac{1}{2}\, m\, (.0164\, v_1)^2$$

Solve for v_1 = 140 m/s. This is the same v_1 we found Problem 4-K, which could support (but not prove) a theory that both bullets were fired from the same gun.

4-53: $F_{ext} = 0$ [the explosion is internal] so $p_f = p_i = 20(+8) = +160$ kg m/s. And $KE_f = KE_i + \Delta KE = \tfrac{1}{2}(20)\, 8^2 + 480 = 1120$ J. If the velocities of the 5 kg and 15 kg pieces are v and V, respectively,

$$+160 \; = \; 5v + 15V \qquad 1120 \; = \; \tfrac{1}{2}\, 5\, v^2 + \tfrac{1}{2}\, 15\, V^2$$

Solve Equation #1: 32 – 3V = v. Substitute into #2 and rearrange: $0 = 12V^2 - 192V + 576$. Solve the Q-equation: V = +12 or +4. {Do you see what would cause these two possibilities? Think about it, then read on.} Substitute V back into #1: (v = –4, V = +12) or (v = +20, V = +4). Of these possibilities, only the first has the large block moving faster, so it is the correct answer. This problem is the same as 4-##, but with different "given information".

If the 15 kg part was "in front", 5/15, the explosion makes it go faster than its initial +8 m/s. But if it was in back, 15/5, its → velocity is decreased to +4 m/s. {V changes by either +4 (+8 to +12) or –4 (+8 to +4). And v changes by –12 (+8 to –4) or +12 (+8 to +20).}

4-54:: If block/spring/block is a system, F_{spring} is F_{int} and mv is conserved. If the surface is frictionless or the collision Δt is very short, mechanical energy (KE + PE) is also conserved.

$$5(+20) + 15(+4) \; = \; (5+15)\, v_f$$
$$\tfrac{1}{2}(5)\, 20^2 + \tfrac{1}{2}(15)\, 4^2 + \tfrac{1}{2}\, k\, 0^2 \; = \; \tfrac{1}{2}(5+15)\; v_f^2 + \tfrac{1}{2}(3840)\; x_f^2$$

Solve the p-equation: $v_f = +8$ m/s. Substitute this into the energy equation and solve: $x_f = .50$ m.

This "final" situation is like the initial situation for Problems 4-## and 4-##, but instead of chemical PE in the firecracker there is now 480 J of PE stored in (and then released from) the spring.

4-55: 3-B: For all-blocks-together, the only F that doesn't cancel (== but this is making an internal-external distinction) is

==[what is this for?

4-##:

==[what is this at end for?

Just before the bag hits the cart, the system's KE is $\tfrac{1}{2}(200\text{ kg})(10\text{ m/s})^2 + (50\text{ kg})(9.8\text{ m/s}^2)(1.5\text{ m}) = 10735$ J. After the collision, $KE = \tfrac{1}{2}(250\text{ kg})(8\text{ m/s})^2 = 8000$ J. The system loses 2735 J of KE.