Chapter 15

Light Waves

Chapter 15 is finished, but is not in camera-ready format. All diagrams are missing, but here are some excerpts from the text with omissions indicated by .....

After 15.1, read 15.2 (single-slit diffraction), 15.3 (thin film interference), or 15.4 (polarization of light).

As explained in the introduction to Chapter 14, other light-related topics are in Chapters 9, 14, 16, & 17.

15.1 Wave Diffraction and Interference in Double-Slit and Multiple-Slit Gratings

The first picture below shows the familiar behavior of light as it passes through a large opening like a window. The area in front of the opening is illuminated, but there is a dark "shadow" on the sides where light doesn't reach.

The second picture shows the surprising behavior of light when it passes through a tiny slit that is roughly the same size as the light's wavelength; light waves radiate outward from the opening into the "shadow region". This process, which is shown below, is called *diffraction.* When light that moves forward from the slit in Wall #1 reaches Wall #2 the same thing happens; each slit becomes a "source" from which light waves radiate outward.

[pictures of walls-and-diffraction will be here]

{ Many characteristics of diffraction can be explained using *Huygens' Principle,* which states that every point on a wave front can be considered to be a new source of spherical waves that spread out in all directions with the same speed and frequency as the original waves. Huygens' Principle is probably discussed in detail in your textbook. }

The diagram below will help you understand what happens to light that goes through the *double slits* in Wall #2. **d** is the center-to-center separation of the slits, **L** is the length of the *center line* between the wall and a screen, **x** is the distance from the center line to a point (like ∆) on the screen, and **θ** is the angle between the center line and that screen-point.

The wiggly line to the right of the screen is a graph that shows the light intensity at each point on the screen. At • and ∆, intensity is a maximum (MAX). Between these two points, the intensity reaches a minimum (MIN).

.....[diagram will be above]..... It is easy to explain this variation in light intensity, using the principles of *wave superposition* from Section 9.2. If two waves are in-phase, *constructive interference* occurs and light intensity is a *maximum.* If the waves are out-of-phase by exactly $\frac{1}{2}$ cycle, *destructive interference* occurs and intensity is a *minimum.* If the phase difference is between these extremes, intensity is between zero and maximum.

Light waves from the slit in Wall #1 start moving outward in all directions at the same time. If the slits in Wall #2 are both the same distance from the Wall #1 slit*, waves that reach the Wall #2 slits are in-phase and waves that move out from these slits are in-phase. The *center point* • is an equal distance from each slit, so when the waves reach • they are still in-phase and intensity is a maximum.

The "o" point is further from the lower slit than it is from the upper slit. If the difference in path length is $\frac{1}{2}$ λ the lower-slit wave will, like a runner who runs extra distance in a race, fall behind the upper-slit wave by $\frac{1}{2}$ wave cycle. This "relative phase change" (of one wave with respect to the other) puts the waves out-of-phase by $\frac{1}{2}$ cycle, so they cancel each other to produce an intensity minimum.

At the Δ point the lower-slit wave has traveled an extra distance of 1λ, so it has fallen behind by 1 cycle and is back in phase with the upper-slit wave, to produce an intensity maximum.

Two sources (A & B) are *coherent* if their waves have a phase difference (between the A-wave & B-wave) that stays constant with time. Two sources can form an interference pattern only if they are coherent. With a 2-slit apparatus the double slits are coherent sources.

Your textbook may show how a *laser* produces *coherent light,* and how this light differs (in some ways) from the *incoherent light* that is produced by common sources like the sun, light bulbs, ...

Do you see why a maximum occurs when the path difference is zero or 1λ or 2λ or..., and why a minimum occurs when path difference is $\frac{1}{2}$ λ or $1\frac{1}{2}$ λ or...? Here is a mathematical summary of this idea: the path difference is "$m\lambda$" for a maximum, and "$(m + \frac{1}{2})\lambda$" for a minimum, where m can be any integer (0, 1, 2, 3, ...).

It can be shown that the path difference is "d sinθ" when, as is usually the case, L is much larger than d.

By combining this information with the previous paragraph's logic, you should be able to see why these equations are true:

For a maximum, $\quad d \sin\theta = m\lambda$

For a minimum, $\quad d \sin\theta = (m + \frac{1}{2})\lambda$

If θ is small, x is much smaller than L, and $\sin\theta$ [$= x/\sqrt{L^2 + x^2}$] is approximately equal to $\tan\theta$ [which is $x/L = x/\sqrt{L^2 + 0^2}$]: $\sin\theta \approx \tan\theta$.

Substituting "$\sin\theta \approx x/L$" into the maximum & minimum equations gives:

$$d\,\frac{x}{L} \approx m\lambda \qquad\qquad d\,\frac{x}{L} \approx (m + \tfrac{1}{2})\lambda$$

PROBLEM 15-A

Yellow light, wavelength 589 nm, shines on the 3-slit apparatus described above. The double slits in the wall are separated by .10 mm. A screen is .50 m away. Find the location (angle & distance from center) of the "third order maximum" (the third bright spot to the side of the center-line maximum).

What happens to this location if the slits are moved closer together? if the screen moves further away? if you look for the fourth bright spot? if red light is used?

Where is the third dark (not bright) spot?

SOLUTION 15-A

The problem doesn't refer to any letters directly, but you can translate words into equation-letters: $\lambda = 589 \times 10^{-9}$ m, $d = .10 \times 10^{-3}$ m, $L = .50$ m, $m = 3$. You are asked to find θ and x.

To solve, just choose the appropriate equation, substitute and solve.

$$d \quad \sin\theta \quad = \quad m \quad \lambda \qquad\qquad d \quad \frac{x}{L} \quad = \quad m \quad \lambda$$

$$(.10 \times 10^{-3}) \;\sin\theta \;=\; 3(589 \times 10^{-3}) \qquad\qquad (.10 \times 10^{-3}) \;\frac{x}{.50} \;=\; 3\;(589 \times 10^{-3})$$

$$\sin\theta \;=\; .01767 \qquad\qquad x \;=\; 8.8 \times 10^{-3}$$

$$\theta \;=\; 1.01° \qquad\qquad x \;=\; 8.8 \text{ mm}$$

To answer the questions in the second paragraph, use ratio logic on $d\sin\theta = m\lambda$: three of the changes ($d\downarrow$, $m\uparrow$, or $\lambda\uparrow$) cause $\sin\theta$ and θ to increase. But there is no L in this equation, and $L\uparrow$ does not affect θ; by studying the diagram you'll see that an $L\uparrow$ makes a larger ___ triangle (with larger L and x) but doesn't change θ.

{ Section 14.1 discusses the spectrum of visible light and the fact that red light, with $\lambda \approx 400$ nm, has a longer wavelength than yellow light. }

Then use ratio logic on $d\,x/L = m\lambda$ (or on rearranged versions like "$d\,x = L\,m\,\lambda$" or "$x = m\,L\,\lambda\,/\,d$") to find that each of the four changes will cause x to increase.

Bright spots are maxima, dark spots are minima. The third dark-spot minimum has a path difference of $2\frac{1}{2}\,\lambda$, but because the $\frac{1}{2}$ is supplied by the formula's "$m + \frac{1}{2}$" it only has $m = 2$. To find the minimum's location, substitute into "$d\sin\theta = (m + \frac{1}{2})\lambda$" and "$d\,x/L = (m + \frac{1}{2})\lambda$", then solve for $\theta = .84°$ and $x = 7.4$ mm.

Conservation of Energy

Wave interference does not produce or destroy energy. It just redistributes the light energy that comes through the double slits to different parts of the screen.

Multiple-Slit Diffraction Gratings

A *diffraction grating* is a large number of equally-wide, equally-spaced slits.

A grating is usually made by cutting shallow "grooves" on a glass surface, instead of cutting "slits" all the way through it. { In a similar way, a *reflection grating* can be made by cutting lines on a metal surface. }

The interference pattern of a multiple-slit grating is similar to the pattern formed by double slits, and is described by the same equations: maxima & minima occur when "$d\sin\theta = m\lambda$" and "$d\sin\theta = (m + \frac{1}{2})\lambda$". { The "center line" of a multiple-slit pattern, from which θ is measured, is at the center of the illuminated part of the grating. }

The pictures below show the intensity patterns formed by light passing through double-slit and multiple-slit walls with the same center-to-center slit separation:

[two pictures will be here]

for a DOUBLE-SLIT PATTERN   for a MULTIPLE-SLIT PATTERN

What causes this difference in the width of maximums? With only two slits, waves reaching the screen at • (a short distance away from a maximum) are "almost in phase", so intensity is almost the same as at the maximum. With multiple slits, at • the light from one slit is only slightly out of phase with its neighbor, but is $\frac{1}{2}$ cycle out of phase with a slit that is hundreds of slits away, because hundreds of "slightly out of phase" differences have accumulated. The light waves from these two out-of-phase slits cancel each other. Most light gets canceled in this way, except at (or very near) the maximums where neighboring slits are exactly in-phase. Since energy is conserved by wave interference, light energy that does not go to points between maximums shows up in the maximums, thus making them brighter. The grating also has more light to be distributed because light comes through a large number of slits, not just two.

Multiple slits form bright sharp maximums at locations that depend on light-wavelength, so a grating spreads "white light" into a continuous spectrum of colored components. { Spectrum formation is analyzed in Problem 15-#. }

Optional: Section 15.93 discusses the *dispersion* and *resolving power* of gratings.

15.2 Single-Slit Diffraction

Diffraction was introduced in Section 15.1. Now we'll examine it in another context.

The diagram below shows what happens when light radiates outward from a single slit; more light goes in some directions than in others. Notice that the *central maximum* is much higher than the *secondary maximums,* and is also twice as wide. .....[diagram will be here].....

Here are formulas for the location of single-slit *secondary* maxima and minima:

For a maximum,	**For a minimum,**
$\mathbf{w \ \sin\theta \ = \ (m_w + \frac{1}{2})\,\lambda}$	$\mathbf{w \ \sin\theta \ = \ m_w \ \lambda}$

where **w** is the slit width, and $\mathbf{m_w}$ is a non-zero integer (1, 2, 3,...).

Let's compare the double-slit and single-slit formulas. The left sides are similar, with $d\sin\theta$ replaced by $w\sin\theta$. m is replaced by m_w; the reason for the w-subscript is explained in Problem 15-B. A major difference is that "m" and "m + $\frac{1}{2}$ " are reversed; single-slit maxima and minima occur at m+$\frac{1}{2}$ and m, not at m and m+$\frac{1}{2}$.

A single-slit intensity pattern is caused by interference. But if there is only one slit, how can interference occur? According to Huygens' principle, every part of a wavefront is a wave-source; this allows waves from different parts of a slit-opening to interfere with each other! { Your text or teacher may explain the details of how single-slit interference produces the diffraction pattern that is observed. }

PROBLEM 15-B: The Combined Effects of Diffraction and Interference

The diagram below shows the intensity pattern that forms when 500 nm light shines on two .002 mm wide slits whose centers are .010 mm apart. The **single-slit diffraction** pattern forms an imaginary "envelope", shown by - - - -, that governs the size of the **double-slit interference** maximums.
Use formulas (and a link between them) to show that 9 interference maxima occur within the central diffraction maximum. ..... [diagram will be here] .....

SOLUTION 15-B

To find the minima on each side of the central diffraction maximum, substitute w, $m_w = 1$, and λ into "$w \sin\theta = m_w \lambda$" and solve for $\theta = 14.5°$; you can do the algebra yourself. Then use the θ-link;

substitute d, $\theta = 14.5°$, and λ into "d $\sin\theta = m\lambda$", and solve for $m = 5$. Look at the diagram above, and you'll see that intensity is zero at the 14.5° point where you expect to find the fifth order (m = 5) double-slit interference maximum, because single-slit diffraction interference doesn't let light come to that point. The 9 double-slit maximums have m = 4, 3, 2, 1, 0, 1, 2, 3 and 4.

Most textbooks use the same letter "m" for the double-slit and single-slit formulas. Do you see why this is not a good idea?

The extent to which diffraction "bends light into the shadow region" depends on the ratio of w/λ. To find the angular width of the central maximum, substitute "$m_w = 1$" into "w $\sin\theta = m_w \lambda$" and solve for $\theta_{c\text{-}max}$. If λ = 400 nm and w = 4000 nm (10 times larger than λ), $\theta_{c\text{-}max}$ is 5.7°. But if w = 800 nm (2 times larger than λ), $\theta_{c\text{-}max}$ is 30°. And if w is 400 nm (the same size as λ), $\theta_{c\text{-}max}$ is 90°.

As w decreases, the central maximum becomes wider and a larger fraction of the light is diffracted into the shadow region, but there is less total light; for example, a 4000 nm slit lets in 10 times as much light as a 400 nm slit.

For a very wide slit, like a door with .70 m width, an extremely small amount of diffraction occurs at the edges of the slit, but most light passes straight through. If parallel light rays strike the door opening straight-on, there is a .70 m wide bright region (this is not the "central diffraction maximum") directly in front of the door; if all reflection could be eliminated, the shadow region would be almost dark.

Problem 15-# will help you learn to use *Rayleigh's criterion* for solving problems.

15.3 Thin-Film Interference

As described in Section 14.3, when light reaches an interface where "n" changes, some light reflects backward and some continues forward. When light reflects from an interface where n increases (like at 1.00/1.60 in the first picture, and 1.00/1.33 or 1.33/1.60 in the second picture), the reflected wave "flips" and its phase is changed by $\frac{1}{2}$ cycle. But when light reflects from an interface where n decreases (like at the 1.60/1.00 interface in the first picture), the wave's phase does not change. ..... [pictures here] .....

In the first picture there are two types of reflected waves: "air waves" reflect from the top interface, and "glass waves" travel through glass before reflecting from the bottom interface. The air-wave has its phase flipped by $\frac{1}{2}$ cycle, but the glass-wave doesn't, so reflection flips put the air and glass waves out of phase by $\frac{1}{2}$ cycle.

The glass-waves travel an extra round trip distance of "2t", where t is the glass thickness. If 2t equals $\frac{1}{2}\lambda$, the glass-waves change phase by $\frac{1}{2}$ cycle with respect to the air-waves. This puts the air & glass waves back in phase, and reflected light has maximum intensity. Reflection maximums occur when 2t equals $\frac{1}{2}\lambda$, $1\frac{1}{2}\lambda$, $2\frac{1}{2}\lambda$,...; maximums occur when $2t = (m + \frac{1}{2})\lambda$.

But if 2t equals 1λ, 2λ, 3λ,..., the air and glass waves remain out of phase by $\frac{1}{2}$ cycle, and reflected light has minimum intensity; minimums occur when $2t = m\lambda$.

As explained in Section 14.2, when light waves move through glass their speed and wavelength both decrease by a factor of $1/n_{glass}$. Our equations must use λ_{glass}, the light's wavelength in glass, where $\lambda_{glass} = \lambda_{air} / n_{glass}$.

By combining these ideas, we can derive the equations in the left-side column:

For the first picture, flips put the waves out-of-phase.

Maximums occur when

$$2t = (m + \tfrac{1}{2}) \frac{\lambda_{air}}{n_{glass}}$$

and minimums occur when

$$2t = m \frac{\lambda_{air}}{n_{glass}}$$

For the second picture, flips put the waves in-phase.

Maximums occur when

$$2t = m \frac{\lambda_{air}}{n_{water}}$$

and minimums occur when

$$2t = (m + \tfrac{1}{2}) \frac{\lambda_{air}}{n_{water}}$$

In the right-side picture, reflection-flips change both waves by $\frac{1}{2}$ cycle; this puts the waves in-phase. If $2t = m\lambda$, the waves remain in-phase to produce a maximum. But if $2t = (m + \frac{1}{2})\lambda$, the waves become out-of-phase to produce a minimum.

Study the two pictures and four equations. Do you see why each equation makes logical sense, and why n_{glass} must be changed to n_{water}?

Notice that there are only two equation-forms: one has "m", the other has "$m + \frac{1}{2}$ ".

Here is a strategy that will help you solve most thin-film problems:

1) Read, draw a picture, and find two reflected waves that will interfere.

2) Decide the phase flip ($\frac{1}{2}$ cycle or none) of each reflected wave and decide if these flips, by themselves, put the reflected waves in-phase or out-of-phase.

3) Does the problem ask you about a maximum (bright) or minimum (dark)? Write an appropriate equation [it is usually one of the equations from above], using this logic: **round-trip distance = extra distance needed to make the reflected waves be in-phase (for a maximum) or out-of-phase (for a minimum)**. Substitute and solve.

4) Answer the question that was asked.

PROBLEM 15-C

600 nm light shines straight down on a film of oil (n = 1.45) that floats on water. At a certain point on the film (whose thickness varies) the amount of reflected light is a maximum. What is the smallest thickness the film could have at this point?

When 450 nm light shines on this point from straight above, the reflection of 450 nm light is at a minimum. What is the actual thickness of the film?

If 450 nm and 600 nm light shines on this point with equal intensity, which light has a larger intensity when viewed from underneath the oil film?

SOLUTION 15-C

Draw a picture, as at the left below. There is a reflected air-wave ($\frac{1}{2}$ cycle flip) and oil-wave (no flip). Flips put the waves out-of-phase, so a reflection maximum occurs when the oil-path difference is $\frac{1}{2}\lambda$, $1\frac{1}{2}\lambda$, $2\frac{1}{2}\lambda$,... To answer "What is the smallest thickness...", we need the solution for $\frac{1}{2}\lambda_{oil}$. Write the equation, substitute & solve:

.....[picture appears above-left, equation is "2t = ..." and solving gives "t = 1.03×10^{-7} m"].....

A reflection maximum occurs if the oil-wave's round trip is $\frac{1}{2}\lambda_{oil}$, $1\frac{1}{2}\lambda_{oil}$, $2\frac{1}{2}\lambda_{oil}$,... (these can be written as $\frac{1}{2}\lambda_{oil}$, $\frac{3}{2}\lambda_{oil}$, $\frac{5}{2}\lambda_{oil}$,...), which means that the thickness of the oil layer can be $1(1.03 \times 10^{-7})$,

$3(1.03 \times 10^{-7})$, $5(1.03 \times 10^{-7})$, ...

It is possible to write an equation to find the actual thickness (see Problem 15-#), but to do one problem it may be easier to use a *guess-and-check strategy*. We want to find a value of " t " that is consistent with a 600 nm maximum (possible t-values are described in the previous paragraph) and a 450 nm minimum. To do this, we write the "450 nm minimum-equation", substitute a "possible 600 nm t-value" and check to see if, as required by interference principles, m is an integer. As shown below, the smallest possible thickness gives a non-integer m, so it can't be the actual thickness. But a layer 3 times this thick gives an m that is very close to an integer (it is close enough), so we conclude that the oil thickness is 3.09×10^{-7} m.

$$2\,(1 \times 1.03 \times 10^{-7}) = m\,\frac{450 \times 10^{-9}}{1.45} \qquad\qquad 2\,(3 \times 1.03 \times 10^{-7}) = m\,\frac{450 \times 10^{-9}}{1.45}$$

$$.664 = m \qquad\qquad 1.99 = m$$

NO! $\qquad\qquad$ OK

If you look upward from under the film, you see light that is transmitted through the oil. Light reflection is minimum for 450 nm, maximum for 600 nm. Less 450 nm light is reflected, so (if absorption is negligible at both wavelengths) more 450 nm light is transmitted.

Comments: 1) At both wavelengths, most light is transmitted; "reflection maximum" doesn't mean "all light reflects" or even "a majority of light reflects". 2) As emphasized in Section 15.1, energy is not produced or destroyed by interference, it is just redistributed.

Problems 15-# to 15-# show more examples of light interference: a lens coating, soap bubble, wedge-shaped air space, and interferometer.

15.4 Polarization of Light

Light coming from the sun (or an incandescent bulb or a candle) is *unpolarized.* As shown below, the E-vectors of unpolarized light can point in any direction that is $\perp$ to the wave's velocity. When unpolarized light passes through *polarizing sheet* #1, the vertical E-field components pass through but the horizontal E-field components are absorbed, so the wave's intensity is cut in half.

When it travels between sheets #1 and #2, light is *plane polarized.* It looks like the picture of an EM wave in Section 14.1; the E-field oscillates only in the y-direction and the B-field oscillates only in the z-direction. When this light passes through sheet #2, whose *polarizing direction* is oriented at angle "θ" away from the vertical polarizing direction of sheet #1, light intensity is reduced by a factor of $\cos^2\theta$ and the E-field oscillates in the direction that is allowed by sheet #2. ..... [there will be a picture] .....

A polarizing sheet can also be called a polarizing filter, polarizer, Polaroid sheet, Polaroid, ...

This section describes "perfect" polarizing sheets that pass 100% of one component and 0% of the other component. Real sheets will, of course, fall short of this ideal.

PROBLEM 15-D

Unpolarized light with intensity I_o passes through two perfect polarizing sheets: the polarizing direction of #1 is vertical, while #2 is horizontal. What is the light intensity after sheet #2? Why does this result occur?

If unpolarized light with intensity I_o passes through three sheets (#1 is vertical, #2 is 70° away from vertical, and #3 is horizontal), what is the intensity after #3?

SOLUTION 15-D

After #1, $I = .50\,I_o$. The angle between #1 (vertical) and #2 (horizontal) is 90°, so I-after-#2 is $(.50\,I_o)(\cos 90°)^2 = 0$. Polarizer #1 absorbs the horizontal components of the original light, while #2 absorbs the vertical components: nothing survives!

With three filters, I-after-1 is $.50\,I_o$, I-after-2 = $(.50\,I_o)(\cos 70°)^2$ = $.0585\,I_o$, I-after-3 = $(.0585\,I_o)(\cos 20°)^2 = .0516\,I_o$. { Do you see why the angle between #2 and #3 is 20°? } Or you can multiply all 3 factors at once: I-after-3 = $I_o(.50)(\cos^2 70°)(\cos^2 20°)$.

Your textbook or teacher may share other interesting topics, like how a polarized sheet (and some life-chemicals like amino acids & sugars) polarize light, or why the sky polarizes sunlight and how bees use this to navigate.

It may also discuss the red, white and blue of "light scattering", which explains why a sunset is red, clouds are white, and sky is blue.

15.90 Memory-Improving Flash Cards (are on the next page)

15.90 Memory-Improving Flash Cards

15.1	Huygens' Principle says that each __ is a __ These can __ and __ .	part of a wavefront, new wave-source bend into shadows, interfere
15.1	Diffraction can occur if __ is __ . Interference can occur if sources are __ (if __).	λ of wave, roughly equal to width of slit coherent, their phase-Δ doesn't vary with t
15.1	If a wave travels extra distance of $\frac{1}{2}$ λ, it __ .	changes "relative phase" by $\frac{1}{2}$ cycle
15.1	Double-slit maximums occur at __ . Double-slit minimums occur at __ . A multiple-slit dark spot occurs __ .	points where waves are perfectly in-phase points where waves are totally out-of-phase even if waves are only slightly out-of-phase
15.1	Angular $\rightarrow$ linear: if __ , then __ .	$x \ll L$, $\sin\theta \approx \tan\theta = x / L$
15.1	Wave interference does not __ , it just __ . Maxima: multiple slits (>2) make them __ .	produce or destroy light, redistributes energy sharper [narrower] and brighter
15.2	Central maximum is __ and __ as __ maxima.	more intense, twice as wide, secondary
15.2	d is slits' __ , w is slits' __ .	center-to-center separation, width
15.2	Equations: single & double slit maxima use __ . Variables: __ are the same, but __ aren't.	$m+\frac{1}{2}$ (single) & m (double) θ and λ , d-w and $m\text{-}m_w$
15.2	Single slit : interference occurs between __ .	waves from different parts of the slit
15.2	As w $\downarrow$, __ and __ .	$\theta_{central\ maximum} \uparrow$, total light intensity $\downarrow$
15.3	At n_1 / n_2 interface, reflected light __ if __ .	has its phase changed by $\frac{1}{2}$ cycle, $n_2 > n_1$
15.3	In glass, 2t is __ , and __ must be used.	round-trip path, $\lambda_{glass} = \lambda_{air} / n_{glass}$
15.3	A useful thin-film strategy is __ .	draw & find waves, decide flip & difference, equation (choose m or $m+\frac{1}{2}$), solve & answer
15.3	If 2t is __ , relative phase change is __ . (2)	is $m\lambda$, zero ; is $(m + \frac{1}{2})\lambda$, $\frac{1}{2}$ cycle
15.4	Light from __ , __ or __ is __ .	sun, light bulb, candle, unpolarized
15.4	When __ light passes thru a perfect polarizer, intensity changes by a factor of __ . (2 answers)	unpolarized, $\frac{1}{2}$ polarized, $\cos^2 \theta$

Eventually, Chapter 13 will be "finished" in a camera-ready format.