Chapter 14

Light and Optics

Read Section **14.1** when your class studies *electromagnetic waves,* **14.2-14.3** when you study *light refraction,* and **14.4-14.5** when you study *optics (lenses & mirrors).*

Optional: Section **14.93** discusses light intensity and Poynting vectors.

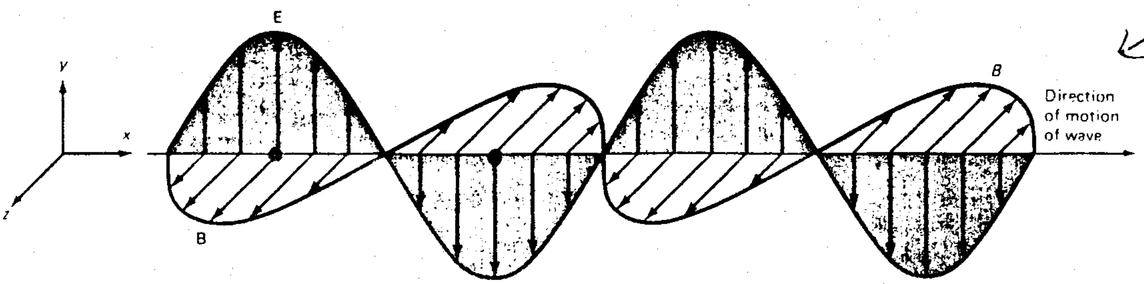
Some light-related topics are covered elsewhere: basic wave behavior (Chapter 9), wave diffraction & interference (Chapter 15), relativity & Doppler Effect (Chapter 16), wave-particle duality & photons (Section 17.1). When you study any of these topics, use ==nec? the **Table of Contents** or **Index** to find the corresponding part of this book.

these double ='s are
==[notes to EDITOR], may cut some parts of this: duplicates of standard textbook, no value-added? but maybe necessary for my logical organization?

14.1 Electromagnetic Waves and Light

In Section 12.#, Faraday's Law tells us that a changing magnetic field causes an induced electric field. It is also true that a changing electric field causes an induced magnetic field. It can be shown, using *Maxwell's Equations,* that the symmetric "mutual cause-effect" relationship between changing **E**-fields and changing **B**-fields produces traveling *electromagnetic waves.* The cycle goes something like this: at the same time a changing **E** produces a changing **B**, the changing **B** produces a changing **E**, and they "keep each other going". {Notice that it is the change in **E** and **B** (not **E** and **B** themselves) that keeps the cycle going.}

The picture below shows an electromagnetic wave moving in the +x direction. The **E** field oscillates in the y-direction and the **B** field oscillates in the z-direction. Notice that **E** and **B** are maximum at the same time, and are zero at the same time.



from Giancoli. I don't want to bother trying to draw it now

Optional: Can you discover a relationship between the directions of the **E**, **B** and **v** vectors? Does it work at both of the • points? Hint: use analogy to a right-hand rule from Chapter 12.
{After you've thought about it, compare your ideas with Problem 14-#.}

The *speed of electromagnetic waves* in free space [a vacuum] can be derived from Maxwell's equations, and expressed in terms of the physical constants of electricity and magnetism, ε_0 and μ_0: wave speed = $1/\sqrt{\varepsilon_0 \mu_0}$ = $1/\sqrt{(8.85 \times 10^{-12})(4\pi \times 10^{-7})}$ = 2.9979×10^8 m/s. This theoretically predicted speed is equal to the experimentally measured speed of electromagnetic (EM) waves, thus supporting (but not "proving") the belief that Maxwell's equations are valid. ==[is it ok to say M's eqtns are valid?]

The "officially accepted" value for the speed of light is 299,792,458 m/s.

Electromagnetic waves cover a wide range of wavelengths. At the left end of the *spectrum* below, EM waves (like AM radio) have long wavelength & low frequency. At the right end, the EM waves (like gamma rays) have short wavelength & high frequency. Beneath the ⇓ is a detailed version of the *visible light* spectrum, which spans a wavelength range of approximately 400 nm (for red light) to 750 nm (for violet light). In physics, *light* can mean either "visible light" or "any type of EM wave".

long wavelength low frequency ←						→ short wavelength high frequency	
AM radio	FM radio, television	radar, microwave	infrared light	**visible light**	ultraviolet light	x-rays	gamma rays

⇓

(red ≈400 nm orange yellow green blue violet ≈750 nm)

The sun and incandescent light bulbs (light bulb) produce *white light* with a wide range of frequencies, beginning in the infrared and continuing through the visible range (red to violet) and on into the ultraviolet. In a rainbow you see the results of white light that has been split into an almost-continuous spectrum of component colors.

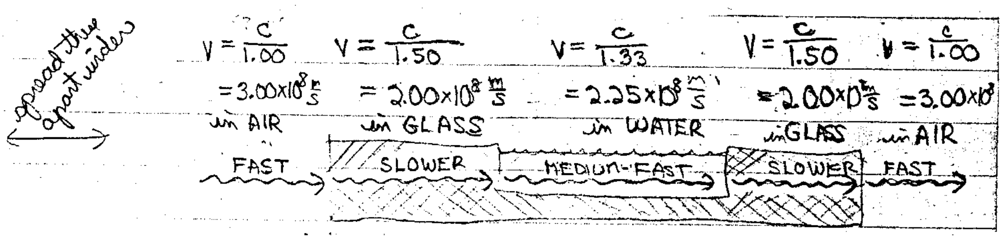
In a vacuum, all EM waves have the same speed, independent of wavelength or frequency. To interconvert frequency & wavelength, use **$v = f\lambda$** from Section 9.1.

14.2 Speed of Light, Index of Refraction

When light travels through a *vacuum* (empty space), its speed is 2.998×10^8 m/s. This speed doesn't depend on frequency; it is the same for all electromagnetic waves and is abbreviated "c". But when light moves through water, interaction between light and the water molecules makes light slow down to 2.249×10^8 m/s.

The ratio of these two speeds is called the water's *index of refraction,* abbreviated "n": $\mathbf{n \equiv c/v}$. For water, $n_{water} = c/v_{water} = (2.998 \times 10^8 \text{ m/s})/(2.249 \times 10^8 \text{ m/s}) = 1.333$.

The pictures below show light-speed as it passes through air (with n = 1.00), glass (n = 1.50), water (n = 1.33), glass, and air. To find the speed through each substance, substitute c & n into $v = c/n$ (a rearrangement of $n = c/v$).

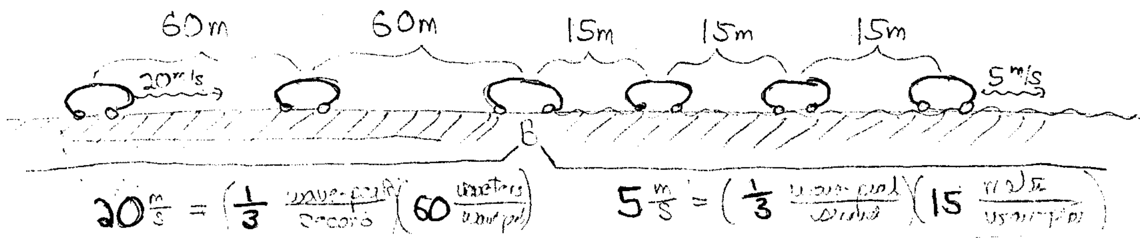


Light speed doesn't depend on the light's "history". In the example above, light travels through air with a speed of 3.00×10^8 m/s, even after it has been moving at a slower speed through water and glass.

The following analogy will help you understand the behavior of light when it moves from one medium into another.

Imagine that a long line of cars travels at 20 m/s on a smooth road (to the left of a boundary spot marked "B") but when the cars reach a rough road (to the right of B) their speed immediately drops to 5 m/s. If cars are spaced 60 m apart on the fast road a different car will, because $\Delta t = \Delta x/v$, pass spot "B" once every 3 seconds. A car leaves B once every 3 s, and at 5 m/s it travels 15 m before the next car reaches B, so on the slow road the car spacing is only 15 m.

This line of cars is analogous to a series of traveling wave-crests with a constant frequency of 1/3 wave-crest per second. Do you see why the "wavelength" of the car-line decreases by a factor of 4 (from 60 m to 15 m) when its speed decreases by a factor of 4 (from 20 m/s to 5 m/s)?



These principles can be generalized to electromagnetic waves. When EM waves move from a vacuum into a medium with index of refraction "n", their speed and wavelength decrease by a factor of 1/n, but frequency stays the same:

$$v_n = \frac{v_{vacuum}}{n} = \frac{c}{n} \qquad \lambda_n = \frac{\lambda_{vacuum}}{n} \qquad f_n = f_{vacuum}$$

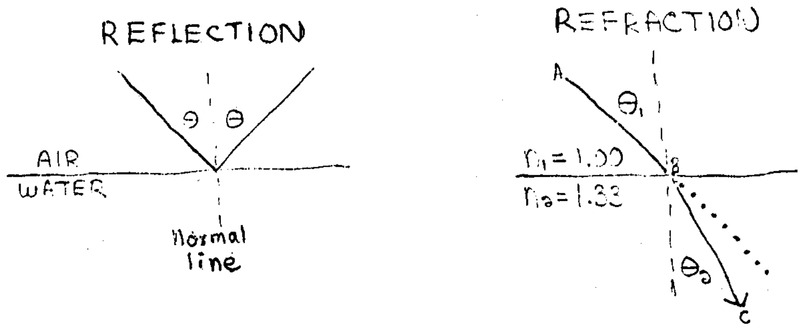
The "n" of a substance varies with the frequency of light. For example, quartz glass has n = 1.4561 for red light (with λ = 670 nm), and n = 1.4636 for blue light (with λ = 480 nm). In a vacuum, red light and blue light travel at the same speed, but in quartz glass they move at different speeds. {Which color moves faster?}

14.3 Reflection and Refraction

At the *interface* where light moves from one material into another, part of the light is *reflected* backward and part of it is *transmitted* forward.

As shown below, when light strikes a smooth surface (like calm water) the reflection is symmetric: **the angle of incidence equals the angle of reflection.**

The right-side picture shows that transmitted light doesn't continue in a straight line; instead, it gets "bent". This change of transmittance-direction, which is called *refraction,* is described by *Snell's Law:* $n_1 \sin\theta_1 = n_2 \sin\theta_2$.



$\theta_{INCIDENCE} = \theta_{REFLECTION}$ $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Important: <u>"θ" is always measured starting from the *normal* (perpendicular) direction</u> that is shown by - - - -, not from the surface of the water.

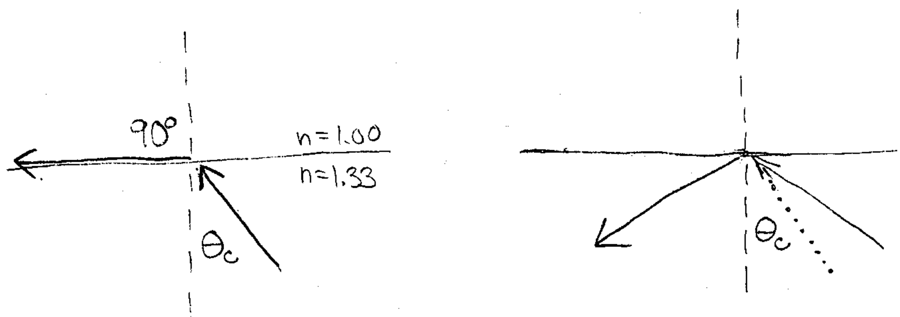
Light paths are reversible. For example, if a mirror placed at C makes the light reverse its direction 180° so it returns to B, it will then be refracted back to A.

REFRACTION RATIO LOGIC: n and sin θ are multiplied in "$n_1 \sin\theta_1 = n_2 \sin\theta_2$", so when one is large the other is small. If n_2 is larger than n_1 (as when light goes from air into water), θ_2 is smaller than θ_1 and light is bent "toward the normal". But if n_2 is smaller than n_1 (when light moves in the reverse direction, from water into air), θ_2 is larger than θ_1 and light is bent "away from the normal".

Are blue and red light refracted at the same angle? If not, which color is "bent" more? Do you see how this difference could be used to separate white light into a spectrum of component colors? {Spectrum formation is studied in Problem 14-#.}

If light begins in water at $\theta_2 = 48.6°$, you can substitute this and the n-values (try it yourself) and solve for $\theta_1 = \sin^{-1}[(1.333)(\sin 48.6°)/(1.000)] = \sin^{-1}[1.00] = 90°$.
{A calculator tip: use the $\sin^{-1}$ button. Punch " 1.333 x sin 48.6 ÷ 1.000 = $\sin^{-1}$ ".}

For a water-to-air interface, 48.6° is called the *critical angle* θ_c because Snell's Law predicts that a refracted light ray would "skim" along the water surface with $\theta = 90°$. If light comes from an angle larger than θ_c, no light emerges into the air and all light is reflected. This is called, quite logically, *total internal reflection.*

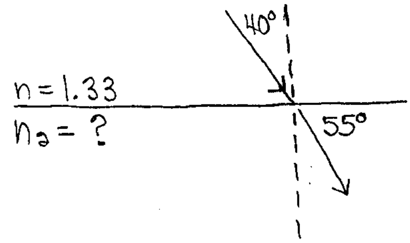


PROBLEM 14-A

For the situation at the right, find n_2.

To get total internal reflection, should light come from the top layer or bottom layer? What range of angles can be used?

Bonus: What happens if you try to solve the equation with an angle that is larger than θ_c?



SOLUTION 14-A

Substitute, solve for n_2. Because θ is the angle-from-normal, $\theta_2 = 90° - 55° = 35°$.

For total internal reflection, light must start in the layer with the larger n-value; this is the bottom layer. To find θ_c, use "$n_2 = 1.49$" from the first solution (in the left column below), set θ_1 equal to 90° and solve for θ_2. **{ I'm using 1 & 2 subscripts for the top & bottom layers, respectively. If you want, 1 & 2 can represent the layers where light begins & goes to. Both systems work, as long as you're consistent. }**

Total internal reflection occurs when the medium where light originates has a large n (compared with the other medium's n) and large θ (when light gets closer to "skimming"). The "total internal reflection equation" can be written as:

$$n_{incident} \sin\theta_{incident} > n \sin 90°$$

Bonus: If light comes from an angle larger than 63° (like 70°) and we substitute $\theta_2 = 70°$ into Snell's Law, we cannot solve "$\sin\theta_1 = 1.05$" (because sinθ is never larger than 1) to find θ for the transmitted light. This lack of a θ-solution corresponds to the fact that there is no transmitted light.

$n_1 \sin\theta_1 = n_2 \sin\theta_2$	$n_1 \sin\theta_1 = n_2 \sin\theta_2$	$n_1 \sin\theta_1 = n_2 \sin\theta_2$
$(1.33)(\sin 40°) = n_2 (\sin 35°)$	$(1.33)(\sin 90°) = (1.49) \sin\theta_c$	$(1.33) \sin\theta_1 = (1.49)(\sin 70°)$
$1.49 = n_2$	$1.12 = \sin\theta_c$	$\sin\theta_1 = 1.05$
	$.893 = \theta_c$	$\theta_1 = ?$

The laws of light reflection and refraction can be derived by using 1) *Huygens' principle* for wave behavior, 2) *Maxwell's equations,* or 3) *Fermat's principle.* This book won't discuss the derivations using Huygens' principle or Maxwell's Equations. But Problem 14-## shows how to explain refraction with *Fermat's principle of least time,* which says that when light travels between two points it follows the path that requires the least possible time.

Geometry Tips

To solve homework & exam problems, use the principles from this section and improvise using Section 1.1's geometry tools: ΔXYZ, draw extra lines, sum-of-parts, similar triangles. Be careful when you draw "normal" lines; if a surface is slanted (like V) the normal-line will also be slanted, it won't be horizontal or vertical.

Use the *brainstorm-and-edit* strategy of Section 20.7. Be creative first [use pencil to minimize fear of mistakes, try different kinds of extra lines, explore possibilities] and then check to be sure you're doing everything correctly.

Reflection versus Transmission

When light reaches an interface between two materials, the amount of light that is reflected depends on the change in "n" and on the angle of incidence "θ". As the n-change or θ-angle become larger, so does the % of light that is reflected.

For example, if normal (straight-on, θ = 0°) light strikes an air-water interface where n changes from 1.00 to 1.33, only 2% of the light is reflected; the other 98% is transmitted. Interfaces of air-to-glass [1.00-to-1.50] and air-to-diamond [1.00-to-2.42] have larger n-changes, and reflect more normal light: 4% and 17%, respectively.

Common sense leads you to expect that more light is reflected for ↘↗ than for ↑. This is true; as θ increases, so does the reflection-%, until all light is reflected when light "skims" the surface with θ = 90°. If light begins in the water, reflection-% again increases as θ increases until, as discussed earlier, all light is reflected when θ is equal to or larger than the critical angle.

14.4 The Principles of Optics: understanding how a lens or mirror forms images.

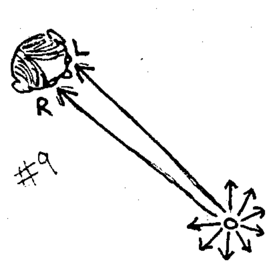
In most optics problems the visual logic & math are easy if you know the physical principles. Therefore, the goal of this section is to help you understand how and why images form.

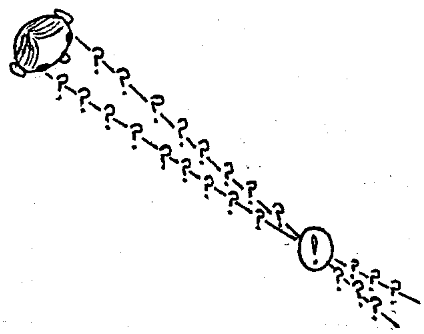
Whether your class studies lenses or mirrors first, read this section in the order it is written. Basic principles are the same for lenses or mirrors, and it is easier to learn optics as a "whole" because this lets you see how everything fits together.

Image Formation by a Plane Mirror

As shown in the first bird's-eye picture below, light moves out in all directions from a tiny source of light "o", but only light that travels in the two directions shown reaches the left & right eyes (L & R) of observer "Q".

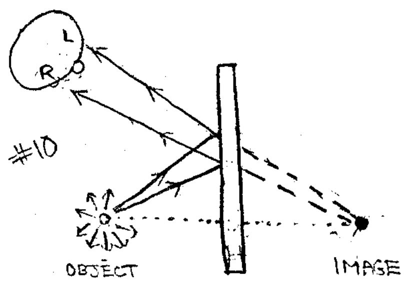
In the second picture, the ?'s show that each eye knows light is coming from a certain direction, but can't tell whether it is coming from near (a "?" close to the eye) or far (a "?" far from the eye). There is only one place where both eyes agree "the light could have come from this spot"; it is the ? that is circled. The observer's brain automatically performs this *triangulation logic* that lets him determine the object's location. {Other location-cues are discussed in Problem 14-##'s "==Optical Illusions".}

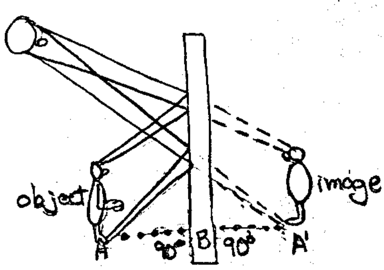




In the first picture below, light coming from a tiny "object" reflects from a plane mirror. Even though no light actually comes from behind the mirror, it looks as if it was coming from there. This is shown by backward extensions (- - - -) of the light paths. By using the triangulation logic described above, Q concludes that light is coming from the spot labeled "image".

The second picture shows that light coming from every part of an object forms its own image, so Q sees an *image* of the entire *object*. It can be shown, using simple geometry, that object & image are directly opposite each other (as shown by the 90° angles) and are equal distances from the mirror (AB and BA' are the same length).





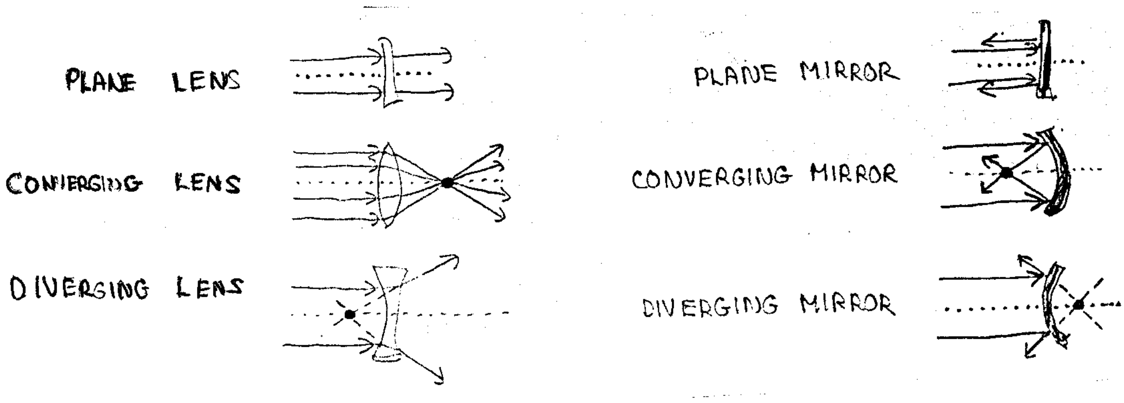
The meaning of Converging and Diverging

The pictures below show 3 kinds of lenses and 3 kinds of mirrors. Notice that each lens (or mirror) is vertical, and there is a horizontal *center line* ···· drawn perpendicular to the vertical axis of the lens (or mirror) and passing through its center. Also notice that I've only drawn rays that are horizontal, parallel to the center-lines.

The two top pictures show that "parallel-to-the-center-line" light passes straight through a *plane lens* and bounces straight back from a *plane mirror.*

If a *converging lens* is shaped "just right", all parallel light rays (like the four that are shown) are bent toward the *center line* ···· and pass through the *focus* •. Similarly, all parallel light rays that hit a "perfectly shaped" *converging mirror* are reflected toward the center line and passes through the focus.

When parallel light hits a *diverging lens* it is refracted away from the center line. If a lens is shaped "just right" and the paths of the refracted light rays are extended backward, as shown by - - - -, they pass through the focus •. Similarly, all parallel light that hits a "perfectly shaped" *diverging mirror* is reflected away from the center line, and the extension of each reflected ray passes through the focus.



Notice the characteristic shape of lenses; a converging lens is fat in the middle, a diverging lens is skinny in the middle. In each case, light is bent toward the fat part of the lens. For mirrors, use common sense; imagine that you throw a ball at the mirror and ask "Will the ball bounce straight back, be directed toward the center line, or away from it?".

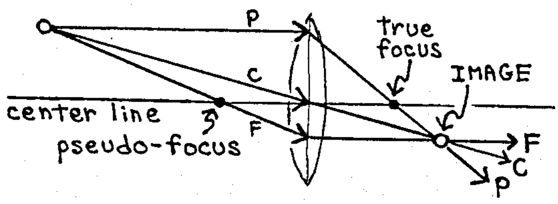
Image Formation by a Converging Lens

In the Far Example below, a tiny object emits light in all directions, but we'll only trace the path of 3 light-rays. One ray, which I've labeled "P", begins parallel to the center-line; the lens refracts it through the *true focus.* Another ray (C) goes through the lens-center; it continues straight with no change of direction. A third ray (F) goes through the *pseudo focus* [which is exactly opposite the true focus]; if you compare the paths of the P and F rays, you'll see that one ray is "parallel, then focus" while the other is "focus, then parallel".

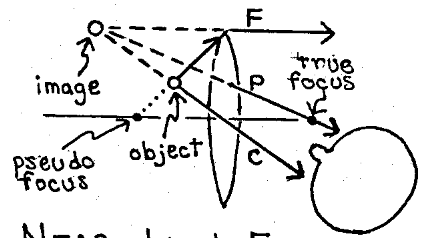
Notice that these 3 rays come back together at the spot labeled "image".

When you draw P & F rays, make the ray go straight until it hits the vertical center-axis of the lens, even though this isn't the real light path (the ray actually refracts when it reaches the curved edge of the lens). Then draw the ray as a straight line that begins at the axis-line.

The Near Example traces these same 3 rays: parallel (P), center (C), focus (F). The P and C rays are drawn as before. Because of the object location the F-ray cannot go forward through the pseudo-focus, so it is drawn as if it was coming from the pseudo-focus; this is shown by the ···· line. The lens then refracts it, as before, parallel to the center-line. By using the triangulation logic discussed earlier, concludes that light is coming from the point labeled "image", because the backward extensions of the rays (- - - -) meet at that point.



FAR-object EXAMPLE



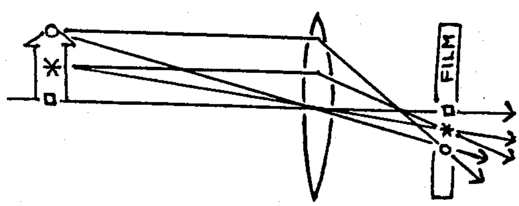
NEAR-object EXAMPLE

In the Far Example, light emitted in different directions goes to the same point. In the Near Example, light observed from different locations (at the observer's left and right eyes) seems to come from the same point. These are both called "images" but they are very different!

Real Images and Virtual Images

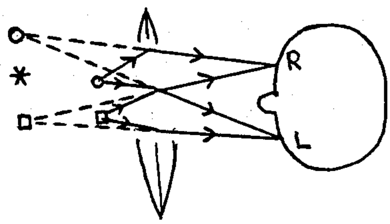
When real light rays (→) from an object all meet at the same point, they form a *real image*. When the imaginary backward extensions of rays (- - - -) meet at the same point, a *virtual image* is formed.

The picture below is like the Far Example above, but there are now three light sources [o * □] at the top, middle and bottom of an arrow-shaped object. The lens causes each source to form its own image. Like the 3 sources, the o, * and □ images are in a line, but □ is now on top. If we turn the lens into a *camera* by placing film as shown, a *photograph* forms because all of the o-light that is gathered by the lens (not just the P, C & F rays) focuses to one point on the film, all *-light goes to another point, and all □-light goes to a third point. {A "perfect lens" would make all o-light go to one point, but a real lens cannot do this because of various technical difficulties. A good camera can, however, make "almost all" of the light from each source go to the correct point on the film, thus producing a relatively sharp photographic image of the object.}

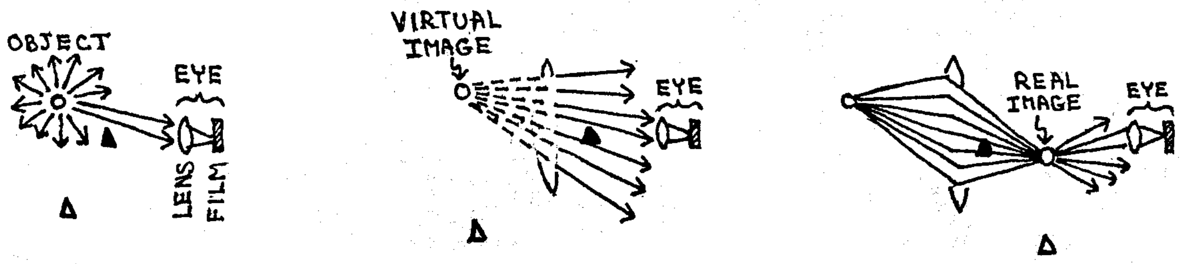


If the lens is taken away, some o-light goes to every part of the film, and so does *-light and □-light. Instead of a photograph of 3 individual lights, you'll get a piece of film that has been exposed to all 3 lights over its entire surface.

The picture below is like the Near Example. No matter where ◁O is (I've put him on the center-line) the rays from "o" that reach his left & right eyes (these are not usually the P, C or F rays) look as if they come from the same "virtual image" spot, where the - - - - extensions come together. Similarly, the rays from □ (directly below o) seem to come from an image-spot that is directly below o's image-spot.



As shown in the first picture below, a human eye is like a miniature camera that uses a converging lens to form a real image on "biochemical camera film" at the back of the eyeball. In each picture, look at the light that comes into the eye. Because light radiates outward in many different directions from an original object, a virtual image or a real image, the eye is able to see each of them.



In the pictures above, will an eye at Δ or ▲ be able to see the object (in the first picture), the virtual image (second picture), or the real image (third picture)? If the eye [combination of lens and film] is replaced by a piece of film, will a photograph be formed? The answers are in Problem 14-#.

A *magnifying glass* is a common example of a converging lens. When it is used in the usual way the m-glass is moved close to an object, as in the Near Example above, so the object is between the lens and the focus. When you look from the other side of the m-glass you see the virtual image, which is an enlarged version of the object.

Another way to use the m-glass is to gather and focus light from an object that, as in the Far Example above, is further away than the focal point. For example, if most of the sunlight energy that falls on a large magnifying glass is focused into a very small area, the energy intensity can be great enough to set paper on fire, even though ordinary sun intensity makes the paper only slightly warm.

Diffuse Reflection

Many surfaces that look smooth are rough (like ~~~~~~~~) on the microscopic level. The reflection law of Section 14.3, incident angle = reflection angle, is still true when light shines on a "microscopically rough" surface. But the surface is oriented in many different directions, so the light is reflected in many different directions. This many-direction reflection is called *diffuse reflection.*

When light shines on an ⥣-shaped object covered with cotton cloth, light reflects diffusely from every part of the ⥣; this reflected light lets us see the ⥣. When a room is totally dark we don't see the ⥣ because there is no light that can be reflected into our eyes, but if every part of the ⥣ is covered with tiny light-bulbs we can see the ⥣.

The principles of optics are the same whether light is direct (coming from tiny sources glued to an object) or indirect (initially coming from another source and then being reflected from the object's surface).

The ***lens (or mirror) equation*** shows the relationship between d_o (distance from lens to object), d_i (distance from lens to image) and **f** (distance from lens to focus):

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

{Instead of the symbols "d_o, d_i and f", some textbooks use "p, q and f" or "o, i and f".}

This equation (and the "± rules" given below) can be used for lenses or mirrors.
{For a mirror, d_o, d_i & f are, of course, distances from the mirror to the object, image & focus.}

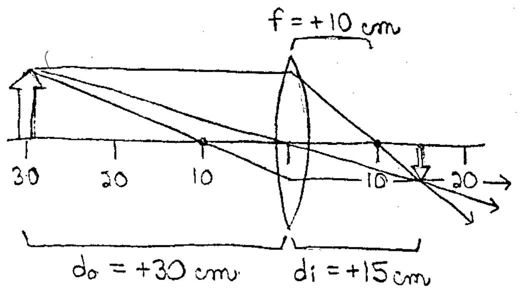
± Signs: d_o and d_i and f are + for "real" quantities, are – for "virtual" quantities.

d_o is real if, as is usually the case, light is coming from the side of the lens where the object is located. {As discussed in Problem 14-#, a *virtual objects* (with a d_o that is negative) can occur in a "two lens" or "lens and mirror" situation.}

d_i is + for a real image, – for a virtual image. {By thinking about the fact that a lens transmits, a mirror reflects, and a real image is formed "on the side where light goes to", you can logically conclude that a real object & real image are on opposite sides of a lens, but on the same side of a mirror. For a virtual image it is reversed: a real object & virtual image are on the same side of a lens, but on opposite sides of a mirror.}

f is + if P-rays actually pass through the focus; this occurs for a converging lens or mirror. {A memory trick: <u>co</u>nverging and <u>po</u>sitive both have an "o".} **f** is – if the backward extensions (- - - -) of P-rays pass through the focus; this occurs with a diverging lens or mirror.

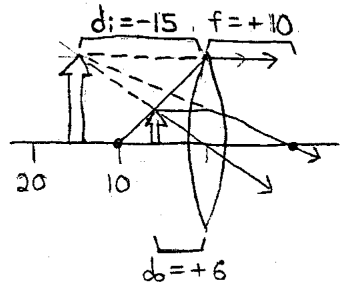
An example: the two converging lens examples we used earlier are drawn below, but now the distances are included. The lens equation predicts that images should be formed at +15 cm (real, 15 cm beyond the lens) and at –15 cm (virtual, 15 cm before the lens). These are the same image-locations we get by "tracing rays"!



$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{+30} + \frac{1}{d_i} = \frac{1}{+10}$$

$$d_i = +15$$



$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{+6} + \frac{1}{d_i} = \frac{1}{+10}$$

$$d_i = -15$$

Be consistent. Use the same units (but they don't have to be SI; you can use cm) for d_o, d_i and f. Use your calculator's "1/x button". To solve the first equation, punch: 10 1/x – 30 1/x = 1/x.

Magnification "m" is a ratio that shows the size of an image compared with the original object: $m \equiv h_i / h_o$, where h_i & h_o are the heights of image & object.

Look at the left-side picture above and notice the similar triangles formed by the object & image arrows (with lengths h_o & h_i), the center-line that connects the bases of these arrows (the lens splits this line into lengths d_o & d_i), and the C-ray that forms the hypotenuses). Each Δ has a 90° angle, and equal angles are formed by the "x-ing" of the center-line and C-ray, so the Δ's are similar and their side/side ratios are equal: $h_i/h_o = -d_i/d_o$. **h_i and d_i are proportional**; if an image is far from the lens, it is a large image. The negative sign is needed to give correct results for the inversion (or non-inversion) of the image; this is illustrated later.

3 equations ($m = h_i/h_o$, $m = -d_i/d_o$, $h_i/h_o = -d_i/d_o$) are in this 3-sided equation:

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

h_o and h_i are + for a right-side up object or image (like ⇑), but they are − if the object or image is inverted (like ⇓). The ± signs for d_o and d_i (+ for real, − for virtual) were discussed earlier.

For the first converging-lens example, $m = -d_i/d_o = -(+15)/(+30) = -1/2$; the minus sign shows that the image is "inverted" with respect to the object. Look at the first diagram above; is the image half as large as the object, and is it flipped upside down?

For the second example, $m = -d_i/d_o = -(-15)/(+6) = +5/2$. Does the second diagram show that the image is $2\frac{1}{2}$ times as big as the object and, as indicated by the + sign, rightside up?

When an image is produced by one lens or by one mirror, a real image is always inverted (flipped upside down) with a negative "m", while a virtual image is always erect (right side up) with a positive "m". Any well-formed image, either virtual or real, will have the same shape as the original object.==cut?[incorrect? convex car mirrors do distort (because of imperfections? distorts 3-D object because things at different distances aren't shown with correctly scaled distances in the image??]

LINKS: d_o and d_i are in the Lens Equation and Magnification Equation.

Angular magnification is discussed in Section 14.5.

Image Formation by a Diverging Lens

On the diagram below, $d_o = +25$ cm, $f = -10$ cm. The image location is found by tracing rays or (you can do this yourself) by using the lens equation. As usual, the C-ray is easy to draw. Since this is a diverging lens, the P-ray is bent away from the true focus, and thus away from the center line, not toward the pseudo focus. If you try to draw the F-ray through the true focus, the outgoing parallel ray is bent toward the center line; this is not what a diverging lens does! Instead, you aim the F-ray as if it was going through the pseudo-focus (this is shown by the ···· extension); when this ray is refracted to become parallel, it is bent away from the center-line.

Every lens has two focuses: true and pseudo. When you draw the P-ray or F-ray, try both focuses and decide which one should be used.

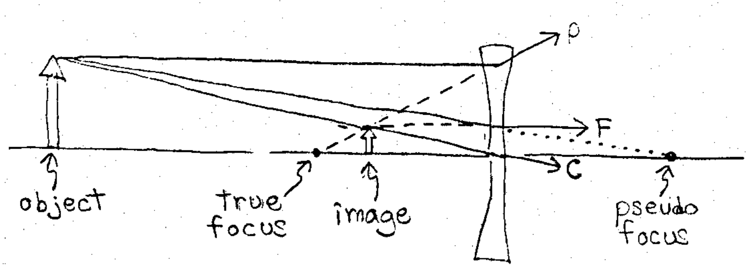


Image Formation by Converging and Diverging Mirrors

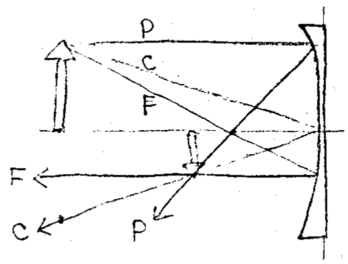
The pictures below show ray-tracing for a converging mirror (with $d_o = +30$ cm, $f = +10$ cm), and a diverging mirror (with $d_o = +25$ cm, $f = -10$ cm).

Mirrors have only one focus, so the P-ray and F-ray are easy to draw.

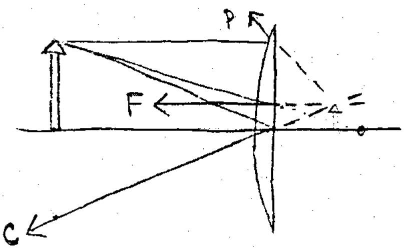
For a lens the C-ray goes straight, just like it would through a flat plane-lens. Similarly, for a mirror the C-ray reflects symmetrically, just like it would from a plane-mirror. Before you trace the C-ray, find the object's "mirror image on the other side of the center line"; it is shown by •. The C-ray goes from the object to the center of the mirror, and then reflects through the •-point.

At the left below, the 3 rays from the arrow-tip meet to form a real image. At the right, backward extensions of the rays meet to form a virtual image of the arrow-tip.

If the 3 rays (or their extensions) don't meet in one spot, you've made a mistake. It is easier to find the error if you use the "$1/d_o$ + ..." equation to find where the image should be. If you want, you can substitute facts from the first paragraph and check: does the equation predict the same image locations that are found by the ray-tracings below?



CONVERGING MIRROR



DIVERGING MIRROR

If only one lens [or mirror] is used, which insures that the object is real*, a converging lens [or mirror] forms a real image when the object is farther away than the focal point, and a virtual image when the object is nearer than the focal point.

But a single diverging lens [or mirror] can only form a virtual image; no matter where the object is located, a real image cannot be produced.

If only one lens [or mirror] is used, a real image is always inverted (flipped upside down), while a virtual image is always erect (right-side up).

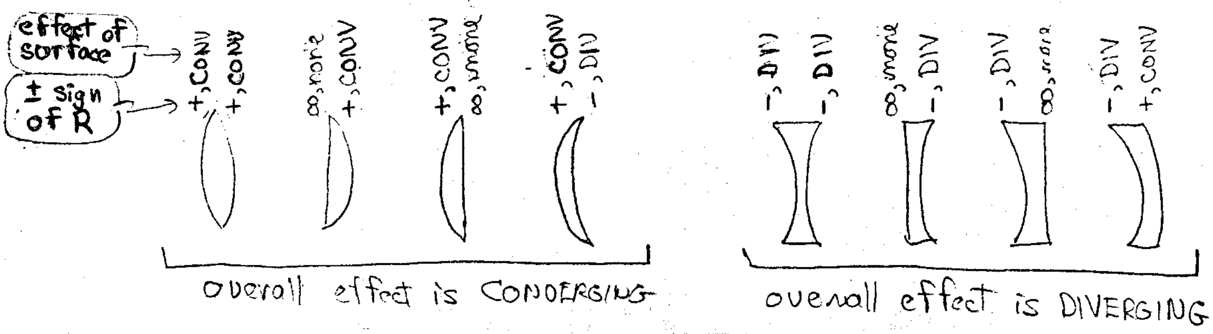
* As discussed in Problem 14-#, if the first lens [or mirror] is converging it can cause a second lens [or mirror] to have a "virtual object". When this occurs, these inversion-principles are reversed for the second lens [or mirror]; a real image is inverted (compared with the virtual object) but a virtual image is erect (if the virtual object is ⇓, the virtual image is also ⇓).

14.5 The Lens (and Mirror) Maker's Equation. Cameras, Microscopes and Telescopes.

The **LENS MAKER'S EQUATION**, which can be derived by using Snell's Law and geometry, gives the approximate focal length of a <u>thin spherical</u> lens <u>in air</u>:

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

where **f** is the focal length of the lens, **n** is the index of refraction of the lens material, R_1 and R_2 are the radii of each of the lens-surfaces. If a surface is convex (see the diagrams below) its R is +, and it helps the lens converge light. A concave surface has an R that is −; it helps the lens diverge light. A flat surface has R = ∞, so $1/R = 1/\infty \approx 0$; it is "optically neutral", neither converging nor diverging.



Another version of the lens maker's equation is "$1/f = (n - 1)(1/R_1 - R_2)$", with the + changed to −. When it is used correctly, this equation gives the same results as the lens maker's equation above.

The **MIRROR MAKER'S EQUATION** gives the relationship between the focal length "f" of a mirror (either converging or diverging) and its spherical radius "R": $f = \frac{1}{2} R$.

A spherical-shaped mirror works fairly well for light rays that hit near the center of the mirror. But a parabolic-shaped mirror does a better job at making all parallel rays (or all rays from a tiny point-object) reflect through the same focal point, no matter what part of the mirror the rays hit.

In Section 14.4, I refer to lenses & mirrors that are shaped "just right". Actually, it is impossible to make a lens (or mirror) that works perfectly under all conditions: for objects at all distances, in all off-center directions, for all wavelengths of light. Your textbook may discuss some causes of *optical aberration* and describe clever designs that minimize the adverse effects of these aberrations.

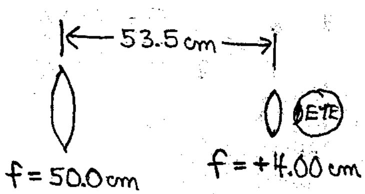
Cameras, Magnifying Glasses, Microscopes, Telescopes, Binoculars,...

The basic principles of *cameras* were discussed in Section 14.4. The human eye is a familiar example: it is an extremely well designed == "motion picture camera" with automatic focusing (from a few inches away on out to infinity), self developing and self renewing color film (with more than 100 million light-receptive cells), plus a sophisticated computer to process images in real time and provide 3-D stereoscopic data that is combined with sensory information from your body (shoulder, elbow and wrist joints) to give eye-hand coordination. And visual images provide stimulation for what you are doing right now — thinking! [minimum maintenance? at ==? probably not nec;self/renewing & self/developing]

You can learn the details of magnifying glasses, telescopes,... from your textbook and teacher. This section just illustrates a few key principles (that will help you understand the details) with an Aesop's Problem.

PROBLEM 14-B: A Two-Lens Telescope

A 10.0 m tall tree is 1000 m away. It is viewed through the two-lens combination shown at the right. How tall is the final image (that is seen by the eye), and how far from the second lens is it?



SOLUTION 14-B

Step 1 (of 5 steps): Substitute into the lens equation for the first lens, solve for d_i.
Step 2: Substitute d_i into one of the magnification-equations, and solve it for h_i.

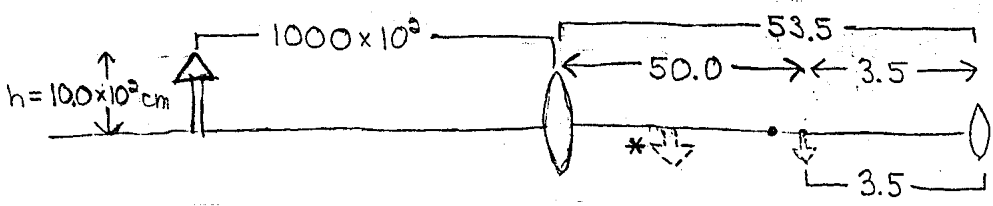
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \qquad\qquad \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$\frac{1}{+1000 \times 10^2} + \frac{1}{d_i} = \frac{1}{+50.0} \qquad\qquad \frac{h_i}{+10 \times 10^2} = -\frac{+50.0}{+1000 \times 10^2}$$

$$d_i = +50.0 \text{ cm} \qquad\qquad h_i = -.50 \text{ cm}$$

The image formed by the first lens becomes the "object" for the second lens.

Step 3: Draw a diagram like the one below, then use "total = sum of parts" logic to find that the first-lens image (the second-lens object) is 53.5 – 50.0 = 3.5 cm from the second lens, which thus has d_o = +3.5 cm. The ± sign is + because light comes from this "object" and continues moving rightward through the second lens. {Unless you are "tracing rays", the distances in your diagram don't have to be drawn in correct proportion.}



Steps 4 & 5 are analogous to Steps 1 & 2. Solve for d_i, substitute it and solve for h_i.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \qquad\qquad \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$\frac{1}{+3.5} + \frac{1}{d_i} = \frac{1}{+4.0} \qquad\qquad \frac{h_i}{-.5} = -\frac{-28}{+3.5}$$

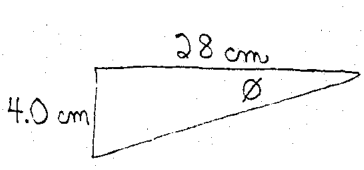
$$d_i = -28 \text{ cm} \qquad\qquad h_i = -4.0 \text{ cm}$$

Your eye sees an image (represented by ∗⇓ in the diagram above) that is virtual (as shown by the – sign of d_i) and located 28 cm from the second lens. This image is 4.0 cm high and (as shown by the – sign of h_i) is upside down compared with the original object (the tree).

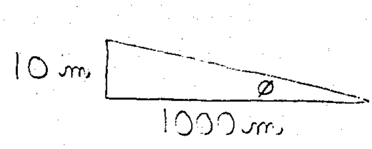
The "inversion principles" at the end of Section 14.4 correctly predict that the final image is ⇓. The first-lens image is real so it is inverted; ⇑ flips to become ⇓. The second-lens image is virtual so it isn't flipped; ⇓ remains ⇓. {As shown in Problem 14-##, these principles cannot be used if the second lens has a "virtual object".}

As shown in the first picture below, your eye sees an image that is 4.0 cm high and 28 cm away. The *angular size* of the image is $\text{Ø}_{image} = \tan^{-1}(4.0/28) = 8.13°$. Compare this with the second picture, which shows that looking at the tree without the "two-lens telescope" gives a smaller angular size: $\text{Ø}_{object} = \tan^{-1}(10/1000) = .573°$.

ANGULAR SIZE OF IMAGE



ANGULAR SIZE OF OBJECT



The image is an upside-down tree with an angular size of 8.13°. If you draw the appropriate triangle, you'll find that in order to see an 8.13°, 10.0 m tree, you would have to be 10/tan(8.13°) = 70 m from the tree. The telescope seems to bring the tree "closer" to you, from 1000 m away to only 70 m.

Angular magnification "M" is defined as:

$$M \equiv \frac{\text{angular size of the image that is formed by an optical device}}{\text{angular size of the object as it is seen by the unaided eye}} = \frac{\text{ø}_{image}}{\text{ø}_{object\ (unaided)}}$$

Our telescope's angular magnification is $\text{ø}_{image} / \text{ø}_{object}$ = (8.13°)/(.573) = 14.2. {This is approximately equal to the ratio of 1000m/14m, which is 14.3.}

Compare this with the *magnification* (also called *lateral magnification)* defined in Section 14.4: $m \equiv h_i/h_o = (-.04\text{ m})/(10\text{ m}) = -.004$. This would seem to imply, if we interpret "magnification" with its everyday non-technical meaning, that the tree looks smaller! But we know, as shown by the analysis above, that the telescope makes the tree look larger. Do you see why "angular magnification" (it is 14.2) gives a more intuitively meaningful description of what the telescope actually does?

For a telescope that magnifies faraway objects, ø_{image} and ø_{object} are calculated as in Problem 14-B: $\text{ø}_{image} = \tan^{-1}(h_i/d_i)$, and $\text{ø}_{object} = \tan^{-1}(h_o/d_o)$ where d_o is the distance to the <u>original</u> object. {In 14-B, d_{obj} is 1000 m (eye to tree), not the 3.5 cm (second lens to first-lens-image) that is used as d_o in "$1/d_o + 1/d_i = 1/f$" for the second lens.}

For a microscope that magnifies small nearby objects, the "unaided eye" cannot focus on an object that is closer than the eye's *near point.* {The near point varies from one person to another, and increases with age, but is usually taken to be ≈25 cm.} For calculating ø_{object} we assume that the object is at the closest distance where a clear, in-focus image can form, at the near point, and $\text{ø}_{object} = \tan^{-1}(h_o/d_{near\text{-}point})$.

For a telescope,

$$M = \frac{\tan^{-1}\frac{h_i}{d_i}}{\tan^{-1}\frac{h_{original\ object}}{d_{original\ object}}}$$

For a microscope,

$$M = \frac{\tan^{-1}\frac{h_i}{d_i}}{\tan^{-1}\frac{h_{original\ object}}{d_{near\text{-}point}}}$$

14.90 Memory-Improving Flash Cards

14.1	EM waves survive and move because ___.	ΔB → E (and ΔE) while ΔE → B (and ΔB)
14.1	___ and ___ give the same light-speed.	theory-prediction, experiment-measurements
14.1	White light contains ___. Red vs. blue: Red has ___ and ___ (λ, f).	a wide range of frequencies (and colors) larger λ, smaller f
14.2	When n ↑, v ___, f ___, λ ___.	↓, stays same, ↓
14.2	Light speed doesn't depend on ___. ==cut?	the light's "history" (its previous speed)
14.2	n & v of red vs. blue: In vacuum, red has ___. In glass, red has ___.	same n (≡ 1.00) and same v smaller n and faster v

14.3	*Refraction* is the __ of __ light at __ .	direction change, transmitted, interface
14.3	θ is __ , so light that "skims" has $\theta =$ __ .	angle-away-from-normal, 90°
14.3	n_1 vs. n_2: If , __ . If , __ . n and $\sin\theta$ are __ , so if n is large, $\sin\theta$ __ .	n_2 is larger (like air-to-water), n_2 is smaller inversely proportional, is small (and so is θ)
14.3	Moving either direction, light paths are __ .	the same (they are "reversible")
14.3	To get __ , light must start in the __ and have a θ that is __ than the __ .	total internal reflection, larger-n medium larger, critical angle θ_c
14.3	Geometry: to solve problems, use __ .	ΔXYZ, extra lines, total = sum, similar Δs
14.3	To reflect lots of light, use __ interface, __ angle. If $\theta = 0°$ and n changes from 1.0 to 1.5, __ .	large n-change, large-θ 4% of light is reflected
14.3	If a surface is __ , light reflects in __ . (2)	smooth, 1 direction; rough, many directions
14.4	Our eyes use __ to find the location of __ .	triangulation logic, object or image
14.4	In a plane mirror the image is __ , __ the object.	behind the mirror, directly opposite
14.4	__ lens or mirror bends light __ center-line. (2)	converging, toward; diverging, away from
14.4	Three easily-traced rays are __ (__), __ (__) and __ . A lens has __ , so you must __ .	C (straight thru center), P (parallel, focus) F (focus, parallel); two focuses, choose
14.4	A __ image is formed when gathered light __ . A __ image is formed when gathered light __ .	real, goes to the same point virtual, seems to come from the same point
14.4	An eye (or camera) can see __ .	an object, real image, virtual image
14.4	Optics principles are same whether light is __ .	direct (from source) or indirect (reflected)
14.4	To determine the ± sign for d_i, d_o and f : __ .	+ for real quantities, – for virtual quantities
14.4	__ and __ give the same image location.	ray-tracing, lens (or mirror) equation
14.4	One converging lens (or mirror) can form __ . One diverging lens (or mirror) can form __ .	either real or virtual images only virtual images
14.4	If a real object is ⇑, a __ image is __ . (2) If a virtual object is ⇓, a __ image is __ . (2)	real, inverted to ⇓; virtual, ⇑ (not inverted) real, ⇓ (not inverted); virtual, inverted to ⇑
14.5	The focal length of a mirror depends on __ . The focal length of a lens depends on __ .	its radius of curvature its n and radii of curvature
14.5	The human eye is a __ camera, with __ film, linked to a __ that provides __ pictures and __ .	motion picture, self-developing & renewing, computer, 3-D color, eye-hand coordination
14.5	In lens-lens (or lens-mirror) combinations, __ .	first lens-image becomes second-lens object
14.5	Regular & angular magnification are __ of __ . Your eye estimates size by __ , not __ .	ratios, heights (h_i/h_o) & angles ($ø_i/ø_2$) angular magnification, regular magnification