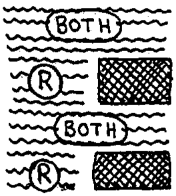
Chapter 11

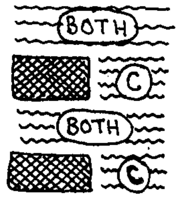
Direct-Current Circuits: Resistors and Capacitors

Sections 11.1 to 11.7 cover the essentials of direct current circuits.
Many principles apply to resistors **and** capacitors; these take up the full page width.
Tools that are used only for resistors (or only for capacitors) occupy half of the page.

When you study resistor circuits, read the full-width parts and left columns.



When you study capacitor circuits, read the full width parts and right columns.



Section 11.8 is a good summary of CAPACITOR FORMULAS. It is independent from Sections 11.1 to 11.7. Before you read 11.1, look at 11.8 just to see what is there, then study the 11.8 formula summary whenever you think it will be useful.

Optional Resistor Topics: Kirchoff's Junction & Loop Rules (11.9), Current Density & Drift Velocity (11.92), how resistance varies with temperature (Problem 10-#), Voltmeters & Ammeters (Problems 10-## to 10.##).

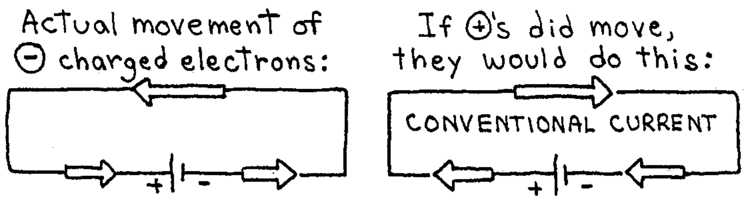
RC Circuits, which contain resistors **and** capacitors, are studied in Section 11.10.

11.1 Batteries, Charge Movement and Buildup. Resistors & $\underline{\Delta V = I\,R}$, Capacitors & $\underline{Q = \Delta V\,C}$.

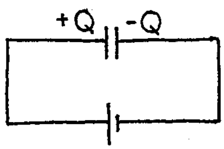
Sections 10.5 & 10.7 defined what *electric potential (voltage)* is and explained the relationship between $F_{electric}$, E and ΔV. In this chapter, we'll study what ΔV does.

Most circuit diagrams use "⊣⊢" to represent a ΔV-producing battery. The large & small vertical lines show the battery's + and – terminals, respectively. Chemical reactions inside the battery produce an *electric potential difference* "ΔV" between its + and – terminals. { Some battery details, including electromotive force and terminal voltage, are discussed in Section 11.5.} This ΔV "makes things happen" in a circuit:

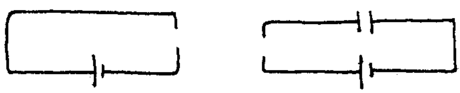
1) In a metal wire, electrons are free to move. They have – charge, so they're repelled away from the – terminal and attracted toward the + terminal. As shown below, the *conventional current* that is drawn on most circuit diagrams goes from the + to – terminal; this is the direction + charges would move if they did move, which they don't. The reason for this backwards definition (Benjamin Franklin Plays With a Kite) is discussed in Problem 10.#. Or just accept the fact that current flow is defined this strange way; if you analyze circuits by imagining that + charges are moving, you'll get correct answers.



2) If two parallel metal plates (which form a *capacitor*, represented by ⊣⊢) are placed in a circuit, the battery-caused movement of charge makes + and – charge build up on the plates that are connected to the battery's + and – terminals. { Charge accumulates on the plates because it usually can't leap across the air gap between them. When the plates have a certain charge, no more charge moves onto them because of the following "push-of-war" balance: the battery tries to push + charge onto the + plate (and – charge onto the – plate), but this is prevented by the electrostatic repulsive force caused by charge that is already on the plates.}



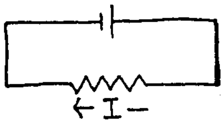
The circuits below don't make a "complete loop" from the + to – terminal, so current won't flow, and charge won't build up on the ⊣⊢'s plates:



The formulas below explain the basic equations of circuit analysis.
Read only the column for the subject (either resistors or capacitors) that you are studying now.

Resistors: $\Delta V = I R$

If a metal wire (like copper, gold,...) is connected across a battery's terminals, a large current flows. To reduce and control the amount of current flow a *resistor*, represented by -/\/\/- , is used.



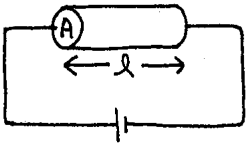
The amount of *current* **I** flowing through a resistor "-/\/\/-" depends on the *potential difference* ΔV between its ends, and its *resistance* **R**: $\Delta V = I R$.

SI units: ΔV is in Volts (V), I is in Amperes (Amps, A), R is in ohms (Ω).

If 5 Coulombs of charge move past a point during 1 second, I is 5 C/s. This flow of charge is called "5 Amperes": 5 C/s ≡ 5 Amperes = 5 Amps = 5 A.

Some circuit elements (like *transistors*) don't follow "ΔV = IR". I won't be discussing transistors (or "semiconductors").

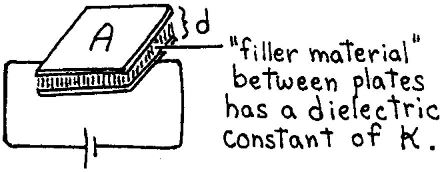
A resistor's "resistance" depends on its structure. $R = \rho \ell / A$, where ρ is the *resistivity* of the -/\/\/-'s material (most textbooks have tables of ρ values; as discussed in Problem 11-##, ρ usually increases slightly as temperature increases), ℓ is the -/\/\/-'s length, and **A** is its cross-section area:



Capacitors: $Q = \Delta V\, C$

A capacitor's "capacity" to hold charge (which is its *capacitance*, abbreviated **C**), depends on its surface area **A**, plate separation **d**, and the *dielectric constant* κ of the material between its plates: for a parallel-plate capacitor,

$$C = \epsilon_0 \kappa \frac{A}{d}$$



If air, which has $\kappa = 1$, is between the plates, the C-magnitude formula is $C = \epsilon_0 A/d$. Other dielectrics are discussed in Problem 11-#.

ϵ_0 is, as stated in Section 10.1, a constant with a value (in SI units) of 8.85×10^{-12}.

Capacitors can have other shapes: cylindrical, rolled, spherical, ... {I won't give magnitude formulas for any of these shapes.}

The amount of charge **Q** that builds up on a capacitor's plates depends on the ΔV between them and the capacitor's *capacitance* **C**: $Q = \Delta V\, C$.

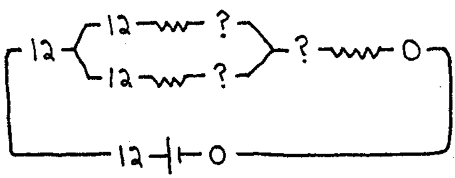
SI units: Q is in Coulombs (C), ΔV is in Volts (V), C is in Farads (F).

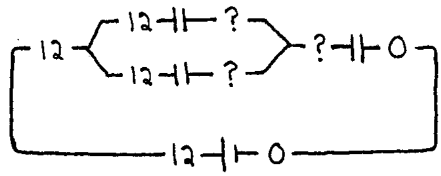
There are two ways to define R (or C).
One is based on "structure", what a -/\/\/- or -||- <u>is</u>: $R = \rho l / A$, and $C = \kappa \varepsilon_0 A / d$.
The other formula describes what a -/\/\/- or -||- <u>does</u>: $\Delta V / I = R$, and $Q / \Delta V = C$.

11.2 Voltage Logic for Circuits

If we arbitrarily define the low-V (negative) terminal of a 12 Volt battery to be V ≡ 0, the high-V (positive) terminal, which is 12V higher because of the chemical reactions occurring within the battery, will be at +12V.

V-TRACING LOGIC : For all ⊣⊢ circuits and most -ww- circuits*, every point on a wire has the same voltage, until it is interrupted by a ⊣⊢ or -ww- . If V is +12 (or 0) at a battery terminal, then it will be +12 (or 0) everywhere the wire goes:

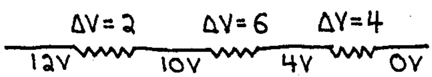


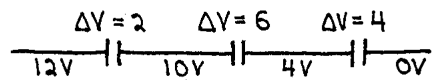


The ?'s show that you can't trace V through a ⊣⊢ or -ww-.
When a wire splits, like at the —< , both branches have the same V.

* If a wire's resistance is negligible compared with the circuit's total resistance (because this is usually true, it is assumed in the rest of this chapter), all wire-points have approximately the same V.

ΔV LOGIC : If the voltages on the diagram below are known (12, 10, 4 and 0), you can find the ΔV across each -ww- or ⊣⊢ by using "$\Delta V = V_{high} - V_{low}$". For the first resistor, ΔV = 12 − 10 = 2. Do you see how the other ΔV's (of 6 and 4) are calculated?



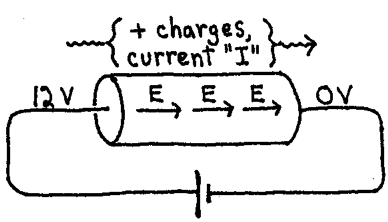


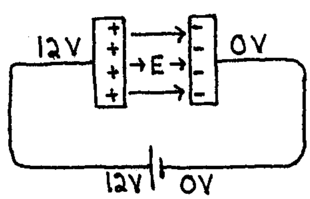
For the first -ww- or ⊣⊢ above, ΔV = 2 (not the actual V's of 12 or 10) is substituted into V=IR or Q=VC. ΔV (the <u>change</u> of V) is always used for "ΔV = IR" or "Q = ΔV C", even when equations are written in abbreviated form as "V = IR" or "Q = V C".

Section 10.## shows the relationships between $F_{electric}$, E and ΔV. In the first picture below, if + charges move rightward the E field must point toward the right, in the direction of decreasing V. **Conventional current, the imaginary movement of + charge, always goes "downhill" in number-line voltage, from high V to lower V.**

Section 10.3 states that E = 0 inside an *electro<u>static</u>* conductor, but a current-carrying conductor is not electrostatic so it can (and always does) have a non-zero E field.}

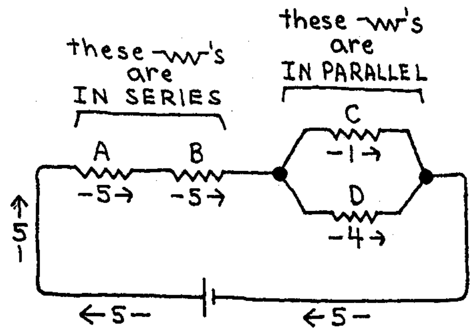
The second picture shows the relationships between V, E and charge separation; **+ charges build up on the high-V ⊣⊢ plate (which is connected to the + terminal of the battery) and – charges build up on the low-V plate.**

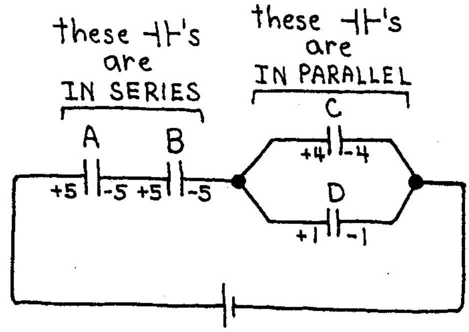




11.3 <u>Series</u> and <u>Parallel</u> Circuit Connections

Imagine that you're walking clockwise around either of the circuits below. You must walk across both A and B; they are *in <u>series</u>*. When you get to a *junction* (marked "•"), the circuit splits so you can choose to go across either C or D; they are *in <u>parallel</u>*. Do you see, using 11.2's V-tracing logic, that each "in parallel" -/\/\/- (or -| |-) will have the same ΔV across it?

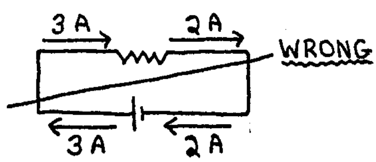




If resistors are in **series**, the **same current** (in this case, 5A) flows through each -/\/\/-, like the steady flow of water through a pipe. {The analogy to familiar flowing water may help you clearly visualize the impossible-to-see, yet very real, flow of charge through a wire.}

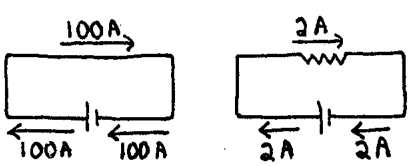
When the circuit splits at a **parallel junction, the I coming in equals the I going out**. In the example above, 5A comes into the first •-junction and 5A (the sum of 1A + 4A) goes out.

The picture below shows a WRONG idea of "resistance", that I is reduced by the -/\/\/- (from 3 A to 2 A) and is then increased back to 3 A by the battery:



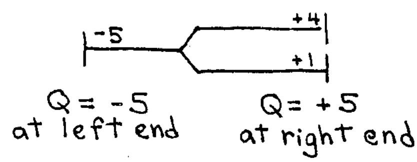
Do you see why this kind of charge flow cannot occur? {Hint: the reason is in Section 10.1.} This question is discussed in Problem 11-#.

The pictures below show the CORRECT idea of resistance. Without -/\/\/-, the circuit's R is small and I is large. The -/\/\/- reduces I to 2 A, but 2 A flows through every part of the circuit.



A capacitor's + and − plates always have equal-and-opposite charges. This is shown above: +5 & −5 are equal-and-opposite, so are +4 & −4, and +1 & −1.

Each section of a circuit that is not connected to the battery has equal-and-opposite charges at its + and − ends. Why? If a section is originally neutral (this is the usual assumption) and no charge is given to it by an external object or by charge leaping across the -| |- gap from an opposing plate, the section will stay uncharged. For example, in the diagram above the |—|-shaped section has −5 & +5 at its left & right ends. |—< also has −5 & +5 (the sum of +4 and +1) at its left & right ends:



These facts about plates and sections lead to the following conclusions:

SERIES CAPACITORS have the same charge. {Above, A & B each have Q = 5.}

PARALLEL CAPACITORS have enough charge, when added together, to satisfy the requirement that charges at ends of a "section" must be equal-and-opposite. {Above, C & D are in parallel; their combined Q (+1 and +4) balances the −5 at the left end.}

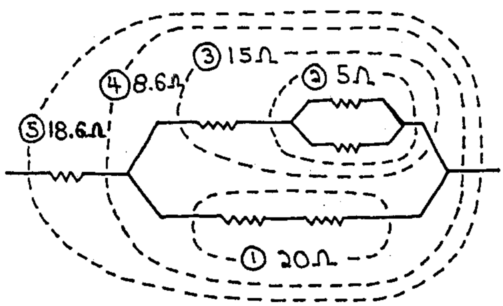
The capacitor example above shows the two ways to think of a *metric prefix* that are described in Section 1.6. For calculation of C_{total}, "n" is treated as if it was part of the "nF" unit; 5 nF is thought of as 5 <u>nF</u>. If every C is in nF units, you know that C_{total} will also be in nF units (it is 1.28 nF) so you don't have to punch the 10^{-9}'s into the calculator.

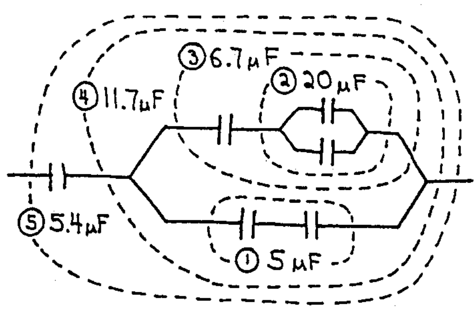
But for Q=VC, "n" is treated as part of the number, not as part of the unit. C = 1.28 nF = <u>1.28 n</u> F, and C is substituted (without units) as "1.28 n", which means "1.28 x 10^{-9}". Similarly, when Q=VC is written without units, Q = <u>23 n</u> means that Q = <u>23 n</u> Coulombs = <u>23 x 10^{-9}</u> C.

Complex circuits can usually* be separated into groupings that are pure series or pure parallel. These groups can then be recombined to get R_{total} (or C_{total}).

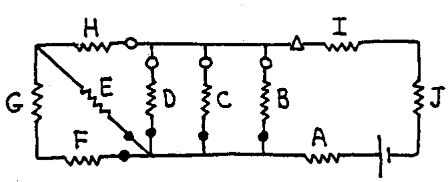
* Some circuits cannot be split into series & parallel groups; if such a circuit contains resistors, it can be analyzed using "Kirchoff's Rules", as explained in Section 11.8.

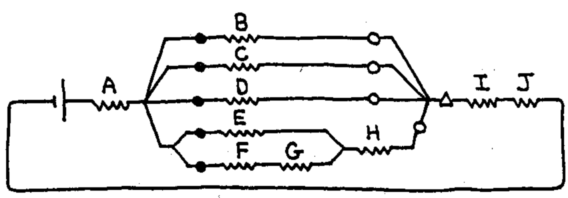
In the diagrams below, each -WW- is 10 Ω, and each -||- is 10 μF. For Steps 1 to 5, the R (or C) for the ( )-group is shown. For practice, do these calculations yourself:





With circuits in "rectangular" format, it is difficult to see whether components are in series or parallel. Sometimes it helps to redraw a circuit in a visually logical "tournament bracket" style that is easier to analyze. Below, a circuit is re-drawn to make series & parallel relationships clear. { -||- circuits use the same re-drawing principles }





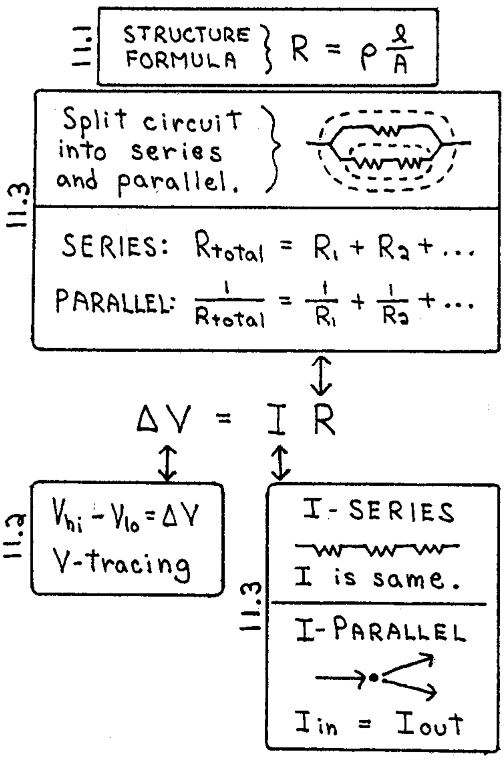
Series combinations (like F & G, or I & J) are easy to see. Imagine that you are walking around the circuit; if you must walk through both -WW-'s, they are in series.

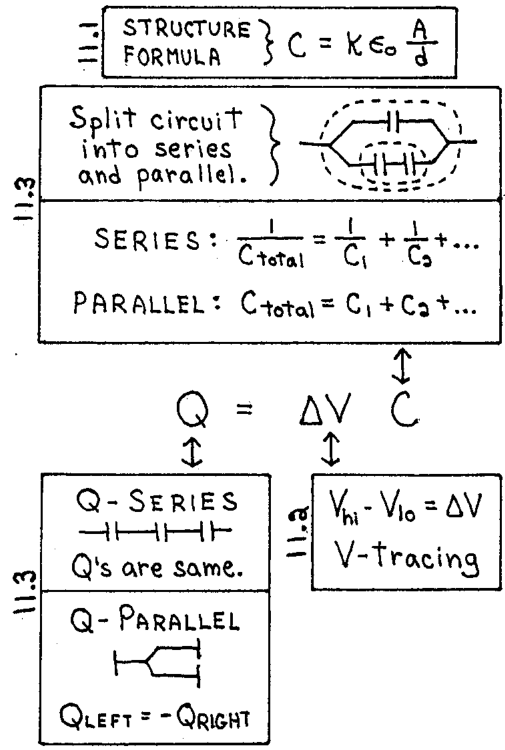
<u>Use V-tracing</u> to find parallel combinations. The five spots marked "•" have the same V; they are at one end of a parallel group. The four o's are at the other end of this group. The Δ point has the same V as the o's, but the o's are before the parallel junction (after going through the "previous" -WW-'s: H,D,C or B) while Δ is after the junction (before going through the "next" -WW-, I). Do you see the difference between Δ and the o's? A similar "before and after" difference occurs at every parallel junction, like between G/E and H.

I use **colors** for V-tracing; if the wire-after-A is red (traced over the original black), and the wire-before-I is green, it's easy to see that all 4 parallel branches have the same "red-to-green" V change.

11.4 A Strategy for Analyzing Circuits

These summaries show how to "funnel" information into V=IR (or Q =VC):





V=IR (or Q =VC) can be used for each individual -ΛΛ- (or -||-), or for any combination.

If you know any two variables in V=IR (or Q =VC), you can find the third.

LOOK FOR LINKS! Each time you solve for something, use it in every possible place, <u>on the circuit diagram</u> or <u>in one of the many V=IR (or Q =VC) equations</u>:

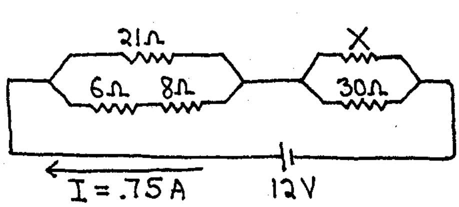
SOLVE ⇒ USE ⇒ SOLVE ⇒ USE,... until you've solved the problem.

Problems 11-AR & 11-AC are more complicated than most you'll have to solve, but they're worth studying closely because they show how to use all 6 of the main tools from the summary above:

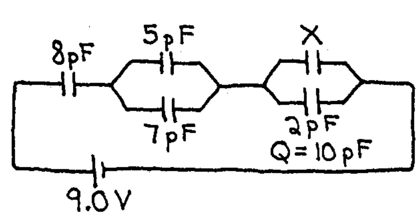
Finding R_{total}. **Solving V = IR.**
V logic: **$\Delta V = V_{hi} - V_{lo}$** and **V-tracing.**
I logic: **Series-I** and **Parallel-I.**

Finding Q_{total}. **Solving Q = VC.**
V logic: **$\Delta V = V_{hi} - V_{lo}$** and **V-tracing.**
Q logic: **Series-Q** and **Parallel-Q.**

PROBLEM 11-A$_R$: V= IR Analysis
Find R for the resistor marked "X".



PROBLEM 11-A$_C$: Q = VC Analysis
Find C for the capacitor marked "X".

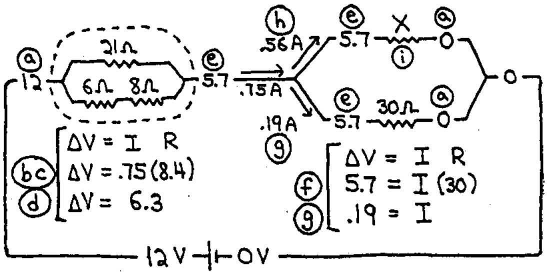


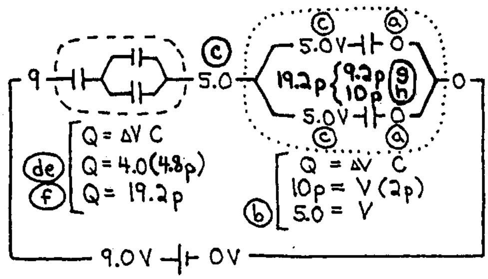
SOLUTION 11-A_R: As you read each step, look at the corresponding letter on the diagram below. { You'll find it easier to follow my explanation if you draw your own diagram and make step-by-step changes on it. }

a) Trace V's of 12 & 0 away from ⊣⊢.
b) Series-I: the I through ( ) is .75 A.
c) Rtotal for the ( ) group is 8.4 Ω .
d) Use V=IR to find ΔV across
e) Use ΔV = Vhi – Vlow to find V= 5.7 on other side of ( ), and trace V's.
f) Use ΔV = Vhi – Vlow to find ΔV across the "X" and 30 Ω resistors.
g) V=IR gives I through the 30Ω resistor.
h) Parallel-I: the rest of the .75 Amps (.75 – .19 = .56A) goes through X.
i) V=IR: $R_X = \Delta V/I = 5.7/.5.6 = 10\ \Omega$.

SOLUTION 11-A_C: As you read each step, look at the corresponding letter on the diagram below. { You'll find it easier to follow my explanation if you draw your own diagram and make step-by-step changes on it. }

a) Trace V's of 9 and 0 away from ⊣⊢.
b) Q=VC gives ΔV across the 2 pF ⊣⊢.
{"10.0 p" means "10.0 x 10^{-12}", p/p = 1.}
c) Use ΔV = Vhigh – Vlow to find V= 5.0 on left side of 2pF ⊣⊢, and trace V's.
d) ΔV = Vhi – Vlo gives ΔV across ( ).
e) Find Ctotal of the ( ) group.
f) Use Q=VC to find the Q of ( ).
g) Series-Q: ( ) and ( ) are in series, so each holds the same Q of 19.2 pC.
h) Parallel-Q: The 2 pF ⊣⊢ holds 10 pC, so (to get 19.2 pC total) X has 9.2 pC.
i) ΔV = Vhi – Vlo gives ΔV across X.
j) Q=VC: $C_X = Q/\Delta V = 9.2p/5 = 1.8$ pF.





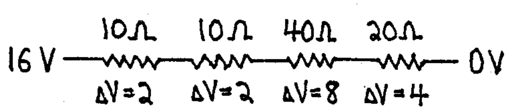
Notice the **links**; each "next step" depends on using the previous step's conclusion. Do you see the connections between diagram-information and equation-information?

Hints: **Draw circuits larger** than those shown above, and **use colors** to show different kinds of information. For example, use black to draw the circuit, black for R's (or C's) and for V=IR's (or Q=VC's), red for V's and ΔV's, and green for I's (or Q's).

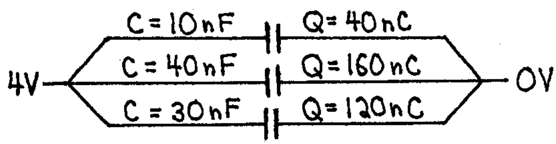
11.5 Other Circuit-Analysis Tools

Problems 11-BR and 11-BC show a useful "shortcut trick" for analyzing circuits:

PROBLEM 11-B_R: Can you find a relationship between R and ΔV?



PROBLEM 11-B_C: Can you find a relationship between C and Q?



SOLUTION 11-B_R

For resistors in series,
ΔV is proportional to R.

For example, ΔV across the 40 Ω -\/\/\- is 2 times as large as ΔV across the 20 Ω -\/\/\-.

Each series -\/\/\- has the same I. It is more difficult to push this I through a -\/\/\- that has large R, so a large-R -\/\/\- requires more ΔV.

It can be shown (as in Problem 11-##) that
R-fraction = ΔV-fraction.
For example, the 20 Ω resistor contributes 1/4 of the total R (20 Ω of the total 80 Ω), so it gets 1/4 of the total ΔV (4V of the total 16V).

A useful "ratio trick" for parallel resistors is explored in Problem 11-2#.

SOLUTION 11-B_C

For capacitors in parallel,
Q is proportional to C.

For example, Q across the 30 nF -||- is 3 times as large as Q across the 10 Ω -\/\/\-.

Each -||- has the same ΔV, and Q is proportional to C (as shown by Q=VC), so the -||- with the largest C has the largest Q.

It can be shown (as in Problem 11-##) that
C-fraction = Q-fraction.
For example, the 10 nF -||- has 1/8 of the total C (10 nF of the total 80 nF), so it gets 1/8 of the total Q (40 nC of the total 320 nC).

A useful "ratio trick" for series capacitors is explored in Problem11-2#.

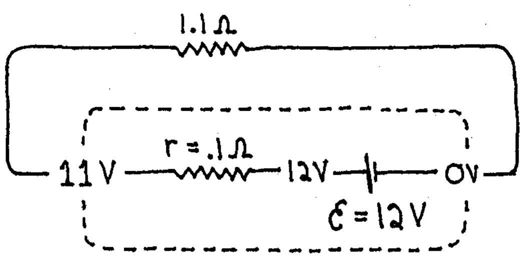
Internal Resistance of a Battery

As electric current moves **through** a battery to complete a circuit loop, it encounters *internal resistance* "r".

When I = 0, chemical reactions in the battery below produce an *electromotive force* "ℰ" of 12 V. But some ΔV is used to push "internal current" through the battery; if I = 10 A and r = .1 Ω, this ΔV is $I_{internal} R_{internal} = (10)(.1) = 1$ Volt. Because of this 1V loss, the battery has a *terminal voltage* (the ΔV between its – and + terminals, ΔV that is available to push charge through external -\/\/\- s) of only 11V:

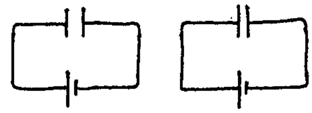
$$\Delta V_{terminal} = \mathcal{E} - I_{internal}\ r_{internal}$$
$$\Delta V_{terminal} = 12 - (10)\ (.1)$$

The entire battery, enclosed by [ ], is drawn as a combination of -||- (an EMF source) and -\/\/\- (internal resistance).

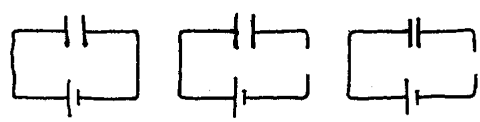


Changing a Capacitor-Circuit

Initially, the -||- below has a certain ΔV and Q. If its plates are moved closer together (which changes C) while the battery is connected, **ΔV stays the same** but Q changes:



If the battery is disconnected before the plates are moved, ΔV can change. But **Q stays the same** because charge cannot escape (unless a spark leaps from one plate to the other) when the circuit-loop is incomplete:



A circuit change* can be done in (at least) two different ways: at constant-V, or at constant-Q.

* By changing a -||-'s plate separation or surface area or "dielectric material" (as in Problem 11-2#), or by reconnecting -||-'s in a different way (Problem 11-##).

11.6 Power, Energy Storage

The principles of *energy transformation and storage,* discussed in Section 7.#, can be applied to electrical circuits. A battery uses up *chemical potential energy* to make electrons move through a circuit containing a resistor or capacitor. When electrons move through a resistor, the -/\/\/-'s temperature increases and, as explained in Section 7.#, some **energy is transformed** into *heat and light*. When a battery causes the separation of + and – charge across a capacitor's plates (this is an "unnatural action" that must be forced, like rolling a ball to the top of a hill), **energy is stored** as *electric potential energy.* Gravitational PE is "released" when a ball rolls downhill; a ⊣⊢'s electrical PE can also be released, as shown in Problem 11-C_C.

To find the amount of energy that is transformed or stored, use $W = -q\,\Delta V$:

When +50 C of charge moves through a resistor with $\Delta V = -10$ (like 12V -/\/\/- 2V), $W_{electric} = -q\,\Delta V = -(+50)(-10) = +500$ J. If this occurs during 20 seconds, Power = $W/\Delta t$ = +500 J / 20 s = 25 Watts.

A -/\/\/-'s power-transformation is $P = W/\Delta t = (Q\,\Delta V)/\Delta t = I\,\Delta V$, because $Q/\Delta t = I$. By using $V = IR$ (try these substitutions for yourself),

$$\mathbf{P = I\,V = I^2 R = V^2/R}$$

If you know any 2 of these 4 variables (V, I, R, P), you can find the other 2 by solving two of these formulas:

$V=IR$, $P=IV$, $P=I^2R$, or $P=V^2/R$.

The P that is transformed by a -/\/\/- is a valuable *linking-tool* that can be used in Section 11.4's circuit-analysis strategy.

While a ⊣⊢ is being charged by a battery with voltage "V", the ⊣⊢'s ΔV goes from 0 (when Q = 0) to "V" (when the ⊣⊢ is fully charged). During this process, $V_{average} = \frac{1}{2}(0 + V)$, and $|W_{electric}| = q\,\Delta V_{average} = Q\,[\,\frac{1}{2}V\,]$.

By using Q=VC (do this on your own), you can derive these three formulas for the potential energy stored in a ⊣⊢ :

$$\mathbf{PE_{el} = \tfrac{1}{2}QV = \tfrac{1}{2}V^2C = \tfrac{1}{2}Q^2/C}$$

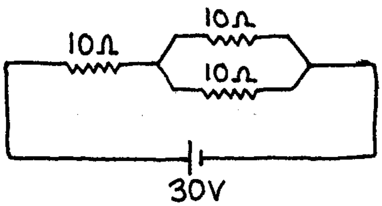
If you know any 2 of these 4 variables (Q, V, C, PE), you can find the other 2 by solving two of these formulas:

$Q=VC$, $PE=\frac{1}{2}QV$, $PE=\frac{1}{2}CV^2$, $PE=\frac{1}{2}Q^2/C$.

The PE that is stored by a capacitor is a valuable *linking-tool* that can be used in Section 11.4's circuit-analysis strategy.

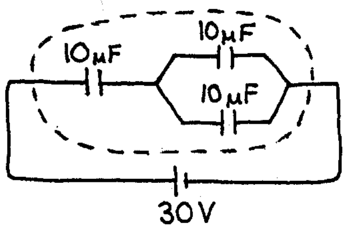
PROBLEM 11-C_R: Power Analysis

Find the power transformed by each -/\/\/- below, the power delivered by the battery, and the Q moved and energy transformed during 2 minutes.



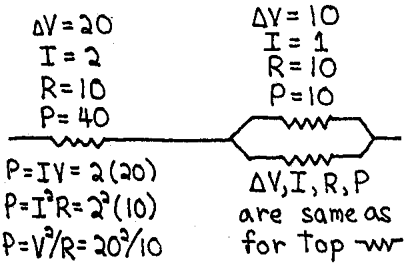
PROBLEM 11-C_C: Stored Energy

Find the energy stored by each ⊣⊢ below. If the ⊣⊢ is replaced by a -/\/\/-, how much Q moves through the -/\/\/-, and how much energy is transformed?



SOLUTION 11-C$_R$

Use Section 11.4 methods to find ΔV and I for each -/\/\/-, then solve for their P's. The left -/\/\/-'s power is calculated in three ways; each P-formula gives the same result, of course.



Each second, the total energy transformed by the -/\/\/-s is 40 J + 10 J + 10 J = 60 J. Total power can also be calculated by combining all three -/\/\/-s into a single -/\/\/-: $P_{total} = I_{total}\,V_{total} = (2)(30) = 60$ Watts, or $P_{total} = (I_{total})^2\,R_{total} = (2)^2(15) = 60$ Watts, or $P_{total} = (V_{total})^2/R_{total} = (30)^2/(15) = 60$ Watts.

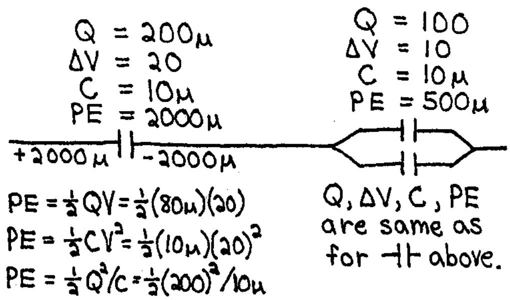
This energy comes from the battery, whose power output is 60 Watts.

Like all "ratios", I and P can be used in standard *conversion-factor equations* :

$$Q = I\,t \qquad W = P\,t$$
$$(\underline{Q}\text{ C}) = (\underline{I}\ \tfrac{\text{C}}{\text{s}})(\underline{t}\text{ s}) \qquad (\underline{W}\text{ J}) = (\underline{P}\ \tfrac{\text{J}}{\text{s}})(\underline{t}\text{ s})$$
$$Q = (2\ \tfrac{\text{C}}{\text{s}})(120\text{ s}) \qquad W = (60\ \tfrac{\text{J}}{\text{s}})(120\text{ s})$$
$$Q = 240\text{ Coulombs} \qquad W = 7200\text{ Joules}$$

SOLUTION 11-C$_C$

To find the total stored PE, add the PE's of individual -||-s : $PE_{total} = 2000\mu J + 500\mu J + 500\mu J = 3000\mu J = 3000 \times 10^{-6}\,J = 3 \times 10^{-3}\,J = 3\,mJ = .003\,J$. Or calculate PE for the whole ( ) combination, using any of the 3 PE-formulas: for example, $PE_{total} = \frac{1}{2}\,QV = \frac{1}{2}\,(200\mu)(30) = 3000\,\mu J$.



If the battery is replaced by a -/\/\/-, 200 μC of + charge moves* from the left -||-'s + plate, through the -/\/\/-, to the – plates of the two parallel -||-s. Energy is conserved during this movement; 3000 μJ (all of the capacitors' stored PE) is transformed into heat and light.

*As stated in Section 11.10, negative-charged electrons really move, but circuits are usually analyzed as if the + charges were moving.

Optional **ENERGY DENSITY:** If your class uses the equation PE density = $\frac{1}{2}\,\varepsilon_0\,E^2$, study Problem 11-#2, where it is derived and explained.

11.7 Ratio Logic for Resistors and Capacitors

The first time you read this, focus on the device you're studying, either resistors or capacitors. The second time through, when you're familiar with both V=IR and Q=VC, pay special attention to how (and why) their "ratio effects" differ.

Let's look at factors that affect the amount of charge movement (I) or storage (Q).

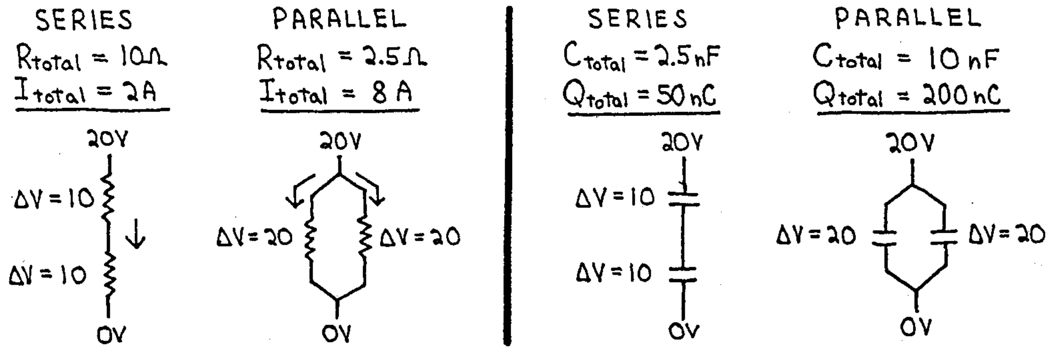
ΔV is on top of "$I = \Delta V / R$" and "$Q = \Delta V C$"; as ΔV increases, so do I or Q. When ΔV is large it produces more "action", and more charge will move or accumulate.

R is on the bottom of "$I = \Delta V/R$", so as R ↑, I ↓. As its name implies, *resistance* measures how much a -W- resists (reduces) the flow of charge. A -W- with large R reduces I a lot, causing I to have a small value.

But C is on the top of "$Q = \Delta V C$", so as C ↑, Q also ↑. *Capacitance* measures the capacity of a ⊣⊢ to hold charge. A ⊣⊢ with large C can hold a large Q.

In the circuits below, every -W- has $R = 5\ \Omega$, and every ⊣⊢ has $C = 5$ nF.

In the series circuits, only half of the 20 V appears across each -W- (or ⊣⊢). But in the parallel circuits, the entire 20 V appears across each -W- (or ⊣⊢), so each -W- (or ⊣⊢) has more I (or Q). The parallel -W-s also offer two paths for current flow, so I is quadrupled, not just doubled. Similarly, 4 times as much charge builds up at the left & right ends of parallel ⊣⊢ combinations, because each of the parallel ⊣⊢s has twice as much ΔV (and twice as much Q) as its series counterpart.



Do Section 11.3's formulas for R_{total} and C_{total} agree with these conclusions? Yes.

For series -W-s, R_{total} ($= R_1 + R_2 + ...$) is larger than the R of any individual -W-. But for parallel -W-s, R_{total} is smaller than individual R's. As R ↓, I ↑, so parallel -W-s (which have smaller R) allow more current.

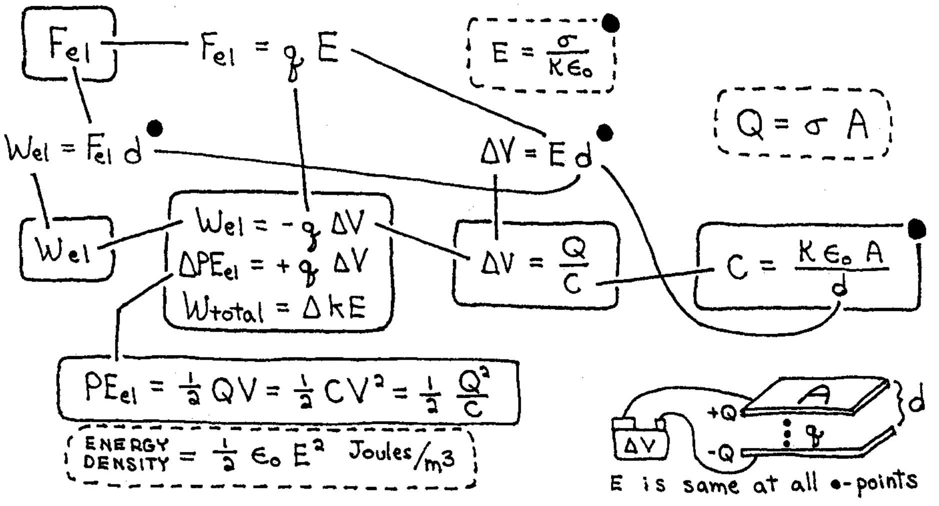
For parallel ⊣⊢s, C_{total} ($= C_1 + C_2 + ...$) is larger than the C of any individual ⊣⊢*. As C ↑, Q also ↑, so parallel ⊣⊢s (which have larger C) hold more charge. For series ⊣⊢s, C is smaller than any of the individual Cs. * Parallel -W-s have a small R_{total} (which allows large I), while parallel ⊣⊢s have a large C_{total} (which holds a large Q).

11.8 A Summary of Capacitor Formulas

This summary organizes capacitor equations gathered **from previous sections**: $Q = VC$ and $C = \kappa\varepsilon_0 A/d$ {from 11.1}, $PE = \frac{1}{2}QV = \frac{1}{2}V^2C = \frac{1}{2}Q^2/C$ {11.6}, $F = qE$ and $W_{el} + W_{total} = \Delta KE$ {10.4}, $W_{el} = -qV$ {10.5}, $W = Fd$ {4.1}, and $\Delta V = Ed$ {10.7}.

The equations marked "•" can be used only for parallel-plate capacitors.

The ⟨⁚⟩ equations are optional: $Q = A\sigma$ (from 10.94), $E = \sigma/\kappa\epsilon_0$ (Problem 10-#), PE density $= \frac{1}{2}\epsilon_0 E^2$ (11.6). If you aren't using these equations, cross them out and ignore them.

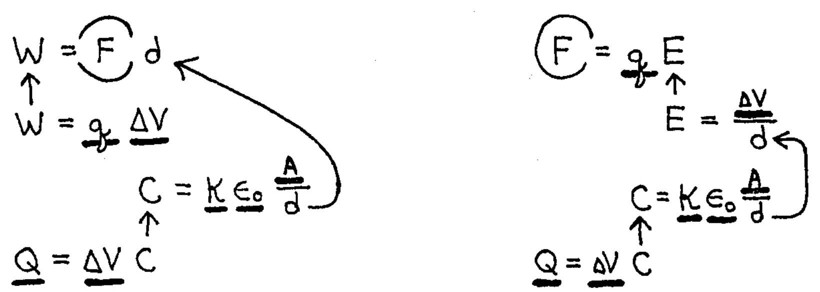


LINK-LINES: The lines running between the equations show *links* that are often the key to problem-solving; learn these links! For example, you should know all equations that contain ΔV; then if you need ΔV in one equation, you'll know where to find it. Also learn the equations that contain C, and E, and...

To help you learn the links, use visual organization [as in this summary], and know the meaning of each letter: F, E, V, W & PE (from Chapter 10), Q, q & σ (Q is charge on plates, q is a "free charge" in the gap between the plates, the optional σ is charge density), C, A, d & κ (these are related to ⊣⊢ structure), and ϵ_0 (a physical constant with an SI value of 8.85×10^{-12}).

If you use the optional formulas, draw your own "link lines" from them to the other equations.

This example shows how to use "links". If you know q, ΔV, Q, κ and A, and want to find the $F_{electric}$ acting on q, you can use either of these equation-link strategies:



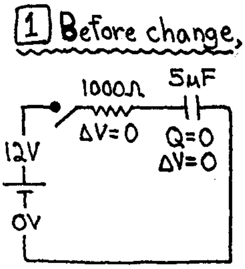
11.9 Kirchoff's Junction & Loop Rules

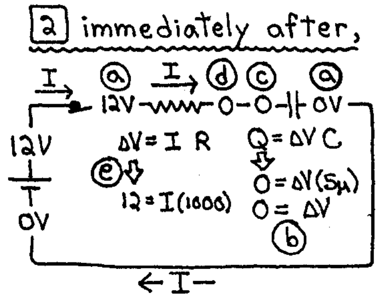
This section is not finished yet.
It will be "page 230" (it will be approximately one page in length, maybe a little more.

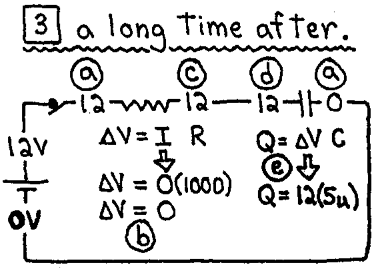
11.10 RC Circuits

CHARGING A CAPACITOR

The diagrams below, and the step-by-step explanations under them, show the changes in ΔV that occur while an initially uncharged capacitor is being "charged".







BEFORE THE BATTERY IS CONNECTED: Q =VC and Q = 0, so ΔV across the ⊣⊢ is 0.

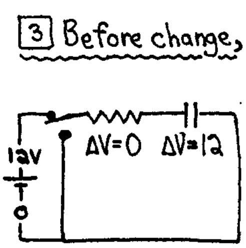
IMMEDIATELY AFTER THE SWITCH IS CLOSED: a) V-changes occur almost instantly (they travel at almost the speed of light), so the battery's V can be traced to the near sides of the -ʌʌʌ- and ⊣⊢. b) As described in the "bathtub analogy" below, it takes time for charge to accumulate across the plates of a capacitor, so ⊣⊢ still has the same Q and ΔV (both are zero) it had an instant earlier, just before the switch was closed. c) Use "$\Delta V = V_{high} - V_{low}$" to find V on the far side of ⊣⊢, d) trace this V to the -ʌʌʌ-. e) ΔV across -ʌʌʌ- is $12-0 = 12$ Volts, and you can solve V=IR for I = .012 A.

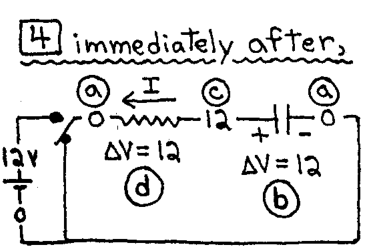
The instant after water begins running into an empty bathtub, the tub is still almost empty, even if water is flowing in at a rapid rate. Similarly, it takes time for the plates of a capacitor to fill with charge, even if current (the rate of charge-flow) is large.

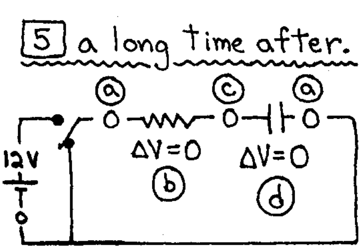
AFTER A LONG TIME: a) Trace V's. b) If charge doesn't leap across the capacitor gap, a "dead end" occurs. When no more charge can move onto the ⊣⊢ because it is fully charged, current stops flowing through -ʌʌʌ-; I = 0 and V=IR and $R \neq 0$, so ΔV across the -ʌʌʌ- has to be zero. c) Use "$\Delta V = V_{high} - V_{low}$", d) trace V, e) find ΔV across ⊣⊢, then use it in Q =VC to solve for $Q = 60 \times 10^{-6}$ Coulombs.

DISCHARGING A CAPACITOR

The diagrams and explanations below show what happens to ΔV across -ʌʌʌ- and ⊣⊢ while a capacitor is being "discharged".







BEFORE THE SWITCH IS CHANGED, the situation is the same as in the "Charging" Picture #3. The ⊣⊢ is fully charged, with ΔV = 12V.

IMMEDIATELY AFTER THE SWITCH IS CLOSED: a) Trace the battery's V. b) It takes time for charge to drain from the ⊣⊢, so it has the same Q (and thus ΔV) it had an instant earlier, before the switch was moved. c) V=0 on the ⊣⊢'s right side, and ΔV across it is 12, so V in the wire between the -/\/\- and ⊣⊢ is 12. d) ΔV across -/\/\- is $12-0 = 12$ Volts, and I = .012 A.

The current in Pictures 2 (charging) and 4 (discharge) are in opposite directions, but in each case the "conventional current" is in the direction of decreasing number-line voltage. { While "charging", current is caused by the battery's ΔV; during "discharge" it is caused by the capacitor's ΔV. In both cases, negative-charged electrons move away from the – thing (terminal or plate) and toward the + thing (terminal or plate). }

AFTER A LONG TIME: a) Trace V. b) Charge continues to move between the ⊣⊢'s + and – plates until they are "neutralized", with Q = 0 and thus $\Delta V = 0$. c) Use "$\Delta V = V_{hi} - V_{lo}$"; V in the wire connecting -/\/\- and ⊣⊢ is zero. d) ΔV across -/\/\- is zero, so I through it is zero.

Equations for EXPONENTIAL INCREASE & DECREASE

During the capacitor-charging process, ΔV across -/\/\- decreases from 12 to 0; this is called a *decay*. But ΔV across ⊣⊢ increases from 0 to 12; this is a *buildup*. When ⊣⊢ is discharged, ΔV across -/\/\- and ⊣⊢ both decrease (decay) from 12 to 0.

For a resistor and capacitor connected "in series", as in the circuit above, the progressive change of ΔV with time is described by these equations:

EXPONENTIAL DECAY:	EXPONENTIAL BUILDUP:
When ΔV is decreasing,	When ΔV is increasing,
$\Delta V = (\Delta V_{max})\, e^{-t/RC}$	$\Delta V = (\Delta V_{max})\{1 - e^{-t/RC}\}$

where ΔV_{max} is the maximum ΔV (this occurs at the start of a decay or the end of a buildup), ΔV is the V-difference after the circuit has been changed for **t** seconds, **R** is the resistance for current going to the ⊣⊢ (Problem 11-# gives an example of R that is not included), **C** is the ⊣⊢'s capacitance, and **e** is (as described in Section 19.6) the base of natural logarithms.

These formulas describe ΔV_R (ΔV across a -/\/\-) or ΔV_C (ΔV across a ⊣⊢). If ΔV's are replaced by IR (because V=IR) or Q/C (because Q=VC), the equations will contain I or Q instead of ΔV. For example,

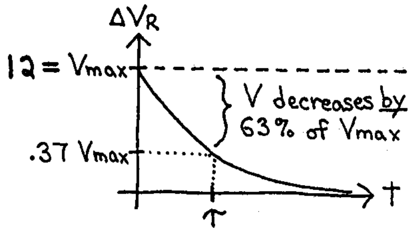
$$\Delta V = (\Delta V_{max})\, e^{-t/RC} \qquad \Delta V = (\Delta V_{max})\, e^{-t/RC}$$
$$IR = (I_{max} R)\, e^{-t/RC} \qquad Q/C = (Q_{max}/C)\, e^{-t/RC}$$
$$I = (I_{max})\, e^{-t/RC} \qquad Q = (Q_{max})\, e^{-t/RC}$$

I prefer "ΔV equations" because they make it easy to analyze circuits using V-logic (V-tracing, $\Delta V = V_{high} - V_{low}$). When it is necessary, V-equations can be quickly transformed into I- or Q-equations by replacing ΔV's with I or Q.

The calculations and graphs below show ΔV across the -W- and -||- at 3 times: when t is close to 0, at $t = RC = (1000)(5 \times 10^{-6}) = .005$ seconds, and at $t = 10RC = .050$ s.

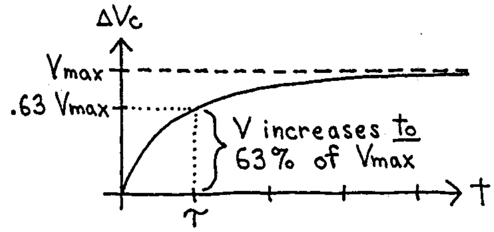
<u>DECAY: ΔV across -W- decreases.</u>

$$12e^{-\frac{0}{RC}} \qquad 12e^{-\frac{RC}{RC}} \qquad 12e^{-\frac{10RC}{RC}}$$
$$12\ e^{0} \qquad 12\ e^{-1} \qquad 12\ e^{-10}$$
$$12\ (1) \qquad 12(.37) \qquad 12(.00005)$$



<u>BUILDUP: ΔV across -||- increases.</u>

$$12[1-e^{-\frac{0}{RC}}] \qquad 12[1-e^{-\frac{RC}{RC}}] \qquad 12[1-e^{-\frac{10RC}{RC}}]$$
$$12[1-e^{0}] \qquad 12[1-e^{-1}] \qquad 12[1-e^{-10}]$$
$$12[1-1] \qquad 12[1-.37] \qquad 12[1-.00005]$$
$$12(0) \qquad 12(.63) \qquad 12[.99995]$$



As shown on the graphs above, when $t = RC$, ΔV has either dropped to 37% of its maximum value (because $e^{-1} = .36788$), or increased to within 37% of its maximum value. This amount of time is called the circuit's *lifetime,* abbreviated τ ; $\tau \equiv RC$.

A time that is "immediately after" a circuit change can be more clearly defined as "when t is much less than RC, abbreviated $t \ll RC$ or $t \ll \tau$ ". Similarly, "a long time after" is "when t is much greater than RC, abbreviated $t \gg RC$ or $t \gg \tau$ ".

Whether a time is short or long depends on the size of a circuit's "RC". For example, .050 seconds is a "long time" if RC = .005 s, because .050 s is 10 lifetimes and current has dropped to .00005 of its maximum value. But .050 second is a "short time" if RC = 50 s, because .050 s is now only .001 RC, a small fraction of a lifetime.

PROBLEM 11-D: A series RC circuit has a 12V battery, $R = 1000\ \Omega$, and $C = 5\ \mu F$. A switch is closed and the capacitor starts charging; .0005 second later, what is I? After .010 s, what is the -||-'s charge? When the fully charged -||- is discharged, as in Pictures 3 to 5, how long does it take for 20% of the capacitor's charge to escape? When t = .0005 second, find ΔV across the -W-, and ΔV across the -||-.

SOLUTION 11-D for **CHARGING**: ΔV across -W- decays, ΔV across -||- builds up.

$$\Delta V = \Delta V_{max}\ e^{-t/RC}$$
$$I = I_{max}\ e^{-\frac{.0005}{1000(5\mu)}}$$
$$I = \left(\frac{12V}{1000\Omega}\right) e^{-.1}$$
$$I = (.012A)(.90)$$
$$I = .011 \text{ Amps}$$

$$\Delta V = \Delta V_{max}\ [1 - e^{-t/RC}]$$
$$Q = Q_{max}\ [1 - e^{-\frac{.010}{1000(5\mu)}}]$$
$$Q = (12V)(5\mu F)[1 - e^{-2}]$$
$$Q = (60\mu)\ [1 - .135]$$
$$Q = 52 \times 10^{-6} \text{ Coulombs}$$

DISCHARGE: When 25% of the original charge (which is Q_{max}) is gone, 75% remains. The sentence "Q is 75% of Q_{max}" can be turned into the equation "$Q = .75\ Q_{max}$".

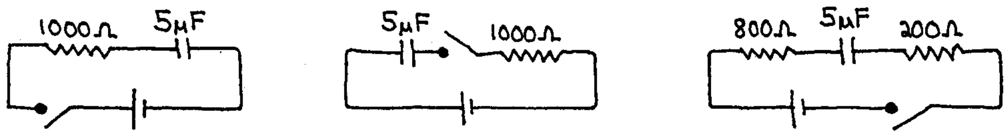
To "liberate" t from the exponent of $e^{-t/RC}$, take the *natural logarithm* ("ln") of both equation-sides; the reason for this strategy is explained in Section 19.6.

$$
\begin{aligned}
\Delta V &= \Delta V_{max}\, e^{-t/RC} \\
Q &= Q_{max}\, e^{-\frac{t}{1000(5\mu)}} \\
.75\,\cancel{Q_{max}} &= \cancel{Q_{max}}\, e^{-t/.005} \\
\ln(.75) &= \ln\left(e^{-t/.005}\right) \\
\frac{-.288}{1} &= \frac{-t}{.005} \\
.0014\,s &= t
\end{aligned}
$$

$$
\begin{aligned}
\Delta V &= V_{max}\, e^{-t/RC} \\
\Delta V &= 12\, e^{-\frac{.0005}{1000(5\mu)}} \\
\Delta V &= 10.9 \text{ Volts}
\end{aligned}
$$

[The -ww- and -||- both have the same ΔV of 10.9V.]

In a series circuit, the order of battery, switch, resistors & capacitors doesn't matter. For example, each circuit below behaves the same during the process of charging:

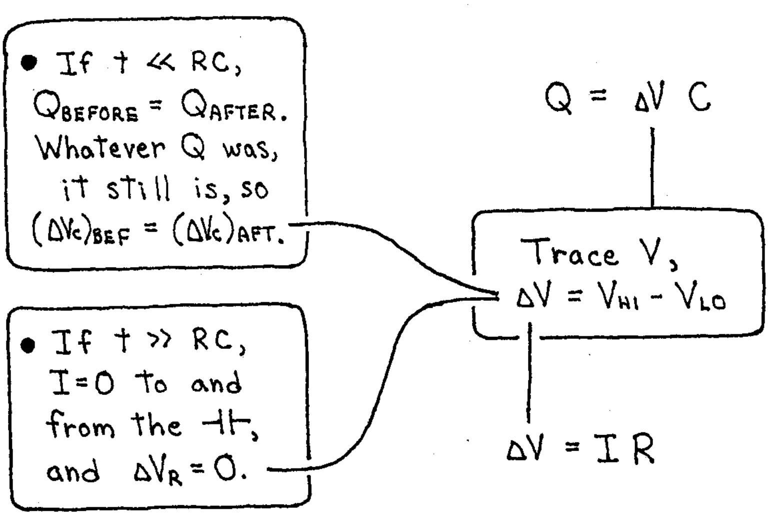


A simple *parallel RC circuit* is analyzed in Problem 11-##, using V-logic principles. If your class studies "parallel" circuits or if you're curious, look at this problem.

The principles of RC-analysis are visually organized in the Chapter 11 Summary.

Here is a ROUGH preview of the "RC Summary" for Section 11.10. It will be revised some (I will PROBABLY change the placement of Q = ΔVC, for example). There will also be an SHORT explanation, referring back to the CHARGE + DISCH problems that began 11.10.

I don't have flash cards finished.



for exponential decrease, $\Delta V = (\Delta V)_{max}\left\{\ e^{-t/RC}\ \right\}$

for exponential increase, $\Delta V = (\Delta V)_{max}\left\{ 1 - e^{-t/RC} \right\}$