Chapter 10

Electrostatics

When your class studies electric charge & force & field, read Sections 10.1-10.3.
Optional: if your class studies electric fields with "Gauss's Law", use Sections 10.92-10.94.

Later, when you study electrical work & potential energy, voltage & electric potential, use Sections 10.4-10.7 and the 10.8 Summary.

10.1 Electric Force and Electric Field

ELECTRIC CHARGE : All objects contain huge numbers of tiny particles, *protons* and *electrons*, which have an interesting property called *electric charge.* Protons and electrons have charges of $+1.602 \times 10^{-19}$ and -1.602×10^{-19} Coulombs, respectively; a *Coulomb,* abbreviated "C", is the SI unit of electric charge.

If an object contains unequal amounts of + and – charge, some of it won't "cancel". This leftover charge gives the object a + or – *net charge.*

ELECTRIC FORCE

A *point-object* is assumed to be so small that it occupies only one point (·) in space. If two point-objects each have a net charge, they will exert an *electric force* (which can also be called *electrostatic force*) on each other.

The DIRECTION of this $F_{electric}$ vector is along a line between the two point charges.
If Q & q are alike in sign (+ + or – –), $F_{electric}$ is "repulsive"; it pushes them apart.
If Q & q have opposite charges (+ –), $F_{electric}$ is "attractive", pulling them together.

$\Leftarrow +\quad\quad + \Rightarrow$	$+ \Rightarrow\quad\quad \Leftarrow -$
ALIKE CHARGES REPEL	OPPOSITE CHARGES ATTRACT

Problems 10-# to 10-# discuss Static Electricity & Ben Franklin's Kite, how charged objects respond to each other (attraction, repulsion,...), conductors & induced charge, electroscopes, and "grounds".

The MAGNITUDE of $F_{electric}$, the electrostatic force vector, is

$$F_{electric} = k \frac{Q\,q}{r^2}$$

where **k** is a constant-of-nature with an SI value of 8.99×10^9, **Q** & **q** are the net charges (in Coulombs) of the two point-objects, and **r** is the distance (in meters) between them. { k is sometimes written as "$1/4\pi e_o$", with e_o having an SI value of 8.85×10^{-12} }

You can draw $F_{electric}$ on a force-diagram, just like any other force, and analyze the motion of a charged object using "$F_{total} = ma$" and the tvvax equations. The work-energy & impulse-momentum relationships [$F\,\Delta x = \Delta(\frac{1}{2}mv^2)$ & $F\,\Delta t = \Delta(mv)$] can also be used. The work done by electric force is studied in Section 10.4.

ELECTRIC FIELD

With rope tension or friction, it is easy to see the "contact" that causes the force: a rope that is attached to the object, or the surfaces that are touching. The cause of electric force isn't as obvious, because electricity can't be "seen" or "touched", but some imaginative visual thought will help you understand it.

Here is a useful way to interpret the electric force exerted on q by Q. First, imagine that Q's charge affects the space around it by producing an *electric field,* which I'll label "E_Q"; E_Q has a magnitude defined to be kQ/r^2. Any other charge (like q) will be affected by Q's field: q feels an $F_{electric}$ with MAGNITUDE $F_{electric} = q(E_Q)$, because "q E_Q" is just another way to write "kQq/r^2". Notice that the (⋯)-grouping is E_Q:

$$F_{electric} = \left(k\frac{Q}{r^2}\right) q$$

$$F_{electric} = E_Q \; q$$

Notice that E_Q (which is kQ/r^2) depends only on Q; the electric field produced by Q doesn't depend on the presence of q. But electric force is caused by the interaction between q & Q, as shown by "$F_{electric} = q\,E_Q$" or "$F_{electric} = kQq/r^2$".

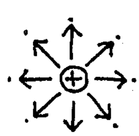
Because $F/q = E$, E is the amount of force (measured in Newtons) "per" Coulomb of q-charge that is being affected. The SI unit for E is Newtons/Coulomb, or N/C.

SYMMETRY: **Every charge (including q) produces its own electric field, which then exerts an electric force on all other charges.** Here are "F=qE" interpretations of the "Newton's Third Law" equal-and-opposite forces that Q and q exert on each other:
the $F_{electric}$ exerted on q by Q = (charge of q)(E-field produced by Q) = q E_Q = q (kQ/r^2),
the $F_{electric}$ exerted on Q by q = (charge of Q)(E-field produced by q) = Q E_q = Q (kq/r^2).

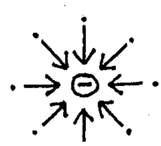
A charge is not affected by its own E-field. For example, Q does not feel an $F_{electric}$ caused by its own "E_Q" field, and q does not feel an $F_{electric}$ caused by E_q.

At any location in space, the DIRECTION of the E_Q vector
is along the *radial line* connecting that location with Q.

In the drawings below, E-field direction is shown at each of the 8 locations marked by " • ",



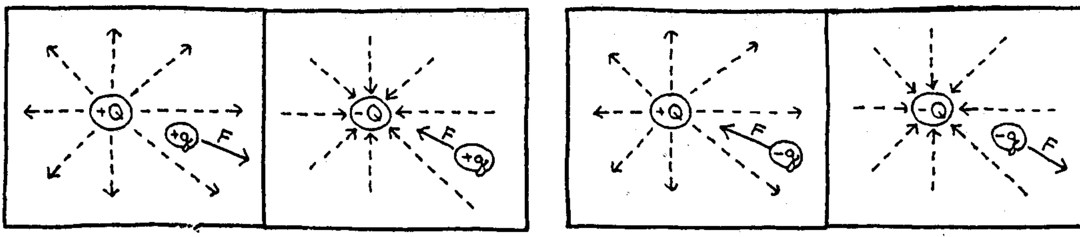
If Q is + ,
E_Q points radially
outward away from Q.



If Q is – ,
E_Q points radially
inward toward Q.

Remember that reality is 3-Dimensional; the diagrams above show a flat 2-D "cross section", but the real E-lines form a spherical (not circular) pattern.

In the pictures below, compare the directions of E & F. Do you see their relationship?



If q is +, the $F_{electric}$ acting on q will point in the same direction as E.
If q is −, the $F_{electric}$ acting on q will point in the direction opposite to E.

Just remember that $\mathbf{F} = q\,\mathbf{E}$ is a *vector equation.* The ± sign of q determines whether F and E point in the same direction or in opposite directions.

10.2 The Principle of Superposition, for F and E

The E-field produced by Q exists independently of q (or any other charge); E_Q is not affected by the presence of q. Similarly, E_q is independent of Q. In an "isolated system" where Q and q are the only charges, the total E-field at any location is the vector sum of $E_Q + E_q$.

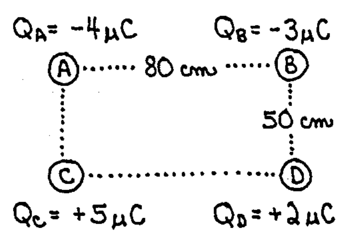
If we bring in a third charge "J", J will feel two electric forces: one caused by Q, another caused by q. The total F-vector acting on J is the vector sum of these forces:
$F_{total} = F_{caused\ by\ Q} + F_{caused\ by\ q} = J\,E_Q + J\,E_q = J\,(E_Q + E_q) = J\,(E_{total})$.

PROBLEM 10-A: Superposition.
Answer Part 1 now. Do Parts 2 & 3 later, when you study Section 10.6.

Part 1: Q_B is the charge located at point B. What electric force acts on Q_B? What is the electric field at B? Section 1.6 discusses prefixes like "μ".

Part 2: How much work is needed to move Q_B at constant speed from point B to the middle of the rectangle?

Part 3: What total voltage do Q_A, Q_C and Q_D produce at point B? at the rectangle's center? Now use "$W_{electric} = -q\,\Delta V$" to answer the question in Part B.

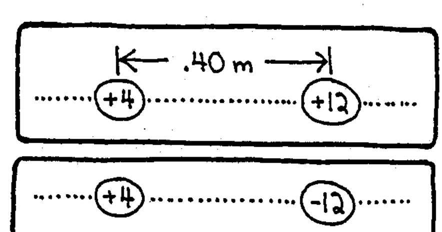


PROBLEM 10-B: A Comparison of E and V.
Do the E-Problem now, the V-Problem later.

E: At what location(s) on the dotted line is E zero for the top picture? for the bottom picture?

V: At what location(s) on the dotted line is V zero for the top picture? for the bottom picture?

For substitution into the E=0 equation, how do you decide ± signs? for the V=0 equation?



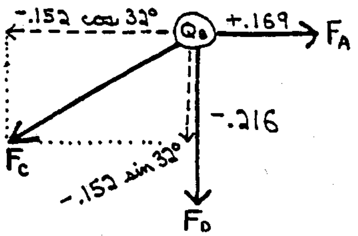
SOLUTION 10-A. Q_B feels 3 forces: F_A, F_C & F_D, caused by the charges at A, C & D. Find the magnitude & direction of each F. Then, using the *split-combine-reconstruct* method from Section 1.3, add them together as vectors to get the F_{total} vector.

Step 1: $F_A = kQq/r^2 = (9 \times 10^9)(4 \times 10^{-6})(3 \times 10^{-6}) / (.80)^2 = .169N$ [– –, repulsion]

$F_C = (9 \times 10^9)(5 \times 10^{-6})(3 \times 10^{-6}) / (\sqrt{.8^2 + .5^2})^2 = .152N$ [+ –, attraction]

$F_D = (9 \times 10^9)(2 \times 10^{-6})(3 \times 10^{-6}) / (.50)^2 = .216N$ [+ –, attraction]

Step 2: SPLIT



Step 3: COMBINE

Add x's and y's separately.

$F_x = (-.129) + (0) + (+.169)$

$F_x = +.040$ Newton

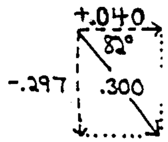
$F_y = (-.081) + (-.216) + (0)$

$F_y = -.297$ Newton

Step 4: RECONSTRUCT

SIZE $= \sqrt{(.040)^2 + (.297)^2}$

ANGLE $= \tan^{-1}(.297/.040)$



Do you see where the 32° in Step 2 comes from? Hint: look at the charge-rectangle!

Notice that the ± signs of Q & q are not substituted into $F_{el} = kQq/r^2$, because this equation only gives F-magnitude. To find the ± signs for the x & y components of F, use this "visual logic": look at the direction of F on your diagram, and decide whether F_x points in the + or – direction, and whether F_y points in the + or – direction.

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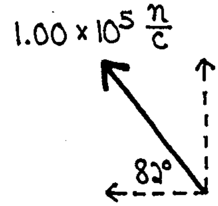
There are two ways to find E_{total}. A) Use the same 4-step method as above: find the magnitude & direction of each E, using $E = kQ/r^2$, then split-combine-reconstruct.

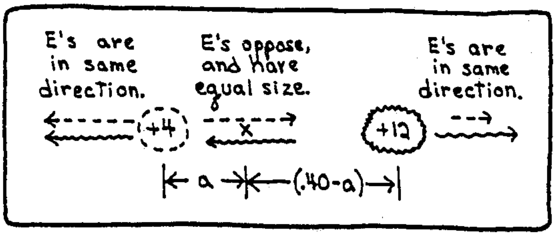
B) If you already know that $F_{total} = .300N$, you can find E_{total} by solving "$E = F/q = (.300)/(-3 \times 10^{-6})$". Q_B is –, so E_{total}'s direction is opposite to the $F_{electric}$ that acts on Q_B.

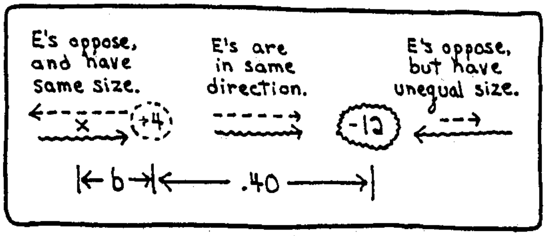


SOLUTION 10-B for E: E points away from +4 and +12, toward –12. At three locations on the pictures below, I've drawn the direction of E caused by each charge. If these E's are opposite in direction and equal in magnitude, they cancel to cause $E_{total} = 0$; this occurs only at the locations marked "x".

LARGE versus CLOSE: Look at the second picture. To the left of +4 there is a spot where $E = 0$, but there is no $E = 0$ spot to the right of –12. Why? To answer this question, let's consider the two factors that affect E. To the left of +4, the LARGE factor favors E_{12}, but the CLOSE factor favors E_4. At the proper distance, which is found by solving "$E_4 + E_{12} = 0$", these two factors cancel and the E-magnitudes are equal. But at every location to the right of –12, E_{12} is larger than E_4 (so they cannot cancel to cause $E = 0$) because the CLOSE & LARGE factors both favor E_{12}.

In the first picture, the $E = 0$ spot is closer to +4 than to –12. Can you explain this fact by using "close versus large logic"?





Compare the diagrams above with the equations below, and notice that **the ± sign of E isn't necessarily the same as the ± sign of Q.**
For example, to the left of +4, E_{+4} points ← so it is –.
But to the right of +4, E_{+4} points → so it is +.

$$E_{of\,+4} + E_{of\,+12} = 0$$
$$+k\frac{4}{a^2} - k\frac{12}{(.40-a)^2} = 0$$
$$\frac{4}{a^2} = \frac{12}{(.40-a)^2}$$
$$\frac{2}{a} = \frac{\sqrt{12}}{.40-a}$$
$$.80 - 2a = 3.46\,a$$
$$.1465\ m = a$$

$$E_{of\,+4} + E_{of\,-12} = 0$$
$$-k\frac{4}{b^2} + k\frac{12}{(.40+a)^2} = 0$$
$$\not{k}\frac{4}{b^2} = \not{k}\frac{12}{(.40+a)^2}$$
$$\frac{2}{b} = \frac{\sqrt{12}}{(.40+a)}$$
$$.80 + 2a = 3.46\,b$$
$$.548\ m = b$$

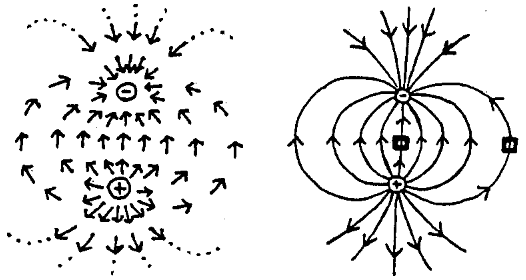
As discussed in Sections 2.7 & 19.7, the √‾-trick lets you avoid the Quadratic Formula.

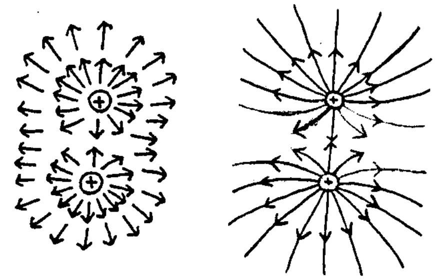
10.3 Electric Field Patterns
Electrostatics for Large "Non-Point" Objects

ELECTRIC FIELD "PATTERNS": Problems 10-A & 10-B show how to calculate the total E at one location when the E's that are produced by several charges are combined. In the first picture below, for + and – charges with equal magnitude, this same *superposition method* is used to estimate* the E-field direction at 59 locations. {* With lots of patience or a good computer program, you could make exact calculations for the E-vector at each of these 59 spots.} Try to estimate E at a few locations; do you see why each → points in the direction shown?

In the second picture, individual →'s are connected together to form continuous lines. These are sometimes called *lines of force,* because the E-line direction at any location shows the direction of $F_{electric}$ that would be felt by a + charge at that spot. {These E-line patterns do not show any kind of "movement" or "flow".}

The last two pictures show the →'s and E-line patterns that are caused by + charges with equal magnitude.





The following principles will help you quickly draw E-field lines.

First, look at the diagrams above and notice that E-lines never cross like this. Compare the overall "pattern shape" formed by opposite charges (+−) and by alike charges (++). E lines point from + to −, as if they were being "attracted". But E lines veer away from an alike charge, as if they were being "repelled".

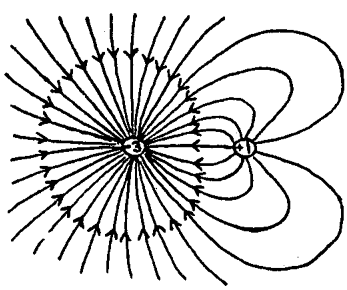
Now consider the E at close, medium and far distances.

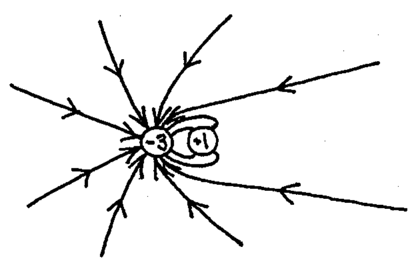
Close to any point charge, E approaches infinite size as r^2 (on the bottom of kQ/r^2) gets close to zero. This huge E overwhelms the E produced by other charges, so E_{total} points along a radial line, either straight toward the charge or straight away from it.

At medium distances, as you move away from a charge, the E's caused by other charges become important. Superposition estimates and a knowledge of "overall pattern shapes" will help you draw E-lines.

Far away from the charges, their separation is negligible compared with the distance to the charges [think about the stereo effects you hear near two speakers, compared with listening from far away]. In the +− and ++ examples above, E_{total} is almost the same as if there were single charges of 0* and +2, respectively. {*If your class studies "Electric Dipoles", you may want to do Problem 10-#, which discusses the far-away E that is produced because equal + and − charges don't totally cancel each other.}

In the first picture below, the −3 charge produces 3 times as many E-lines as the +1 charge. Compared with the equal-magnitude +− situation above, some E-pattern symmetry (but not all of it) is lost. The second picture shows what the pattern looks like when it is viewed from much further away; the −3's "extra lines", which survive cancellation, produce an almost-radial E field at far distances.





at far-away distances

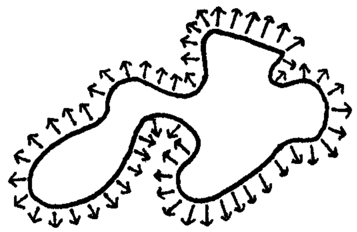
Electrostatics for Large "Non-Point" Objects

Sections 10.1 & 10.2 studied tiny *point charges*. Now we'll look at the electrostatic behavior of large "non-point" charged objects. Here are some basic principles.

The electric field is zero everywhere within an electrostatic conducting object. Why? A metal is a *conductor* of electricity because its electrons are free to move. If there is a non-zero E, electrons feel a "qE" force and they will move. But electrons are not moving* (think about what *electrostatic* means), so E must be zero inside the conductor. * The electrons have *random thermal motion,* but there is no overall movement.

When electrons stop moving, all net charge (if there is an excess of negatively charged electrons an object's net charge is –, if there is a deficiency of electrons the net charge is +) **is on the conductor's outer surface.** I won't try to prove this statement now*, but will just ask you to accept that it is true. * it is proved, using the optional Gauss's Law, in Problem 10-##.

The E produced by this stationary surface charge is ⊥ to the conductor's surface:



Why? Think about what would happen if there was a sideways (non-⊥) E field.
It would produce a sideways force on the electrons, causing them to move.
Logic: charges aren't moving, so there cannot be a non-⊥ E field.

{ A ⊥-to-the-surface E won't push electrons into the metal's interior; but if E is very large, electrons can leap into the air and cause a *spark,* as discussed in Problem 10-#'s "Thunder & Lightning". }

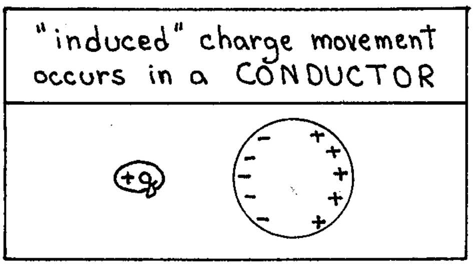
The principles above [$E_{inside} = 0$, all net charge is on surface, E points ⊥ to surface] are only true for *conductors.* They cannot be used for a *nonconducting (insulating)* material like plastic or glass, where electrons cannot move freely.

For any object with a *spherically symmetric charge distribution*, this principle is true: at points outside the object, the E-field is exactly the same as if all charge was located at the object's center. {Problem 10-## shows how to use "spherical symmetry".}

Charged *parallel plates* are discussed in Section 10.7.

Optional: Sections 10.92-10.94 show how symmetric objects (spheres, cylinders, plates,...) can be analyzed using ***Gauss's Law.***

<u>INDUCED CHARGE</u>: When a positive charge "+q" is near a neutral conducting sphere, as shown below, negatively charged electrons are attracted toward the left. This electron movement gives the sphere's left side a net – charge (electron-excess) and the right side a net + charge (electron-deficiency).



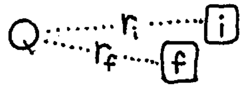
If the sphere is made from nonconducting material, electrons will still feel an attractive force toward the left. But the electrons are not free to move, so the amount of induced charge is, for most purposes, negligible.

10.4 Work done by Electrostatic Force

If you know the basic principles from Sections 4.1-4.3, you'll be able to understand this section.

If q moves from i to f by any path, the MAGNITUDE of the work done on q by $F_{electric}$ is:

$$W_{electric} = -k\,Q\,q \left\{ \frac{1}{r_f} - \frac{1}{r_i} \right\}$$



where r_i & r_f are the distances, at q's initial & final points, between Q and q .

If k, Q, q and r are in SI units, $W_{electric}$ will be in the SI energy units of Joules.

$F_{electric}$ is a *conservative force* so the *change in electrostatic potential energy,* $\Delta PE_{electric}$ or $\Delta U_{electric}$, can be defined as in Section 4.3 by changing the ± sign of W:
$\Delta PE_{el} \equiv -W_{el} = +kQq(1/r_f - 1/r_i)$.

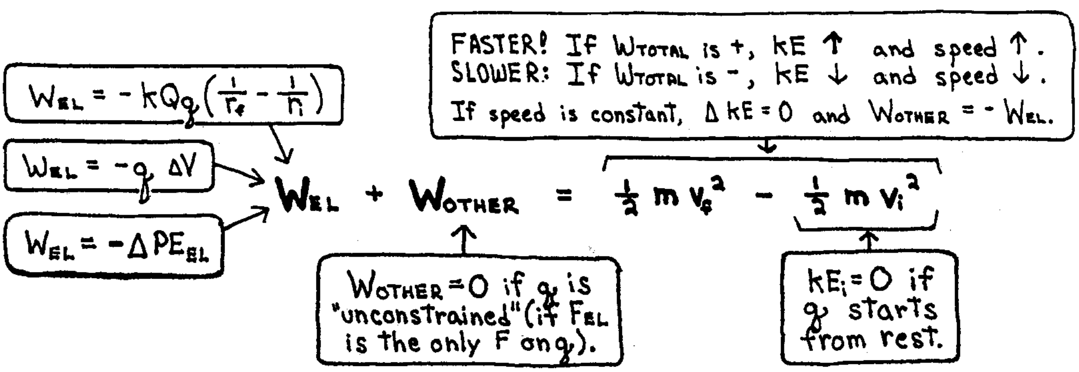
The *gravitational* PE difference between locations i & f, $\Delta PE_{gravity} = mg(h_f - h_i)$, is the work needed to move a mass "m" from h_i to h_f at constant speed.
Similarly, the *electrostatic* PE difference between locations i & f, $\Delta PE_{electric}$, is the work needed to move a charge "q" from r_i to r_f at constant speed.

$W_{electric}$ is not a vector;
W has magnitude and ± sign, but no "direction".

Here are two ways to find the ± sign of $W_{electric}$:

1) Include ± signs when you substitute Q and q into the work-equation, then be careful with ± signs during solution.
2) Use these basic principles from Section 4.2:
   If q moves in the same direction that $F_{electric}$ points, $W_{electric}$ is +.
   If q's movement and $F_{electric}$ are in opposite directions, $W_{electric}$ is –.
   If q's movement is perpendicular to $F_{electric}$, $W_{electric} = 0$.

W_{total} can be split into $W_{electric}$ (the work done by $F_{electric}$)
and W_{other} (work done by all other forces: by you, by gravity,...).
Here is the "$W_{total} = \Delta KE$" equation, and some common substitutions:



10.5 Electric Potential (also called "Voltage")

The *electric potential difference* (the *voltage difference)* between points i & f is "ΔV".
ΔV is derived from W the same way E was derived from F; begin with the W-formula, then group everything together except q (and the – sign) and define it to be "ΔV_Q".

$$W_{electric} = - kQ\, q \left\{ \frac{1}{r_f} - \frac{1}{r_i} \right\}$$

$$W_{electric} = - q \; \Delta V_Q$$

The logic is simple: if $kQ(1/r_f - 1/r_i)$ is defined to be ΔV_Q,
then "$-q\,\Delta V_Q$" is just another way to write "$-kQq(1/r_f - 1/r_i)$".
{ Similarly, $+q\,\Delta V_Q$ is another way to write $+kQq(1/r_f - 1/r_i)$ or $\Delta PE_{electric}$. }

The Q-subscript shows that ΔV_Q is the potential difference caused by Q (not by q), just like the Q-subscript shows that E_Q is the electric field caused by Q (not by q).

ΔV_Q [which is $kQ(1/r_f - 1/r_i)$] depends only on Q; the potential difference produced by Q doesn't depend on the presence of q. But potential energy difference is caused by interaction between q & Q, as shown by "$\Delta PE_{el} = q\,\Delta V_Q$" or "$\Delta PE_{el} = kQq(1/r_f - 1/r_i)$".

Because $\Delta PE/q = \Delta V$, ΔV is the change in potential energy (measured in Joules) "per" Coulomb of q-charge that is being affected.

In most gravitational work-energy problems, some point (usually h_i or h_f) is defined as $h \equiv 0$. In electrostatics, V_Q is usually defined to be zero at infinite (∞) distance away from Q. This makes it possible to define the ***voltage*** caused by Q, to get V_Q instead of just ΔV_Q. If we move from infinitely far away (from $r_i = \infty$, where $V_i \equiv 0$) to a location that is a distance "r" away from Q,

$$\begin{aligned}
\Delta V &= kQ(1/r_f - 1/r_i) \\
V_f - V_i &= kQ(1/r - 1/\infty) \\
V_f - 0 &= kQ(1/r - 0) \\
V_Q &= kQ/r
\end{aligned}$$

Voltage is not a vector. V has magnitude and ± sign, but no "direction".
If Q is +, it produces V_Q that is +. If Q is –, it produces V_Q that is –.

UNITS: The SI unit for voltage is the "Volt", abbreviated "V".
Be careful to interpret "V" correctly.
For example, when "Voltage = 12 Volts" is abbreviated "V = 12 V", the two V's have different meanings.

A commonly used non-SI unit of energy (not voltage!) is the *electronvolt,* "eV".
An eV is the work done when an object with a net charge of 1.602×10^{-19} Coulomb (the charge of a proton or electron) is accelerated through a potential difference of 1 Volt.
$|W| = q\,\Delta V = (1.602 \times 10^{-19}\text{ C})(1\text{ V}) = 1.602 \times 10^{-19}$ Joules; this energy is defined as 1 eV.
Problem 10.## shows how to use electron-volt units.

PROBLEM 10-C: Electrostatic Work-Energy Strategies

This problem uses all of the strategies in the diagram at the end of Section 10.4.

a) To move a proton at constant speed toward a point charge of +5 nC, from 60 cm away to 20 cm away, how much work must you do? What is the proton's ΔPE?

b) Object q, with .05 mg mass and −8 μC charge, is released .25 cm from a −5 nC fixed-position charge. What is q's speed when it is .40 cm from the charge? If the −5 nC charge is free to move, will q's speed at .40 cm separation be different? Why?

c) An electron begins at a location where V = +1200 V, with KE = 4.8×10^{-16} J. When it is at a point where V = +200 V, what is its kinetic energy?

SOLUTION 10-C

a) W_{you} is the only W_{other}, and "constant speed" means "$\Delta KE = 0$". As shown below, $W_{you} = \Delta PE_{el}$; both questions have the same answer, $W_{you} = \Delta PE_{el} = +2.4 \times 10^{-17}$ J.

$$W_{el} + W_{other} = \Delta KE$$

$$-\Delta PE_{el} + W_{you} = 0$$

$$W_{you} = +\Delta PE_{el}$$

$$W_{you} = +(9 \times 10^9)(+5 \times 10^{-9})(+1.60 \times 10^{-19})\{ \frac{1}{.20} - \frac{1}{.60} \}$$

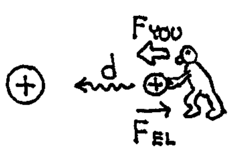
$$W_{you} = +2.4 \times 10^{-17} \text{ Joules}$$

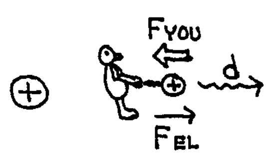
Look at the first picture below. As you push the proton to overcome the repulsive $F_{electric}$, the **d** and $\mathbf{F_{you}}$ vectors point in the same direction, so W_{you} is +. As shown above, ΔPE_{el} (which equals W_{you}) is also +; the movement is "uphill" in PE because the positively charged proton doesn't naturally want to get closer to the +5 nC charge.

W_{you} is negative if, as shown in the second picture, you must "restrain" the proton to keep its speed constant during a movement that is downhill-in-PE.

In the third picture when the proton is "released", $F_{you} = 0$ so $W_{you} = 0$.

The top 3 pictures are for *alike charges.* The bottom 3 pictures show the analogous situations for *unlike charges.* The in-between text summarizes what is happening.

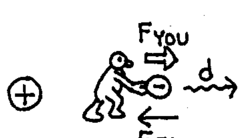


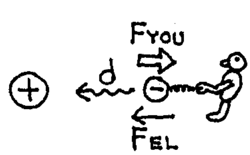


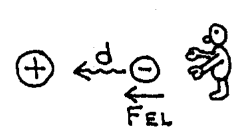
W_{you} and ΔPE are + (you cause an uphill-in-PE movement), W_{el} is −.

W_{you} and ΔPE are − (you restrain a downhill-in-PE movement), W_{el} is +.

$W_{you} = 0$, ΔPE is − (a downhill-in-PE movement causes KE increase), W_{el} is +.







b) "released" means that $v_i = 0$ and $W_{other} = 0$, so (as discussed in Section 4.4) q can do what it wants, like a ball naturally rolling downhill; q's speed and KE increase.

$$W_{el} + W_{you} = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2$$
$$-(9 \times 10^9)(-5 \times 10^{-9})(-8 \times 10^{-6})\left\{\frac{1}{.0040} - \frac{1}{.0025}\right\} + 0 = \tfrac{1}{2}(.05 \times 10^{-3}) v_f^2 - 0$$
$$+.054 = (2.5 \times 10^{-5}) v_f^2$$
$$46.5 \text{ m/s} = v_f$$

A CALCULATOR TIP: It's easy to find W_{el} by using your calculator's "1/x" button. Just punch .0040 1/x – .0025 1/x = x 9 EXP 9 x 5 EXP +/– 9 x 8 EXP +/– 6 = to get "–.054". This becomes +.054 when it is multiplied by the three –'s that were not punched into the calculator, because four –'s is an even number.

c) Use "$W_{el} = -q\,\Delta V$", substitute & solve. The negatively charged electron is moving toward lower (more negative) number-line V. This is an "uphill" movement, against the direction it naturally wants to go, so KE_f is less than KE_i.

$$W_{electric} + W_{other} = \Delta KE$$
$$-\ q\ \Delta V + 0 = KE_f - KE_i$$
$$-(-1.6 \times 10^{-19})[(+200) - (+1200)] + 0 = KE_f - (4.8 \times 10^{-16})$$
$$-1.6 \times 10^{-16} = KE_f - 4.8 \times 10^{-16}$$
$$3.2 \times 10^{-16} \text{ Joules} = KE_f$$

10.6 The Principle of Superposition, for W and V

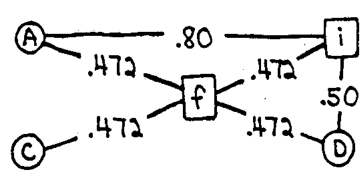
A summary of Section 10.2: the *principle of superposition* states that the $F_{electric}$ felt by any charge is the vector sum of the F's exerted on it by all other charges, and the **E** at any location is the vector sum of the E's produced by all charges.

This principle is also true for W and V. To find the total W_{el} or ΔV, combine the W's or ΔV's caused by the separate charges. It is easy to calculate the total W or ΔV, compared with F or E, because W and ΔV are non-vectors.

Return to Section 10.2 and finish Problems 10-A & 10-B, then look at these solutions.

SOLUTION 10-A, Part 2

Use "$W_{el} = kQq(1/r_f - 1/r_i)$" to find the W done on Q_B by each of the other charges, then combine these to get W_{total}. The diagram at the right shows the initial and final separations of Q_A from Q_B, Q_C & Q_D.



$$W_{EL} = -(9 \times 10^9)(-4\mu C)(-3\mu C)\left[\frac{1}{.472} - \frac{1}{.80}\right] - (9 \times 10^9)(+5\mu C)(-3\mu C)\left[\frac{1}{.472} - \frac{1}{.943}\right] - (9 \times 10^9)(+2\mu C)(-3\mu C)\left[\frac{1}{.472} - \frac{1}{.50}\right]\}$$
$$W_{EL} = -.094 + .143 + .006\ \}$$
$$W_{EL} = +.055 \text{ Joules}$$

"How much work is needed..." really asks "What is W_{you}...". Speed is constant, so $\Delta KE = 0$. The "$W_{el} + W_{other} = \Delta KE$" equation is "$+.055 + W_{you} = 0$, and $W_{you} = -.055$ J. W_{you} is –, so you must "restrain" Q_B to keep its speed from increasing.

SOLUTION 10-A, Part 3: If $V \equiv 0$ at ∞, then each Q produces $V = kQ/r$.

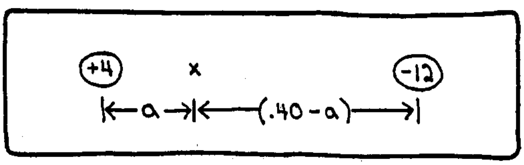
$$V_{at\ "8"} = k\frac{(-4\times10^{-6})}{.80} + k\frac{(+5\times10^{-6})}{.943} + k\frac{(+2\times10^{-6})}{.50} = -45000 + 47720 + 36000 = +38720$$

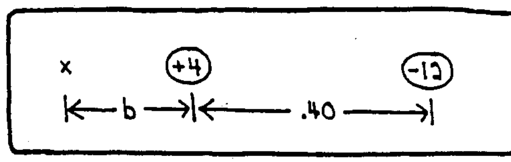
$$V_{CENTER} = k\frac{(-4\times10^{6})}{.472} + k\frac{(+5\times10^{-6})}{.472} + k\frac{(+2\times10^{-6})}{.472} = -76270 + 95340 + 38140 = +57210$$

Q_B is affected by the total V produced by the other 3 charges, but not by its own V. The work done on Q_B is $W_{el} = -q\,\Delta V = -(-3\times10^{-6})[(+57200) - (+38700)] = +.0555$ J. This is, of course, the same result we got in Part 2 using "$W_{el} = -kQq(1/r_f - 1/r_f)$".

To calculate W or ΔV (either individual or total), **you must include all ± signs.** But W and ΔV are non-vectors, so you don't have to split-combine-reconstruct.

SOLUTION 10-B for V: Because +4 and +12 both cause + voltage, V can never be zero in the top picture. { But at points that are extremely far away, V becomes very close to zero. }
In the bottom picture, $V = 0$ at two points. The approximate location of these points can be found using "close vs. large" logic, as discussed in the earlier E-solution. The exact locations are found by solving "$V_{total} = 0$" equations.





$$V_{of\ +4} + V_{of\ -12} = 0$$
$$k\frac{(+4)}{a} + k\frac{(-12)}{.40-a} = 0$$
$$\frac{+4}{a} = \frac{+12}{.40-a}$$
$$.10\ m = a$$

$$V_{of\ +4} + V_{of\ -12} = 0$$
$$k\frac{(+4)}{b} + k\frac{(-12)}{(.40+b)} = 0$$
$$\frac{+4}{b} = \frac{+12}{.40+b}$$
$$.20\ m = b$$

Let's compare the 10-B solutions for E & V, and ask "How do we find the ± sign?".

To find the ± sign of E, look at the picture and decide whether the E-vector points to the right (in the direction I chose to be +) or to the left (in the – direction).

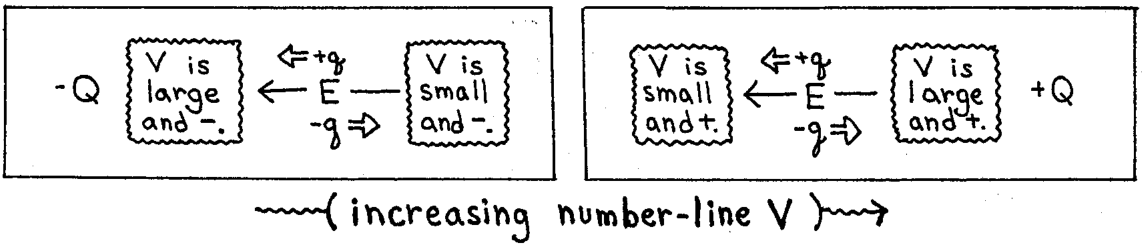
But for V, just substitute the proper charge, either +4 or –12; + charge causes +V, and – charge causes –V.

10.7 The Relationship of F, E and V
Equipotential Surfaces
Parallel Plates

The drawings below* show the V & E produced by Q, and the F's (⇐ or ⇒) felt by q. Do you see the relationship between E & F, between E & V, and between F & V?

* In these drawings (and descriptions of them) +Q,–Q, –q and +q mean that a particular charge is + or –, and not (as + or – mean in an equation) that Q or q is being multiplied by +1 or –1.

The drawings below use the following 3 principles. E_Q points away from +Q, E_Q points toward –Q. +Q produces V that is +, –Q produces V that is –. Close to Q, V_Q, is large; far from Q, V_Q is small.



NUMBER LINES: These numbers (–20, –10, 0, +10, +20) are arranged in "number-line order", with the lower numbers on the left and higher numbers on the right. Their order can be summarized as "large –, small –, zero, small +, large +". A voltage of –50000 Volts has large magnitude (it would be labeled "Warning: Dangerous High Voltage!"), but in number-line terms it is very low voltage.

During the rest of Chapter 10, "V" means "number-line V".

In drawings above, notice that the V's are in correct number-line order, both E's point ←, the F on +q is ⇐, and F on –q is ⇒. Does this help you see the EF, EV & FV relationships?

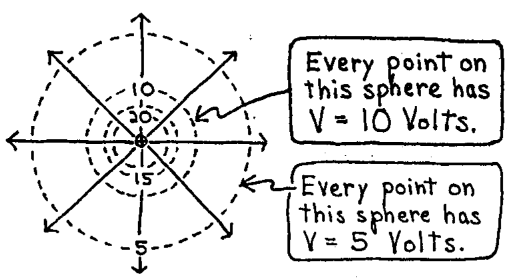
This relationship-summary shows the logic that "if A = B and B = C, then A = C":

<u>If q is +</u>,
F points *in the direction E points.*
In the direction E points, V decreases.
F points in the direction of decreasing V.
{ +q feels force toward "more –" voltage. }

<u>If q is –</u>,
F points *in the direction opposite to E.*
In the direction opposite to E, V increases.
F points in the direction of increasing V.
{ –q feels force toward "more +" voltage. }

EQUIPOTENTIAL SURFACES

The charge in the picture below is +.1 nC. Every point that is 9 cm away will, because $V = kQ/r^2$, have an electric potential of +10 Volts; these points are shown by the - - - circle (which is actually a 3-D sphere). A little closer, there is a ◌ with all points at +15 V. Further away on another ◌, all points have V = +5 V. Each of these - - - surfaces is called an *equipotential* (equal voltage) *surface.*



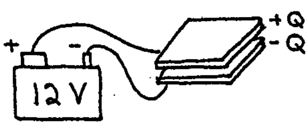
If you move in the direction E points, V decreases. In the direction opposite to E, V increases. If you keep moving ⊥ to E, V doesn't change* and (as you can see by looking at the picture) you will trace out an equipotential surface.

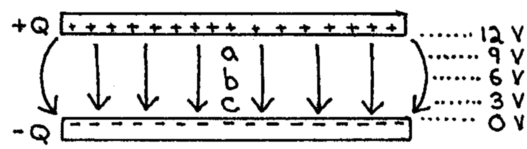
* Why? E is ⊥ to movement, so F is ⊥ to movement. F that is ⊥ to movement doesn't do any work: $W = F\,d\cos\emptyset = F\,d\cos 90° = 0$, $\Delta V = -W/q = -0/q = 0$. Because $\Delta V = 0$, V doesn't change.

Study the picture above. Can you find a relationship between line-spacing and magnitude, for either E or ΔV? {This is discussed in Problem 10-##.}

E and V for PARALLEL PLATES

If a 12 Volt battery is connected to parallel plates, as in the first picture below, charges of +Q and –Q move onto the plates that are connected to the battery's + and – terminals. The second picture shows that E lines point "straight across" from the + plate to the – plate, except near the edge where they "curve". It can be proved {as in Problem 10-#, using Gauss's Law} that <u>E between the plates is constant</u>. For example, E is the same at points a, b & c.





If a point charge q moves a distance "d" straight away from the – plate toward the + plate, E is constant, the F_{el} acting on q is constant, and

$$\Delta V = -(W_{el})/q = -(\mathbf{F}\,\mathbf{d})/q = -(q\mathbf{E}\,\mathbf{d})/q = -\mathbf{E}\,\mathbf{d}.$$

The – sign in "$\Delta V = -\mathbf{E}\,\mathbf{d}$" shows that ΔV is – if the **E** and **d** vectors point in the same direction. As you move away from the – plate, **E** and **d** point in opposite directions so ΔV is +, and V gradually increases (from 0 to 3, 6, 9, 12, as shown on the diagram) until it reaches +12V at the + plate.

The principle that "E always points downhill in number-line V" will (of course) give the same result: you are moving "against" the E-field direction, so you're moving uphill, V is increasing, and ΔV is +.

Important: $\Delta V = -E\,d$ can only be used if E is constant.

{ If E is not constant, you can use a "calculus integral", as explained in Section 18.#. }

"$\Delta V = -Ed$" shows a relationship between ΔV and E, but not between V and E.

For example, there are an infinite number of ways to define " V " for the plates: their V's could be 0 and +12 (as above), +1000 and +1012, –1000 and –988, or ...
The final V changes; it can be +12 or +1012 or –988 or..., but ΔV (which equals –Ed) is always +12.

Using basic principles from Chapter 10, it can be shown (see Problem 10-##) that **every part of an electrostatic conductor has the same electric potential.**

{ This constant V can have any numerical value; ΔV is zero, but V isn't necessarily zero. }

Parallel plate *capacitors* are discussed in Chapter 11; Section 11.7 is a summary of capacitor formulas.

The Chapter 10 Flash Cards will be done later ← FLASH WILL BE pao · 215

== comment: I'll probably cut this next (10.8) section and put the summary only in the "Chapter 10 Sumr
I would refer to it in the Chapter 10 Introduction and in one or two other places during the chapter.

10.8 A Summary of Electrostatic Relationships

The magnitude formulas marked with • are correct only for point-charges or spherically symmetric charge distributions. All other formulas and relationships are always true.

The SI value of "k" is 9.00 x 10^9 Nm^2/s^2.
k can also be written as "$1/4\pi\epsilon_0$", where $\epsilon_0 = 8.85 \times 10^{-12}\ s^2/Nm^2$.

$F = ma$, etc.

• $\vec{F} = k\dfrac{Qq}{r^2}$

Alike charges repel, opposite charges attract.

$F_{on\,q} = q\,E_Q$

If q is +, $\vec{F}$ is in direction of $\vec{E}$.
If q is −, $\vec{F}$ is opposite to $\vec{E}$.
For any q, F = 0 in direction ⊥ to $\vec{E}$.

• $\vec{E}_Q = k\dfrac{Q}{r^2}$

Away from +, toward −.

Directions of $\vec{F}$ (or $\vec{a}$) and movement ($\vec{\Delta x}$ or $\vec{v}$) don't have to be same.

If F_{EL} and Δx are in same direction, W_{EL} is +.
If F_{EL} and Δx are in opposite directions, W_{EL} is −.
If F_{EL} is ⊥ to Δx, $W_{EL} = 0$.

If q is +, F is toward "more −" V.
If q is −, F is toward "more +" V.

$\vec{E}$ points "downhill" in number-line voltage:
In direction E points, V ↓.
Opposite E direction, V ↑.
⊥ to E, V doesn't change.

If E is constant,
$\Delta V = -E\,\Delta x$
(analogous to $W = F\Delta x$)

• $W_{EL} = -k\dfrac{Qq}{r}$

if q begins very far away from Q.

• $W_{EL} = -kQq\left(\dfrac{1}{r_f} - \dfrac{1}{r_i}\right)$

for any r_i and r_f.

$W_{EL} = -q\,\Delta V$

$\Delta PE_{EL} = q\,\Delta V$

• $V = k\dfrac{Q}{r}$ (V of Q)

if $V \equiv 0$ when $r_i = \infty$.

• $\Delta V_Q = kQ\left(\dfrac{1}{r_f} - \dfrac{1}{r_i}\right)$

for any r_i and r_f.

SIGN: If Q is +, V_Q is +. If Q is −, V_Q is −.

MAGNITUDE: Close to Q, V_Q is large. Far from Q, V_Q is small.

$W_{EL} + W_{OTHER} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$
If constant speed, $\Delta KE = 0$
If "unconstrained", $W_{other} = 0$

In the left-side formulas: F and W are caused by the mutual interaction between Q & q, so Q & q both appear in their formulas.

In the right-side formulas: E_Q & V_Q (the E-field and voltage caused by Q) don't depend on q, so neither has q in its formula.

Here are some differences between the top-row and bottom-row formulas.
Top formulas (F and E) have 1/r, are vectors (with magnitude and direction).
Bottom formulas (W and V) have $1/r^2$, are non-vectors (with magnitude and ± sign).

For F and E, ignore the ± sign of Q & q; use visual logic and think "vector direction".
For W and V, substitute the ± sign of Q & q, then be careful as you solve the algebra.

Optional Section 18.# discusses the calculus relationships between F & W and between E & V: $\Delta V = \int E\,dr$, $E_x = dV/dx$, etc.

The <u>PRINCIPLE OF SUPERPOSITION</u>:
the F, E, W & V caused by one charge is independent of that caused by other charges, so F_{total}, E_{total},... is the sum of the F's, E's,... caused by all charges in a system.

F_{total} = sum of F's $\qquad$ E_{total} = sum of E's

W_{total} = sum of W's $\qquad$ $(\Delta V)_{total}$ = sum of (ΔV)'s

To find total **F** or **E**, split-add-reconstruct vectors; for total W or V, use + and − signs.

A charge is not affected by interaction with its own E-field:
the F_{el} acting on a charge = (its own charge)(E-field caused by all other charges).

SI-UNIT SUMMARY: the most common SI units for F, E, W & V are circled below.

F is in (N) $= \frac{kg\,m}{s^2}$ $\qquad$ E is in $\left(\frac{N}{C}\right) = \frac{kg\,m}{C\,s^2} = \left(\frac{V}{m}\right)$

W is in (J) $= N\,m = \frac{kg\,m^2}{s^2}$ $\qquad$ V is in (V) $= \frac{N\,m}{C} = \frac{J}{C} = \frac{kg\,m^2}{C\,s^2}$

I've included combinations like "kg m^2/C s^2" for completeness, but you'll probably never use them*. Instead, just be sure every substitution you make is in SI units, and you'll know that any variable you solve for will be in the appropriate SI units.
{ * For example, a battery's voltage will almost always be given as "12 V", not "12 kg m^2/Cs^2".}

[with some ideas cut and others added]

REVIEWER: This was originally part of Section 10.7. It will be modified some when it is made into a "chapter summary", but many of the main ideas are on these two pages.
{ As usual, each idea in the summary is explained in the regular chapter. }

If your class studies Gauss's Law, this is essential. Otherwise, you can skip it.

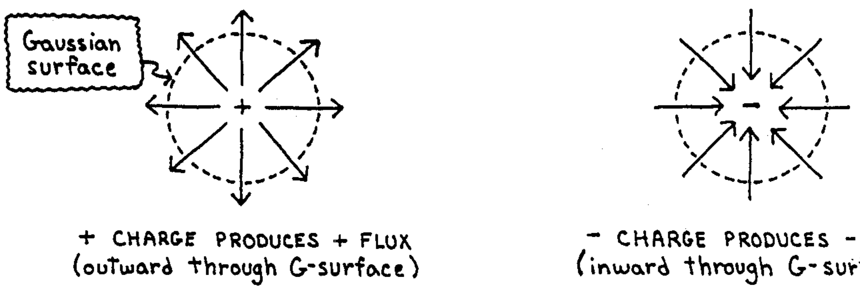
GAUSS'S LAW

In the 1800's, Karl Gauss discovered a new way to calculate electric fields.
Gauss's Law is a clever blending of visual symmetry and math logic.
I think you'll find that it is a fun tool to use.

Read Sections 10.92 to 10.94 in order; they show what flux is and what produces it, summarize useful geometry formulas, and outline a "Gauss's Law strategy".

10.93 The Production and Properties of "FLUX".

E-field lines point away from a + charge, and toward a – charge. When E-lines point through an imaginary "Gaussian surface" surrounding a charge, it is given the name *flux.* E-lines that point outward through a surface are defined as + *flux,* and inward-pointing lines are – *flux,* so (as you can see in the pictures below) <u>+ charge produces + flux</u> and <u>– charge produces – flux</u>.



Gauss's Law is $\Phi = Q_{inside}/\epsilon_0$, where Φ is the flux passing through an imaginary *Gaussian surface,* Q_{inside} is the net charge enclosed within this same G-surface, and ϵ_0 is a constant with an SI value of 8.85×10^{-12}.

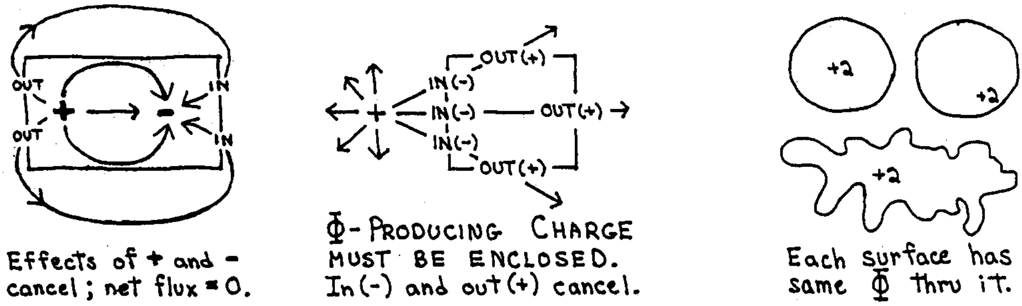
The Gaussian surface is important. It is involved with both variables: the flux "Φ" passing through a G-surface depends on the enclosed charge "Q_{inside}" within that G-surface. { Later, "Gauss's Law Strategy" shows how to choose a useful Gaussian surface. }

Gauss's Law states that flux depends only on <u>net</u> <u>enclosed</u> charge.

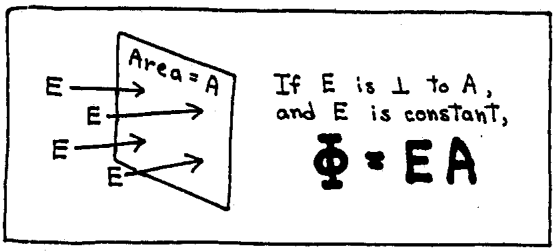
NET: The Φ-effects of equal + and – charges cancel each other. In the first picture below, the + charge's outward flux is canceled by the – charge's inward flux.

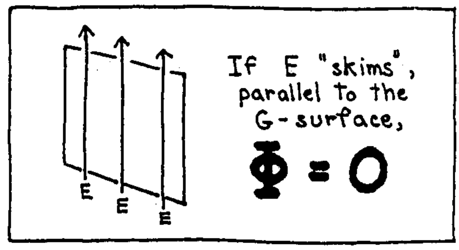
ENCLOSED: As shown in the middle picture, if charge is outside a surface (not enclosed), the inward "– flux" is canceled by outward "+ flux", causing net flux = 0.

Two things that don't matter: as shown in the last picture, flux doesn't depend on the location of enclosed charge, or the shape of a Gaussian surface that surrounds it.



To find Φ for the left side of the G's Law Equation, use these principles:
1) If E points through a surface at a 90° angle, and E is constant over the entire surface area "A", the flux magnitude is "EA". (See the first picture below.)
2) If E "skims" along a G-surface, parallel to it, E doesn't produce any flux **through** the surface. (This is shown in the second picture.)



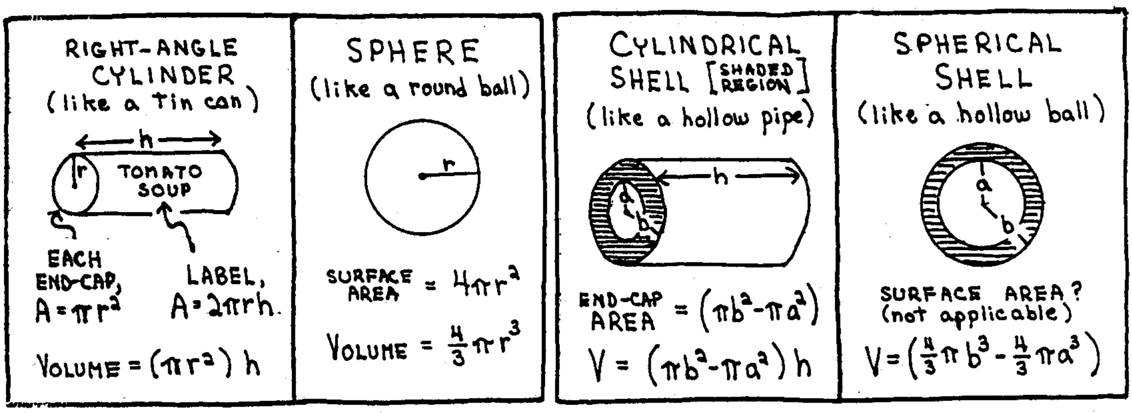


{ If a constant E field points through a G-surface at an angle between 0° and 90°, use the $E_\perp$ component to calculate " $\Phi = E_\perp A$ ". If E isn't constant (for most G's Law problems, E is constant), use the general expression $\Phi = \oint E \cdot dA$, as discussed in Section 18.#'s "optional calculus". }

If you understand what flux is now, that's great! If not, please be patient; when you start using flux in Section 10.94, you'll appreciate its problem-solving value.

10.94 Geometry, Charge Density, Ratio Logic

To find the surface-area and volume of spheres & cylinders, two shapes that are often used as Gaussian surfaces, use these formulas:



CHARGE-DENSITY FORMULAS are often needed to calculate "enclosed charge". There are *3 types of charge density*. Here is what each of them means:

If 5.0 Coulombs of charge is distributed uniformly over the length of a wire that is 10 meters long, the "charge density" is (5.0 C / 10 m) = .50 C/m.

If 5.0 Coulombs of charge is distributed uniformly over the face of a 3m x 4m rectangular metal plate, the "charge density" is (5.0 C / 12 m^2) = .42 C/m^2.

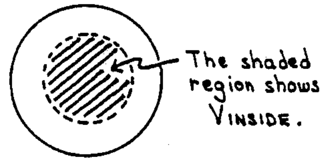
If 5.0 Coulombs of charge is distributed uniformly throughout the volume of a 2m x 3m x 4m rectangular box, "charge density" is (5.0 C / 24 m^3) = .21 C/m^3.

Charge density in Coulombs per m, per m^2, and per m^3 are symbolized by λ, σ and ρ, respectively; they are used in "conversion factor" formulas, as shown at the right. At the far right, these formulas are written without units.

FORMULAS WITH UNITS		SIMPLIFIED
$(\underline{\ell}\ m)(\underline{\lambda}\ \frac{c}{m}) = (\underline{Q}\ c)$	or	$\ell\lambda = Q$
$(\underline{A}\ m^2)(\underline{\sigma}\ \frac{c}{m^2}) = (\underline{Q}\ c)$	or	$A\sigma = Q$
$(\underline{V}\ m^3)(\underline{\rho}\ \frac{c}{m^3}) = (\underline{Q}\ c)$	or	$V\rho = Q$

RATIO LOGIC for non-conductors:

In a *conductor,* charge is free to move; as described in Section 10.3, all net charge moves to the object's outer surface. In a *nonconductor,* charge stays wherever it was initially. When a G-surface is inside a nonconducting object, only part of the object's total charge is "enclosed" by the G-surface.



If charge is distributed uniformly throughout an object's volume, the charge-fraction and volume-fraction enclosed by a G-surface are equal:

Q-fraction = V-fraction

For example, if 2/10 of the total volume is inside a G-surface, 2/10 of the charge is inside that G-surface.

{ This ratio logic is used in Problem 10.##.}

Q-fraction = V-fraction

$$\left(\frac{Q_{INSIDE}}{Q_{TOTAL}}\right) = \left(\frac{V_{INSIDE}}{V_{TOTAL}}\right)$$

$$Q_{INSIDE} = \left(\frac{V_{INSIDE}}{V_{TOTAL}}\right) Q_{TOTAL}$$

10.95 Gauss's Law Problem-Solving Strategy

Step 1: Use symmetry logic to draw the E-lines produced by the charged object, and choose a useful Gaussian surface. A useful G-surface usually "matches" the E-field's symmetry (which is closely related to the object's symmetry).
{ As shown in Problem 10-D, Φ is easy to calculate if every face of a G-surface fits into one of the 3 categories shown below: if E "skims the surface", is constant-and-⊥, or is zero.}

Step 2: Substitute all information into the left and right sides of Gauss's Law, and solve for the unknown. Some common substitutions are shown below.

$$\Phi_{TOTAL} = \frac{1}{\epsilon_0} Q_{INSIDE}$$

Φ_{TOTAL} = sum of Φ thru all surfaces, where Φ = 0 if E "skims" a surface, Φ = EA if E is constant-and-⊥.
E = 0 (so Φ = 0) inside a conductor.

EQUATIONS OF CHARGE DENSITY:
$Q = \lambda \ell$
$Q = \sigma A$
$Q = \rho V$

If Q_{TOTAL} is distributed uniformly throughout the volume of a non-conductor,
(CHARGE FRACTION) = (VOLUME FRACTION)
$Q_{IN} = \left(\frac{V_{IN}}{V_{TOTAL}}\right) Q_{TOTAL}$

If you need A or V (for EA, σA, ρV, or V_{inside}/V_{total}), use a geometry formula. Formulas for spheres and right-cylinders are given in Section 10.93.

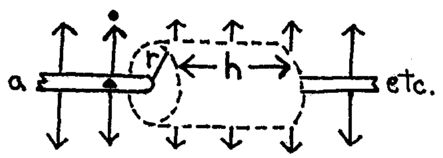
PROBLEM 10-D: Using the Gauss's Law Strategy.
What E field is produced by an infinitely long wire with a net + charge, if the wire's charge density is "λ" Coulombs/meter?

SOLUTION 10-D

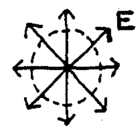
Step 1: Look at the diagram below, and use symmetry logic.

+ charges to the left of the point marked "▲" produce E (at •) with a sideways component toward the right. But the wire is infinitely long, so there is an equal amount of + charge to the right of ▲ ; this charge produces a leftward E. These sideways E-fields cancel each other, leaving only the E-component that points straight away from the wire. If the wire is infinitely long, this same argument can be made for every point on it, so E points straight away from the wire along its whole length, as shown by the ↑'s.

It is easy to calculate total Φ if we choose the Gaussian surface to be a cylinder centered on the wire, because E skims along the endcaps (so $\Phi = 0$) and E through the label is always constant-and-pointing straight-outward* (so $\Phi = EA_{label}$). * The second diagram below, a view of the endcap from point "a", shows that E always points ⊥ to the G-surface.



This end-view shows that E is constant-and-⊥.



Step 2: Substitute (the formulas you need are in the strategy summary) and solve.
(Only the wire length "h" that is enclosed by the G-surface gets counted as Q_{inside}.)

$$\text{TOTAL FLUX} = \frac{1}{\epsilon_0} Q_{INSIDE}$$
$$\Phi_{CAPS} + \Phi_{LABEL} = \frac{1}{\epsilon_0}\left(\lambda \frac{c}{m}\right)(h\ m)$$
$$0 + 0 + EA = \frac{1}{\epsilon_0}(\lambda h)$$
$$E(2\pi r h) = \frac{\lambda \ell}{\epsilon_0}$$
$$E = \left(\frac{1}{2\pi\epsilon_0}\right)\frac{\lambda}{r}$$

This formula gives the E-magnitude at the "label surface" of the Gaussian cylinder. A G-cylinder can have any radius, so this is E at any distance "r" away from the wire.

Ratio logic: "r" is on the bottom of the E-formula, so as r ↑, E ↓. This is probably what you expect, but the result for an infinite charged plane (which is derived in Problem 10.##) may surprise you.

The optional problems show how to apply the essential Gauss's Law strategies to a wide variety of situations. I strongly recommend that you do them.

This will be Section 10.95: optional Gauss's Law Problems.

These problems are already written up, in a January 1987 version. They aren't revised yet, so I won't send them along, but they cover 1) qualitative G's Law logic, 2) a conducting shell with a point charge inside the "hollow", 3) plates (in three parts: an infinite plate, two parallel metal plates, and two parallel nonconductors), 4) a nonconducting sphere. I may do others with just answers and no "solutions": a cylindrical shell, other kinds of nonconducting objects (spherical shell, solid cylinder, cylindrical shell), or ...