Chapter 8

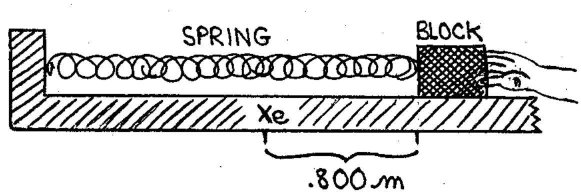
Simple Harmonic Motion

Read Section 8.1 first. At the end of 8.1 and beginning of 8.2, there are "CHOICE" suggestions that will help you decide what to do next.

8.1 Exploring a Cycle of Simple Harmonic Motion

The best way to learn about *simple harmonic motion* is to carefully explore what happens during one full cycle of motion, using your knowledge of spring-force (from Section 3.8), the relationship of v, F and a (from 2.2 & 3.1), the ± sign of work (from 4.2), PE_{spring} (from 4.3), and conservation of energy (from 4.4). {If you quickly review these sections to refresh your memory, it will help you understand Chapter 8.}

Think about what happens if we attach a block (m = 7.29 kg) to a spring (with k = 12.5 N/m), stretch the spring .800 m from its equilibrium position (which is defined to be x ≡ 0) and release it from rest on a horizontal frictionless surface.



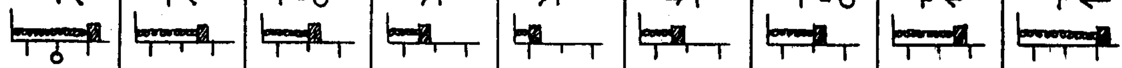
The 9 pictures below are "snapshot photographs" of the block at 8 different positions during one full *cycle of motion.* Notice that the first & last pictures are the same. Study these diagrams, and use principles from the sections mentioned above (3.8, 2.2 & 3.1, 4.2-4.4) to answer the following questions.

Can you explain the **F** vector's magnitude (represented by the arrow length) and direction for each picture? The **v**-arrows also show magnitude and direction at the 9 positions; do these **v** vectors agree with your common sense expectations about the block's velocity? Do you expect **a** to be correlated with **v** or with **F**?

Why does the block's speed increase during the first & third quarter-cycles, and decrease during the second & fourth quarter-cycles?

At what point in the cycle does the system have maximum PE? maximum KE? What are these maximum values of PE and KE? Can you calculate the spring's PE and KE when x = .40 m? At what position(s) during the cycle does the block reach its maximum speed? What is this v_{max}?

Think about these questions, answer them, and then continue reading.



START OF CYCLE	← FIRST QUARTER of cycle →	AFTER 1/4 CYCLE	← SECOND QUARTER of cycle →	AFTER 1/2 CYCLE	← THIRD QUARTER of cycle →	AFTER 3/4 CYCLE	← FOURTH QUARTER of cycle →	AFTER ONE CYCLE
v = 0 a ← F ←	v ← a ← F ←	v ← a = 0 F = 0	v ← → a → F	v = 0 → a → F	→ v → a → F	→ v a = 0 F = 0	→ v a ← F ←	v = 0 a ← F ←
x = +.80	x = +.40	x = 0	x = −.40	x = −.80	x = −.40	x = 0	x = +.40	x = +.80
PE = 4 KE = 0 TE = 4	PE = 1 KE = 3 TE = 4	PE = 0 KE = 4 TE = 4	PE = 1 KE = 3 TE = 4	PE = 4 KE = 0 TE = 4	PE = 1 KE = 3 TE = 4	PE = 0 KE = 4 TE = 4	PE = 1 KE = 3 TE = 4	PE = 4 KE = 0 TE = 4

The variation of **F** with position is explained by "F = −kx" from Section 3.8. Like a homing pigeon, F_{spring} always points toward its equilibrium position x_e. If a spring is far from x_e, F_{spring} is large. If it is close to x_e, F_{spring} is small.

Because F=ma, **a** is directly proportional to **F**.

As emphasized in Sections 2.2 and 3.1, **v** is not directly related to **a** or **F**. If you study the 8 positions above, you can find two positions where F & a have maximum magnitude but v = 0, two positions with maximum-magnitude v but F = 0 and a = 0, two positions where v points the same direction as F & a, and two positions where v points in the direction opposite to F & a.

Now we'll use principles from Section 4.2. During the first quarter-cycle, **F** and **v** both point ←, so W is +, ΔKE ($= \Delta \frac{1}{2} mv^2$) is +, and the block's speed increases. As x decreases and the block gets closer to the spring's equilibrium position, F decreases until it becomes zero when x = 0 at the motion's *center point.* After the block passes the center point and enters the second quarter-cycle, F and v directions are opposite (F points → but v is still ←) so W is −; v decreases until it becomes zero at the far-left *turning point* when x = −.8 m.

As discussed in Section 4.4, if there is no friction or "other forces" (this is true of our system) the *total mechanical energy* is conserved: the sum of $\mathbf{PE_{total} + KE}$ stays constant. $PE_{gravity}$ is constant because the surface is horizontal, so PE_{spring} is the only PE that is changing, and $\mathbf{PE_{spring} + KE}$ is constant.

This energy conservation principle can be used for calculations. When the block is at x = +.8, v = 0 and total energy is TE = PE + KE = $\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}(12.5)(+.8)^2 + \frac{1}{2}(7.29)(0)^2 = 4.0$ J. When the block has moved to x = +.4 and PE = $\frac{1}{2}kx^2 = \frac{1}{2}(12.5)(+.4)^2$ = 1.0 J, the total energy (PE + KE) is still 4.0 J, so KE must be 3.0 J.

Look at the 9 pictures above, and notice that PE & KE keep changing (first they are 4 & 0, then 1 & 3, 0 & 4,...), but they always add to give a TE of 4 Joules.

The block's maximum speed, abbreviated $\mathbf{v_{max}}$, occurs twice during each cycle: at the center point moving ←, and the center point moving →. And twice during the cycle, at the left & right turning points, the block is furthest away from x_e; instead of just calling it x_{max}, this maximum x-magnitude is called *amplitude,* abbreviated **A**.

At either turning point (whether x is +A or −A), v = 0 so KE = 0, and TE = PE + KE = $\frac{1}{2}k(\pm A)^2 + 0 = \frac{1}{2}kA^2$. At either center point (whether v is $-v_{max}$ or $+v_{max}$), x = 0 so PE = 0, and TE = PE + KE = $0 + \frac{1}{2}m(\pm v_{max})^2 = \frac{1}{2}m v_{max}^2$. And at each point in the cycle, the "general equation" is true: TE = PE + KE = $\frac{1}{2}kx^2 + \frac{1}{2}mv^2$.

If there is no friction, total energy is equal at every point in the cycle and we can equate all of these ways to describe TE and form this useful five-sided equation:

PE + KE	=	TE	=	TE at any point	=	TE at turning point	=	TE at center point
PE + KE	=	**TE**	=	$\frac{1}{2}kx^2 + \frac{1}{2}mv^2$	=	$\frac{1}{2}kA^2$	=	$\frac{1}{2}m v_{max}^2$

To find the block's maximum speed, you can substitute into $\frac{1}{2}kA^2 = \frac{1}{2}m\,v_{max}^2$ (which is formed by equating 2 of the 5 equation-sides) and solve for $v_{max} = 1.05$ m/s.

What is Simple Harmonic Motion, and what causes it?

The ninth snapshot is the same as the first; the block is at the start of a new cycle, which will be the same as the first cycle. If friction, air resistance, within-the-spring losses,... could be eliminated (this is impossible, but we can imagine) the block would oscillate back and forth forever. This continuous *cyclic* or *periodic* or *oscillatory* movement is called *simple harmonic motion,* or *SHM.*

SHM occurs whenever there is a *restoring force* that

1) always points toward an *equilibrium position* [which is usually defined as $x \equiv 0$],
2) has a magnitude, if $x_e \equiv 0$, of "kx" where k is a constant,
3) is the only force acting on the object [there is no friction or other forces] so the motion-amplitude and total energy remain constant.

Optional: Section 8.94 examines situations where the third SHM requirement isn't satisfied, where there is a restoring force and either a *damping force* (like friction or fluid drag) that gradually reduces the system's total energy, or a *driving force* that increases the system's energy, or both.
[== there may not be an 8.94; if it is cut, I'll just mention that "your text may discuss ----"]

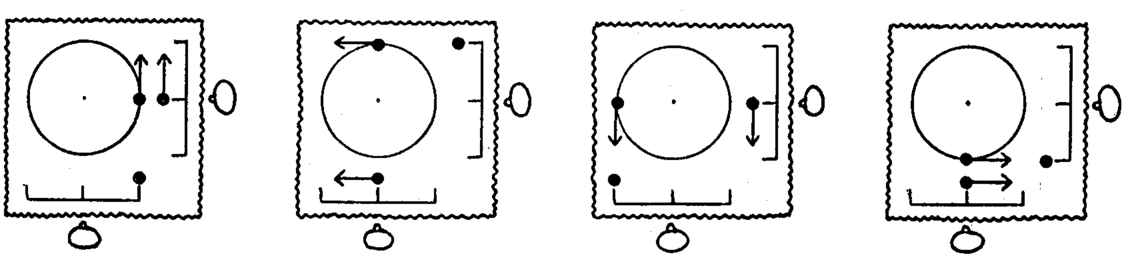
Some SHM examples are a) the horizontal spring-block system [with $F = -kx$ and no friction] we've been studying, b) vertical spring-block system, c) pendulums.

CHOICES: When your class studies either vertical spring-blocks or pendulums, read Section 8.4.

8.2 The Relationship between Simple Harmonic Motion and Circular Motion

CHOICES: Some classes skip the topics in this section. Look at the assigned reading in your text and search for equations with "cos" or "sin" [like $x = A\cos(\omega t)$, $y = A\sin(2\pi f t)$, $x = A\cos(\omega t + \phi)$, $y = A\sin(\omega t + \phi)$]. If you find equations like these, read this section. If not, you can skip to Section 8.3.

The pictures below show bird's eye views, at 4 different times during a cycle, of 3 objects (•,•,•) with 3 kinds of motion that are occurring on the table top: SHM in the east/west x-direction (shows the SHM turning & center points), SHM in the north/south y-direction, and a small race car that is moving around a circle at a constant speed I'll call v_{circle}. These motions are being watched by and .



(The three motions have "matching" amplitude & timing: 1) each SHM amplitude is .8 m and the circle radius is .8 m, 2) each SHM cycle takes 4.8 s and the race car also completes one cycle in 4.8 s.)

Pretend that you are and you have no "depth perception". It can be shown (see Problem 8-#) that you will see the same x-motion whether you watch or O. In the first picture, both •'s are at the far-right point with $v_x = 0$; the race car is moving in the y-direction, but the x-component of v_{circle} is zero. After 1/4 cycle, both •'s are at the center point moving ← at maximum speed; the race car's speed is constant, but at this point in the cycle every bit of v_{circle} points in the x-direction so the x-component of v_{circle} is maximum. Halfway through the cycle in the third picture, both •'s are at the far-left turning point with $v_x = 0$. In the fourth picture, both •'s are at the center point moving → at maximum speed.

Do you see that the x-component motion of an <u>imaginary</u> race car (moving in a circle) accurately imitates the simple harmonic motion of the <u>real</u> block?

At the center point, $\frac{1}{2} kA^2 = \frac{1}{2} m v_{max}^2$ (from Section 8.1), $v_{max} = v_{circle} = A\omega$ (think about the "Pretend..." paragraph above, and $v_T = r\omega$ from Section 5.4). Combining these equations gives a useful relationship: $\sqrt{k/m} = \omega$.

$$\begin{aligned}
\tfrac{1}{2} k A^2 &= \tfrac{1}{2} m (v_{max})^2 \\
k A^2 &= m (A \omega)^2 \\
k A^2 &= m \; A^2 \omega^2 \\
\frac{k}{m} &= \omega^2 \\
\sqrt{\frac{k}{m}} &= \omega
\end{aligned}$$

As explained in Section 5.4, ω (in radians/second) is related to f (in cycles/second) and T (in s/cycle) by: $\omega = 2\pi f$ [because 1 cycle = 2π rads], $\omega = 2\pi/T$ [because $f = 1/T$], and $T = 2\pi/\omega$.

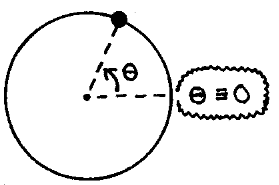
It can also be shown that , if he has no depth perception, will see the same y-motion whether he observes the ↕ SHM or the circular motion's y-component.

{ If your textbook or teacher talks about "projection (of the shadow of the SHM object) onto the x-axis" or "projection onto the y-axis", it has the same meaning as what is described here. }

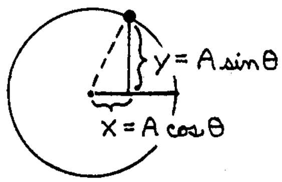
To describe SHM in equations we need to define two variables: θ and ω.

As shown in the first picture below, the race car's angular position "θ" can be described by stating how far it is, measured in radians moving counterclockwise, away from a circle-point we've chosen to be $\theta \equiv 0$. { The $\theta \equiv 0$ point on the circle below is used in most textbooks; it is the same $\theta \equiv 0$ point that is used in math courses.}

As shown in the middle column, at any time "t" the race car's angular position "θ" depends on where it was at the instant we choose to define as $t \equiv 0$ [this initial θ_i position is called the *phase angle,* $\varnothing$] and how much $\Delta\theta$ progress it has made since then, where ($\Delta\theta$ radians) = (ω radians/second)(t seconds) or simply $\Delta\theta = \omega t$.



$$\begin{aligned}
\theta_f &= \theta_i + \Delta\theta \\
&\Downarrow \quad \Downarrow \quad \Downarrow \\
\theta &= \varnothing + \omega t \\
\theta &= \omega t + \varnothing
\end{aligned}$$



The third diagram shows that the car's x-position and y-position can be defined in terms of A and θ: $x = A\cos\theta = A\cos(\omega t + \varnothing)$, and $y = A\sin\theta = A\sin(\omega t + \varnothing)$. These equations describe how the **x**-position or y-position of a SHM object varies with time.

Problem 8-# uses trigonometry and algebra (or calculus) to derive corresponding equations that describe how **v** and **a** vary with time.

Two commonly used formats for time-equations are shown below. {In Chapter 8 I'll use both formats. This is more work for me, but it's easy for you. Just use the set of equations your teacher prefers, either x-format or y-format, and ignore the other set.}

equations with x-format	equations with y-format
$x = A\cos(\omega t + \phi)$	$y = A\sin(\omega t + \phi)$
$v = -\omega A\sin(\omega t + \phi)$	$v = \omega A\cos(\omega t + \phi)$
$a = -\omega^2 A\cos(\omega t + \phi)$	$a = -\omega^2 A\sin(\omega t + \phi)$

In these equations, **A** is the SHM amplitude [which equals the O radius], ω is the O-object's *angular velocity* or *angular frequency* (in radians/s), **t** is the Δt since the instant defined to be $t \equiv 0$, and the *phase angle* **ø** is the O-object's angular position (in radians) at the $t \equiv 0$ instant.

Some textbooks simplify "$\omega t + \phi$" to "ωt". This is correct if $t \equiv 0$ when the O-object is at $\theta \equiv 0$, which makes $\phi \equiv \theta_i = 0$, and $\theta = \omega t + \phi = \omega t + 0 = \omega t$.

8.3 Equations for Simple Harmonic Motion

The most common SHM equations are organized in the Chapter 8 Summary. Study it as you read the following descriptions of five basic equation categories.

- At the top is the <u>5-sided TE conservation equation</u> from Section 8.1. Each box equals every other box, so any two boxes can be equated to make an equation.

- Two of these TE-equations are: $\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2$, $\frac{1}{2}m{v_{max}}^2 = \frac{1}{2}kA^2$.
  Solving these equations gives: $v = \sqrt{k/m}\,\sqrt{A^2 - x^2}$, $v_{max} = \sqrt{k/m}\;A$.
  Because $\sqrt{k/m} = \omega$ [Section 8.2], $\mathbf{v = \omega\sqrt{A^2 - x^2}}$, $\mathbf{v_{max} = \omega A}$;
  these two equations are in the summary.

  {Your textbook may have a different version of the first equation, like $v = v_{max}/A\,\sqrt{A^2 - x^2}$ or $v = v_{max}\sqrt{1 - x^2/A^2}$; these are derived by substitution-and-rearrangement. Each version of the equation gives a different "intuitive description" of the relationships between v, x, A, ω and v_{max}, but for solving problems you only need one version; use the one you find most useful.}

- Angular velocity variables (like ω, f, T) are discussed in Section 5.4. If you know any 1 of [ω, f, T] you can find the others: just equate the appropriate parts of the three-sided equation "$\omega = 2\pi f = 2\pi(1/T)$". And because $\sqrt{k/m} = \omega$, if you know any 2 of these 3 [<u>k</u> , <u>m</u> , <u>ω or f or T</u>] you can find the other one.
  A memory trick: remember $\omega\mathbf{T} = \mathbf{2\pi}$, then use it to get $\omega = 2\pi/T$ or $T = 2\pi/\omega$.

- x-F-a equations are discussed in Section 8.1: $a = F/m = (-kx)/m = -(k/m)x = -\omega^2 x$.

- If your class uses the three "time equations" in the summary, fill the blank spaces with either x or y, cos or sin, + or –. {These choices depend on whether you use x-format or y-format equations.} If your class uses "simplified equations" with no phase angle, omit "ø".

Comments: 1) The amplitude "A" is only half of the overall range of motion; it is the center-to-side [not side-to-side] distance. 2) The units for ω and ø must be rads/s and rads. 3) You cannot use i-to-f "interval equations" like tvvax, FΔx=ΔKE or FΔt=Δp, because **F** and **a** are not constant during SHM.

Three Kinds of Equation-Letters

Some equation-letters vary throughout the SHM cycle. For example, PE is zero at the center point and maximum at the turning points.

Due to "conservation", TE is constant throughout an SHM cycle. Other constant letters (like k) are a property of objects in a particular SHM system, or (like T) only have meaning in the context of the entire cycle: it makes sense to ask "What is PE at the center point?" but not "What is the time-per-cycle at the center point?".

TE or k or T can change from one SHM situation to another, so they aren't the same as true constants like π (which is always 3.14159...) or G (the physical constant for gravity, as used in Section 5.3's $F_{gravity} = GMm/r^2$). It is useful to split equation letters into three categories: G is a *constant,* TE is a *constant-variable,* and PE is a *changing-variable.* {These are my own labels. Your textbook probably won't discuss these variable-categories even though they are, in my opinion, essential for understanding SHM.}

constant-variables:	TE	A	k, m	v_{max}, ω, f, T		ϕ
changing-variables:	PE, KE	x	F, a	v	t	θ

Think about each constant-variable and decide whether it is "conserved", is a property of the system, or is a cycle-property. {Answers are in the Section 8.90 flashcards.}

SHM equations can also be separated into "categories".

In the 5-sided $\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = PE + KE = TE = \frac{1}{2}kA^2 = \frac{1}{2}m v_{max}^2$, the 4-sided $\sqrt{k/m} = \omega = 2\pi f = 2\pi/T$, and $v_{max} = A\omega$, each equation-side is either constant throughout the SHM cycle, or only has meaning in the context of the entire cycle.

Other equations [$F = -kx$, $a = -\omega^2 x$, $\omega\sqrt{A^2 - x^2} = v$] describe relationships between a changing-variable (F, a, v) and the object's x-position. The three "time equations" describe how x, v and a (or y, v and a) vary with time.

As you study the "equation links" in the summary, your intuitive understanding and problem solving skill will improve as you master the connections between the **words** of a problem and the **physical situation** they describe and the **variables** that summarize this situation and the **equations** that relate these variables to each other.

Problems 8-A and 8-B show some common ways to use SHM equations.

PROBLEM 8-A

Part 1) For the SHM system of Section 8.1 (m = 7.29 kg, k = 12.5 N/m, released at .8 m), find F, a, v and KE when x = –.4 m. How long does it take the block to complete one full cycle of simple harmonic motion? What is the block's angular frequency and frequency? What is its maximum speed?

Part 2) To make T half this long, using the same spring, what "m" should the block have? To get this half-T using the original block, how large should "k" of the spring be? Does it make sense that these changes in m or k should decrease T?

Part 3) If you stretch the spring only .4 m before releasing it, what is the system's TE and v_{max}? Has T changed? Why? Find the block's average speed, $v_{average} \equiv$ [distance traveled during one cycle / time elapsed during one cycle].

SOLUTION 8-A

Part 1) Choose the appropriate equation, then substitute and solve.

$$F = -\,k\,x \qquad\qquad F = m\,a \qquad\qquad \sqrt{k/m}\;\sqrt{A^2 - x^2} = v$$

$$F = -(12.5)(-.4) \qquad (+5.0) = (7.29)\,a \qquad \sqrt{(12.5)/(7.29)}\;\sqrt{(.8)^2 - (-.4)^2} = v$$

$$F = +5.0\text{ N} \qquad +.69\text{ m/s}^2 = a \qquad \pm .91\text{ m/s} = v$$

F and a are both +, pointing →,
but **v** can point either → or ←, as shown by ± in the solution of ±.91 m/s.

An option: to find a, solve $a = -kx/m = -(12.5)(-.4)/(7.29) = +.69\ m/s^2$.

After you know $v = \pm .91$ m/s, $KE = \frac{1}{2} mv^2 = .5(7.29)(\pm.91)^2 = 3.0$ J.
Or substitute into "$\frac{1}{2} kx^2 + KE = \frac{1}{2} kA^2$" and solve for KE = 3.0 J.

For time-per-cycle: $\sqrt{k/m} = 2\pi/T$, so $T = 2\pi/\sqrt{k/m} = 2\pi/\sqrt{12.5/7.29} = 4.80$ s/cycle.

"angular frequency" is ω: $\omega = \sqrt{k/m} = \sqrt{12.5/7.29} = 1.31$ rads/s

"frequency" is f: $\omega = 2\pi f$, so $f = \omega/2\pi = 1.31/2\pi = .208$ cycles/s
Here is a check: f should equal 1/T. Does it? f = 1/(4.80) = .208, YES!

$v_{max} = \sqrt{k/m}\,A = \omega A = (1.31)(.8) = 1.05$ m/s, the same answer as in Section 8.1.

Part 2) To get $T = \frac{1}{2}(4.80\,s)$ when k = 12.5 N/m, solve $\frac{1}{2}(4.80) = 2\pi/\sqrt{12.5/m}$ for m = 1.82 kg. An algebra hint: one step in the "cross-multiply and square" process is $2.40^2 (\sqrt{12.5/m})^2 = (2\pi)^2$, so $2.40^2 (12.5)/4\pi^2 = m$. {To understand this, do it yourself!}

If m = 7.29 kg, solve $.5(4.80) = 2\pi/\sqrt{k/7.29}$ for k = 50.0 N/m.

Yes, these changes make sense. During each cycle the spring must accelerate the block from 0 to $-v_{max}$ to 0 to $+v_{max}$ to 0. This acceleration is easier if the block's mass is decreased or if the spring is made stronger. Either change makes the block "respond" more quickly and decreases its time-per-cycle.

To cut time by a factor of 2, m or k must change by a factor of 4 (being multiplied by 1/4 or 4/1, respectively*) because both m & k are inside the $\sqrt{\ }$. {* 7.29 changes to 1.82 kg, 1/4 as large; and 12.5 N/m² changes to 50.0 N/m², 4 times as large.}

Part 3) Just solve $TE = \frac{1}{2} kA^2 = .5(12.5)(.4)^2 = 1.0$ J. This is 1/4 as large as the original KE; less motion, less energy. $v_{max} = \sqrt{k/m}\,A = \sqrt{12.5/7.29}\,(.4) = .52$ m/s.

The T-formula doesn't contain A. k & m are the same as before, and so is T.

Why is T the same? The block doesn't move as far as before, but it isn't moving as fast, either; v_{max} has decreased from 1.05 m/s to .52 m/s. These factors [decreased distance and decreased speed] cancel each other, so T stays the same.

The block travels .4 m (its amplitude) during each quarter-cycle, so it travels 1.6 m during a full cycle: $v_{average} \equiv$ distance / time $= 1.6\,m / 4.80\,s = .33$ m/s.

Problem 8-# derives this formula: $\mathbf{v_{average} = (4/2\pi)\, v_{max} = .637\, v_{max}}$. Using this formula and $v_{max} = .52$ m/s (from above), $v_{average} = .637(.52) = .33$ m/s.

PROBLEM 8-B, to be used if your class is using the equations in Section 8.2.

Part 1) If your class uses "x-format equations" [like $x = A\cos(\omega t + \phi)$,...], write equations for how x, v & a vary with time, for the system of Section 8.1.

If your class uses "y-format equations" [like $y = A\sin(\omega t + \phi)$], write equations for how y, v & a vary with time, if Section 8.1's system is oscillating ↕ on a table and we define t ≡ 0 at an instant when the block is moving ↑ at the center point.

Part 2) To match this SHM motion, what constant speed should a ○-object have?

Part 3) If you use the x-format: When does Part 1's ↔ block first reach x = +.4m?
If you use the y-format: When does Part 1's ↕ block first reach y = +.4m?

Part 4) What is the block's position (x or y) and velocity when t = 2.80 seconds?

Part 5, to be used if your class studies "phase angles") For x-format users: if we define t ≡ 0 when the block is at the center point moving →, find the x-equation.

For y-format users: if the block is at the bottom turning-point when t ≡ 0, find the y-equation.

SOLUTION 8-B

Part 1) Use the "set of equations" you prefer and substitute values for A (= .8 m), ω (= $\sqrt{k/m} = \sqrt{12.5/7.29} = 1.31$ radians/s) and ϕ (which is 0 because at $t \equiv 0$ the O-object is at the $\theta \equiv 0$ point. (The question asks for an equation that shows how x (or y) varies with time, so we want an equation that includes x (or y) and t as letters, but substitutes numerical values for all other variables. A similar strategy is used to get equations for v & a as functions of time.)

$$x = (.8)\cos(1.31t + 0) \qquad y = (.8)\sin(1.31t + 0)$$
$$v = -(1.31)\ (.8)\sin(1.31t + 0) \qquad v = (1.31)\ (.8)\cos(1.31t + 0)$$
$$a = -(1.31)^2\ (.8)\cos(1.31t + 0) \qquad a = -(1.31)^2\ (.8)\sin(1.31t + 0)$$

Part 2) $v_{circle} = v_{max} = \omega A = (1.31)(.8) = 1.05$ m/s, the same as in Part 1 of 8-A.

Part 3) Substitute and solve. Here is a key step: if $.5 = \cos\theta$, then you solve for θ by taking the $\cos^{-1}$ of both equation sides: $\cos^{-1}(.5) = \cos^{-1}(\cos\theta) \equiv \theta$.

For these equations, ω and ϕ must be in radians, so put your calculator into "radian mode" with a button labeled "DRG" or..., then use the $\sin^{-1}$ or $\cos^{-1}$ button.

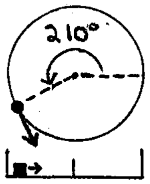
x	=	$A\cos(\omega\ t + \phi)$	y	=	$A\sin(\omega\ t + \phi)$
$(+.4)$	=	$(.8)\cos(1.31t + 0)$	$(+.4)$	=	$(.8)\sin(1.31t + 0)$
$.5$	=	$\cos(1.31t)$	$.5$	=	$\sin(1.31t)$
1.047 rads	=	$1.31t$	.5236 rads	=	$1.31t$
.80 s	=	t	.40 s	=	t

Part 4) x-format: $x = A\cos(\omega t + \phi) = (.8)\cos[(1.31)(2.80) + 0] = (.8)\cos(3.67 \text{ rads}) = -.691$ m, and $v = -\omega A\sin(\omega t + \phi) = -(1.31)(.8)\sin[(1.31)(2.80) + 0] = +.527$ m/s.

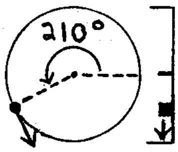
For y-format: $y = A\sin(\omega t + \phi) = (.8)\sin[(1.31)(3.00) + 0] = (.8)\sin(3.93 \text{ rads}) = -.402$ m, and $v = \omega A\cos(\omega t + \phi) = (1.31)(.8)\cos[(1.31)(3.00) + 0] = -.905$ m/s.

A full cycle takes 4.8 s; in 2.8 s, the O-object moves (2.8s/4.8s)(360°) = 210 degrees. Study the picture below and compare it with the ± signs in your answer above (either x is – and v is +, or y is – and v is –). Do the ± signs of the math and picture agree?

For x,



For y,



Part 5) There are two good ways to find ϕ: one method is "visual", the other is "mathematical". Both methods use A and ω, which are the same as in Part 1.

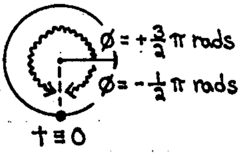
<u>VISUAL</u>: Look at the diagram in the middle column below. Do you see that • shows where the O-object is when $t \equiv 0$? •'s angular position can be described as 3/4 cycle ahead of the $\theta \equiv 0$ point (with $\phi = +.75$ cycle $= +270° = +3\pi/2$ rads $= +4.71$ rads) or as 1/4 cycle behind the $\theta \equiv 0$ point (with $\phi = -.25$ cycle $= -90° = -\pi/2$ rads $= -1.57$ rads). Either ϕ can be used in the equation, as shown below, but ϕ must be in radians.

REVIEWER: the typed columns should be spread further apart.

$x = .8\cos(1.31t + 4.71)$

or

$x = .8\cos(1.31t - 1.57)$



$y = .8\sin(1.31t + 4.71)$

or

$y = .8\sin(1.31t - 1.57)$

MATHEMATICAL: Substitute all known information and solve for ø. Because the O-object passes through each O-point (except the turning points) twice during one cycle, you must check ø to see if the ø-answer given by the calculator is correct. For x-format, the math solution of "ø = +1.57 rads" puts • on top of the O; we know this is wrong, so we use "visual logic" to find the correct ø of +4.71 rads. The y-format math solution gives "ø = –1.57 rads", which is correct. Step 2: after you have found ø, substitute it to get the x-equation or y-equation.

Step 1:

$$\begin{aligned} x &= A \cos[\ \omega\ t\ + ø] \\ 0 &= .8 \cos[1.31(0) + ø] \\ 0 &= \cos[\,ø\,] \\ +1.57 &= ø \end{aligned}$$

Step 2:

$$\begin{aligned} x &= .8 \cos(1.31\,t - 1.57) \\ &\quad \text{or} \\ x &= .8 \cos(1.31\,t + 4.71) \end{aligned}$$

Step 1:

$$\begin{aligned} y &= A \sin[\ \omega\ t\ + ø] \\ -.8 &= .8 \sin[1.31(0) + ø] \\ -1.0 &= \sin[\,ø\,] \\ -1.57 &= ø \end{aligned}$$

Step 2:

$$\begin{aligned} y &= .8 \sin(1.31\,t - 1.57) \\ &\quad \text{or} \\ y &= .8 \sin(1.31\,t + 4.71) \end{aligned}$$

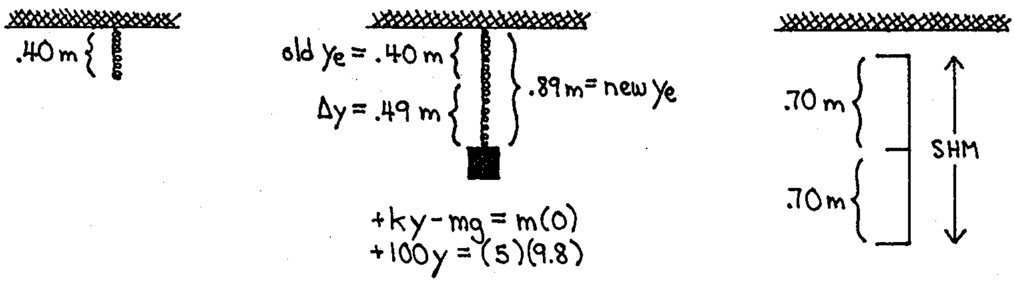
8.4 Vertical Spring-Oscillations and Pendulums

Sections 8.1 to 8.3 illustrate SHM with a horizontal spring-block system, but SHM also occurs in other situations. This section discusses two common SHM systems.

Vertical Spring-Block Oscillations

The first picture below shows a spring, k = 100 N/m, at its y_e of .40 m. The second picture shows what happens when a 5.0 kg block is attached: a new "equilibrium position" (where F = 0 because upward ky cancels downward mg) occurs when the spring is stretched "Δy = .49 m" past its original y_e to make a total length of .89 m.

If the block is lifted up .70 m and released, it oscillates up & down in SHM cycles centered on the "new y_e" at .89 m. (This is proved in Problem 8-#.) The overall range of SHM motion (.70 m above & below y_e) is shown in the third picture.



Notice the four "lengths" in these pictures: the old-y_e of .40m and Δy of .49 m (they aren't used for the new SHM), the new-y_e of .89m (it is ignored after it is defined as $y \equiv 0$) and the amplitude of .70 m that is used in equations like "$\frac{1}{2}kA^2 = \frac{1}{2}ky^2 + \frac{1}{2}mv^2$".

A vertical SHM system is analyzed in Problem 8-#.

Pendulums

Most textbooks explain pendulums well, so I'll just add a few comments.

If you watch a pendulum in action, you'll see that its back-and-forth motion is similar to SHM. At a "turning point" its speed is zero (as in SHM). It builds speed gradually as it moves downhill toward the center point where speed is maximum (as in SHM), then during its uphill movement away from the center it gradually loses speed (as in SHM) until it stops at the other turning point.

Most textbooks show that pendulum motion is mathematically analogous to SHM by deriving an a-equation like the one below, which is compared with the a-equation for spring-block SHM. Does it look like **k & m** are analogous to **g & L**, respectively? If this analogy is correct (and it is), the k & m in a spring-block's ω and T equations can be replaced by g & L to get analogous equations for a pendulum's ω and T:

For spring-block, $a = -(k/m)x$, $\omega = \sqrt{k/m}$, $T = 2\pi/\sqrt{k/m}$.

For a pendulum, $a \approx -(g/L)x$, $\omega \approx \sqrt{g/L}$, $T \approx 2\pi/\sqrt{g/L}$.

where **g** is the "acceleration of gravity", and **L** is the length of the pendulum string.

Notice that a pendulum's angular frequency and time-per-cycle do not depend on m.

The pendulum equations have ≈ instead of =, because their derivation uses an approximation. Problem 8-# examines this approximation and shows why it is usually acceptable for small motion amplitudes.

Your textbook or teacher may discuss other systems that have SHM: physical pendulums, torsional pendulums, atomic vibrations,...

==[mention damped & driven & damped/driven SHM? do as optional 8-#? as 8.94? or just modify the reference to it in 8.1's TE-conservation SHM requirement?]

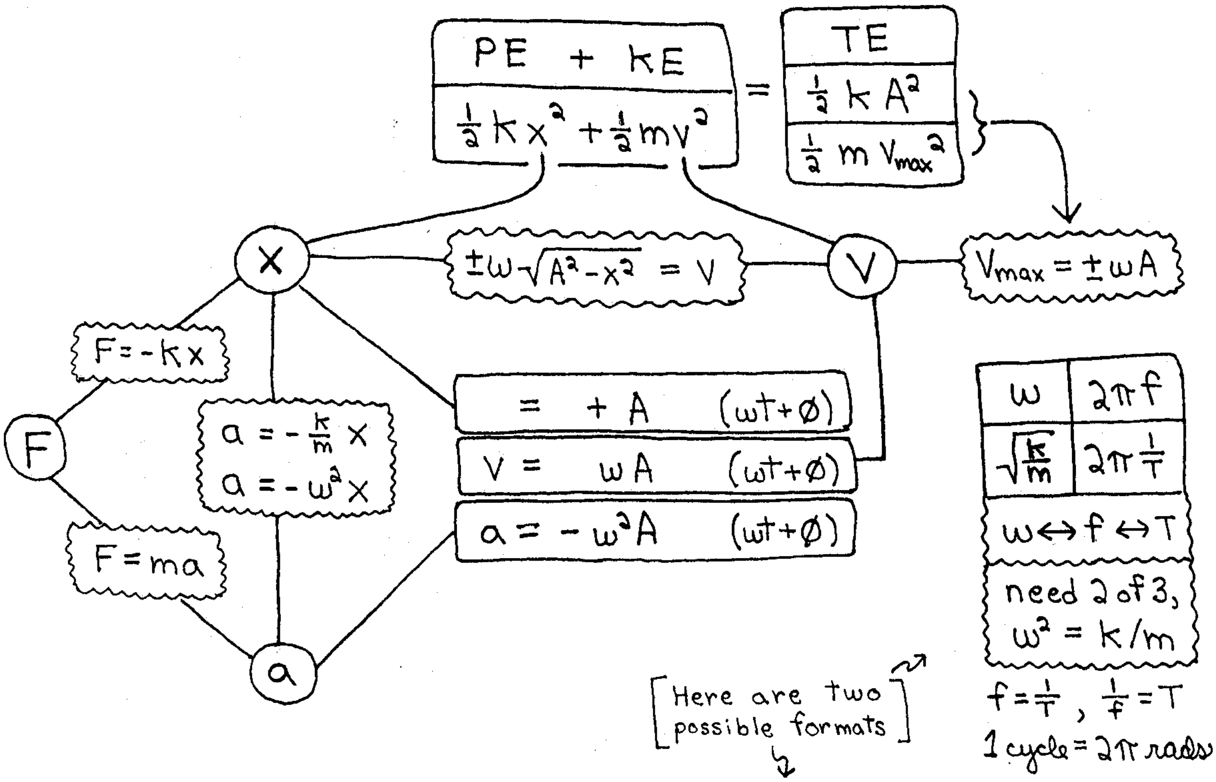
8.90 Memory-Improving Flash Cards

8.1	___ are maximum at the turning points. ___ are maximum at the center point. ___ are related to each other but not to v.	F, a, PE v, KE F and a
8.1	If ___, work is + which causes ___.	F & v have same direction, v & KE increase
8.1	___ direction & sign of ___ change at center point. ___ direction & ___ sign change at turning points.	F and a, W and x v, W
8.1	Energy: ___ and ___ change, but if ___ the ___ (which equals ___) is constant.	PE, KE, $F_{friction}$ (etc.) is zero, TE, $\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$
8.1	SHM occurs if there is ___.	only "–kx" restoring force (points toward x_e)
8.2	SHM is imitated by ___.	the x (or y) component of circular motion
8.2	v_{max} = ___, but the instantaneous v = ___.	v_{circle}, the x (or y) component of v_{circle}
8.2	$\omega t + \phi$, derived from ___, gives ___, has ___ units.	$\Delta\theta + \theta_i$, O-object's angular position, rads
8.2	ϕ, the ___, shows ___ with respect to ___ point.	phase angle, the θ-position at $t \equiv 0$, $\theta \equiv 0$

8.3	__ can be interconverted. You need 2 of __ .	ω , f , t ; ω-f-t , k , m
8.3	SHM amplitude "A" is the __ , which is __ For SHM you cannot use __ because __ .	center-to-side distance, $\frac{1}{2}$ of overall motion i-to-f equations (tvvax,...), F-and-a vary
8.3	An equation-letter can be constant because what it represents is __ , __ , __ or __ .	conserved (TE), object property (k m), cycle property (v_{max} ω f T A ø), true (π G)
8.3	In some equations, the sides are __ or __ . Other equations show the relationships of __ .	constant, "whole-cycle characteristics" changing-variables (F x v a t)
8.3	To get a short time-per-cycle, use __ and __ .	strong spring, small mass
8.3	__ depend on amplitude but __ don't.	{ TE, PE_{max}, KE_{max} , v_{max} }, { ω, f, T }
8.4	Vertical spring-blocks: use only __ , not __ .	new-A , old-y_e or Δy or new-y_e
8.4	In pendulum equations, __ are replaced by __ . A pendulum's ω & T depend on __ but not __ .	k and m, g and L g and L, m

Chapter 8 Summary

This summary is explained in Section 8.3.



$\omega \leftrightarrow f \leftrightarrow T$

2 of 3: $\omega^2 = k/m$

ω	$2\pi f$
$\sqrt{\frac{k}{m}}$	$2\pi\frac{1}{T}$

$f = \frac{1}{T}$, $\frac{1}{f} = T$

1 cycle = 2π rads

constant-variables:	TE	A	k, m	v_{max}, ω, f, T	ø
changing-variables:	PE, KE	x	F, a	v t	θ

$x_{max} = -A$	$x = 0$	$x_{max} = +A$
$F_{max} = -k(-A)$	$F = 0$	$F_{max} = -k(+A)$
$a_{max} = -k(-A)/m$	$a = 0$	$a_{max} = -k(+A)/m$
$PE_{max} = \frac{1}{2}kA^2$	$PE = 0$	$PE_{max} = \frac{1}{2}kA^2$
$KE = 0$	$KE_{max} = \frac{1}{2}m(\omega A)^2$	$KE = 0$
$v = 0$	$v_{max} = \pm\omega A$	$v = 0$