Chapter 1

Math for Physics:

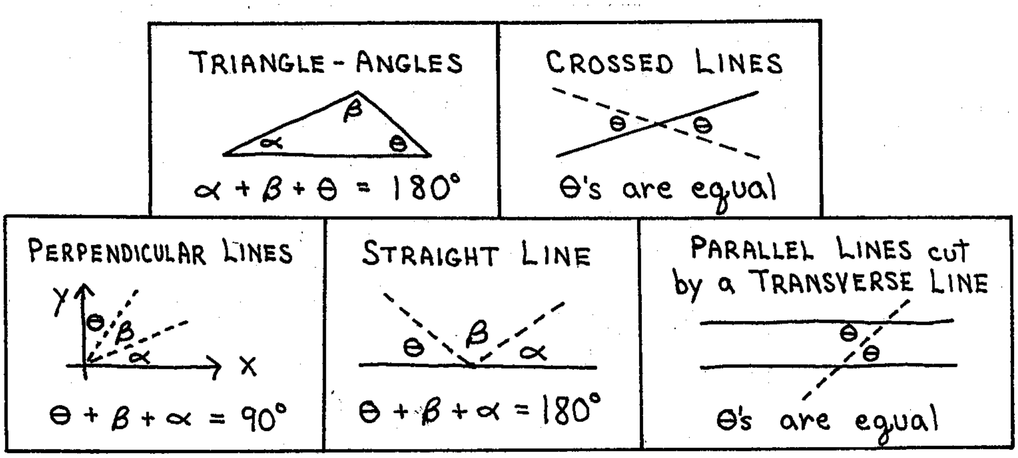
Geometry, Trigonometry, Metric Units, Conversion Factors

Chapter 1 is not really about "physics". It is a collection of math tools you'll need for physics: reviews of Geometry & Trigonometry (in Sections 1.1 & 1.2), and discussions of Metric System Prefixes & Conversion Factors (in 1.3 & 1.4). You can read these sections [one at a time, or all at once] whenever you think it will be helpful.

In addition, **Algebra for Physics** is summarized in Chapter 18, and **Calculus for Physics** (optional) is in Chapter 19.

1.1 Geometry for Physics

Here are the main geometry tools you'll need for physics:

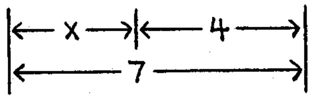


An easy way to remember these tools is $\underline{\Delta XYZ}$: Δ is a triangle, **X** is crossed lines, Y represents the **90°** (˅) and **180°** (ı) rules, and **Z** shows parallel lines.

Many diagrams can be analyzed only if you **DRAW EXTRA LINES**. How do you know what lines to draw? Often, useful extra lines are *extensions* of existing lines, *parallel* to existing lines, *perpendicular* to existing lines, or are needed to *form a right triangle* that can be analyzed using the trigonometry methods of Section 1.3. Be creative and experiment with extra lines. If you're not sure whether you should draw a line, go ahead and draw it; if it isn't needed, you can erase it or ignore it.

Trigonometry and **SIMILAR TRIANGLES** are discussed in Section 1.2.

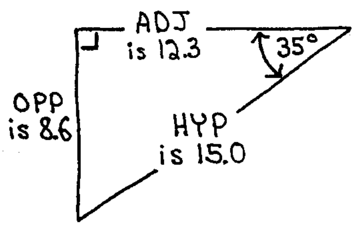
TOTAL = SUM OF PARTS logic can be used to find "x = 3" for this diagram:



1.2 Trigonometry for Physics

This section shows you how to master essential problem solving tools. You'll use these tools often, so it's worth investing a few extra minutes to learn them well.

In the right triangle below, the longest side (15.0) is opposite the 90° angle; this side is the *hypotenuse,* abbreviated HYP. One of the two shorter sides (8.6, OPP) is *opposite* the 35° angle, while the other (12.3, ADJ) is *adjacent* to 35°.
The 3 basic trigonometry functions (sine, cosine, tangent) are side/side ratios, the length of one side divided by the length of another side:



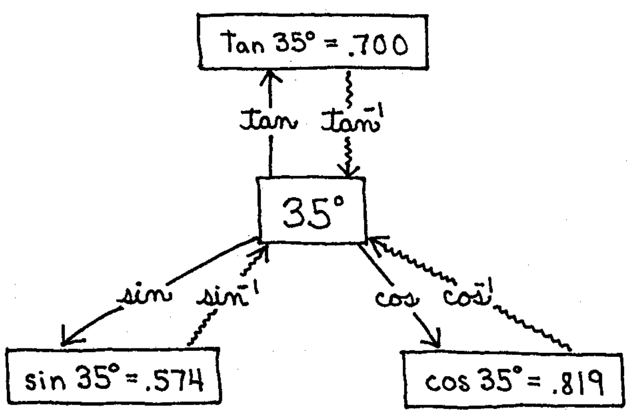
$$\sin 35° \equiv \frac{\text{OPP}}{\text{HYP}} \left(= \frac{8.6}{15.0} = .57 \right)$$

$$\cos 35° \equiv \frac{\text{ADJ}}{\text{HYP}} \left(= \frac{12.3}{15.0} = .82 \right)$$

$$\tan 35° \equiv \frac{\text{OPP}}{\text{ADJ}} \left(= \frac{8.6}{12.3} = .70 \right)$$

These side/side ratios are stored in the memory of your calculator. If you know the angle is 35° and want to find the ratio of OPP/HYP, use the "sin" button: punch " 35 sin ". But if you know this ratio and want to find the angle, use the "$\sin^{-1}$" button: punch " 8.6 ÷ 15.0 = $\sin^{-1}$ ". Practice with your calculator until you know how to use all 6 trig buttons; the diagram below will help you check your answers.

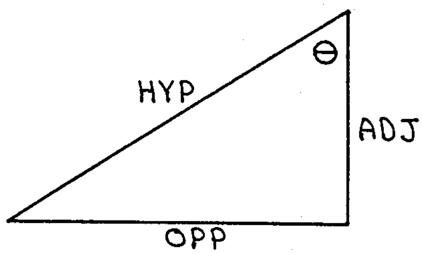
Your calculator has a button (it is often labeled "DRG") that determines whether an angle has units of Degrees, Radians or Grads. If it's in "degree mode", which probably occurs automatically when the calculator is turned on, and you punch "35 sin", it interprets "35" to be 35 degrees (not 35 radians or 35 grads) and gives ".5737..." as the answer. { *Radian* units, which are useful for understanding rotational motion, won't be discussed until they're needed — in Section 5.4. }



Either non-90° angle can be used for trig functions, if adjacent & opposite are defined correctly. The triangle we used earlier has angles of 90°, 35° and (because its angles add to 180°) 55°. Do you see why cos 35° = sin 55° ? Look at the 12.3-side:

12.3 is adjacent to 35°, so 12.3/15.0 = ADJ/HYP (≡ cos 35°) = .82 ,
but 12.3 is opposite 55°, so 12.3/15.0 = OPP/HYP (≡ sin 55°) = .82 .

There are 4 right-triangle variables: ADJ , OPP , HYP , and θ, sin θ, cos θ, tan θ. { θ, sin θ, cos θ and tan θ are really 1 variable, not 4, because if you know one of them you can find the others. } As shown below, there are also 4 equations: 3 trigonometry definitions plus the Pythagorean Theorem, and each equation contains 3 of the 4 variables.



$$\sin\theta \equiv \frac{OPP}{HYP}$$

$$\cos\theta \equiv \frac{ADJ}{HYP}$$

$$\tan\theta \equiv \frac{OPP}{ADJ}$$

$$ADJ^2 + OPP^2 = HYP^2$$

This useful principle will help you plan "the next step" in your problem solution:

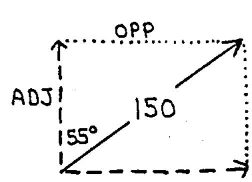
**If you know any 2 of the 4 right-triangle variables,
you can easily find the other 2.**

For example, if you know that θ = 35° and ADJ = 12.3, the equations with θ and ADJ { cos θ = ADJ/HYP , tan θ = OPP/ADJ } can be solved for HYP and OPP, respectively.

SPLITTING A VECTOR INTO COMPONENTS

{ I'm assuming your textbook has discussed vectors, which are quantities that have both **magnitude** and **direction.** }

If a plane flies 150 miles/hour in a direction 55° to the eastward side of due north (abbreviated 55° E of N), find its speed in the eastward and northward directions.



Known: HYP and θ
Unknown: ADJ and OPP

In the drawing above, the velocity vector is a solid arrow-line, and the *components* are dashed arrow-lines. The "extra" dotted lines are parallel to (and the same length as) the components. These ···· lines help form right triangles that can be analyzed.

Each equation that contains both "knowns" can be solved for an unknown:

$$\cos\theta \equiv \frac{ADJ}{HYP} \qquad\qquad \sin\theta \equiv \frac{OPP}{HYP}$$

$$HYP\cos\theta = ADJ \qquad\qquad HYP\sin\theta = OPP$$

$$(150)\cos 55° = ADJ \qquad\qquad (150)\sin 55° = OPP$$

$$86 = ADJ \qquad\qquad 123 = OPP$$

The HYP is always the longest side of a right-triangle; ADJ and OPP are always smaller. The cosθ and sinθ ratios are "cut-down multiplying factors" (like cos 55° = .574, and sin 55° = .819) that tell you how much smaller the ADJ and OPP sides are:

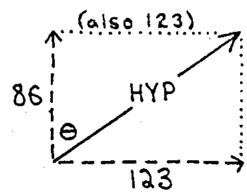
large	**(cut-down factor)**	**=**	**small**	**large**	**(cut-down factor)**	**=**	**small**
HYP	**(cos θ)**	**=**	**ADJ**	**HYP**	**(sin θ)**	**=**	**OPP**

A memory trick: the first letters of ADJ & cos (A,c) are early in the alphabet, while OPP & sin (O,s) come later.

Some textbooks connect cos & sin with x & y, respectively, implying that a vector's x-component is HYP(cosθ), and its y-component is HYP(sinθ). Sometimes, as in the example above, these formulas give the wrong answer. Is this something you need? I strongly recommend that you connect cos & sin with ADJ & OPP (not x & y), so you will always get the correct answer.

RECONSTRUCTING A VECTOR FROM ITS COMPONENTS

If you know an airplane has a northward speed of 86 mi/hr and an eastward speed of 123 mi/hr, can you find the magnitude and direction of its velocity vector?



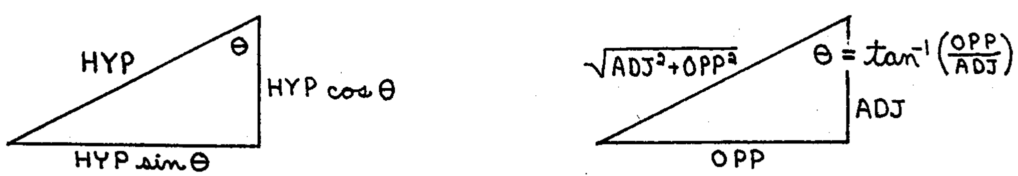
Known: ADJ and OPP
Unknown: HYP and θ

As with "splitting", the 2 knowns appear in equations that can be solved.

$$\tan\theta \equiv \frac{OPP}{ADJ} \qquad ADJ^2 + OPP^2 = HYP^2$$
$$\theta = \tan^{-1}\frac{OPP}{ADJ} \qquad \sqrt{ADJ^2 + OPP^2} = HYP$$
$$\theta = \tan^{-1}\frac{123}{86} \qquad \sqrt{86^2 + 123^2} = HYP$$
$$\theta = 55° \qquad 150 = HYP$$

Another memory trick: to remember "Tanθ = OPP/ADJ", think TOAD.

Here is a visual summary of ***vector splitting*** and ***vector reconstruction:***



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SIMILAR TRIANGLES: If two triangles have the same 3 angles, they have the same "shape" and the <u>same side/side ratios</u>. One of these triangles will be a "scaled up" version of the other triangle. For example, each side in the airplane-velocity triangle (86, 123, 150) is 10 times larger than the corresponding side of the earlier triangle (8.6, 12.3, 15.0) because both triangles have a 35°-55°-90° shape. Do you see why each triangle will have the same side/side ratios? For example, the tan 55° ratios (they are 123/86 and 12.3/8.6, respectively) are both 1.43.

SIMILAR TRIANGLES: If two triangles have the same 3 angles, they have the same "shape" and the same side/side ratios. { One of these triangles will be a "scaled up" version of the other triangle. For example, each side in the airplane-velocity triangle (86, 123, 150) is 10 times larger than the corresponding side of the earlier triangle (8.6, 12.3, 15.0) because both triangles have a 35°-55°-90° shape. Do you see why side/side ratios (like the tan 55° ratios of 123/86 and 12.3/8.6, respectively) are the same for both triangles? }

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Here are two trigonometry relationships you can use, and one you should avoid.

Two correct formulas are: $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$, $(\sin\theta)^2 + (\cos\theta)^2 = 1$.

The relationship below looks like it might be true, using the logic that "if 3(a + b) = 3a + 3b, then we can do the same thing with trig functions like "cos". But with trig functions, this math-technique is wrong, as shown by the ≠ sign:

$$\cos(a + b) \neq \cos a + \cos b$$

In case you ever need them, the correct angle-addition formulas are summarized in Section 18.9#.

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ADDING VELOCITIES (using "graphical" and "splitting into components" methods of analysis) is discussed in Problem 1-7.

RELATIVE MOTION: A common example of vector addition is the behavior of a boat in river current, or an airplane in wind. Such problems are studied in Section 2.10.

VECTOR MULTIPLICATION: Dot-Products and Cross-Products (optional techniques that are used in most calculus-based physics courses) are discussed in Section 19.11.

1.3 Two Interpretations of Metric Prefixes

The *metric system,* which is used in most countries, measures length in **m**eters, mass in **g**rams, and time in **s**econds. These units are abbreviated m, g and s.

PREFIXES: To make smaller or larger units, the metric system uses a combination of prefix & base-unit, as shown in the table below. Any prefix (M, k, c, ...) can be used with any base-unit (m, g, s, ...).

{ *Scientific notation,* like 3.4 x 10^3 being used to represent 3400, is discussed in Section 18.6. }

PREFIX & MEANING			examples	prefix names
M	means	x 10^6	5.7 Ms = 5.7 x 10^6 s	Ms is megasecond
k	means	x 10^3	5.7 kg = 5.7 x 10^3 g	kg is kilogram
c	means	x 10^{-2}	5.7 cm = 5.7 x 10^{-2} m	cm is centimeter
m	means	x 10^{-3}	5.7 mg = 5.7 x 10^{-3} g	mg is milligram
μ	means	x 10^{-6}	5.7 μm = 5.7 x 10^{-6} m	μm is micrometer
n	means	x 10^{-9}	5.7 nm = 5.7 x 10^{-9} m	nm is nanometer
p	means	x 10^{-12}	5.7 ps = 5.7 x 10^{-12} s	ps is picosecond

There are two useful ways to interpret combined prefix/base-unit combinations.

In the table above, the prefix is considered to be part of the number, as illustrated with this underlining: 5.7 cm = 5.7 x 10^{-2} m. For physics problems this is usually the best way to handle prefixes.

But you can also keep the c & m together to form a "cm" unit that is 1/100 as large as a meter: in 5.7 cm , there are 5.7 of these cm-size length units.

1.4 Conversion Factors

A *Conversion Factor* (CF) converts a thing from one unit to another. For example,

$$\underbrace{48 \text{ inches}}_{\text{old unit}} \; \underbrace{\frac{1 \text{ foot}}{12 \text{ inches}}}_{\text{ONE}} = \underbrace{4 \text{ feet}}_{\text{new unit}}$$

1 foot/12 inches = 1 because what is on top (1 foot) is the same as what's on the bottom (12 inches). { This is the same reason that 4/4 = 1, or 12 inches/12 inches = 1. }
When we multiply 48 inches by [1 foot/12 inches], we multiply it by 1. This keeps the 48 inch length the same size; its length-units just change from inches to feet.

A correct CF-Equation is needed, to show the proper relationship between units. This equation must use the correct number ratio ("14 inches = 1 foot" is wrong), and it must match the numbers and units correctly ("12 feet = 1 inch" is wrong).
Proper matching uses one of these types of logic: "1 small-size unit = a fraction of a large-size unit", or "many small-size units = 1 large-size unit". Two examples are:

1 cm = 1×10^{-2} m	100 cm = 1 m
(1 small unit = fraction of large unit)	(many small units = 1 large unit)

The first CF-equation is simple; just use the prefix-meaning of "c" [multiply by 10^{-2}] that is described in Section 1.3. The second CF-equation, after both sides of the first CF-equation are multiplied by 100, is more intuitive because it uses whole numbers; if you look at a meter-stick, it is easy to see that 1 meter contains 100 cm.

Correct Use of CF-Fractions: If a CF-fraction is flipped upside down it still equals 1, because the fraction's top and bottom are still equal. Therefore, one CF-equation produces two CF-fractions. Each CF-fraction is useful in a different situation:

To convert 48 inches to __ feet,

$$48 \cancel{\text{in}} \; \frac{1 \text{ ft}}{12 \cancel{\text{in}}} = 4 \text{ ft}$$

To convert 4 feet to __ inches,

$$4 \cancel{\text{ft}} \; \frac{12 \text{ in}}{1 \cancel{\text{ft}}} = 48 \text{ in}$$

If you choose the wrong CF, units won't cancel and you'll get a mess:

$$\cancel{48 \text{ in} \; \frac{12 \text{ in}}{1 \text{ ft}} = 576 \frac{\text{in}^2}{\text{ft}}}$$

~~square inches per foot ??~~
WRONG!

$$\cancel{4 \text{ ft} \; \frac{1 \text{ ft}}{12 \text{ in}} = .333 \frac{\text{ft}^2}{\text{in}}}$$

~~square feet per inch ??~~
WRONG!

To do a conversion, just multiply by the CF-fractions that are needed to transform the "old units" into the "new units" you want. When 100 km/hour is converted to m/s units, notice that two conversions (km ⇒ m, hour ⇒ s) are needed:

$$\frac{100 \text{ km}}{1 \text{ hour}} \; \frac{10^3 \text{ m}}{1 \text{ km}} \; \frac{1 \text{ hour}}{60 \text{ minutes}} \; \frac{1 \text{ minute}}{60 \text{ s}} = 27.8 \frac{\text{m}}{\text{s}}$$

For some conversions you can use a "substitution shortcut". In this example, "k" and "1 hour" can be replaced by something that is equivalent to them:

$$\frac{100\text{ km}}{1\text{ hour}} \Rightarrow \begin{matrix}\text{"k" means "x }10^3\text{"}\\ \text{and 1 hour = 3600 s}\end{matrix} \Rightarrow \frac{100 \times 10^3\text{ m}}{3600\text{ s}} = 27.8\frac{\text{m}}{\text{s}}$$

To practice conversions, use 1 minute = 60 s, 1 hour = 60 minutes, 1 ft = 12 in, 1 m = 39.37 in and 1 mile = 5280 ft to convert 60 miles/hour to other speed units:

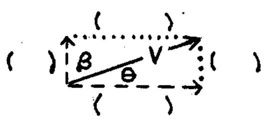
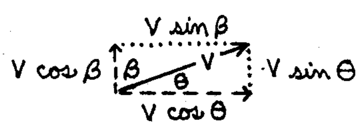
60 mi/hr = 1 mi/minute = 88 ft/s = 26.82 m/s = 96.56 km/hr

1.90 Flash-Card Review for Chapter 1

To use a physics tool, you must remember it. Two good ways to improve your memory are **organization** and **active practice**.

The Chapter Summaries, starting after Chapter 20, provide logical organization. And these end-of-chapter "flash card reviews" are a quick way to do active practice: just cover the answer-column with a card, look at the question and answer it, then slide the card down to check. (Don't worry if you can't get the answer the first time you see a question. This is not an "exam"; it is a way to make many quick reviews.) Repeat the list until you can easily remember all of the tools; it won't take long! If necessary, go back to the appropriate section (listed in the far-left column) for review. If you want, add question-answer pairs of your own.

Section 20.1, "Improving Your Memory", discusses the principles of organization and practice, and also specific techniques: summary notes, flash cards, and more.

	QUESTIONS	ANSWERS
1.1	What are 8 useful geometry tools?	ΔXYZ: Δ, X, 90°, 180°, Z. Draw extra lines, similar-Δ ratios, total = sum of parts.
1.2	A right-triangle's __ variables are ___. If you know __, you can __.	4; HYP, ADJ, OPP, θ/sinθ/cosθ/tanθ any 2 of these, find the other 2
1.2	A vector has __.	magnitude and direction
	↗ can be ___.	split into → and ↑ components
	The SPLITTING formulas are __.	ADJ = HYP cosθ, OPP = HYP sinθ
	The RECONSTRUCTING formulas are __.	HYP = $\sqrt{ADJ^2 + OPP^2}$, θ = $\tan^{-1}$(OPP/ADJ)
1.2	The ( )'s are __. 	
	cos 70° = sin__, so __.	20°, either non-90° angle can be "θ"
1.2	Two "extra" trig-equations are __ and __.	$\cos^2\theta + \sin^2\theta = 1$, sinθ/cosθ = tanθ
1.3	A unit-prefix is a __. 4 km is __ or __.	multiplier; 4 km (4 x 10^3 m), 4 km
1.4	A CF-equation must have correct __ and __. A CF-equation gives __. A CF-fraction = __.	number ratio, number/unit matching two CF-fractions, 1
1.4	Using a __ shortcut, 90 km/hour is __.	substitution, (90 x 10^3 m)/(3600 s)

Chapter 1 Summary

GEOMETRY FOR PHYSICS: ΔXYZ.

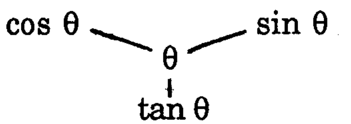
DRAW "EXTRA LINES": extensions of existing lines, parallel to existing lines, perpendicular to existing lines, or to form right-triangles.

TOTAL = SUM OF PARTS: If |← x →|← 3 →| over |← 7 →|, then x = 4.

SIMILAR TRIANGLES: If two triangles have the same angles (and thus the same shape), they have the same side/side ratios.

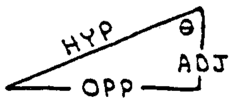
VECTORS have both magnitude and direction.

If you know 1 of [θ, sin θ, cos θ, tan θ], you can find the others. Calculator: θ —("cos")→ ADJ/HYP ratio, and ADJ/HYP ratio —("$\cos^{-1}$")→ θ angle.



$\cos\theta = \sin(90° - \theta)$, and $\sin\theta = \cos(90° - \theta)$

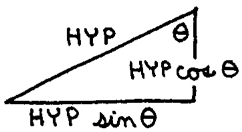
If you know 2 of the 4 right-triangle variables [ADJ, OPP, HYP, θ/sinθ/cosθ/tanθ] you can find the other 2, by solving the equations that contain the knowns.



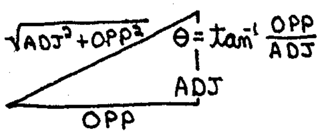
ADJ & OPP are defined "with respect to" θ.

$$\left.\begin{aligned} \sin\theta &\equiv \frac{OPP}{HYP} \\ \cos\theta &\equiv \frac{ADJ}{HYP} \\ \tan\theta &\equiv \frac{OPP}{ADJ} \\ ADJ^2 + OPP^2 &= HYP^2 \end{aligned}\right\}$$

SPLITTING



RECONSTRUCTING



TRUE: $\sin\theta / \cos\theta = \tan\theta$

$\sin^2\theta + \cos^2\theta = 1$

FALSE: $\cos(a + b) \neq \cos a + \cos b$

$\cos(a\,b) \neq (\cos a)(\cos b)$

To add vectors, A) Draw the vectors head-to-tail, like a "relay race",
B) choose axes and split each vector into x & y components,
C) add x-components to get x_{total}, add y-components to get y_{total},
D) use these x & y components to "reconstruct" the *resultant vector*.

There is a difference between *components* (which are always ⊥) and *originals*.

OPTIONAL: Vector multiplication (dot product & cross product) is explained in Sections 18.51-18.53.