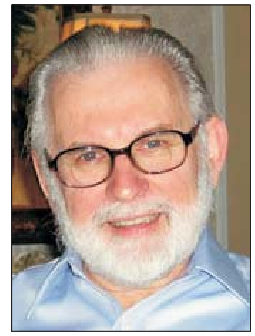
Article

Biblical Longevities: Empirical Data or Fabricated Numbers?

Walter Makous



Walter Makous

Whether the biblical longevities have biological or cultural significance depends on whether they represent actual longevities or are fabricated. As the properties of fabricated numbers differ from those of natural phenomena, this paper examines these properties, particularly in light of those differences. The results show (1) an exponential decline toward contemporary longevities, following approximate constancy at nearly 1,000 years; (2) a Gaussian distribution of deviations from this relationship; (3) no reliable deviations from statistical independence; (4) reliable differences from the properties of fabricated numbers, and instead adherence to Benford's law; and (5) rounding. Results 1 and 4 are difficult to reconcile with fabrication. Result 5 accounts for the inability to reconcile biblical chronologies exactly. Historical records and archeological data appear to conflict with such longevities, but their quality and quantity are insufficient to completely exclude them, perhaps during a brief period in a small subpopulation.

The Hebrew Scriptures are replete with specific numbers, detailing, for example, the patriarchs' ages when their first sons were born, how long they lived afterwards, the ages at which the reigns of kings and judges began, how long they reigned, and at what ages they died. To understand the significance of these numbers, one must know their source. It is possible that they represent just what they purport to represent: natural data on actual ages and longevities; or else they may be artificial, made up to serve a presently unknown purpose. If the numbers are natural, they carry information on ancient history and biological phenomena; if they are artificial, they may reveal something about the people who produced them, such as the numerological as opposed to the numerical significance of numbers in their culture and religion.[1] The purpose of this work is to examine the properties of the numbers, particularly those properties that might shed light on which of these alternatives is more likely to be true.

Numbers have properties not shared by other symbols, such as words, and the properties of numbers that represent natural phenomena tend to differ from those of fabricated numbers. For example, numbers derived from natural phenomena follow Benford's law (described below),[2] they represent systematic processes perturbed by random error, these perturbations tend to be mutually independent,[3] and the distribution of these numbers about their mean values tends

Walter Makous *is Professor of Brain and Cognitive Sciences and of Visual Science at the University of Rochester, where he formerly held the position of Director of the Center for Visual Science. His PhD in psychology is from Brown University. He has had positions in IBM Research (Yorktown Heights, NY), and in the Departments of Psychology and of Physiology and Biophysics at the University of Washington (Seattle). He is a fellow of the American Association for the Advancement of Science and of the Optical Society of America, and has published some seventy peer-reviewed papers, primarily on the functional organization of the visual system. He is a member of the First Bible Baptist Church of Rochester and has recently married and joined his wife, Robbie, in Tallahassee, FL. He can be contacted at walt@cvs.rochester.edu.*

to be Gaussian. The properties of artificial numbers depend on the purposes, knowledge, and skill of those who generate them. However, certain properties are rare in artificial numbers. For example, artificial numbers do not have the properties of randomness unless those generating the numbers (1) understand randomness, (2) desire to make the numbers appear to be random, and (3) benefit either from extensive training[4] or the use of mechanical aids like dice. The following analysis begins with a description of the properties of the numbers and then goes into a discussion of what this tells us about their origin, whether natural or artificial.

Methods and Results

The Data

Method

To find longevities, the entire contents of the NIV were searched electronically (at www.biblegateway.com) for the following words: *died, slept, rested, years, old,* and *age*. In addition, the books, Genesis through 2 Chronicles, were scanned visually for relevant references.

This analysis is based only on the ages of the forty-one males said to have died of natural causes. For example, the NIV uses the word *died* with reference to all those whose ages are given in Genesis except Enoch, who instead "… was no more, because God took him away" (Gen. 5:24, NIV). His age at that time is 12 standard deviations shorter than the longevities of the rest of the first ten named and by any criterion qualifies as an outlier; therefore, although it is plotted in figure 1, it was not used for the computations. The analysis also excludes those who were killed or died in battle, and Ahaziah, who died from a fall (2 Kings 1:2–17). The analysis includes deaths from illness, even when said to be imposed by God, as in the cases of Jehoram (2 Chron. 21:18) and Uzziah (2 Kings 15:5–7; 2 Chron. 26:20–3). A supplementary table containing a list of the judges and kings, their ages at the beginning of their reigns, the duration of their reigns, and the causes of their deaths, is available from the author on request.

Genesis typically gives the ages from the patriarchal era either as the age at death or as two numbers: the age at which a man had his first son, and the number of years he survived after that. Samuel, Kings, and Chronicles typically give the ages at which a man's reign began and the number of years from then until his death. The analysis excludes those whose ages, when they were deposed, are given, but whose deaths were not clearly at the same time (Jehoahaz of Judah [Shallum], Jehoiakim, Jehoiachin, and Zedekiah).

Results

These resulting ages are listed in table 1 (p. 119); those stated directly in the text are in bold print, and those inferred by addition or subtraction are in plain print.

Systematic Properties: Ordinal

Method

Assigning dates to the events listed in table 1 is problematic, and even establishing the dates of the reigns of the kings and judges is complicated by ambiguities and mutual contradictions in the text and by the fact that the list is not exhaustive.[5] None of the attempts to derive a chronology of these events has been completely successful (reviewed by Galil).[6] Thiele's chronology reconciles all the data on the reigns of the kings and judges,[7] but only by making implausible and unconvincing assumptions. The most recent attempt, by Galil,[8] accounts for only about 90% of these data. However, the minimum assumption, on which there is universal agreement, is that the list is in temporal sequence. Therefore one can plot longevity as a function of relative position in time, as in figure 1 below.

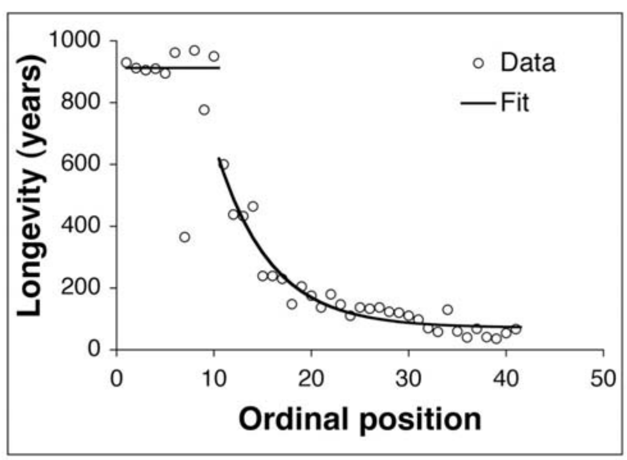


Figure 1. Longevities from different generations, arranged in temporal sequence, from Adam to Manasseh. The points are from table 1, and the curves are maximum likelihood fits of a horizontal line and the exponential decay function specified in the text.

Table 1. Longevities from the Hebrew Bible. “Before” is the age at the birth of the first son (for numbers 1–22) or the age at which the individual’s reign began (numbers 32–41). “After” is the additional number of years lived. “Total” is the total number of years lived.

	No.	Before	After	Total	Source
Adam	1	**130**	**800**	**930**	Gen. 5:3–5
Seth	2	**105**	**807**	**912**	Gen. 5:6–8
Enos	3	**90**	**815**	**905**	Gen. 5:9–11
Cainan	4	**70**	**840**	**910**	Gen. 5:12–4
Mahalaleel	5	**65**	**830**	**895**	Gen. 5:15–7
Jared	6	**162**	**800**	**962**	Gen. 5:18–20
Enoch	7	**65**	**300**	**365**	Gen. 5:21–3
Methuselah	8	**187**	**782**	**969**	Gen. 5:25–7
Lamech	9	**182**	**595**	**777**	Gen. 5:28; 5:30–1
Noah	10	**500**	**450**	**950**	Gen. 5:32; 7:6; 9:28–9
Shem	11	**100**	**500**	600	Gen. 11:10–1
Arphaxad	12	**35**	**403**	438	Gen. 11:12–3
Salah	13	**30**	**403**	433	Gen. 11:14–5
Eber	14	**34**	**430**	464	Gen. 11:16–7
Peleg	15	**30**	**209**	239	Gen. 11:18–9
Reu	16	**32**	**207**	239	Gen. 11:20–1
Serug	17	**30**	**200**	230	Gen. 11:22–3
Nahor	18	**29**	**119**	148	Gen. 11:24–5
Terah	19	**70**	135	**205**	Gen. 11:26–32
Abram	20	**86**	89	**175**	Gen. 16:16; 25:7
Ishmael	21			**137**	Gen. 25:17
Isaac	22	**60**	120	**180**	Gen. 25:26; 35:28
Jacob	23			**147**	Gen. 47:28
Joseph	24			**110**	Gen. 50:22,26
Levi	25			**137**	Exod. 6:16
Kohath	26			**133**	Exod. 6:18
Amram	27			**137**	Exod. 6:20
Aaron	28			**123**	Num. 33:39
Moses	29			**120**	Deut. 34:7
Joshua	30			**110**	Josh. 24:29; Judg. 2:8
Eli	31			**98**	1 Sam. 4:15–18
David	32	**37**	**33**	**70**	2 Sam. 5:5; 1 Kings 2:11; 1 Chron. 29:27
Rehoboam	33	**41**	**17**	58	1 Kings 14:21–31; 2 Chron. 12:13,16
Jehoiada	34			**130**	2 Chron. 24:15
Jehoshaphat	35	**35**	**25**	60	1 Kings 22:42–50; 2 Chron. 20:31; 21:1
Jehorem	36	**32**	**8**	40	2 Kings 8:16–17; 2 Chron. 21:5, 20
Uzziah	37	**16**	**52**	68	2 Kings 14:21; 15:2; 2 Chron. 26:1, 3; 2 Chron. 26:21
Jotham	38	**25**	**16**	41	2 Kings 15:33; 2 Chron. 27:1, 8, 9
Ahaz	39	**20**	**16**	36	2 Kings 16:2; 2 Chron. 28:1, 27
Hezekiah	40	**25**	**29**	54	2 Kings 18:1, 2; 20:21; 2 Chron. 29:1; 32:33
Manasseh	41	**12**	**55**	67	2 Kings 21:1; 2 Chron. 33:1, 20

Results and discussion

Even without knowing the time span between points, one can draw three conclusions from these longevities: (1) they were approximately constant through the 10th number (Noah) with a mean of 912 years; (2) they decrease after the 10th number; and (3) the decrease tends to be progressive (e.g., the Spearman rank correlation coefficient = 0.9998, p < 0.01). These conclusions hold regardless of how the numbers are spaced on the x-axis (i.e., the amount of time between the lives of the individuals listed, or how many generations intervene between samples).

Systematic Properties: Equal Interval

Method

One would hope to have some estimate of the time between points in figure 1 (p. 118). There is little useful information on the dates of the patriarchs before Abraham; then one possible approach is to assume that the time between the individuals listed does not change systematically over time and use the chronologies of Thiele and Galil to check the validity of this assumption over the time span that they cover. (No assumption need be made about the scale of the x-axis, whether it spans 10,000 or 1,000,000 years, for ex- ample.) Figure 2 below shows, for the time span covered by these two chronologies, when each of the reigns ended. It shows that, over this time span at least, the assumption that the reigns are equally spaced over time is a reasonable approximation, and suggests that it may not be a bad assumption for the entire curve. Using either of these chronologies for the abscissae in figure 1 fails to improve the regularity of the data or decrease the error variance.

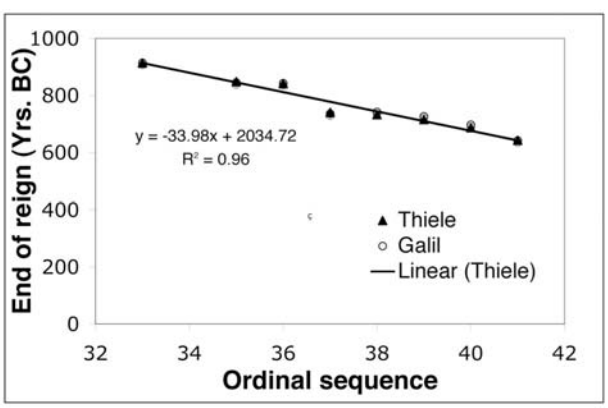


Figure 2. The ends of the reigns of the kings and judges, according to Thiele's (black triangles) and Galil's (white circles) chronologies.

Results and discussion

Use of the assumption that the time between the individuals listed in table 1 (p. 119) does not change systematically over time allows other conclusions. First, it allows quantitative refinement of the conclusions above: (1) there is no detectable trend before Noah (r = 0.13, 7 df excluding Enoch, p = 0.75); and (2) the numbers do decrease after Noah (r = 0.83, 29 df, p < 0.05).

Further, one can draw conclusions about the time-course of the decrease. There are several plausible ways to describe this time-course: (1) it might have been abrupt, as if a tendency to exaggerate ages had ended suddenly; (2) it might simply be linear; (3) it might follow a complicated time-course that could be described by a polynomial; (4) it might be a power function, as reported previously;[9] or (5) it might be an exponential decay, also reported previously.[10] Although this list is not exhaustive, it is a reasonable sample of the leading candidate functions.

Table 2 below shows these functions, the order of the polynomials, the residual squared errors, the F ratio formed by dividing the residual for each function by the residual for the exponential function (explained below), and the probabilities corresponding to the F ratios. The abrupt or discontinuous decrease clearly does not the fit the data, by any criterion. The linear decrease (first order) likewise is a poor fit. The second order polynomial, which has three degrees of freedom, the same number as the power function and the exponential function, is a significantly worse fit than either of those functions. Although the third order polynomial fits as well as the power function and the exponential function, it is less preferred because it requires an added

Table 2. Candidate Functions and Their Goodness of Fit

Function	Order	r	Residual	F	P
Discontinuous	0	0.000	571693	13.94	<<0.0001
Polynomial	1	0.832	175596	4.28	0.0001
Polynomial	2	0.927	80773	1.97	0.0391
Polynomial	3	0.960	44633	1.09	0.4105
Polynomial	4	0.973	30996	0.76	0.7577
Polynomial	5	0.973	30959	0.76	0.7544
Polynomial	6	0.973	30934	0.75	0.7648
Power			39348	0.96	
Exponential			40999		

degree of freedom. Adding more degrees of freedom to form the higher order polynomials does not reliably improve the fit. Although the power function, $y = 1068\ x^{-0.222} - 453$, and the exponential function, $y = 500\ e^{-x/5.5} + 71.7$, fit approximately equally well, the exponential function is preferred for two reasons. First, it describes a relationship often observed in natural science, from heat exchange curves in thermodynamics to learning curves in psychology: a change of external conditions produces changes that follow an exponential time-course whenever the rate of change of the system is proportional to the distance to the final value.[11] Second, the best estimate of the asymptotic longevity of the exponential curve, 71.7 years, is consistent with contemporary values, whereas the asymptote of the power function is negative 453 years, a meaningless figure as a longevity.[12] An exponential time-course for these longevities has been reported before,[13] but the data in these past reports are incomplete and not always accurate.[14]

Consequently, an exponential decay function, fit to the data by maximum likelihood, is used in figure 1 (p. 118):

$$L = 500\ e^{(11 - \gamma)/5.5} + 71.7,$$

where L is longevity, and γ is ordinal sequence. The break between the horizontal line and the exponential curve was chosen between the 10th and the 11th points because it minimized the total residual variance.

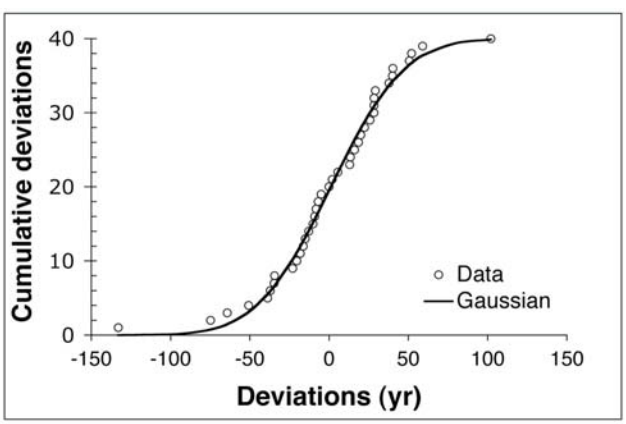


Figure 3. Cumulative deviations as a function of the magnitude of deviations. Deviations (also called errors) are the value of the data minus the corresponding value of the function fitted to the data in figure 1 (p. 118). In this figure, the height (ordinate) of a point at any given deviation on the x-axis (the abscissa of that point) is equal to the number of deviations of that size or less, i.e., the number of points below that point, plus one. (See the section on "Error Distribution.")

Thus, the exponential curve provides a reasonable description of the data, using 3 degrees of freedom to account for 93% of the variance of 31 data.

Error Distribution

It is worthwhile to check the form of the error distribution, i.e. how the numbers in table 1 (p. 119) are distributed about the theoretical curve in figure 1 (p. 118), for it may contain information on the processes that produced them. Natural processes produce only certain kinds of error distributions, such as Gaussian, log Gaussian, and Poisson distributions, each of which is the signature of a different class of process. Artificial numbers can have whatever error distribution its creator chooses, and therefore observation of one of these will not discriminate between natural and artificial origin. However, certain distributions, such as a bimodal distribution or those with gaps or discontinuities, are not characteristic of natural processes, and observation of any of these distributions would be evidence against the naturalness of the numbers. Hence, the test below is warranted to check for any such evidence.

Method

Errors are defined as the value of the data minus the corresponding value of the function fitted to the data in figure 1. The conventional way to form an error distribution is to segment the continuum of error values into discrete intervals, count the number of errors within each interval, and present the counts as a frequency histogram. The disadvantage of this approach is that the shape of the distribution depends on arbitrary choices of size and placement of the intervals, especially when the data are sparse, as they are here. The present analysis avoids these disadvantages by working with cumulative errors. That is, the deviations are arranged from lowest to highest value and plotted left-to-right, each point one unit higher than the preceding one.

Results and discussion

The resulting cumulative error distribution is shown by the symbols in figure 3. The curve is a cumulative Gaussian distribution fit to the data by maximum-likelihood. The data do not differ reliably from the theoretical curve ($p = 0.49$, according to linear interpolation of table 6 of Shapiro and Wilk; $W = 0.971$, $n = 30$);[15] hence, the numbers are well described by a Gaussian distribution. As discussed in the preceding section, this result is consistent with either a natural or artificial origin of the numbers and so fails to reject either.

Distribution of Initial Digits

The human tendency to overuse certain digits and underuse others produces another difference between natural and artificial numbers.[16] For reasons explained below, a particularly important difference lies in the frequency with which different digits occur as the first (most significant) digit in the numbers.

It seems almost self-evident to many (it did to me) that each of the nine, nonzero digits would be equally likely to occur as the first digit (or second or third or any other digit) in natural numbers, producing a uniform frequency distribution. It is the assumption made by C. A. Hill in evaluating biblical longevities, for example, and it survived peer review of her paper.[17] Therefore, one who is trying to mimic natural data might well strive for such a uniform distribution. However, as explained below, this is a fallacy. In figure 4 below, one can compare the frequencies of first digits of the biblical longevities in table 1 (black bars, p. 119), with the frequencies of the uniform distribution (indicated by the horizontal line) required by the false assumption of equal probability. The first digits from table 1 deviate reliably from a uniform distribution ($X^2 = 30.1$, $p < 0.0002$, df = 8). Therefore, either these numbers were not made up, or else whoever made them up was more sophisticated about such probabilities than the typical contemporary scientist.

However, T. P. Hill (not C. A. Hill) has shown that humans attempting to produce random numbers do not tend to produce a uniform frequency distribution; rather, they favor certain numbers, and avoid others.[18] On the basis of the probabilities reported by Hill, the frequencies they would be expected to produce here are shown in figure 4 (gray bars).

The biblical frequencies observed in table 1 deviate reliably not only from a uniform distribution but also from those observed by Hill ($X^2 = 24.1$, $p = 0.002$, df = 8). Therefore, either these biblical longevities were not made up, or else whoever made them up was able to perform better than Hill's students did.

However, contrary to intuition, the frequencies of the first digits of naturally occurring numbers are not uniformly distributed but follow Benford's law;[19] that is, the first digit is more likely to be a low number than a high number. For example, in natural data, the probability that the first digit is a 1 is 0.301, whereas the probability that it is a 9 is 0.046. The numbers humans generate in the attempt to mimic naturally occurring data deviate from Benford's law.[20] As a consequence of this property, Benford's law is used to detect fabrication of data.[21] Hill's data, for example, do deviate from Benford's law ($X^2= 24$, df = 8, $p < 0.002$).

However, when one compares the biblical longevities of table 1 with Benford's law (the black bars versus the gray bars in figure 5 below), the longevities follow the predictions of Benford's law without significant deviation.[22] When the entire data set is considered collectively, the deviations of the overall distribution of the observed frequencies from those of Benford's law do not depart from those attributable to chance ($X^2 = 10.61$, $p = 0.23$, df = 8). And when

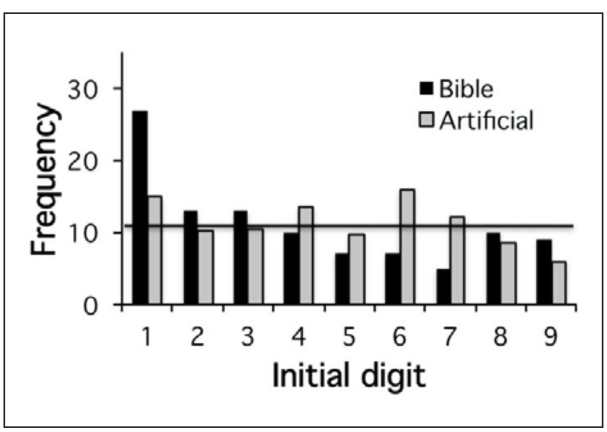


Figure 4. Frequency distribution of first digits from table 1 (black bars, p. 119), compared with those of artificially generated numbers (gray bars) and those of a uniform distribution (horizontal line).

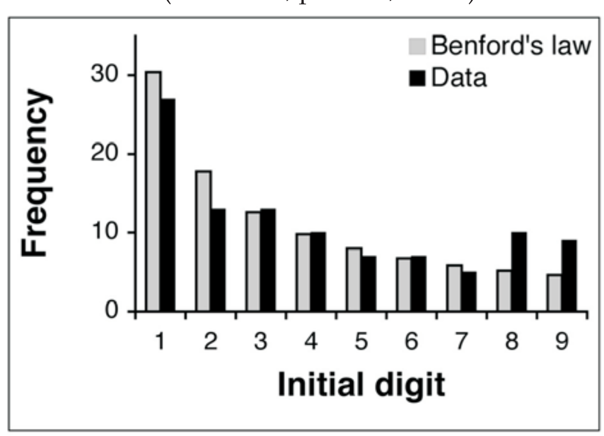


Figure 5. Frequency distribution of first digits from table 1 (black bars, p. 119), compared with Benford's law (gray bars).

the frequency of occurrence of each of the digits (1–9) is considered specifically, they all fall within the 0.95 fiducial limits of the frequency predicted by Benford's law for that digit, as determined by the z-statistic.[23]

Thus, the frequencies of first digits in table 1 differ reliably from the best empirical estimate of the frequencies used by those attempting to generate random numbers, and they differ reliably from those of a uniform distribution, which is almost universally assumed to be characteristic of naturally occurring numbers; however, they do conform to Benford's law, as natural numbers do.

Independence

One way in which artificial and natural numbers differ is that artificial numbers lack several properties of randomness that characterize natural numbers: for example, even when trying to generate random numbers, humans seldom use any given digit twice in succession.[24] So, if one "makes up" the digits in sequence from left to right, then one is less likely to follow a 4 with another 4 than with another digit, and so the number 44 is less likely than other numbers in the 40s. As this form of independence is the most consistent deviation from randomness exhibited by humans,[25] it would seem to be a useful way to distinguish natural from artificial numbers here, but it turns out that the data are too sparse for it to be useful.

Specifically, there are six instances of identical digits adjacent to one another (i.e., 33, 55, 119, 133, and twice in 777) in the 63 bolded numbers of table 1 (p. 119). (Numbers containing zeros were excluded because when a zero was the least significant digit, it could have been produced by rounding, as demonstrated below, and a zero anywhere else in a two- or three-digit number cannot be preceded by a zero.) This is close to the expected frequency (6.45) for a set of numbers that follow Benford's law, as all natural data do. However, to reliably detect such a bias against repetition in a set of 63 numbers, the probability of a repetition would have to be about 0.03 instead of 0.102. As there are no good estimates of this probability, one can only say that this test reveals no evidence of such a bias, but that these data do not allow a powerful test.

Another related form of independence is independence of the magnitude of an error on the magnitudes of the preceding errors. It may be, for example, that a human making up numbers would add variable errors to some theoretical value, and if humans have a tendency to alternate magnitudes,[26] then a large error is more likely to be followed by a smaller error than by another large error. This would show up in autocorrelograms or power spectra. However, owing to problems with autocorrelograms or power spectra,[27] this form of independence was also examined here by plotting the value of each error against the value of the preceding error. The autocorrelogram and power spectrum showed no reliable regularities, nor did the plot just described show any relationship between successive errors. However, again, the sparsity of the data limits the power of the tests; thus, although there is no evidence of characteristically human deviation from independence, the test is too insensitive to warrant any conclusions.

Rounding

Although there is no evidence that these numbers are artificial, they clearly have been rounded. After excluding those numbers from table 1 that have been inferred by addition or subtraction, ten of the 32 remaining two-digit numbers have 0 as the last (least significant) digit; and 17 of the 38 remaining three-digit numbers have 0 as the last digit. Both frequencies are significantly higher (X^2 = 9.88, p = 0.0017, 1 df; and X^2 = 44.6, p = 2^{-11}, 1 df; respectively) than those in a Benford distribution (four expected zeros in both cases, corresponding to probabilities of 0.120 and 0.102, respectively). Although rounding adds to the error variance, its contribution to the total error variance is less than one-tenth of one percent; therefore, its effect on the data in figure 1 (p. 118) is negligible.

Galil argues that one cannot assume "that the data in the Bible regarding the years of reign were rounded off …"[28] However, the analysis here clearly shows that some numbers have been rounded, although one cannot say with confidence that any specific number has been rounded.

To round numbers, of course, is neither misleading nor suspicious. However, such rounding does prevent one from reconciling all the data on biblical chronologies exactly, as Thiele would do;[29] and it invalidates the computation of probabilities based on the assumption that the final digits of these numbers are random.[30]

General Discussion

This is the first general examination of the numerical properties of the longevities recorded in the Hebrew Bible. It confirms previous descriptions of the general shape of the transition from antediluvian to contemporary longevities, but it also refines and corrects errors in those descriptions. It shows that the deviations from this systematic, temporal trend form a Gaussian distribution. Neither the magnitudes of these deviations nor the sequence of digits in the numbers representing longevities show detectible departures from stochastic independence. The longevities also have been rounded. Finally, and most important, the properties of the numbers representing longevities differ reliably from those characteristic of the two most likely forms of fabrication, and the numbers conform to Benford's law.

However, the main purpose of the analysis was to shed light on the origin of these numbers. Either these numbers represent natural phenomena, or at least some of them are fabricated. As one cannot prove the null hypothesis, one can prove one hypothesis only by disproving its alternative (or all the alternatives, if there be more than one). Here we use the differences between the properties of natural and fabricated numbers to evaluate these two alternatives.

Natural Origin

Specifically, numbers derived from natural phenomena follow Benford's law,[31] they represent systematic processes perturbed by random error, these perturbations tend to be mutually independent,[32] and the numbers tend to be normally distributed about their mean values. If biblical longevities lack any of these properties, they cannot be true, and at least some must be fabricated. All efforts to show that the numbers lack the properties of natural numbers failed; therefore, one cannot reject the hypothesis that the numbers have a natural origin. This, of course, does not prove a natural origin; it simply fails to disprove it.

Arguments against Natural Origin

It seems surprising that natural origin is so difficult to disprove, since it seems so improbable. But why does it seem improbable? It is worthwhile to review here the reasons.

1. Inconsistency with contemporary longevities. The main reason that these longevities seem implausible is that they are inconsistent with contemporary human longevities: the longest documented life so far is 122 years and 164 days.[33] Moreover, there is a consensus that even under optimal living conditions, and with all fatal diseases cured, life expectancy at birth is not likely to exceed about 90 years.[34] However, the assumption that conditions that limit longevity were the same in antediluvian times as they are now is not necessarily true. The principal process that limits longevity in humans is aging,[35] and the fact that some animals show no evidence of aging (i.e., show *negligible senescence*),[36] means that aging is not necessarily a universal and immutable characteristic of living organisms, necessarily applicable to all humans under all conditions. The possibility of such longevities is supported by de Grey's argument that human life expectancy can, even now, be extended to a thousand years, although the strength of his argument is mitigated by its dependence on technologies that are only now being developed or have yet to be developed.[37] In any case, the argument against these biblical longevities on the basis of contemporary experience is not conclusive.

2. Dearth of corroborating records. Another reason why these longevities are implausible is that the more reliable surviving texts tell of longevities that are consistent with, or shorter than, contemporary data.[38] However, the records considered reliable do not extend back far enough in time to be relevant here. Many of those records that do go back far enough lend support to the biblical longevities but are considered unreliable. In some cases, such as that of the Weld-Blundell prism,[39] the argument may be circular, for the main reason that they are considered unreliable is that they tell of unbelievably long lives. Josephus says, "All who have written antiquities … relate that the ancients lived a thousand years," and he lists eleven specific authors as examples.[40] But Josephus's accuracy, particularly with respect to numbers, has been widely challenged on other grounds.[41] This epitomizes the dilemma here, for, unreliable as Josephus may be, he may be "more reliable than most historians of his day."[42] The historical record, then, is of little help in resolving this issue.

3. Absence of supporting archeological evidence. The absence of archeological evidence of skeletal remains of extremely aged individuals may seem to be the strongest evidence against their existence, but here other problems arise. The first lies in the limitations of the methods. According to Acsádi and Nemeskéri,

> The methods of age determination generally employed in historical anthropology include … the closure of cranial sutures, abrasion of teeth, and regressive signs in the external morphology of skeletal bones at young adult, middle adult and old adult ages … [J]ustifiable doubts may arise on the accuracy of age determination based on the closure of cranial sutures and on the so-called classical anthropological methods in general (and) … in historical anthropology usually only the basic distinction between child, juvenile or adult is made.[43]

One can see from their table 30 that closure of the cranial sutures approaches an asymptote in middle age, and the variability is so great that one cannot reliably discriminate a 30-year-old from the oldest specimens examined. As for abrasion of the teeth, they state,

> The abrasion of teeth at adult age … depends primarily on living conditions … Stomatologists have shown that the degree of abrasion is more often indicative of the individual's way of eating than of the period of time during which the teeth have been used.[44]

A second problem follows from excavators' practice of culling the skeletal evidence, discarding all but the best skeletons.[45] The skeletons of the very old, lacking teeth, for example, may be excluded from the sample; an individual who has lived for a millennium may not have many surviving teeth.

Third, there are no data from the relevant time and place. As Finch has pointed out,

> We cannot know the actual trajectory of change (in human life spans), which could have included fluctuations with decreases, as well as increases, in life span during these several hundred thousand generations of Darwinian selection.[46]

To exclude the biblical account, one must exclude the possibility that there was a time, perhaps brief in historical perspective, in which a particular subpopulation, including but not necessarily limited to the Hebrews, enjoyed extraordinarily long life. The most nearly relevant data are from Anatolia[47] and Jericho,[48] but both populations differ from the long-lived Hebrews of the Bible, who lived in Mesopotamia.[49]

Finally, use of skeletal evidence of aging begs the question. As Acsádi and Nemeskéri point out, archeologists can measure only biological age, not chronological age.[50] Longevities of several centuries could have been achieved only if biological aging were somehow retarded.

Thus, while these objections carry considerable weight, none of them conclusively confutes the possibility that the reports of the biblical longevities are true.

Possible Explanations Consistent with Natural Origin

Several efforts have been made to preserve the validity of these numbers without accepting implausible longevities.

1. Changes in the ways of expressing or measuring time. For example, attempts have been made to account for biblical longevities by changes in the way of expressing or measuring time.[51] However, no such scheme can work. To be conservative, suppose that the biblical life span of 969 biblical years (Methusaleh) were actually equal in time to the contemporary life span of 122 contemporary years.[52] Then a biblical year would have to be 122/969 = 0.126 contemporary years. If so, Saleh, Peleg, and Serug would each have to have been less then 4 years old (30 x 0.126 = 3.8) when their first sons were born, and Mahalaleel and Enoch would have to have been about 8 (65 x 0.126 = 8.2) when their first sons were born. These consequences of this hypothesis are at least as implausible as a 969-year life span.

2. Dynasties, not individuals. Others have suggested that the ages refer not to individuals but to "an individual and his direct line by primogeniture."[53] For example, Adam and his direct line are supposed to have held sway for 930 years, after which Seth and his family assumed control for the next 912 years. Archer points out, however, that as Seth was the oldest surviving son of Adam aside from exiled Cain, there was no other son to carry on Adam's line until Seth's line took over.[54] Borland lists some eight problems with such dynastic theories that render them untenable.[55] Moreover, there are not enough plausible gaps between the individuals listed to account for the required lapsed time, and there are too many instances of coexistence of individuals that, according to this explanation, would have to have been successive dynasties.[56]

3. Physical explanations. Yet other attempts have been made to make these longevities plausible by offering physical explanations. There may once have been a set of conditions that fostered great longevity in humans. If so, figure 1 (p. 118) suggests that these conditions changed abruptly at about the time of Noah to bring longevities progressively closer to contemporary values. The question is, then, what changed? Unfortunately, it is hard to say.

One suggestion is that a protective canopy protected antediluvian humans from harmful radiation.[57] A gradual decay of the canopy could explain the gradual shortening of the duration of life. However, this is entirely speculative and adds nothing to the description of the phenomenon except the idea that radiation is what limits contemporary longevities.

Ross has suggested that "higher telomerase activity in concert with other slight biochemical adjustments, combined with a just-right diet (low calorie, low oxidant, high antioxidant) and the avoidance of (radioactive) igneous rocks …" may help explain the long lives of the first humans, and that irradiation of Earth by the remnants of a recent supernova may explain the subsequent shortening of life.[58] The Monogem supernova may be a possible candidate, remnants of which even now account for some 60% of the cosmic irradiation of Earth at the "knee" of the cosmic ray spectrum, but its timing, 86,000 years ago, is problematical.[59] In any case, this hypothesis leaves much unexplained. Astronomical explanations such as this and the one in the preceding paragraph are inconsistent with the possibility that these extraordinary longevities occurred only in a small subpopulation of humans.

As mentioned above, aging constitutes the principal limitation on longevity; hence, a difference in the genes controlling aging seems a necessary condition for extreme longevity.[60] Although the *principle* of extending life by manipulation of genes has been demonstrated, retardation of the aging of humans by gene manipulation has yet to be demonstrated, and such effects as have been observed in animals so far are modest compared to the requirements here. What might have caused such genes to change also remains unknown. Nevertheless, the limits on the magnitude of such effects are unknown, and so the possibility that changes in genes account for the putative changes in longevities remains open.

Artificial Origin

The properties of artificial or fabricated numbers depend on the conditions and purposes of fabrication, and therefore one cannot disprove fabrication in general but only specific forms of fabrication. The first possibility to consider is that the author or authors of the numbers deliberately intended to mimic natural data, and here there are three levels of sophistication with which the task might have been approached.

Deliberate Mimicry

1. Naïve mimicry. Mathematically naïve humans trying to mimic natural data show specific preferences for the first digits of numbers.[61] The present analysis shows that the biblical longevities do not follow those preferences; therefore, it is unlikely that these longevities resulted from a mathematically naïve effort to mimic natural data.

2. Sophomoric mimicry. A more sophisticated yet fallacious approach to fabrication is based on the false assumption that in natural data all digits are equally probable. Actually, as described above, these probabilities follow Benford's law. However, biblical longevities do not follow the assumption of equal probability either (and do follow Benford's law), and therefore it is unlikely that these longevities arose from this more sophisticated attempt to mimic real data.

3. Sophisticated mimicry. The most sophisticated form of fabrication would be to mimic Benford's law. However, one can exclude that possibility because there is no known way to mimic Benford's law without knowing about it, and what has come to be called Benford's law was not discovered until 1881, by Newcomb,[62] millennia after these longevities were recorded.

It follows, then, that one can exclude all three forms of intentional mimicry of natural data. However, one cannot entirely exclude the possibility that other, unknown forms of intentional mimicry might exist.

Time-Course of the Changes

The gradualness of the progression of longevities from antediluvian to contemporary values is entirely consistent with natural processes, but it does impose stringent constraints on hypotheses based on fabrication. The nature of the constraints depends on whether a single individual fabricated all the longevities, whether different individuals independently fabricated each of the longevities, or whether some combination of the two is responsible for them.

1. A single author. If a single individual fabricated all the longevities, that individual would be subject to biased numerical preferences, such as those documented by T. P. Hill, as discussed above. However, that individual's biases may not necessarily be the same as those of the contemporary college students used in Hill's study, and, without knowing that individual's biases, one cannot entirely exclude the possibility that a single individual fabricated these longevities.

However, this hypothesis does require the fabricator to have a particular function in mind to mimic and to have a motive for mimicking it. According to a consensus among historians of mathematics,[63] exponential and related functions were not suggested until the fourteenth century, and therefore the fabricator could hardly have had it in mind while fabricating the numbers. However, this conclusion is vitiated by the existence of cuneiform tablets from Mesopotamia containing tables of exponential series,[64] including those entailing the use of fractional exponents,[65] the consensus of mathematical historians notwithstanding. These, along with the reciprocal numbers also found among the tabulated numbers, theoretically could have been used to generate an exponential function such as that in figure 1 (p. 118). These mathematical advances appear to have been made at about 1800 BCE,[66] after the time of Abraham[67] and after most of this exponential decay had occurred, but possibly before the numbers were generated.

The most likely motivation for attempting to mimic such a function is to make them appear real, an act of deliberate deception that conflicts with the view many have of the Bible and the motives of its authors. Thus, the time-course of the change in longevities does not allow one to exclude a single fabricator, but it does place heavy demands on such a fabricator.

2. Multiple independent authors. Different individuals independently fabricating different longevities could not, of course, intentionally mimic any particular decay function. There is no known mechanism by which numbers generated by different individuals could produce an exponential-like function.

3. A combination of these two possibilities. That is, different individuals could have fabricated the numbers, but some could have fabricated more than a single number. This possibility is subject to both of the above sets of constraints to a varying extent, depending on the particular combination.

Thus, the time-course of the change poses substantial challenges for any hypothesis based on fabrication, but it does not entirely exclude it.

Fabrication without the Intention to Mislead

Even if the numbers were not deliberately fabricated, one must consider the possibility that some process other than true longevities might have given these numbers these properties. Among such proposals is the idea that the numbers were not meant to convey quantitative information but were instead intended to have cultural significance. For example, large numbers may have conferred honor on the individuals with whom they were associated, or the numbers might have had numerological rather than numerical significance.

Such hypotheses are hard to evaluate without a specific interpretation of the meaning of the numbers, although the systematic properties evident in figure 1 are not consistent with a set of numbers completely devoid of numerical significance. The most specific and best supported hypothesis in this category is the idea that the Mesopotamians preferred the numbers 60 and 7, considering them sacred, for example.[68] Such numbers are artificial, then, instead of representing natural data, but they do not represent an effort to fake or mimic natural data.

In general, such numerological arguments are unconvincing. To show why, take a specific example cited by C. A. Hill, the most rigorous proponent of a numerological interpretation of these ages. She points out that each of the ages in Genesis from before the flood is equal to the sum of a multiple of

5 years (60 months) and a multiple of 7 years, and she attributes this to the special significance of these numbers for Mesopotamians. However, numerological arguments are hard to evaluate without considering the relevant probabilities. In this case, the fact that all these ages are the sum of multiples of 5 and 7 is irrelevant, since this is true of *all* numbers from 24 to 1000. For example, 24 = 2 x 5 + 2 x 7; 25 = 5 x 5; 26 = 5 + 3 x 7; 27= 4 x 5 + 7; and similarly up to and beyond 999 = 5 x 197 + 2 x 7.[69] In other words, no matter what the ages, they would nevertheless all be equal to the sum of a multiple of 5 and a multiple of 7. The numbers 5 and 7 are not unique in this respect: for example, all the numbers above 20 are equal to the sum of multiples of 3 and 11. Possibly any pair of prime numbers has analogous properties.

The present example notwithstanding, Hill did significantly advance the rigor of this line of argument by actually comparing the probabilities of occurrence of the numbers representing the ages of the patriarchs to their frequency of occurrence.[70] For example, she analyzed the 60 numbers describing, for each of the first twenty patriarchs, his age when his first son was born, his remaining years of life, and his total years. She states that none of these numbers ends in 1 or 6, "a chance probability of one in about one-half million."[71] Unfortunately, she is not clear about what the probabilities refer to or how she arrived at them, and her computations err in several ways.

First, the probability that two specific final digits (1 and 6, in this case) would fail to occur by chance is 1.5325(10^{-6}), or one chance in 652,530, close to the "chance of one in about one-half million" that she states. However, the appropriate probability to apply here is that *any* two digits would fail to occur; if, for example, 3 and 8 had failed to appear as final digits she would have drawn the same conclusions. Since 10 digits can form 45 different pairs, the probability that any of those pairs would fail to occur is 6.89623(10^{-5}) or 1 in 14,500, not 1 in about half a million.

Although this difference does not vitiate her conclusion that one can exclude chance as an explanation of such numbers, it typifies computational errors in this paper. Moreover, the conclusion is moot because the statement that none of the numbers ends in 1 or 6 is false: Abraham's age was 86 when his first son, Ishmael, was born. Hill excludes this number by breaking her own rule, using Abraham's age at the birth of his second son, Isaac, instead of his age at the birth of his first son, Ishmael.

Second, Hill's computations are based on the false assumption (p. 244) that the ages should be random numbers; instead, they should conform to Benford's law (explained above). Also, her computations do not take into account the effects of rounding (demonstrated above). Finally, her choice of which patriarchs to include in her sample (the first twenty) and which subset to select for separate analysis (the first ten) is arbitrary and post hoc, and it therefore inflates the significance of her probabilities.

However, estimating the probabilities of the final digits of these numbers, as Hill has attempted, is unnecessary in this case, for her point—that the final digits of these numbers are not natural—clearly follows from the fact that some have been rounded (shown above). Moreover, not all the numbers have been rounded. Choice of which numbers to round allowed whoever did the rounding wide latitude in determining the properties of the remaining final digits. Whether such choices were guided by their supposed sacredness or by other considerations is not clear from present data.

Note (see the discussion of rounding above) that any manipulation of these least significant digits would have a negligible effect on the systematic properties of the numbers, the error distribution, the independence of the numbers, and conformance to Benford's law; that is, these properties are insensitive to the values of the least significance digits, which, by definition, have relatively small effects. Therefore, the fact that the least significant digits have been manipulated does not affect the conclusion that the numbers represent natural phenomena; nor, conversely, does the fact that these numbers have these natural properties exclude the possibility that the least significant digits have been manipulated.

Conclusions

These biblical longevities admit to but two possibilities: at least some are true, or all are false. The frequencies of first digits in table 1 (p. 119) differ from the frequencies used by those attempting to generate random numbers, but do conform to Benford's law.

Also, the time-course of the longevities is difficult to reconcile with deliberate fabrication or with any other form of fabrication. Like any empirical finding, the results are not absolutely conclusive, but the mathematical properties of these numbers favor natural origin. In other words, the biblical longevities, as a set, are likely to be true.

Acknowledgments

I thank Carol A. Hill, James C. Makous, David R. Williams, Hugh G. Gauch Jr., and an anonymous reviewer for their most helpful comments on earlier versions of this paper.

Disclosure Statement

No competing financial interests exist.

Notes

[1]Carol A. Hill, "Making Sense of the Numbers of Genesis," *Perspectives on Science and Christian Faith* 55, no. 4 (2003): 239–51.

[2]F. Benford, "The Law of Anomalous Numbers," *Proceedings of the American Philosophical Society* 78 (1938): 551–72; Mark John Nigrini, *Digital Analysis Using Benford's Law: Tests and Statistics for Auditors* (Vancouver, BC: Global Audit Publications, 2000).

[3]W. A. Wagenaar, "Generation of Random Sequences of Human Subjects: A Critical Survey of Literature," *Psychological Bulletin* 77 (1972): 65–72; M. Bar-Hillel and W. A. Wagenaar, "Perception of Randomness," *Advances in Applied Mathematics* 12 (1991): 428–54.

[4]Allen Neuringer, "Can People Behave 'Randomly?': The Role of Feedback," *Journal of Experimental Psychology: General* 115, no. 1 (1986): 62–3.

[5]Gleason L. Archer, *A Survey of Old Testament Introduction* (Chicago: Moody Press, 1994), 211–2.

[6]Gershon Galil, *The Chronology of the Kings of Israel and Judah*, Studies in the History and Culture of the Ancient Near East, vol. 9 (New York: E. J. Brill, 1996).

[7]Edwin R. Thiele, *A Chronology of the Hebrew Kings* (Grand Rapids, MI: Zondervan, 1977).

[8]Galil, *The Chronology of the Kings of Israel and Judah.*

[9]John C. Sanford, *Genetic Entropy and the Mystery of the Genome*, 2d ed. (Lima, NY: Elim Publishing, 2005), 152.

[10]Hugh Ross, *The Genesis Question* (Colorado Springs, CO: NavPress, 1998), 122; P. M. Holladay and J. M. Watt, "De-Generation: An Exponential Decay Curve in Old Testament Genealogies." Paper presented at the 52d National Conference of the Evangelical Theological Society, Nashville, TN, 2000, 4–7; Sanford, *Genetic Entropy and the Mystery of the Genome*.

[11]Eli Maor, *e: The Story of a Number* (Princeton, N J: Princeton University Press, 1994), 98–103.

[12]The upper 95% confidence interval on this estimate does extend to positive 54 years; the 95% confidence interval for the asymptote of the exponential curve is from -95 to 132.2 years.

[13]Ross, *The Genesis Question*; Holladay and Watt, "De-Generation: An Exponential Decay Curve in Old Testament Genealogies."

[14]Holladay and Watt, "De-Generation: An Exponential Decay Curve in Old Testament Genealogies," 152.

[15]S. S. Shapiro and M. B. Wilk, "An Analysis of Variance Test for Normality (Complete Samples)," *Biometrika* 52, no. 3/4 (1965): 591–611.

[16]Wagenaar, "Generation of Random Sequences of Human Subjects: A Critical Survey of Literature"; Bar-Hillel and Wagenaar, "Perception of Randomness."

[17]See C. A. Hill, "Making Sense of the Numbers of Genesis," 244.

[18]Theodore P. Hill, "Random-Number Guessing and the First Digit Phenomenon," *Psychological Reports* 62 (1988): 967–71.

[19]Benford, "The Law of Anomalous Numbers"; Nigrini, *Digital Analysis Using Benford's Law: Tests and Statistics for Auditors*; Simon Newcomb, "Note on the Frequency of Use of the Different Digits in Natural Numbers," *The American Journal of Mathematics* 4 (1881): 39–40.

[20]T. P. Hill, "Random-Number Guessing and the First Digit Phenomenon," 970.

[21]Nigrini, *Digital Analysis Using Benford's Law: Tests and Statistics for Auditors*.

[22]Ibid., 14.

[23]Ibid., 42–3, 73–6.

[24]Wagenaar, "Generation of Random Sequences of Human Subjects: A Critical Survey of Literature," 70.

[25]Ibid.

[26]Alphonse Chapanis, "Random-Number Guessing Behavior," *The American Psychologist* 8, no. 332 (1953).

[27]Wagenaar, "Generation of Random Sequences of Human Subjects: A Critical Survey of Literature," 68–9.

[28]Galil, *The Chronology of the Kings of Israel and Judah*, 10.

[29]Thiele, *A Chronology of the Hebrew Kings*.

[30]C. A. Hill, "Making Sense of the Numbers of Genesis," 244.

[31]Benford, "The Law of Anomalous Numbers"; Nigrini, *Digital Analysis Using Benford's Law: Tests and Statistics for Auditors*.

[32]Wagenaar, "Generation of Random Sequences of Human Subjects: A Critical Survey of Literature"; Bar-Hillel and Wagenaar, "Perception of Randomness."

[33]Jean-Marie Robine and Michel Allard, "The Oldest Human," *Science* 279, no. 5358 (1998): 1831.

[34]S. Jay Olshansky, Bruce A. Carnes, and Christine Cassel, "In Search of Methuselah: Estimating the Upper Limits to Human Longevity," *Science* 250 (1990): 638; D. Harman, "Aging: Overview," *Annals of the New York Academy of Science* 928 (2001): 2.

[35]Ibid.; also Leonard Hayflick, "The Future of Ageing," *Nature* 408, no. 6809 (2000): 267–69.

[36]Caleb E. Finch, "Variations in Senescence and Longevity Include the Possibility of Negligible Senescence," *Journal of Gerontology* 53A, no. 4 (1998): B235–9; ———, *The Biology of Human Longevity: Inflammation, Nutrition, and Aging in the Evolution of Life Spans* (Burlington, ME: Academic Press, 2007), 47–8; Hayflick, "The Future of Ageing," 268.

[37]Aubrey J. N. J. de Grey, "Escape Velocity: Why the Prospect of Extreme Human Life Extension Matters Now," *Public*

Library of Science Biology, no. 6 (2004): 5 www.plosbiology.org/article/info:doi/10.1371/journal.pbio.0020187 (last accessed April 26, 2011); ———, "A Strategy for Postponing Aging Indefinitely," *Studies in Health Technology and Informatics* 118 (2005): 218; ——— and Michael Rae, *Ending Aging: The Rejuvenation Breakthroughs That Could Reverse Human Aging in Our Lifetime* (New York: St. Martin's Press, 2007), 327–34.

[38]Edwin M. Yamauchi, "Attitudes toward the Aged in Antiquity," *Near East Archeological Society Bulletin* 45 (2000): 2–7.

[39]Patrick Moore, *Atlas of the Universe* (New York: Rand McNally, 1970), 146.

[40]Josephus, *Antiquities*, 3.9.

[41]Paul L. Maier, "Introduction," in *The New Complete Works of Josephus* (Grand Rapids, MI: Kregel Publications, 1999), 14.

[42]Ibid.

[43]Gy Acsádi and J. Nemeskéri, *History of Human Life Span and Mortality*, trans. K. Balás (Budapest: Akadémiai Kiadó, 1970), 101.

[44]Ibid., 104.

[45]J. Lawrence Angel, "The Length of Life in Ancient Greece," *Journal of Gerontology* 2 (1947): 19.

[46]Finch, *The Biology of Human Longevity: Inflammation, Nutrition, and Aging in the Evolution of Life Spans*, 373.

[47]M. S. Senyürek, "The Longevity of the Chalcolithic and Copper Age Inhabitants of Anatolia," *Belleten* 15 (1951): 447–68.

[48]G. Kurth, "Vorbericht über Anthropologische Beobachtungen bei der Jerichograbung," *Homo* 6 (1955): 145–56.

[49]Carol A. Hill, "A Time and a Place for Noah," *Perspectives on Science and Christian Faith* 53, no. 1 (2001): 28–38.

[50]Acsádi and Nemeskéri, *History of Human Life Span and Mortality*, 102.

[51]James A. Borland, "Did People Live to Be Hundreds of Years Old before the Flood? Yes," in *The Genesis Debate: Persistent Questions about Creation and the Flood* (New York: Thomas Nelson Publishers, 1986), 170–1; C. A. Hill, "Making Sense of the Numbers of Genesis," 239.

[52]Galil, *The Chronology of the Kings of Israel and Judah*, 10.

[53]Archer, *A Survey of Old Testament Introduction*, 211; see Borland, "Did People Live to Be Hundreds of Years Old before the Flood? Yes," 173–7.

[54]Archer, *A Survey of Old Testament Introduction*, 212.

[55]Borland, "Did People Live to Be Hundreds of Years Old before the Flood? Yes," 174–6.

[56]Ibid., 175; C. A. Hill, "Making Sense of the Numbers of Genesis," 240.

[57]John C. Whitcomb and Henry M. Morris, *The Genesis Flood: The Biblical Record and Its Scientific Implications* (Philadelphia, PA: Presbyterian and Reformed Publishing, 1961), 399–405.

[58]Ross, *The Genesis Question*, 117–22; ———, "New Clues to a Genesis Mystery," *Connections* 7, no. 1 (2005).

[59]S. E. Thorsett, R. A. Benjamin, W. F. Brisken, A. Golden, and W. M. Goss, "Pulsar Psr B0656+14, the Monogem Ring, and the Origin of the "Knee" in the Primary Cosmic-Ray Spectrum," *Astrophysical Journal* 592, no. 2 (2003): L71–L73.

[60]M. C. Haigis and L. P. Guarente, "Mammalian Sirtuins–Emerging Roles in Physiology, Aging, and Calorie Restriction," *Genes and Development* 20, no. 21 (2006): 2913–21; Matt Kaeberlein and Pankaj Kapahi, "Aging Is Rsky Business," *Science* 326 (2009): 55–6.

[61]T. P. Hill, "A Note on Distributions of True Versus Fabricated Data," *Perceptual and Motor Skills* 83 (1996): 777.

[62]Newcomb, "Note on the Frequency of Use of the Different Digits in Natural Numbers," 38–40.

[63]Maor, *e: The Story of a Number*, 9.

[64]Otto Neugebauer, *Mathematische Keilschrift-Texte, Herausgegeben und Bearbeitet von O. Neugebauer*, ed. O. Neugebauer, J. Stenzel, and O. Toeplitz. 3 vols., Vol. 1, *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik* (Berlin: J. Springer, 1935), 77.

[65]Otto Neugebauer and A. Sachs, *Mathematical Cuneiform Texts*, American Oriental Series, vol. 20, ed. S. Harris Zellig, Murray B. Emeneau, and George A. Kennedy (New Haven, CT: American Oriental Society and the American Schools of Oriental Research, 1945), 35–6.

[66]Otto Neugebauer, *The Exact Sciences in Antiquity*, 2d ed. (New York: Dover, 1969), 29, 34.

[67]Archer, *A Survey of Old Testament Introduction*, 227.

[68]For summary, see C. A. Hill, "Making Sense of the Numbers of Genesis," 241–3.

[69]One can easily check this for oneself. In MS Excel, list the numbers from 0 to 200 in column A, starting in row 2. List the numbers from 0 to 4 in the first row, starting in column B. Type "=5*$A2+7*B$1" in cell B2, and copy it in all the empty cells of the 6 by 201 cell matrix; this forms a set of numbers consisting only of the sums of multiples of 5 and 7. Change the cell entries from formulae to values (i.e., copy the entire matrix and select "Paste special" from the "edit" menu and select "values" from the submenu). Delete row 1 and column A, leaving a 5 by 200 matrix consisting only of sums of multiples of 5 and 7. Copy column B and paste it at the end of column A. Copy column C and paste it at the end of column A. Do the same with columns D and E, so that all the numbers are in column A. Sort column A in ascending sequence. The result is that in cells A12 to A988 all the numbers from 24 to 1000 are arranged in sequence. Therefore, all the numbers from 24 to 1000 can be formed by the sum of multiples of 5 and 7.

[70]C. A. Hill, "Making Sense of the Numbers of Genesis."

[71]Ibid., 244.