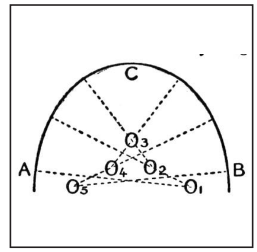
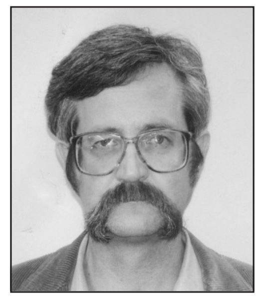


The Limitations of Mathematics in Assessing Causality

Ben M. Carter

From its inception in the sixteenth century, natural science has sought to construct a complete mathematical model of physical reality. This goal was based on three assumptions: (1) that mathematics was equal to the task; (2) that humans, insofar as they perceived the world, perceived it as it is; and (3) that the universe would reveal itself to be fundamentally fairly simple. Today we recognize that not only are all three of these assumptions flawed, their flaws are interrelated and, because of that, formulating a complete mathematical model of physical reality may be beyond our ability. In this paper, I discuss this development in light of William Wharton's work and close with a comment on what this might mean for scientists who are also Christians.



Ben M. Carter

> Right into old age I have had the incorrigible feeling that if, like my schoolmates, I could have accepted without a struggle the proposition that $a = b$, then mathematics might have fooled me endlessly—just how much I only began to realize at the age of eighty-four.
>
> Carl Jung[1]

Science looks for an underlying coherence in the various processes, properties, and outcomes of nature, many of which lack obvious relationship. In other words, science is based on two fundamental intuitions: (1) that the universe is orderly and (2) that its order can be discovered. What is more, scientists since the publication of Newton's *Mathematical Principles of Natural Philosophy* in 1687, have generally been committed to the proposition that the structure of the physical world can be formulated mathematically as laws which demonstrate their validity by being predictive. Initially scientists believed these laws could be forged into a seamless network that would describe the universe completely at a certain level of detail, define what is and is not possible, and preclude certain outcomes. Now they recognize that the laws—and such a goal—have limits set by the uncertainty principle.[2]

But even had its most optimistic agenda been achievable, it would have meant only that science purposed to describe a framework of rules by which it could evaluate certain types of data. Given its own presuppositions, science did not pretend to be able to provide an exhaustive description of what actually occurs, in part because mensuration must always remain approximate, in part because almost everything that happens or has happened remains unobserved, and in part because mathematics itself, which is or has been the preferred means of scientific formulation, might prove inadequate to the task. These limits on the descriptive powers of science are a consequence of its empiricism, the contingent nature of material reality, and constraints inherent in mathematics; and they mean that the descriptions science constructs are primarily inductive.[3] One problem with this, of course, is that conclusions based on inductive reasoning are not unique. One can always hypothesize alternatives.

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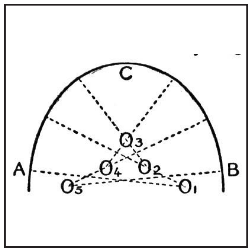

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William Wharton and I have discussed these limitations in the scientific method's capacity to model reality as they relate to his own theory that causal chains with the ability to move backward in time at the level of the microworld can resolve the apparent conflict between quantum mechanics and special relativity. This paper is one result of our dialogue. I will briefly analyze methods science employs in interpreting the world and discuss the role that mathematics and intuition can and cannot play in the process of interpretation. I also will discuss the thesis that a reality external to our minds exists even if we cannot fully grasp it, particularly as that thesis relates to physics.

In an earlier paper, I argued that mathematics, like other forms of human reasoning, may have only limited abstractive value, that it may not be decisive when it comes to answering our questions about nature.[4] In making this argument, I relied primarily on the work done by George Lakoff and Rafael Núñez that interpreted mathematics as metaphor. However, the argument can be illustrated in three other ways.

First, the idea that mathematics can be used to depict natural systems abstractly has been confounded by the intractable complexity of many such systems. To a degree, the computer revolution has rectified this problem by making it possible to model unstable systems with unprecedented accuracy, but predicting specific outcomes is contingent on the exactness of the measurements of such systems' initial conditions. Even slight imprecision quickly corrupts projections as the unstable system is expressed. While infinitely precise measurements could, in theory, make chaos models predictive, such exactitude is, in principle, impossible to achieve. Significantly, as Stephen Wolfram has pointed out in *A New Kind of Science*, it has only recently been feasible to design models that can help us understand the phenomenon of complexity itself.[5] However, these models go beyond traditional mathematical formulas. They instead are based on computer programs that embody more general types of rules. Thus the advent of computers has not only enhanced the power of mathematics, it has allowed us to go beyond traditional mathematics and forge a new intellectual structure for science. Of course, these claims by Wolfram have yet to be fully evaluated by the scientific community. Whether his thesis stands or falls, what is significant is his realization that, in order to address nature as it really is, we need to get beyond the kind of mathematical formalism that has characterized scientific theorizing to date. The complexities of nature highlight the deficiencies of the traditional approach.

Second, it is significant that Immanuel Kant is the philosopher neurobiologists most frequently cite to illustrate the nature of their conclusions. Kant argued that the mind is organized in a particular way and because of that constructs a specific kind of world out of restricted stimuli provided by the sense organs. Neurobiology has shown that a limited ranged of outside influences activate the sense organs to transmit signals, part chemical and part electrical, to various regions of the brain. These regions are only able to process a fraction of the total information they receive, but they are coordinated so that they integrate what they do process into the unified whole, the "virtual reality," that we experience as the external world. This coordination need not imply physical contact among all of the neural systems. Rather our perception of an external world seems to emerge as increasingly higher level systems in the brain edit and splice the various bits lower level systems provide. One consequence of this is that we have no assurance that we experience the world as it is. Rather we experience a world of our own making. Of course, our virtual world enables us to interact successfully with the real world, but the process that results in that virtual world gives us no grounds for supposing that, by using our virtual world as a standard, we can model the actual world in any genuinely exhaustive way.

Third, the above suggests that our intuitions themselves may be unreliable. This is a concern because we know, since Kurt Gödel formulated his famous theorem in 1931, that mathematics ultimately rests on intuitions that cannot be proved. It is significant in this regard that Gödel was himself "a very strong Platonist,"[6] because Plato's concept of a reality that lies beyond this universe, is unaffected by it, and yet shapes it and makes it intelligible, is the only compelling alternative remaining to those who wish to resurrect from the wreckage of formalism the classicist's claim that mathematics is grounded in objective truth. Intuitions, I would argue, rest upon one's (often preconscious) model of the world. When, in our experience of the world, we see how an event that might otherwise be inexplicable fits into our naive world view or corrects or overturns it, we have an intuition. But given such a definition, intuitions cannot be unaffected by this universe. Instead they are generalizations based upon the way we imagine the universe to be. Thus the realm of necessary truth, if it exists, remains opaque to them.

That computers have enabled us to go beyond traditional mathematical formulations, that the world we perceive is distinct from the world that is, and that formalism has failed to secure the necessary truths of mathematics leaving them rooted in system-bound intuitions suggest that unraveling the truth about our universe might require tools more powerful than mathematics can provide.

These three illustrations: that computers have enabled us to go beyond traditional mathematical formulations, that the world we perceive is distinct from the world that is, and that formalism has failed to secure the necessary truths of mathematics leaving them rooted in system-bound intuitions, suggest that unraveling the truth about our universe might require tools more powerful than mathematics can provide. After all, the original perception that the laws of the universe might be exhaustively expressed mathematically was itself an intuition based on a belief that the universe would ultimately be revealed as both comprehensible and relatively simple, two assumptions currently in question. And this suggests that while it is perhaps desirable to have a fully developed mathematical model to explain a scientific theory, such a model itself might not be sufficient or even finally necessary.

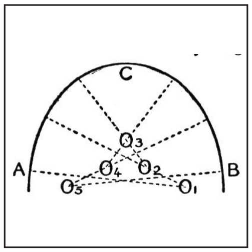
We see this problem in quantum mechanics (QM). QM is mathematically elegant and consistent but the universe it reveals is incomprehensible. That incomprehensibility suggests that the quantum world is not fully explained by the equations physicists used to depict it. Is this because the universe itself at the microlevel really is indeterminate, or does the fault lie with the equations, in the shape of the brain that thought them up, in both, or in something else? If there is a dilemma here, I believe it grows out of our own epistemological limitations, epistemological limitations that would include mathematics, and I believe that the concept of decoherence in QM can help us see that this is so.

First let us review a little history. In 1913 Niels Bohr, while working with Ernest Rutherford in the University of Manchester, began to explore the notion that instead of imagining electrons as analogous to little planets orbiting nuclear suns, it was better to think of them as confined to specific levels or shells around a nucleus and as moving between levels or shells as they absorbed or released specific bits or quanta of energy. After returning to Copenhagen, Bohr, at the urging of Rutherford, published his idea which in time became known as the Copenhagen interpretation.

In his doctoral thesis in 1923, Louis de Broglie argued that subatomic particles, rather than behaving as specific points, act like standing waves and that these waves have frequencies that are simultaneously specific and dissimilar. Later in that decade, Erwin Schrödinger, while reflecting on de Broglie's work, developed his famous equation to describe how such waves might function. Max Born reasoned that Schrödinger's equation was best interpreted in terms of probabilities, but that insight left many people, including Schrödinger himself, uncomfortable since it meant that randomness was built into the very fabric of nature.

According to the Schrödinger equation, as Born understood it, a card perfectly balanced on its edge will not stand forever as predicted in classical physics. Instead, it will fall, but when it falls, it will fall face down and face up at the same time. In other words, the card when it falls will obey a continuous and smooth wave function that is called "unitary" and will create two realities that exist in superposition. However, when we observe the card, our act of observation causes the wave function to "collapse" so that only one part of it survives. Thus we see the card randomly falling face up or face down. We do not see it doing both. We do not see the cards in superposition.[7]

In 1957 Hugh Everett III, while a doctoral candidate at Princeton University, argued that in fact the universe evolves in a unitary way and that the wave function does not collapse. Instead the observer and the card continue to exist in two different places, each place corresponding to a part of the wave function. Everett's idea, formally known as the relative-state formulation, became known popularly as the many worlds interpretation of quantum mechanics. The idea, though initially ignored, has been confirmed via experiments first proposed in 1978 by John Archibald



The Copenhagen interpretation of quantum mechanics has produced three very different views of reality: Zeh's and Zurek's idea of decoherence, Wharton's idea of causal chains which can go backward in time at the quantum level, and Tegmark's idea of a banal universe devoid of much "real" information. Each view is consistent within its set of assumptions but plainly they contradict one another.

Wheeler and successfully conducted in 1984. The experiment, which showed that a single photon could be in two places at once, has been successfully repeated with atoms, small molecules, and most recently with sixty atom buckyballs. Thus they seem to support Everett's prediction. The obvious question is: if these alternative worlds exist, why do we not perceive them?

Here we introduce the idea of decoherence as it was developed by H. Dieter Zeh, Wojciech H. Zurek, and others during the later decades of the twentieth century. These men argued that the ideal superposition created by the falling card is coherent but that the coherent state can be maintained only so long as it is isolated from the rest of the world. It is the environment itself which destroys coherence and makes it impossible to observe superposition. Thus because it is impossible for us to keep large objects isolated so as to prevent decoherence, and because our brains are themselves part of the environment, we never see superposition. Though from a technical standpoint the wave function created by the falling card never collapses, decoherence creates a situation that is indistinguishable from a collapse.[8] This means that QM does not predict decoherence. Instead the idea is added to the theory in an attempt to explain what is happening.

Wharton has developed another interpretation of the data. Based on the premise that time does not flow, that it is rather a coordinate of measurement, Wharton argues that causal chains, that is, an interconnection of events that assume a direction from cause to effect, flow either forward or, at the quantum level, also backward in time. In his theory, decoherence marks the beginning of new causal chains which are created as a *state vector* of unrealized potential that interacts with its macroscopic environment. Such interaction causes the changeable properties of the two particles, which exist as potential within the state vector, to become actualized. Interaction with the macroscopic environment causes the actualized particles to behave quite differently from one another. The particle that interacts with the macroscopic environment, that is, the particle that has been measured, becomes disentangled from its distant twin, but that twin, because it has not interacted with the macroscopic environment, remains entangled with the potentiality of the disentangled particle. Furthermore, because causal events at the quantum level can go backward in time, what impacts the disentangled particle also impacts its entangled twin as causality races to the inception point of the two particles, then rebounds forward in time on the alternative path to affect the sister particle. This ability of the effect to go backward in time creates the impression that it moves faster than light speed, but it does not.

Measurement, or more generally interaction with the macro world, is key here because the causal chains that trace to the common origin of the two particles are terminated by the decoherence occasioned when one particle is measured. A measurement then is a beginning and an ending of causal chains that go backward or forward in time between the twin particles, and also the beginning of a causal chain that goes forward in time from the measured particle. Thus a single measurement terminates one causal chain and creates two, but the two that it brings to reality are different causal chains. Furthermore, the measurement usually acts as a barrier between the causal chains, thus enforcing their decoherence. For this reason, it can be treated as a first cause since it acts as a beginning for new causal chains. Only if the measurement is determined with one hundred percent certainty by an existing causal chain, may it lack this attribute.[9]

Max Tegmark, physicist at the University of Pennsylvania, has developed yet another idea to account for the data. He begins by arguing that the universe can be compared to a Mandelbrot set which, though it appears to contain a huge amount of information, can be expressed in a simple sentence. Thus, he maintains, most of what we see as real information is illusion. To make his argument, Tegmark begins by assuming that the big bang was very simple. However, this initial simple state involved slight fluctuations in various fields. Gravitation, the electromagnetic force, and the strong and weak nuclear forces worked in a nonlinear way to transform these fluctuations in the simple state into a state that expressed various kinds of complexity. Tegmark goes on to argue that the current wave function of the universe is a superposition of a large number of macroscopic states that are both extremely different from one another and unperceived by us. We live in this complexity which we perceive as an information rich environment, but what we recognize as information is, in fact, a mental construct predicated on our very limited perspective. Tegmark concludes by claiming that the universe which we imagine as grand and glorious is a very banal place containing almost no information. He uses astronomy to drive his point home, saying that the libraries of data astronomers have collected on their subject contain no "real" information, only information in the eye of the beholder.[10] In another context he says dramatically, "[N]ot even the pines and the Big Dipper of our world would exist if neither we nor any other [self-aware subsets] were here to perceive them."[11] In other words, the universe that we see is itself an example of decoherence created by our very act of seeing it. Such a conclusion, if true, reduces empirical science to the study of an extremely subtle and profound illusion.

Thus the Copenhagen interpretation of QM has produced three very different views of reality: Zeh's and Zurek's idea of decoherence, Wharton's idea of causal chains which can go backward in time at the quantum level, and Tegmark's idea of a banal universe devoid of much "real" information. Each view is consistent within its set of assumptions but plainly they contradict one another. In Zeh's and Zurek's imagination, trillions of interrelated data rich universes multiply themselves as they evolve simultaneously from fixed pasts across the sweep of eternity. In Wharton's imagination, there is only one universe made indeterminate on the quantum level by either a lack of causal chains or causal chains going backward in time from first causes, which have not yet occurred. In Tegmark's imagination, the universe is a data-poor banality, a screen for our intellectual illusions.

These three alternatives are the fruit of induction and as such they suggest that the true nature of our physical world is hidden from us. Indeed, given the failure of formalism coupled with our recognition that the world we perceive is distinct from the world as it is, an awareness that points starkly to the limits of our intuitive powers, we may interpret such contractions as an indication that a final exhaustive description of reality is forever beyond our abilities. Perhaps we have no hope of educing in any comprehensive way principles that could help us determine the true nature of reality and the best we can hope for is an analysis of a human construct that comes into existence as we observe the universe. That universe at its quantum level could be characterized by indeterminacy, or perhaps that indeterminacy is an illusion created by our epistemic limitations. The point is we might never know for sure, perhaps because science rests upon metaphysical assumptions that lie beyond the realm of mathematics and, as metaphysical assumptions, are themselves closed to scientific investigation.

As a Christian I believe that I am made in the image of God and that God, who created the universe, is truth. But I also believe that God's thoughts and ways differ from mine. Hence I am not at all dismayed by such a conclusion. What some may see as a frustrating impasse, I view an illustration of our fundamentally religious nature. God gives us not only reason, God also gives us faith. The two must work in tandem. Those who walk by the light of their own fire, as Isaiah says, will know only torment (Isa. 50:11).

Notes

1 C. G. Jung, (recorded and edited by Aniela Jaffe), *Memories, Dreams, Reflections* (New York: Random House, 1965), 28.

2 Stephen Hawking, *A Brief History of Time* (New York: Bantam Books, 1988), 166; ——, *The Theory of Everything* (Beverly Hills, CA: The New Millennium Press, 2002), 161. One might ask how the uncertainty principle sets such limits. Briefly, the principle means that observation must necessarily affect what is observed. That means the neutral observer assumed by science does not exist. It is generally granted that the effect of an observer has no significant influence in the world of our everyday experience, but it does have a significant impact on events at the subatomic level since it means that events on that level cannot be measured accurately. And that means not only that accurate knowledge of such events will forever elude us, but also that statements about them are devoid of meaning. In this way the uncertainty principle frustrates both our ability to formulate laws describing events on the subatomic level and to develop a complete scientific picture of the world.

3 Scientists often argue that their discipline also employs deduction that the theories or laws of science act as major premises in a deductive argument. I will not dispute that. But it remains the case that those theories and laws are derived inductively which is why they can be countermined by additional evidence.

4 Ben M. Carter, "Mathematics and Metaphysics," *Perspectives on Science and Christian Faith* 55, no. 3 (September 2003): 159.

5 Stephen Wolfram, *A New Kind of Science* (Champaign, IL: Wolfram Media, 2002).

6 Roger Penrose, *The Emperor's New Mind* (New York: Penguin Books, 1991), 113.

7 Max Tegmark and John Archibald Wheeler, "100 Years of Quantum Mysteries" *Scientific American* 284, no. 2 (February 2001): 54–61. Schrödinger, in protest against such a conclusion and intending to illustrate its silliness, proposed his famous thought experiment or gedanken in which a cat sealed in a container might be both dead and alive at the same time since no observer had yet looked into the container to collapse the wave function and establish the condition of the cat (Dennis Overbye, "Quantum Theory Tugged, And All of Physics Unraveled" *The New York Times*, *Science Times*, section "Quantum Wars," p. D4). Of course, one might counter that the cat itself qualified as an observer of its own condition and thus collapsed the wave function moment by moment.

8 Tegmark and Wheeler, "100 Years of Quantum Mysteries"; and David Deutsch, *The Fabric of Reality* (New York: Penguin, 1997), 50. While Deutsch admits that the "multiverse" interpretation remains the minority opinion among physicists, he argues that this is true for reasons that have much more to do with philosophy than science. *The Fabric of Reality* provides what is probably the finest defense currently available for the proposition that quantum theory is best explained by assuming the existence of a multiverse and why at a minimum at least a trillion (I assume Deutsch means a British trillion) universes run parallel to our own.

9 William Wharton,"Understanding Time and Causality is the Key to Understanding Quantum Mechanics," http://xxx.lanl.gov/abs/quant-ph/0310131

10 Max Tegmark, *Foundations of Physics, Letters* 9, no. 1 (1996): 25–41.

11 ——, "Is 'The Theory of Everything' Merely the Ultimate Ensemble Theory?" *Annals of Physics* 270 (1998): 46.